Quantifying Effect of Uncertainty Sources on Optimum in Tolerance Optimization

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1. Abstract

In cost based manufacturing tolerance optimization, multiple sources of uncertainty are present in the cost model inputs that leads to uncertainity in the outputs. The main output of interest is the cost associated with the optimal tolerance value. The cost model used in this paper combines the multiple objectives of manufacturing, quality and performance (reflected in value of weight savings). Combining these costs allows finding a tolerance value that balances the value of weight saving and production cost objectives. Sensitivity analysis showed that main sources of uncertainty are manufacturing error data, cost of useful load (hence value of weight savings) and mean cost of quality review. We propagated uncertainities present in the input variables by Monte Carlo simulation and explored uncertainty reduction startegies that would lead to maximum uncertainty reduction in the output i.e. total cost. It was found that uncertainty reduction in the cost of useful load leads to significant reduction in the total cost uncertainty, and uncertainty reduction in the manufacturing error data has the least effect.

2. Keywords: Tolerance optimization, uncertainty reduction, aircraft tolerance.

3. Introduction

In aircraft industry, manufacturing tolerance allocation plays an important role in balancing the conflicting objectives of structural weight and production cost. Minimizing structural weight requires tighter tolerance values, while reducing production cost is served by relaxation of tolerances. The production cost consists of quality and manufacturing cost and for given manufacturing technology the only input from manufacturing is the material cost. In such situations, a qualitative trade-off existing between these players is shown in Figure 1.

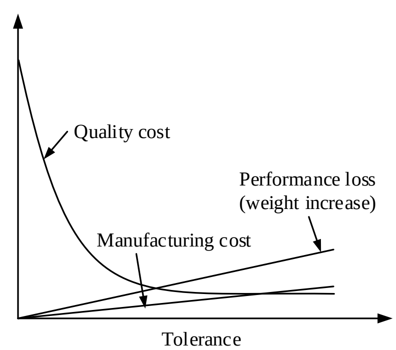

Figure 1: Qualitative trade-off between disciplines

Most tolerance allocation techniques only focus on minimizing the manufacturing cost without considering the quality cost [1, 2]. A few methods allocate tolerance by balancing both manufacturing and quality costs [3, 4]. Even fewer techniques account for the concurrent effect of tolerance on the performance, quality and manufacturing costs. Curran *et al*. [5] investigated the influence of tolerance on the direct operating cost (DOC) of an aircraft from the study performed on engine nacelle structure, and showed that relatively small relaxation in the tolerances resulted in reduced costs of production that lowered the DOC without unduly penalizing the parasite drag (performance). The authors [6] explored a similar approach on a business jet's wing spar designed under fatigue and damage tolerance constraints, and estimated the optimal tolerance by balancing the quality, manufacturing and performance (structural weight) objectives. The study concluded by identifying the input variables (cost variables and manufacturing error data) that could significantly impact the optimal tolerance and corresponding total cost.

In aircraft design, manufacturing tolerances are mostly decided early in the design phase when cost and manufacturing error data pertinent to the new aircraft are not accurately known. So, for tolerance optimization the obvious first choice is to use data available from similar aircraft that use similar manufacturing techniques. It introduces uncertainty in the optimal cost and optimal tolerance due to lack of accurate information, but as the new aircraft program moves closer to production, more accurate information is available for some of the cost variables (e.g. cost of useful load). When the aircraft is in production, more accurate manufacturing error data and quality cost data becomes available but at that stage it would be very expensive to re-design the tolerances. So, it is of interest for an engineer to estimate the uncertainty in an optimal point (i.e. optimal cost and optimal tolerance) followed by appropriate steps taken to reduce the uncertainty in output (i.e. optimal cost). Park *et al.* [7] did a similar study where they investigated the effect of uncertainty reduction measures (i.e. coupon and element tests) on uncertainty in element failure stress.

In this paper we only propagate the epistemic uncertainties due to cost model input variables (i.e. due to uncertainties in the cost of useful load and mean cost of quality review) and sampling epistemic uncertainty (i.e. uncertainty due to finite sample of manufacturing error data) using Monte Carlo Simulation. We then quantify the effect of uncertainty reduction measures (URMs) on the optimal cost and optimal tolerance. Aleatory uncertainty associated with the quality cost is not considered as it is negligible in comparison to the epistemic uncertainties.

4. Design of a Lap Joint for Damage Tolerance

We consider the design of a wing spar lap joint for damage tolerance (i.e. fatigue resistance) as a demonstration for the manufacturing tolerance optimization procedure. Lap joints are widely used on aircraft structures for attaching various parts together by using fasteners. A real example of such a lap/splice joint is shown in Figures 2-3 that connects the wing spars (from left and right wing) with the help of strap and fasteners. Such joints are typically in double or triple shear loading but for simplicity we have assumed it to be in a single shear. An idealized cross-sectional geometry representative of the front spar is shown in Figure 3. We use this idealized geometry to perform our analysis. See reference [6] for further design details pertaining to the idealized spar geometry.

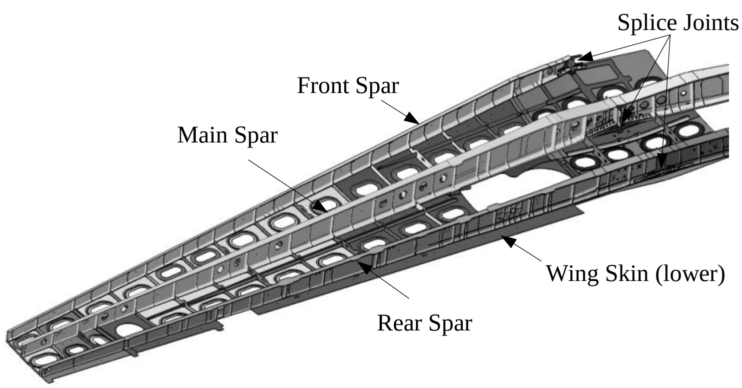

Figure 2: Wing assembly of a business jet showing wing spars attached at splice joints and to the wing skin

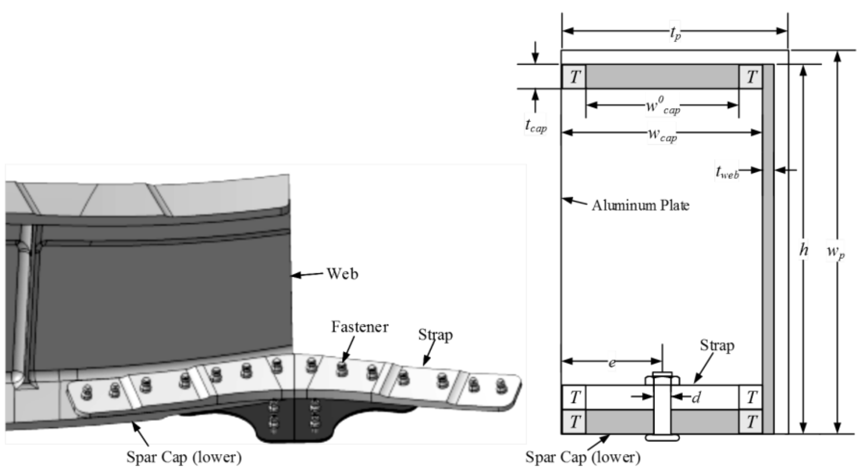

Figure 3: Front spar (C-channel) lap/splice joint (left), and idealized spar geometry (right)

4.1 Damage Tolerance and Manufacturing Errors

The objective of the damage tolerant design is to ensure that cracks (e.g. present at the fastener holes) do not grow to a size that could impair flight safety during the expected lifetime of an aircraft. It is done by specifying structural inspection intervals so that cracks could be found and the component replaced or repaired. A manufacturer has to show that the specified inspection intervals would satisfy the safety requirement by performing a crack growth analysis at every fastener hole.

Due to manufacturing errors, fastener holes can get mislocated by Δe and/or oversized by Δd as shown in Figure 4, which may lead to nonconformance with the desired inspection interval constraints. Generally, multiple inspection intervals are defined throughout the service life of an aircraft, but we only consider here the initial structural inspection interval I^* that is assumed to be set at 12,000 flight hours (FH). In actuality, a manufacturer identifies such deviations and checks if they violate the inspection interval constraints. If they do violate a constraint then a repair or part scrapping may be needed. So, we simulate this procedure in the presence of simulated manufacturing errors (shown in Figure 4) by executing crack growth analyses at a fastener hole. See reference [6] for more information on crack growth analysis.

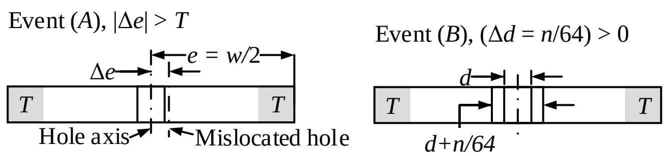

Figure 4: Common fastener hole manufacturing errors, Event (*A*) represents mislocated hole, and Event (*B*) represents an oversized hole.

If any or both of the above events occur, we assume that a quality review (QR) will be initiated. It represents criteria that airframe manufacturers often use to identify such errors, which may result in constraint violation. A major task accomplished under QR is the analysis and resolution of a quality problem by the concerned engineers. If the outcome of an engineering analysis (crack growth analysis in our case) does not lead to constraint violation then an easy repair is carried out. Otherwise, depending upon the severity of error an intricate repair might work or part (e.g. front spar) may have to be scrapped. A few examples of easy repairs are,

1. Plug and relocate the fastener hole while maintaining the specified edge distance.
2. Clean the hole to next available fastener diameter size and install the fastener.

Whereas an example of an intricate repair would be to cold work a hole and install a fastener. However, for our analysis we assume that constraint violation will always lead to part scrapping. Such an assumption is expected to add reasonable conservatism in the analysis because such intricate repairs are very rarely employed. In addition, as we will see from the results, even the cost of this drastic measure contributes relatively little to the total cost.

5. Manufacturing Error Data and Distributions

In this paper we model the two types of manufacturing deviations/errors (shown in Figure 4) as random variables with distributions based on collected data. The edge distance deviation Δe is a continuous random variable, while hole diameter deviation Δd is a discrete random variable. The physical data for the two random variables were collected from the wing assemblies of a business jet with the help of Cessna Aircraft Company. These data are used to estimate the distributions for each random variable in order to estimate the probability of initiating a quality review P_{QR} and probability of violating an inspection constraint P_{CV}. These probability estimates are used in the cost model to calculate the expected value of the quality cost C_Q.

5.1 Edge distance distribution

The edge distance deviation data Δe were collected from the lower spar caps of the 8 wing assemblies of a business jet. A total of 8164 samples were fitted with 12 continuous parametric distributions available in MATLAB (Normal, Lognormal, Logistic, Log Logistic, Weibull, Beta, Generalized Extreme Value, Gamma, Inverse Gaussian, Nakagami, Rician, and Birnbaum-Saunders). The logistic distribution was found to be a reasonably good fit with respect to other distributions. A probability plot of the logistic fit is shown in Figure 5. The maximum likelihood estimates (MLE) of the logistic distribution parameters is listed in Table 1. The cumulative distribution function of the logistic distribution are given as follows,

$$F(x) = \frac{1}{1 + e^{-\frac{x-\mu_{LF}}{s_{LF}}}}, \tag{1}$$

The standard deviation is related to scale parameter by following relationship,

$$\sigma = \frac{\pi s_{LF}}{\sqrt{3}} \tag{2}$$

Table 1: Maximum likelihood parameter estimates of logistic fit

Logistic fit	
Parameters	MLE (inch)
Location (μ_{LF})	-5.467×10^{-4}
Scale (s_{LF})	0.01378

The logistic distribution was used in the sensitivity analysis [6] to illustrate the impact of uncertainty in edge distance deviation data on the optimal tolerance by simply varying the scale parameter (s_{LF}) of the distribution. Therefore, we have used logistic distribution to model the uncertainty in the edge distance deviation data. Although a more accurate fit was found to be a semi-parametric distribution [6], but in this paper we continue with logistic distribution as it allows easier investigation of the effectiveness of uncertainty reduction measures

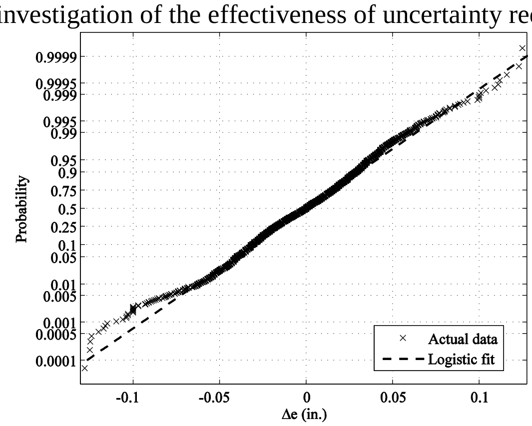

Figure 5: Probability plot of logistic fit and actual Δe data

5.2 Hole diameter deviation distribution

The hole diameter deviation data (Δd) were collected from the wing assemblies of about 110 airplanes with sample size of 650,642. Aerospace fasteners are generally available in $\Delta d = n/64$" standard increments, i.e. if d = 8/32" fastener gets oversized due to manufacturing error then the next available fastener size is 8/32"+1/64" and so on. This makes Δd a discrete random variable that can be modeled by using a simple histogram. The frequencies associated with 13 subsequent fastener increments are listed in Table 2.

Table 2: Fastener oversize (hole diameter deviation) distribution

Δd (inch)	Frequency	Δd (inch)	Frequency	Δd (inch)	Frequency
0/64	649520	5/64	43	10/64	2
1/64	100	6/64	64	11/64	1
2/64	666	7/64	17	12/64	2
3/64	90	8/64	7	13/64	1
4/64	119	9/64	10	-	-

The probability that a fastener will be oversized is,

$$P(\Delta d > 0) = P(\sum_{n=1}^{13} F_n) = \frac{1122}{650,642} = 1.724\times10^{-3} \tag{3}$$

6. Estimating Probabilities

The probability of quality review and probability of constraint violation are estimated by using the distributions estimated above to calculate the expected value of quality cost. The procedure for estimating these probabilities is discussed next.

6.1 Probability of Quality Review (P_{QR})

As discussed earlier that quality review is initiated if the criteria shown in Figure 4 are satisfied i.e. when edge distance deviation exceeds the allocated tolerance value $|\Delta e| > T$ and/or hole diameter deviation is greater than zero $\Delta d > 0$. It is assumed that the two events are independent of each other. Therefore, it allows the calculation of P_{QR} (per fastener) by using the following formula,

$$P_{QR} = P(|\Delta e| > T) + P(\Delta d > 0) - P(|\Delta e| > T)P(\Delta d > 0) \tag{4}$$

where, $P(|\Delta e| > T)$ is estimated from the edge distance distribution and $P(\Delta d > 0)$ is estimated from the hole diameter distribution. Notice that former has direct dependency on the tolerance and later does not. It means that P_{QR} is expected to gradually decrease with increase in tolerance and will tend to 1.724×10^{-3} (the probability of oversizing a hole) for very large T. Such a trend is shown in Figure 6,

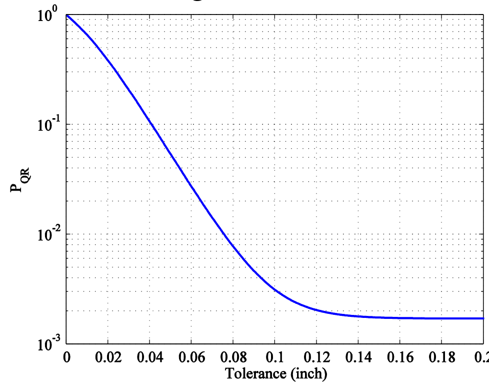

Figure 6: P_{QR} variation with tolerance

6.2 Probability of Constraint Violation (P_{CV})

As mentioned before that if quality review shows that initial inspection interval constraint is violated then spar would be scrapped. The P_{CV} is calculated by Monte Carlo Simulation (MCS) that simulates the quality review, where a given combination of fastener deviation (Δe, Δd) are checked for the possibility of constraint violation by executing a crack growth analysis. For the given combination of deviation samples an initial inspection interval is calculated by dividing the total crack growth life (CGL) by a factor of two i.e. $I = CGL/2$. A sample combination (Δe, Δd) fails to meet the initial inspection constraint of $I^* = 12{,}000$ flight hours (FH) if,

$$I - I^* < 0, \text{ or } {CGL}/{2} - 12{,}000 < 0 \tag{5}$$

A total of 10 million (n_{total}) samples generated from their respective distributions are used in the estimation of P_{CV}. The P_{CV} also decreases with the increase in tolerance as shown in Figure 7.

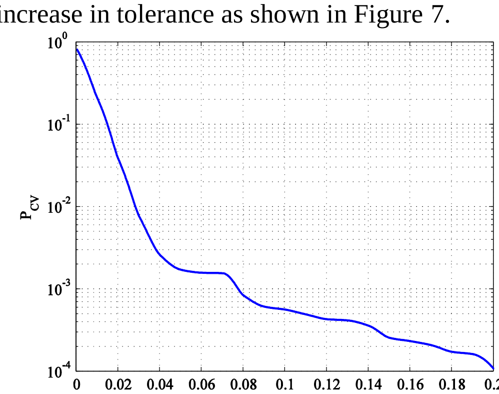

Figure 7: P_{CV} variation with tolerance

7. Cost Model

The total cost is the sum of three components i.e. quality cost, manufacturing cost and performance cost.

7.1 Quality Cost (C_Q)

The quality cost is incurred due to quality review and constraint violation/part scrapping. For the optimization of the tolerance we use the expected value of the quality cost is the sum of expected quality review cost C_{QR} and constraint violation/scrap cost C_{CV},

$$C_Q = C_{QR} + C_{CV} \tag{7}$$

The quality review cost C_{QR} captures the cost associated with resolving the two types of manufacturing errors discussed earlier. Cost of materials and tools used in easy repairs is negligible in comparison to labor cost, i.e. engineering time required to review and specify a repair and labor used to execute a repair. The following equation is used to estimate the repair cost,

$$C_{QR} = n_f P_{QN} C_{QRperFas}\ ,\ C_{QRperFas} = \mu_{ET}\mu_{EC} + \mu_{LT}\mu_{LC} \tag{8}$$

where, $C_{QRperFas}$ is the quality review cost for a single fastener; μ_{EC} (100 \$/hr.) and μ_{LC} (65 \$/hr.) are the mean hourly engineering and labor rates; μ_{ET} (3/4 hr.) and μ_{LT} (1/2 hr.) are the mean engineering and labor time involved in completing a quality review; P_{QR} is the probability of quality review and n_f is the total number of fastener holes to be drilled in a spar. The cost of constrain violation is mainly the scrap cost that is estimated by the following expression,

$$C_{CV} = 2P_{CV} W_p C_{Al}\ ,\ W_p = w_p t_p l \rho_{Al} \tag{9}$$

where, C_{Al} is the cost for a pound of aluminum alloy (5.50 \$/lb.); W_p is the weight of aluminum plate; P_{CV} is the probability of violating an inspection interval constraint. A factor of 2 is used in the equation because scrap cost is generally twice the raw material cost for the high valued parts (such as wing spar).

7.2 Manufacturing Cost (C_M)

The two main constituents of C_M are tooling cost and material cost. As tooling cost is assumed to be constant (i.e. jigs, tools and technology remains the same), it does not vary with the tolerance and therefore it can be taken out from the following equation,

$$C_M = \cancel{C_{tool}} + C_{Mat} \tag{10}$$

Then, material cost is proportional to the weight increase ΔW_p of the raw aluminum plate multiplied by the cost per pound of the aluminum alloy C_{Al}. The increase in weight is calculated with respect to the zero tolerance design. It leads to the following expression for C_M,

$$C_M = C_{Mat} = \Delta W_p C_{Al}\ ,\ \Delta W_p = 2hlT\rho_{Al} \tag{11}$$

7.3 Performance Cost (C_P)

It is the extra money that customers have to pay due to increased structural weight attributable to the addition of tolerance. It serves as a penalty parameter that penalizes the total cost function C_{total} due to increase in structural weight with addition of tolerance.

The maximum takeoff weight of an airplane can be divided into operational empty weight (OEW) and useful load. We have slightly modified the breakdown of the OEW to put everything that does not take flight loads (inclusive of engines) under non-structural weight. The tolerance is added to the structural weight, and useful load is the sum of full fuel load and full fuel payload (i.e. passengers, crew, baggage etc.).We have assumed that maximum take-off weight (MTOW) of an airplane remains constant i.e. addition of tolerance increases the structural weight leading to reduction in the useful load capacity. Conversely, weight savings due to tolerance optimization decreases the structural weight leading to increase in the useful load capacity.

The useful load of an aircraft is as an important characteristic that customers care about. The following equation expresses the performance cost as a function of increase in the spar weight ΔW_s due to addition of tolerance

T and cost of useful load C_{UL}. Again, weight increase is measured with respect to zero tolerance weight that leads to the following relationship for C_P,

$$C_P = \Delta W_s C_{UL} = 4 t_{cap} l T \rho_{Al} \left(\frac{S_{price}}{W_{useful}} \right) \tag{12}$$

Where, C_{UL} is the expected cost that customers pay for a pound of useful load. A reasonable measure of this cost can be calculated by dividing the sales price S_{price} [8] of an aircraft with useful load W_{useful} for the existing airplane models. A plot of C_{UL} calculated for various business jets is shown in Figure 8 with airplanes arranged according to the increasing useful load capacity (weight data extracted from their respective websites). For our calculations we have used the mean value of 1,200 $/lb.

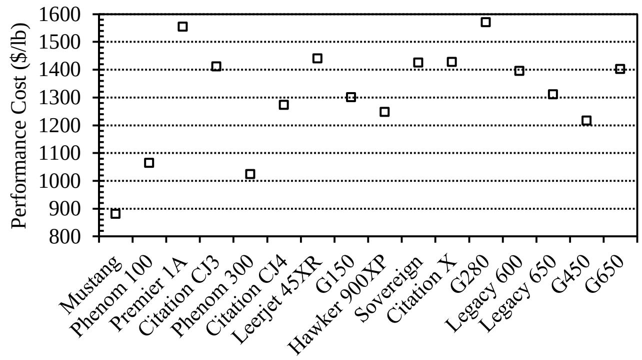

Figure 8: Performance cost estimated for various business jets

7.4 Total Cost (C_{total})

It is used to represent the integrated cost function that combines all the individual cost objectives into a single cost objective that is used to optimize the tolerance is expressed by the following equation,

$$C_{total} = \underbrace{C_Q + C_M}_{C_{prod}\text{ (Production cost)}} + C_P \tag{13}$$

8. Uncertainty Sources and Respective Distributions

The significant sources of uncertainty identified by performing local sensitivity analysis [6] were edge distance deviation data, cost of useful load and mean cost of quality review. The data for edge distance deviation (Δe) were collected from 8 wing assemblies (8164 samples) that only represent about 2.3 % of the expected number of airplanes (350) to be produced. The finite sample means that there is uncertainty about the scale (s_{LF}) and location (μ_{LF}) of the estimated logistic distribution, which can lead to errors in the estimation of the P_{QR} and P_{CV}. It is important to quantify the impact of such uncertainties on the optimal tolerance and total cost. The two possible sources of uncertainties are limited amount of data and measurement errors in ascertaining the edge distance of the fastener holes.

In this study we limit ourselves to uncertainty in edge distance data due to limited sample size and we model this uncertainty by determining a distribution of location (μ_{LF}) and scale parameter (s_{LF}) of the logistic fit. This is done by bootstrapping; i.e. 10,000 times of resampling (with replacement) from original 8,164 samples and fitting a logistic distribution to each sample. We find that the distributions of μ_{LF} and s_{LF} are normal With means and standard deviations given in Table 3.

Table 3: Mean (μ) and standard deviation (σ) of the distribution of μ_{LF} and s_{LF}

	Bootstrap estimates (in.)	Lower 95%	Upper 95%
$\mu_{\mu LF}$	-5.47×10^{-04}	-5.52×10^{-04}	-5.41×10^{-04}
$\sigma_{\mu LF}$	2.71×10^{-04}	2.67×10^{-04}	2.75×10^{-04}
μ_{sLF}	1.3780×10^{-02}	1.3778×10^{-02}	1.3783×10^{-02}
σ_{sLF}	1.205×10^{-04}	1.189×10^{-04}	1.222×10^{-04}

The uncertainty in the location ($\sigma_{\mu LF}$) and scale parameters (σ_{sLF}) can be reduced by collecting more data i.e. in order to reduce $\sigma_{\mu LF}$ and σ_{sLF} by 50%, 4 times more data is needed (8164×4 = 32,656).

The exact value of the cost of useful load is not accurately known for a new aircraft early in the design phase. So, it is estimated by dividing the sales price by useful load for the current similar aircrafts. The C_{UL} ranged between 800-1600 $/lb for the current aircrafts as shown in Figure 8. We have used 1200 $/lb (nominal value) to perform the optimization, but it is possible that the actual value of C_{UL} for the aircraft under consideration might be different. Therefore, it is important to evaluate the impact of uncertainty/error in the estimation of C_{UL} on the optimal cost and optimal tolerance.

We have assumed that C_{UL} follows a normal distribution with mean (μ_{CUL}) of 1200 $/lb and standard deviation (σ_{CUL}) of 100 $/lb that is estimated from the bounds of C_{UL} derived from Figure 8 as follows,

$$4\sigma_{CUL} = 1600 - \mu_{CUL},\ \sigma_{CUL} = \frac{1600-1200}{4} = 100 \text{ \$/lb} \tag{14}$$

The cost of quality review represents the average cost involved in reviewing and resolving the two types of quality problems for a single fastener. It captures the cost incurred due to utilization of engineering and labor resources, and was estimated to be $ 107.5 (nominal) per quality review. However, there can be some errors/uncertainty involved with the estimation of C_{QR} as engineering and labor times involved in quality review are not accurately known. So, it is important to evaluate the impact of uncertainty in C_{QR} on the total cost at optimal tolerance value.

We have assumed C_{QR} to follow a normal distribution with mean (μ_{CQR}) of $ 107.5 and standard deviation (σ_{CQR}) of $ 18. The value of standard deviation is carefully chosen based on the assumed maximum engineering and labor time involved with resolving a single quality review. The maximum expected C_{QR} is determined to be $ 161.25 and is considered to represent $\mu_{CQR} + 3\sigma_{CQR}$ value

Uncertainty Propagation and Quantification

At first, optimization is executed by considering the nominal values of all the input variables that gives mean total cost curve ($\mu_{totalcost}$) as a function of tolerance as shown in Figure 9, and optimal point is found at 0.0727" with total cost of $ 2,457. Then uncertainty is propagated by Monte Carlo Simulation for the fixed value of tolerance (i.e. 0.0727") by generating random samples from the distributions estimated in Section 8. It basically gives a distribution of total cost about optimal point as shown in Figure 9 (right). The initial uncertainty in total cost at optimal tolerance is determined to be $\sigma_{totalcost}$ = $ 166 i.e. 3σ (99.9%) bounds are $1,977 and $ 2,973. Also it is noticed that uncertainty is minimal at optimal point as shown in Figure 9 (left).

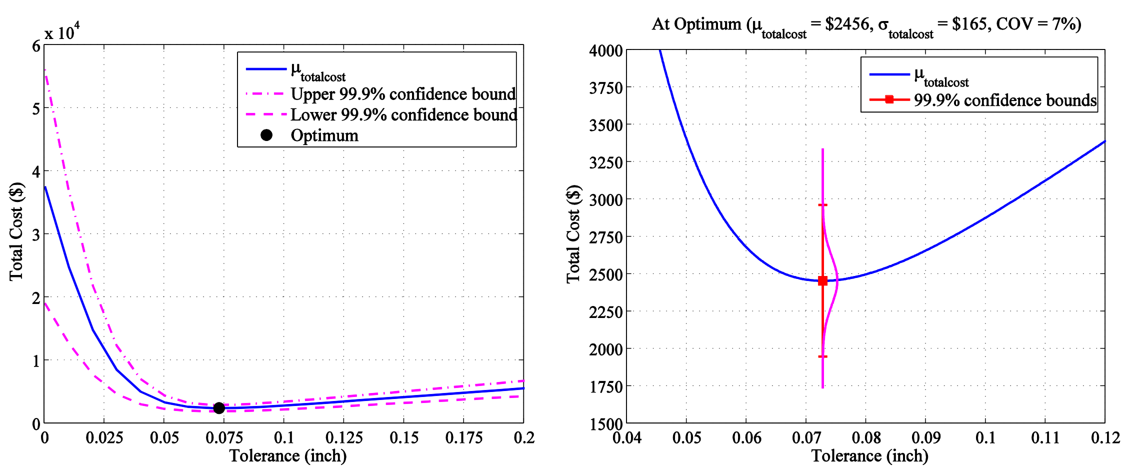

Figure 9: Initial uncertainty in the total cost

Next, we explored uncertainty reduction measures and their impact on the total cost uncertainty. The uncertainty in edge distance data is basically due to limited sample size, so it is easy to reduce the uncertainty in the location (μ_{LF}) and scale parameters (s_{LF}) of the logistic fit by collecting more data. We assumed that by collecting 4 times more samples would reduce the parameter standard deviations $\sigma_{\mu LF}$ and σ_{sLF} by 50% and their updated values are given as follows,

Table 4: Updated standard deviation of location and scale parameters of logistic distribution

	Bootstrap estimates (in.)	Lower 95%	Upper 95%
$\mu_{\mu LF}$	-5.381×10^{-04}	-5.407×10^{-04}	-5.355×10^{-04}
$\sigma_{\mu LF}$	1.336×10^{-04}	1.318×10^{-04}	1.355×10^{-04}
μ_{sLF}	1.3813×10^{-02}	1.3811×10^{-02}	1.3814×10^{-02}
σ_{sLF}	6.075×10^{-05}	5.993×10^{-05}	6.161×10^{-04}

We further assumed that uncertainty in the C_{UL} could be reduced by having more accurate information of the sales price and useful load of the aircraft and can update the $4\sigma_{CUL}$ bounds of the C_{UL} distribution and could reduce the uncertainty by 50% (σ reduces from 100 $/lb to 50 $/lb). On the other hand, uncertainty in the C_{QR} could be reduced by simply collection more accurate time reporting data. It is assumed that it is possible to reduce the uncertainty in the C_{QR} by 50 % i.e. σ_{CQR} reduces from $18 to $9.

Out of the three uncertainty reduction measures discussed above, 50% in σ_{CUL} leads to 35.85% reduction in the uncertainty of optimal cost (see Table 5); 50% reduction in $\sigma_{\mu LF}$ *and* σ_{sLF} leads to only 0.96% reduction in optimal cost uncertainty and 50% reduction in σ_{CQR} leads to about 8.81% reduction in optimal cost uncertainty. Whereas 50% reduction in σ_{CQR} and σ_{CQR} together results in about 49% reduction in the total cost uncertainty. Therefore, the best way to reduce the uncertainty in optimal cost is through accurate prediction of cost of useful load and mean cost of quality note. Whereas, collection of more edge distance deviation/error data will have negligible effect.

On the other hand, the uncertainty in optimal tolerance (on horizontal scale about optimal point) is reduced by 36.8% by 50% reduction in σ_{CQR} and only about 4.6% with 50 % reduction in σ_{CUL}. Again, 50% reduction in $\sigma_{\mu LF}$ and σ_{sLF} only leads to about 1.1% reduction in optimal tolerance uncertainty.

Table 5: Uncertainty reduction in inputs and corresponding uncertainty reduction in optimal cost

Input uncertainty reduction			Optimal Cost	
$\sigma_{\mu LF}$, σ_{sLF}	σ_{CUL}	σ_{CQR}	$\sigma_{totalcost}$	Reduction
0%	0%	0%	166.8	-
50%	0%	0%	165.2	0.96%
0%	50%	0%	107.0	35.85%
0%	0%	50%	152.1	8.81%
0%	50%	50%	84.7	49.22%
50%	50%	50%	83.0	50.24%

The final uncertainty in total cost at optimal tolerance is determined to be $\sigma_{totalcost}$ = $ 83 i.e. 99.9% confidence bounds are $ 2,212 and $ 2,710 as shown in Figure 10.

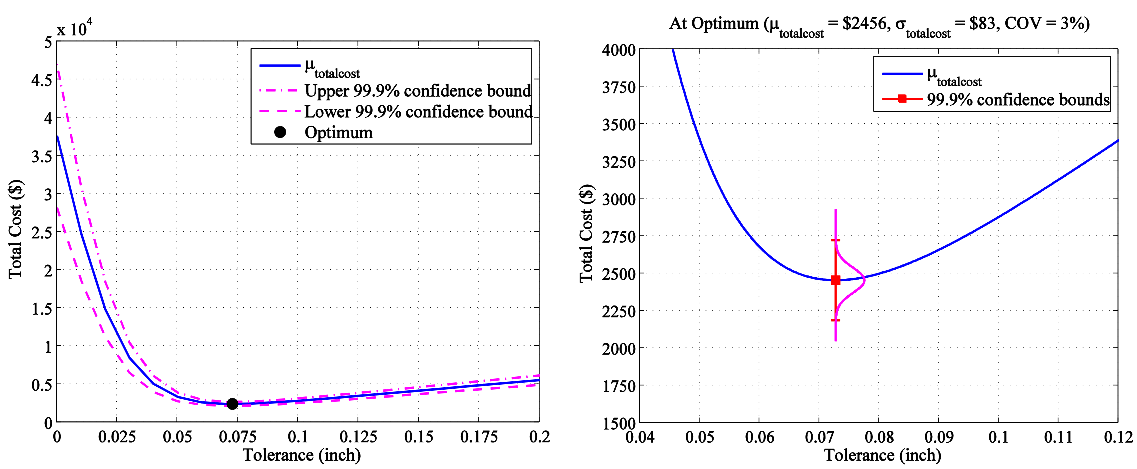

Figure 10: Uncertainty reduction in total cost by reducing uncertainty in all the inputs

9. Concluding Remarks

The main sources of uncertainty identified by sensitivity analysis were edge distance deviation data, cost of useful load and mean cost of quality review. We propagated uncertainities present in the input variables by Monte Carlo simulation and explored uncertainty reduction startegies that would lead to maximum uncertainty reduction in the output i.e. total cost. It was found that uncertainty reduction in the cost of useful load leads to significant reduction in the total cost uncertainty, and uncertainty reduction in the manufacturing error data has the least effect.

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