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Direct Fully Stressed Design for Displacement Constraints

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1. Abstract
This paper presents a new fully stressed design method for the solution of the general weight optimization problem applicable to determinate trusses subject to displacement constraints. Fully stressed design (FSD) has been a popular method to perform structural optimization when only stress constraints are considered. It is attractive because it reaches the optimal design following a few iterations which minimizes computational effort. For this reason, it is one of the earliest optimization techniques and was used prior to the advent of computers. However, FSD needs to be modified when displacement constraints are considered. This is known as fully utilized design (FUD). FUD has been implemented with several variations, at times utilizing energy methods. In this study, it is shown how there is the possibility to apply FSD directly to determinate trusses, even in the presence of displacement constraints. The method does not require the modification of FSD to FUD, but rather it manipulates the stresses to directly account for displacement constraints. This new technique can be easily automated using structural analysis software in conjunction with a spreadsheet. Presently, it is limited to determinate structures. However, it is quite appealing for its ease of implementation. A representative truss example is presented showing how the optimal design of trusses subject to displacement constraints is attained. Results compare favorably with those solved with other well-known optimization techniques as, for example, genetic algorithms.
2. Keywords: Optimization, structural design, trusses.

3. Introduction
Traditional optimization techniques encompass:
(1) mathematical programming
(2) optimality criteria
(3) approximation methods, and
(4) fully stressed design and fully utilized design.

Optimal structural design has been implemented for many years [1-4]. Fully stressed design is one of the first methods that was implemented for optimization. The reason being its ease of application and the low computational demand. Patanik et al. [5] argue that mathematical programming and optimality criteria were developed to overcome the shortcomings of fully stressed design. In fact, fully stressed design is apt to address only stress constraints. Even its modifications, such as fully utilized design, were leading to unsatisfactory results in the presence of displacement constraints. New optimization techniques were sought, where the domain is searched using the gradient of the objective function. A limitation arises when the function is not continuous and it is not possible to calculate its gradient. In the 1960s and 1970s genetic algorithms and neural network methods were conceived and developed [6].

The minimization of truss volume, subject of this paper, has been extensively studied. For example, optimization of large trusses using traditional algorithms was presented by Schmit and Lai [7]. Ghasemi et al. [8] have demonstrated the suitability of GA to address large trusses with many uncertain variables.

4. Genetic Algorithms
A brief introduction on Genetic Algorithms (GAs) is given here since GAs are used to compare the optimization method proposed in this manuscript with an established optimization technique. GAs are a heuristic method. They are of simple implementation, as they mimic natural selection. An initial truss population, randomly chosen, is ranked based on desired characteristics. For example a light truss which does not violate many constraints will achieve high rankings. Only half of the population is retained. Trusses, just like living beings, mate and produce offspring. The new population is ranked and the cycle starts again. When no improvement in the objective function is found, convergence occurs. GAs result in demanding several iterations prior to reach convergence, even for simple problems. The problem solved with the method of GAs is unconstrained. In the GAs the constraints cannot be expressed as a function of the design variables. Instead, the constraints are introduced by assigning a penalty to the structural volume of those trusses that exceed allowable stresses and displacements. Trusses that violate the constraints will not be able to mate.

5. Fully Stressed Design

One of the most popular optimality criteria methods is the fully stressed design. Optimality criteria methods are widely used in structural design. They resort to optimality criteria to arrive to a solution. They are mostly effective when a limited number of constraints, with respect to the number of design variables, is considered.

5.1. Fully stressed design is an intuitive optimality criterion

Fully stressed design (FSD) has been attractive to aerospace engineering. The high demand in reducing the structural weight for aircrafts and other vessels spur the research in the optimization field. The fully stressed design is categorized among the intuitive optimality criteria as it is based on a simple principle. This criterion is stated as:
"For the optimum design, each member of the structure that is not at its minimum gage must be fully stressed under at least one of the design load conditions." [4]
This enunciation is self-explanatory. In fact, FSD can handle problems subject to stress and minimum weight constraints. When a structural member does not reach its allowable stress, its area is reduced in order to make it "fully stressed." A minimum gage is imposed to not lose any member and compromise the stability of the structure. Problems arise when the stress in a member affects the other components of the structure. In this case, reducing a member size so that its stress equals the allowable value, might increase significantly the stress in other members. This situation is typical of indeterminate structures. The convergence of FSD can be guaranteed only through several iterations. An optimal solution must accept that some members are not fully stressed to reach the best possible weight for the entire structure [9].

5.2. Characteristics of FSD

The considerations made in the previous section allow listing some advantages and disadvantages in employing FSD for structural design. Favorable to FSD are its ease of implementation, and the few iterations required to reach an optimum. A fully stressed design is often near the true optimal solution. Even in the case where the actual optimum is not achieved, FSD allows an appreciable improvement with respect of the traditional design. Traditional is intended in the sense that satisfies safety without maximizing savings. In addition, it is notable that FSD does not require derivatives to search the feasible domain. The main disadvantage of FSD is that it cannot handle multiple constraints and highly indeterminate structures. Another drawback in implementing FSD include the fact that at times convergence to an optimum is not obtained.

5.3. Minimal volume truss optimization

This paper introduces later examples of volume minimization for truss optimal design. The structural optimization problem can be stated as in Eq. (1):

$$\text{Minimize } V(A_i, P_k) \text{ such that}$$

$$\sigma_i(A_i, P_k) \le \sigma_{i0}$$

$$x_j(A_i, P_k) \le x_{j0} \tag{1}$$

Where

- V is the volume expressed as a function of the design parameters, i.e. the cross-sectional areas (A_i). V depends also on the external loads (P_k)
- i = number of members, j = number of degrees of freedom, and k = number of nodal loads
- $\sigma_i(A_i, P_k)$ and $x_j(A_i, P_k)$ are the constraints, i.e. the stresses and the displacements respectively
- σ_{i0} and x_{j0}, are the allowable values for constraints $\sigma_i(A_i, P_k)$ and $x_j(A_i, P_k)$.

5.4. Implementation of FSD in Structural Design

The algorithm that allows implementing FSD is based on assigning the area to each member according to their allowable stress. The method must assume that each area carries a constant force, dependent only on the external loads. In other words, the internal force of one member is supposed not to affect the forces in other members. A "re-sizing" technique is used assuming that F_i, the internal force in the i^{th} member, is constant. F_i can be expressed by stress σ_i times the cross-sectional area, A_i. This leads to the subsequent formulation, Eqs, (2 and 3):

$$F_i = \sigma_{i,new}\, A_{i,new} = \sigma_{i,old}\, A_{i,old} \tag{2}$$

$$A_{i,new} = A_{i,old} \frac{\sigma_{i,old}}{\sigma_{i0}} \tag{3}$$

For determinate structures the assumption that each internal force is independent from the others is exact. The same does not hold true for indeterminate structures where iteration is necessary. Convergence is achieved when a pre-established tolerance is met.

5.5. The Fully Utilized Design (FUD) Method

FSD is easy implemented when stress constraints alone are present. To be able to apply it to problems encompassing also displacement constraints, FSD must be modified. First, FSD is applied until convergence occurs accounting only for stress constraints. If the maximum displacement of the structure exceeds the allowable, it is necessary to increase the areas of the structural member. The easiest path is to augment all the areas using a uniform scaling factor. This method is illustrated in the following Eq. (4):

$$A_{i,FUD} = A_{i,FSD} \frac{x_{\max,FSD}}{x_0} \tag{4}$$

This implementation is simple but often produces a redundant design, which is not the true optimum. The method can be improved by adopting different scaling factors for each structural member.

5.6. Direct Fully Stressed Design for Displacement Constraints

A different approach to problems with displacement constraints is adopted here. This method will be called hereafter Direct Fully Stressed Design (DFSD). The displacement is viewed in terms of its effect on the truss internal forces. The structural response is manipulated in order to distribute the areas in a way to satisfy simultaneously the stress and displacement constraints. This method is under investigation and is being developed. However, some preliminary results have been obtained. They are very promising for determinate structures, but the method has not been proven effective for indeterminate structures yet.

6. Examples

A twenty-five bar aluminum truss was chosen as a representative example for the direct fully stressed design method accounting for both stress and displacement constraints.

6.1. Geometry, material properties and load condition.

Figure 1 shows the loading condition and the geometry of the truss. Vertical loads are affecting the top chord. The value of P_1 is 30 Kip (133.4 kN). Aluminum has a modulus of elasticity equal to 10^4 ksi ($69 * 10^6$ kPa). The truss is determinate, and simply supported at its ends. There are six-bays each 360 in (9,144 mm) long for a total length of 72 ft (approximately 22 m). The height of the truss also measure 360 in (9,144 mm).

6.2. The structural optimization problem.

The objective function to be minimized is the volume of the structure. The design variables are the areas of the members. The stresses cannot exceed the limit of 25 ksi ($172 * 10^3$ kPa) for all members in tension or compression. No minimum gage value for the cross sectional area is set for this example. The allowable displacement is set at 2 in. (50.8 mm). The optimization is displacement sensitive given the loading condition and the large truss span. The maximum displacement is vertical and occurs at mid-length, at 36 ft (11 m) from each support at the bottom chord joint D. Since this joint experiences the largest displacement for the given loading, it is chosen to be constrained.

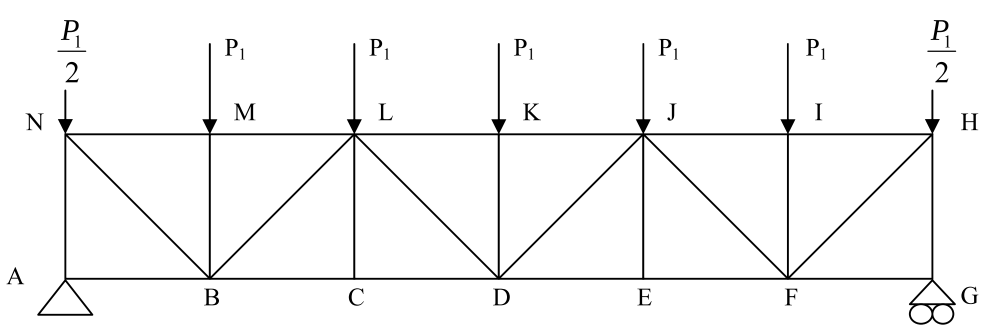

Figure 1: 25-bar truss

6.3 Results

The problem is solved employing the DFSD method which requires only one iteration. The optimization is also shown for comparison when genetic algorithms are employed. For the 25-bar truss results obtained with the DFSD are shown alongside those obtained using GAs. From Table 1, it can be noticed that the comparison is excellent, with DFSD yielding a slightly better volume. This could be due to the fact that no minimum gage was imposed in the solution using DFSD.

Table 1: 25-bar truss areas

Member	*DFSD Areas / in^2*	*GA Area / in^2*
AB	0	0.118
BC	12.60286	13.659
CD	12.60286	13.344
DE	12.60286	13.156
EF	12.60286	11.553
FG	0	0.114
GH	7.50136	9.004
HI	6.90136	7.825
IJ	6.90136	7.240
JK	17.104	16.087
KL	17.104	16.364
LM	6.90136	7.067
MN	6.90136	8.395
NO	7.50136	7.344
NA	9.760096	10.382
NB	1.2	1.208
MB	8.063	7.241
BL	0	0.102
LC	6.365984	3.960
LD	1.2	1.202
KD	6.365984	5.765
DJ	0	0.113
EJ	8.063	8.397
JF	1.2	1.210
FH	9.760096	10.730
Volume (in^3)	28691.23	28919.6

Note: 1 in^2 = 6.45 cm^2, 1 in^3 = 16.39 cm^3

7. Discussion on DFSD

This paper has set the foundation for improving the application of FSD where in the presence of displacement constraints. However more research is necessary to validate the success of the proposed methodology. The fundamental next steps to advance this study are:

- Utilize a structural analysis routine embedded in the iteration of FSD to unite the structural analysis software and the optimization one. This would automate the re-sizing of the cross sectional areas. This step is of easy implementation and would require minimum programming. Alternatively, the structural design could be carried out using matrix analysis and a mathematical package such as MATLAB or Mathcad.
- More examples can be solved for determinate structures. This would prove that the method can handle larger structures with more than one active displacement constraints.
- The most challenging step would be to extend this study to indeterminate structures. This would require further developing the theory and the implementation of DFSD.

8. Conclusion

A modification of the well-known fully stressed design method has been presented to account for displacement constraints. The proposed methodology is efficient since it requires minimum calculation effort but is limited in application. In fact, DFSD displays an attractive solution to determinate trusses and it is worth pursuing more research to generalize the method to larger and indeterminate structures.

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