10th World Congress on Structural and Multidisciplinary Optimization
May 19 -24, 2013, Orlando, Florida, USA

Structural Optimization of Reinforced Concrete Building Grillages considering Cardinality Constraints

Moacir Kripka[1], Guilherme F. Medeiros[1], Afonso C.C. Lemonge[2]

[1] Engineering Graduate Program, University of Passo Fundo, Passo Fundo, Brazil, mkripka@upf.br, guifleith@upf.br
[2] Department of Applied and Computational Mechanics, Federal University of Juiz de Fora, Juiz de Fora, Brazil, afonso.lemonge@ufjf.edu.br

Abstract

Despite the advent of commercial software programs, which automate several design stages, the initial sizing of structural elements requires an engineer's direct and intellectual work, and the solution found is very unlikely to be the best among the several options compliant with safety and usage requirements. Nevertheless, by using optimization strategies coupled to the structural design, one seeks to find the best solution by means of a systemic search, based on a well-defined mathematical model, with the definition of objective functions, parameters, and constraints. Regarding reinforced concrete structures, the cost minimization in compliance with the ultimate and serviceability limit states and with other technical regulations, could mean remarkable cost savings, enabling construction companies and, especially, structural design offices, to stay ahead of their competitors. The aim of this work is to show the application of optimization strategies for the cost of beams in reinforced concrete buildings. In order for this goal to be met, an optimization software program was developed. The program combines the analysis of structures by the grid model, reinforced concrete sizing, and the simulated annealing optimization heuristic. Sizing is compliant with the NBR 6118 Brazilian standard, according to which flexural, shearing, torsion, and web reinforcements, and serviceability limit states (deflection and crack width limitation) are checked. Since the number of beams with different sizes must be limited due to practical reasons, cardinality constraints are also considered, aiming to identify the ideal grouping of elements. Some results obtained to different number of member groups are presented in order to illustrate the proposed procedure.

Keywords: Optimization, Beams, Reinforced Concrete, Grid Model, Simulated Annealing.

1. Introduction

Usually, pre-sizing is one of the first stages in the development of a structural design. After finding a solution that meets stability and functionality requirements, the designer often ceases his/her quest for alternatively more economical solutions, and sticks to the best result obtained so far. However, the structural sizing process is iterative and relies ultimately on the intuition and experience of the designer, who has to choose from the several available options and be very sensitive to the proposed initial solution, for achieving a satisfactory outcome. Despite the advent of commercial software programs, which automate several design stages, the initial sizing of structural elements requires an engineer's direct and intellectual work, and the solution found by him/her is very unlikely to be the best among the several options compliant with safety and usage requirements. Nevertheless, by using optimization strategies coupled to the structural design, one seeks to find the best solution by means of a systemic search, based on a well-defined mathematical model, with the definition of objective functions, parameters, and constraints. In structural optimization, the smallest weight and the lowest cost are the main goals to be attained.

Cost minimization of reinforced concrete structures, in compliance with the ultimate and serviceability limit states and with other technical regulations, could mean remarkable cost savings, enabling construction companies and, especially, structural design offices, to stay ahead of their competitors. The lack of resources and the need to lower the consumption of raw materials and thus have a more sustainable development should also be taken into account. Structural optimization plays a key role in this process by allowing for more rational projects in an efficient and relatively quicker fashion. This is a field that has been neglected in practice, but it has large potential for application. To do that, it is necessary to fine-tune structural optimization with the practical characteristics of the project, seeking to describe the actual situations faced by designers and their complexity, as well as classic and trivial examples of optimization of structures, widely reported in the technical literature. As far as reinforced concrete is concerned, structural optimization will be attractive when the examples used are closely related to the conditions commonly found in usual constructions with typical floors and with realistic geometry and loading conditions, and when technical regulations are effectively followed.

There are numerous works in the literature that address optimal sizing of reinforced concrete structures, in which calculations are made using several classical optimization techniques. In most studies, the objective is to minimize costs of concrete section so as to fulfill functional constraints based on calculation standards and to meet the constraints involving strength criteria. In these studies, the cross-sectional dimensions of the beams are usually grouped in order to reduce the number of design variables, also reducing the fabrication costs.

The aim of this work is to show the application of optimization strategies for the cost of beams in reinforced concrete buildings. In order for this goal to be met, an optimization software program was developed. The program combines the analysis of structures by the grid model, reinforced concrete sizing, and the simulated annealing optimization heuristic. Sizing is compliant with the NBR 6118 Brazilian standard [1], according to which flexural, shearing, torsion, and web reinforcements, and serviceability limit states (deflection and crack width limitation) are checked. Since the number of beams with different sizes must be limited due to practical reasons, cardinality constraints are also considered, in order to identify the ideal grouping of elements and its influence regarding optimal results. Cardinality constraints (CC), can limit the number of different design parameters in order to achieve economies of bulk purchasing and to simplify construction, leaving to the optimization algorithm the task of deciding how to group members and/or design variables [2].

This work is an extension of former studies of the authors regarding the optimization of reinforced concrete building structures [3] and the usage of cardinality constraints to steel structures [4]. Some results obtained to different number of member groups in a building floor are presented in order to illustrate the proposed procedure.

The remainder of the paper is structured as follows. Section 2 give describes Simulated Annealing Method, Section 3 shows the formulation of the Optimization Problem, Section 4 presents an example of optimization, and, finally, Section 5 concludes this work.

2. Simulated Annealing Method

In structural engineering, optimization techniques have been constantly applied to a wide range of problems, allowing the use of the best sets of material, topology, geometry and/or dimensions of cross-sections in different types of structural systems [5].

The algorithms used to solve an optimization problem can be deterministic or probabilistic. Deterministic optimization methods, also known as classical methods, in which mathematical programming methods are included, are often based on the calculation of first-order derivatives or of second-order partial derivatives. Conversely, heuristic methods, based on probabilistic algorithms, add stochastic data and parameters to the optimization process, solving the problem from a probabilistic perspective.

Mathematical programming methods have some limitations such as their difficulty in finding global optimal solutions, as they rely on the starting point, difficulty in employing discrete variables, and difficulty in utilizing non-differentiable functions. A *sine qua non* condition for the application of classical methods is that the objective function must be continuous and differentiable in the search space. However, this does not occur in most of practical engineering problems, thus preventing their application. Heuristic methods do not calculate derivatives, but they directly search for solutions in the feasible space. Nevertheless, these methods require a larger number of evaluations of the objective function value, and are therefore computationally more expensive than mathematical programming methods. Thus, they should not be used injudiciously, but only to solve problems for which mathematical programming techniques show limitations. Heuristic methods include a large number of algorithms such as genetic algorithms, simulated annealing, ant colony algorithm, bee colony algorithm, harmony search, particle swarm optimization, among others. Genetic algorithms and simulated annealing are the most popular of these methods [5], [6].

Simulated annealing is a heuristic method based on statistical mechanics which dates back to the annealing process, and was introduced by Kirkpatrick et al. [7]. In the physical process of solid hardening, a material is quickly heated and slowly cooled so that its structural flaws can be eliminated. If cooling is sufficiently slow, the final configuration of the material will correspond to the minimum energy state. On the other hand, quick cooling will result in a metal with weak and brittle structure. In brief, in simulated annealing, a single neighboring state s' of current solution s is randomly generated in each iteration. The difference (Δ_f) between the quality of the new solution *s'* and the quality of the current solution *s*, (Eq. (1)) is calculated to assess the acceptance of this new solution *s'*:

$$\Delta_f = f\left(s'\right) - f\left(s\right) \tag{1}$$

In a minimization problem, if the value of Δ_f is less than zero, the new solution *s'* is automatically accepted and can substitute *s*. Otherwise, the acceptance of the new solution s' depends on the probability established by the Metropolis criterion:

$$p = exp\left(\frac{-\Delta_f}{T}\right) \tag{2}$$

As temperature drops throughout the process, there is a higher probability of acceptance of new solutions in the initial stages, even if this eventually worsens the current solution. This probability decreases throughout the process, reaching the point (when temperature is close to zero) at which only those movements that improve the cost function are accepted.

Several works, published in the past few years, successfully used simulated annealing for structural optimization. Hasançebi and Erbatur [8] used this heuristics and optimized a 942-member truss tower, an 18-member truss, and a 47-member plane truss tower. In the latter two cases, the geometry of the models was optimized along with the cross-sections. Discrete variables were used. By comparing the results with those of other studies, the proposed simulated annealing algorithm outperformed genetic algorithms. Park and Ryu [9] proposed altering the parameters in order to improve the heuristics. They optimized the weight of two structures usually found in structural optimization problems. Both discrete and continuous variables were used. The authors concluded that the number of necessary iterations in the new simulated annealing algorithm was significantly smaller than that of the conventional algorithm. Kripka [10] optimized plane and spatial trusses to discrete variables, and compared the obtained results with those of different methods. In all cases, the optimal solution provided by simulated annealing was equal to or better than the others. Dagertekin [6] optimized the section of steel frames using simulated annealing and genetic algorithms. Three simulations were carried out, and Simulated annealing had a slight advantage in all simulations. Payá-Zaforteza et al. [11] optimized reinforced concrete frames used in the construction of buildings following five heuristic methods, including simulated annealing and genetic algorithms. Initially, the different methods were tested using a model made up of two bays and four floors. Of these methods, simulated annealing was more efficient in the search for an optimal solution, showing an intermediate processing time. Later, several models were used to fine-tune the method: two-bay frames with two, four, six and eight heights. Suji et al. [5] optimized fiber-reinforced concrete beams. The reinforcement of beams was used as rationally as possible, given the costly prices of this material. González-Vidosa et al. [12] optimized four reinforced concrete frames. The first model consisted of a soil barrier system. The second and third models consisted of frames used in road construction, and the fourth optimized model consisted of a 20-member plane frame commonly used in buildings. Payá-Zaforteza et al. [13] conducted multiobjective optimization using simulated annealing. In addition to assessing cost minimization, they evaluated three other objectives: maximization of model constructability, minimization of environmental impacts, and maximization of global structural safety. The model used consisted of a 20-member reinforced concrete plane frame of a 4-story building. Their results indicated that, with a small increase in optimal cost, it is possible to have structures with higher constructability, larger sustainability and better global structural safety. Hasançebi, Çarbas and Saka [14] proposed the improvement of the simulated annealing algorithm. To test the alterations, the authors optimized plane and spatial steel frames with 304 and 132 members, respectively. The simulated annealing algorithm had the lowest weight for both structures, compared to the results of two other heuristic methods: harmony search and tabu search. Sonmez [15] optimized truss weight using discrete variables and different methods, including simulated annealing. The best result was obtained for the classical model with a 10-member plane truss with simulated annealing, bee colony algorithm, and ant colony algorithm. The simulated annealing algorithm had the best result for the 25-member spatial truss, compared to other heuristic methods. Particle swarm optimization and genetic algorithms were the other methods used in the analyses. Finally, Bordignon and Kripka [16] used the method to minimize the cost of reinforced concrete columns sections, achieving a great economy regarding genetic algorithms and mathematical programming.

3. Formulation of the Optimization Problem

The aim of this paper is the cost minimization of reinforced concrete beams, taking into account the influence of formwork, of concrete, and of transverse and longitudinal reinforcements. The major optimization problem variable was the cross-sectional height of beams. The cost of each material was obtained by multiplying the respective amounts by the unit costs of each material. Steel was quantified in mass (kg), concrete was expressed in volume (m³) and formwork as area (m²). Therefore, the objective function is given by Eq. (3), where C_t corresponds to the overall cost of the analyzed structure, P_A, P_{Asw}, A_F, V_C refer to the amounts of material (500 MPa steel, 600 MPa steel, formwork, and concrete, respectively), while C_A, C_{Asw}, C_F and C_C stand for the unit costs of each material. The latter ones were calculated based on the compositions and mean values recommended for the southern Brazilian region.

$$C_t = [(P_A + P_{Asw}).C_A] + (A_F.C_F) + (V_C.C_C) \tag{3}$$

NBR 6118 (2007), Brazilian standard for the design and execution of reinforced concrete structures, was used for the sizing and detailed description of structural elements. The optimization problem constraints related to the serviceability limit states are the maximum deflection of each element, taking into account long-term effects, and the maximum crack width. Constraints were also imposed regarding flexural, shearing and torsional reinforcements, according to NBR 6118 (2007) standard. The detailed formulation of the problem can be found in Medeiros and Kripka [3].

In addition to the constraints related to limit states design listed above, in this work cardinality constraints, or *CC*, were considered. According to this, the *maximum* number of different cross sections is limited by *Ng*, or number of groups. After that, each beam is randomly assigned to a group. It can be noticed that the original problem is recovered when *Ng* is set equivalent to the total number of beams.

4. Example

In order to illustrate the application of the proposed formulation, some results obtained to a quite traditional typical floor, shown in Fig. 1 and adapted from Ref [3], are presented. The software developed analyzes the beams of the floor by applying the grid model and using simulated annealing as an optimization tool. In the grid model, loads are applied perpendicularly to the plane formed by the floor beams. In this example, the beams were considered as being simply supported by columns.

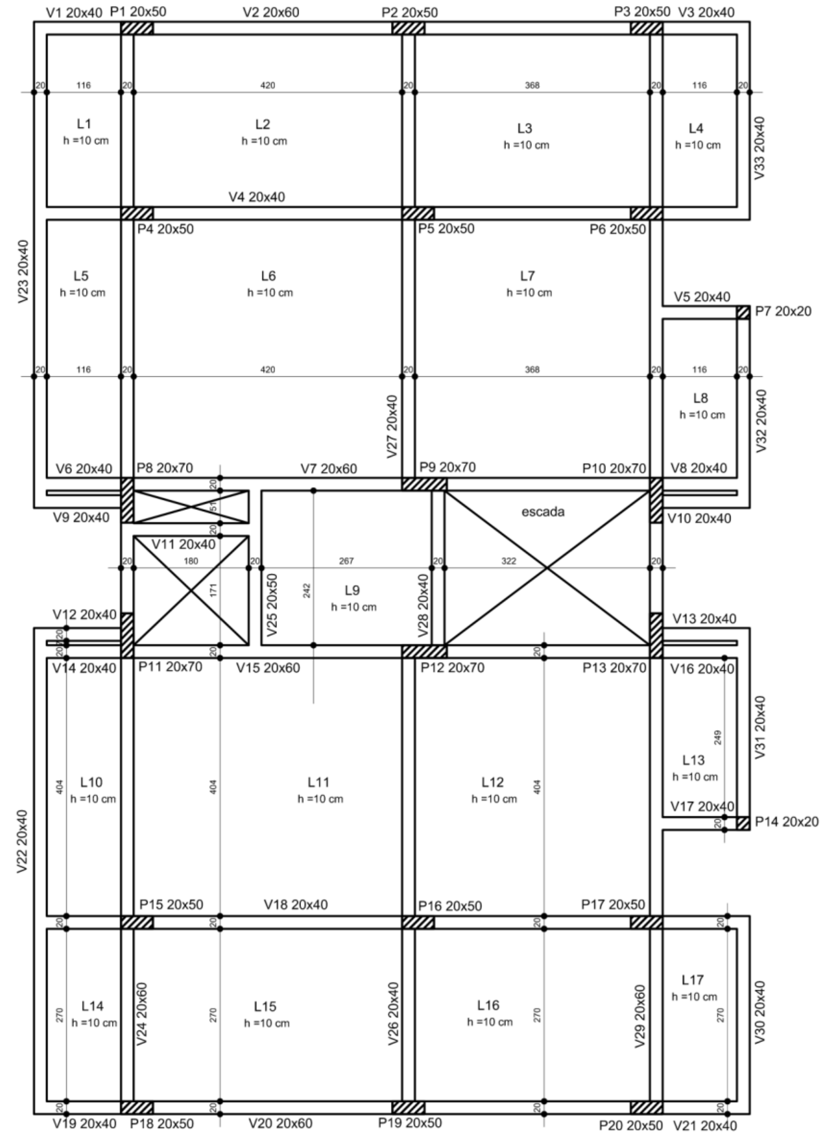

Figure 1: Formwork for typical floors with 33 beams (dimensions in cm)

At first, the same three original groups of beams in the example were used. Later, the beams were categorized into 5 and 10 groups. And finally, the heights of all 33 beams were regarded as distinct variables in the optimization process. Table 1 shows the monetary values of the overall cost (in Brazilian Reais, R$) and the percentage variation of optimal costs relative to the originally proposed structure.

Table 1: Results for typical floors with 33 beams

Assessed item	**Initial (3 section types)**	**Optimized (3 section types)**	**Optimized (5 section types)**	**Optimized (10 section types)**	**Optimized (33 section types)**
Overall cost (R$)	7,111.93	5,441.68	5,244.79	5,089.09	4,682.88
Variation in optimal cost relative to the original structure (%)	-	-23,9	-26,3	-28,4	-34,1

The results obtained for the same three groups of sections of the original structure demonstrate remarkable savings of 23.9% in the cost of the typical floor. It can be observed that, although the other situations analyzed led to a bigger economy (up to 34,1%), they represent larger difficulties in terms of construction.

Similar analyses to 3,5 and 10 groups were performed considering the cardinality constraints (CC). In these analyses, cardinality constraints are introduced to find alternative member groupings of beams. One can observe clearly that such a grouping procedure affects the final results and that its effectiveness depends crucially on the designer's experience in assigning members/variables to a group. As a result, it can be advantageous to the designer:

1) to limit the number of different design parameters (such as width and heights of the beams) aiming at i) achieving economy of fabrication, and ii) simplifying construction,
2) to leave to the optimizer algorithm the task of deciding how to group the beams, and
3) to reach the best or optimum solution within a given computational budget.

Objectives 1) and 2) can be achieved by introducing a cardinality constraint. Objective number 3) can only be attained with a careful formulation of the optimization problem on the part of the designer.

Table 2 compares the results obtained with and without cardinality constraints.

Table 2: Results for typical floors with 33 beams (with and without cardinality constraints)

Assessed item	**3 section types**	**5 section types**	**10 section types**
Optimize cost (R$) without CC	5,441.68	5,244.79	5,089.09
Optimized cost (R$) with CC	4,770.39	4,700.60	4,686.04
Additional reduction in optimal overall cost (%)	12,3	10,4	7,9

The results listed in Table 2 indicate an additional economy regarding the situation when the groups were not defined automatically by the optimizer. It can also be noticed that the optimized cost obtained with the usage of cardinality constraints to 3 section types is very similar to that which allows each beam to assume a different cross-sectional dimension (i.e., 33 section types).

5. Conclusions

Former results obtained by the authors from the optimization of reinforced concrete building floor grids allowed

confirming the efficiency of the software program in minimizing the cost of more complex structures, with remarkable savings in comparison with the initial solution. In the study that maintained the same number of cross-section types, which does not represent difficulties in terms of construction, savings amounted to 23,9%.
On the other hand, the consideration of cardinality constraints is quite easy, and can lead to a more economical structure, since the definition of member grouping is independent of the designer's skills.

6. Acknowledgements

To CAPES for financial support granted to the first author (AEX 1292/13-7), and to CNPq (306815/2011-7) and FAPEMIG (PPM 528-11) for the support to the third author.

7. References

[1] Brazilian Association of Technical Standards, Procedures for the design of reinforced concrete structures. NBR 6118, Rio de Janeiro, 2007 (in Portuguese).

[2] H.J.C., Barbosa, A.C.C Lemonge and C.C.H. Borges, A genetic algorithm encoding for cardinality constraints and automatic variable linking in structural optimization, *Engineering Structures*, v. 30 (12), p. 3708–3723, 2008.

[3] G.F. Medeiros and M. Kripka, M., Structural Optimization and Proposition of Pre-Sizing Parameters for Beams in Reinforced Concrete Buildings. *Computers and Concrete*, v.11, n.3, p. 1-18, 2013**.**

[4] A.C.C. Lemonge, H.J.C. Barbosa, A.L.G.A. Coutinho, A.L.G.A and C.C.H. Borges, Multiple cardinality constraints and automatic member grouping in the optimal design of steel framed structures. *Engineering Structures*, v. 33 (2), p. 433–444, 2011.

[5] D. Suji, S.C. Natesan, R. Murugesan and R.S. Prabhud, Optimal design of fibrous concrete beams through simulated annealing**.** *Asian Journal of Civil Engineering (Building and Housing*), v. 9, n. 2, p. 193-213, 2008.

[6] S.O. Degertekin, A comparison of simulated annealing and genetic algorithm for optimum design of nonlinear steel space frames. *Structural and Multidisciplinary Optimization*, v. 34, p. 347–359, 2007.

[7] S. Kirkpatrick, C.D. Gelatt and M.P.Vecchi, Optimization by Simulated Annealing. *Science*, v. 220, n. 4598, p. 671-680, 1983.

[8] O. Hasançebi, F. Erbatur, On efficient use of simulated annealing in complex structural optimization problems. *Acta Mechanica*, v. 157, p. 27-50, 2002.

[9] J. Park and M.. Ryu, Optimal Design of Truss Structures by Rescaled Simulated Annealing. *KSME International Journal*, v. 18, n. 9, p. 1512-1518, 2004.

[10] M. Kripka, Discrete optimization of trusses by simulated annealing. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Rio de Janeiro, v. 26, n. 2, p. 170-173, 2004.

[11] I. Payá-Zaforteza, V. Yepes, F. González-Vidosa and A. Hospitaler, Multiobjective Optimization of Concrete Frames by Simulated Annealing. *Computer-Aided Civil and Infrastructure Engineering*, v. 23, p. 596-510, 2008.

[12] F. González-Vidosa, V. Yepes, J. Alcalá, M. Carrera, C. Perea, I. Payá-Zaforteza, Optimization of Reinforced Concrete Structures by Simulated Annealing. In: TAN, C. M. *Simulated Annealing*. Áustria: I-Tech Education and Publishing, 2008.

[13] I. Payá-Zaforteza, V. Yepes, F. González-Vidosa and A. Hospitaler, On the Weibull cost estimation of building frames designed by simulated annealing. *Meccanica*, v. 45, p. 693-704, 2010.

[14] O. Hasançebi, S. Çarbas and M.P. Saka, Improving the performance of simulated annealing in structural optimization**.** *Structural and Multidisciplinary Optimization*, v. 41, p. 189–203, 2010.

[15] M. Sonmez, Discrete optimum design of truss structures using artificial bee colony algorithm. *Structural and Multidisciplinary Optimization*, v. 43, p. 85–97, 2011.

[16] R. Bordignon and M. Kripka, Optimum Design of Reinforced Concrete Columns Subjected to Uniaxial Flexural Compression, *Computers and Concrete*, 9(5), p. 345-358, 2012.