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CADMM applied to Hybrid Network Decomposition

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Abstract

As system design problems increase in complexity, researchers seek approaches to optimize such problems by coordinating the optimizations of decomposed subproblems. The original problem may either be decomposed according to the involved disciplines or constitutive components. However, a mixed decomposition is obtained when a new discipline (component) is added to a problem decomposed by components (disciplines) to reflect the inherent dynamics within the design process. The resulting problem becomes a non-hierarchical, network optimization problem that requires a suitable Multi-Disciplinary Optimization (MDO) and Multi-Level Optimization (MLO) coordination approach. In this paper, two hybrid decompositions are used for a micro-accelerometer benchmark problem. In these hybrid problems, each subproblem is fully coupled with the other subproblems. Consensus optimization via Alternating Direction Method of Multipliers (CADMM) is proposed for modeling and solving the decomposed optimization problem. The numerical results error of CADMM is within 3% of the All-In-One reference solution, which indicates that the consensus mechanism in CADMM for dealing with coupling variables performs well on the hybrid network problem. This is, to our knowledge, the first application of CADMM to hybrid decompositions in which subproblems represent disciplines and/or components.

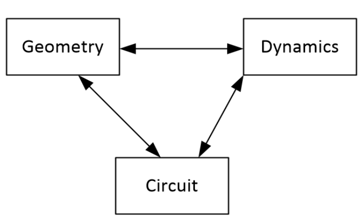
Keywords: Network problem; Consensus optimization; Micro-accelerometer problem; Hybrid decomposition

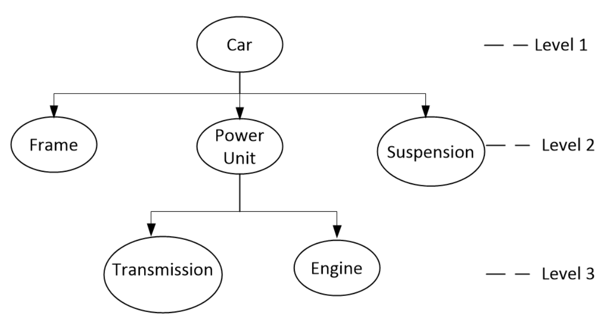
1. Introduction

In our modern society, people are depending more and more on high technology products. From cell phone to automobile, from aircraft to satellites, all these engineering products can be considered complex systems. Even consumer products can be considered complex if one considers all the components and disciplines involved in solving the problem. Such systems involve interactions among several disciplines (mechanical, electrical, etc.) and consist of many interacting components.

To design and optimize such complex systems, optimization techniques have been developed. Multidisciplinary Design Optimization (MDO) [1] decomposes problems into different disciplines that are optimized autonomously; Multilevel Optimization (MLO) decomposes problems according to levels in a hierarchy. Unlike single-level methods which have a single, centralized decision-making process, MLO distributes the decision-making tasks over each subproblem [2]. Both MDO and MLO have been applied to various engineering problems such as vehicle design and aircraft design [3][4][5].

In the practical engineering design process, it is common to modify a decomposition during the design process. The introduction of a new component (discipline) to a discipline- (component-) based decomposition reflects the dynamics of the design process as requirements and criteria evolve. This forms a hybrid network optimization problem in which the interacting nodes are either components or disciplines [6]. Some methods such as Analytical Target Cascading (ATC) [7][8] imply a hierarchical structure and may not be directly applicable to this kind of hybrid network optimization problem





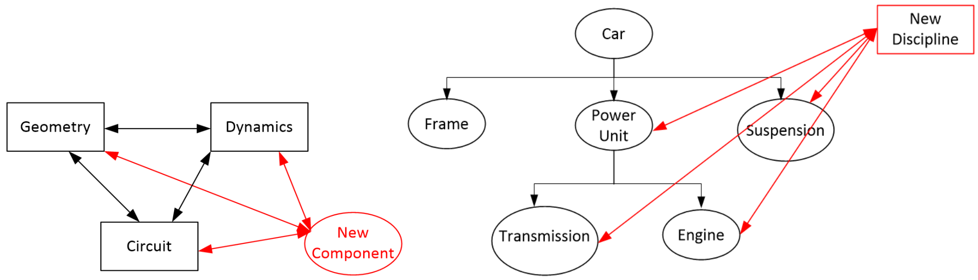


Figure 1: top left - discipline-based decomposition; top right – component based decomposition; bottom – hybrid decompositions (rectangle stands for discipline, circle stands for component)

Consensus Optimization via the Alternating Direction Method of Multipliers (CADMM) for Network Target Coordination (NTC) is a distributed coordination method designed to optimize subsystems that are decomposed in a nonhierarchical fashion [9].Through the consensus variables, coupled systems are solved concurrently and the method of multipliers is used to efficiently reduce the inconsistency between the linking variables. Its efficacy has been proved by some engineering examples [9][11].
In this paper, we adopt CADMM to solve two hybrid nonhierarchical formulations of the ADXL150 micro-accelerometer design problem [10]. Both problems involve three disciplines (Structures, Dynamics, Electrostatics) and several components (Proof mass, U-spring, Comb, etc.). By adding and removing disciplines and/or components, the flexibility of the hybrid network (nonhierarchical) is explored. The results are discussed and conclusions drawn with respect to the modeling capabilities and computational efficiency of the proposed approach.

2. Consensus Optimization via the Alternating Direction Method of Multipliers (CADMM) for Network Target Coordination (NTC)

Assume the following problem formulation

$$\begin{aligned} &\min_{\mathbf{x}_i,\mathbf{y}} \sum_{i=1}^{3} f_i(\mathbf{x}_i,\mathbf{y}) \\ &\textit{subject to } \ \mathbf{g}_i(\mathbf{x}_i,\mathbf{y}) \le \mathbf{0}, \textit{ for } i=1,\dots,3 \\ &\qquad\qquad\quad \mathbf{h}_i(\mathbf{x}_i,\mathbf{y}) \le \mathbf{0}, \textit{ for } i=1,\dots,3 \end{aligned} \tag{1}$$

where x_i is the local design variable for each subproblem, $\boldsymbol{y}$ is the coupling variable, f_i is the local objective function and g_i, h_i are local constraint functions. Problem (1) is decomposable into three subproblems as shown in Figure 2 (left). For subproblem 1 the coupling variables are $\mathbf{y}_1 = [\mathbf{y}_{12}\ \mathbf{y}_{13}]$, for subproblem 2 $\mathbf{y}_2 = [\mathbf{y}_{12}\ \mathbf{y}_{23}]$, and for subproblem 3, $\mathbf{y}_3 = [\mathbf{y}_{13}\ \mathbf{y}_{23}]$.
To decouple the subproblems, we introduce copies for $\mathbf{y}_{12}$, $\mathbf{y}_{13}$, $\mathbf{y}_{23}$ at each subproblems $[\mathbf{y}_{12}]_1$, $[\mathbf{y}_{12}]_2$, $[\mathbf{y}_{13}]_1$, $[\mathbf{y}_{13}]_3$, $[\mathbf{y}_{23}]_2$, $[\mathbf{y}_{23}]_3$, and let $\mathbf{y}_1 = [[\mathbf{y}_{12}]_1\ [\mathbf{y}_{13}]_1]$, $\mathbf{y}_2 = [[\mathbf{y}_{12}]_2\ [\mathbf{y}_{23}]_2]$, $\mathbf{y}_3 = [[\mathbf{y}_{13}]_3\ [\mathbf{y}_{23}]_3]$. The consensus variables $\mathbf{z}_i$ are adopted to ensure all these copies to be consistent $\mathbf{z}_i - \mathbf{y}_i = \mathbf{0}$.

The decomposed formulation obtained introducing the consensus variables is shown in Figure 2 (right).

$$\begin{aligned} &\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}_3} \sum_{i=1}^{3} f_i(\mathbf{x}_i,\mathbf{y}_i) \\ &\textit{subject to } \ \mathbf{g}_i(\mathbf{x}_i,\mathbf{y}_i) < \mathbf{0}, \textit{ for } i=1,\dots,3 \\ &\qquad\qquad\quad \mathbf{h}_i(\mathbf{x}_i,\mathbf{y}_i) \le \mathbf{0}, \textit{ for } i=1,\dots,3 \\ &\qquad\qquad\quad \mathbf{h}(\mathbf{z}_i,\mathbf{y}_i) = \mathbf{z}_i - \mathbf{y}_i = \mathbf{0}, \textit{ for } i=1,\dots,3 \end{aligned} \tag{2}$$

The following formulation is obtained relaxing the consistency constraints to the objective function

$$\min_{x_i, y_i, z_i} \quad f_i(\mathbf{x}_i, \mathbf{y}_i) + \mathbf{v}_i^{\mathrm{T}}(\mathbf{y}_i - \mathbf{z}_i) + \frac{\rho}{2}\|\mathbf{y}_i - \mathbf{z}_i\|^2$$
$$\textit{subject to} \;\; \mathbf{g}_i(\mathbf{x}_i, \mathbf{y}_i) \le \mathbf{0}, \textit{for } i = 1, \ldots, m \tag{3}$$
$$\mathbf{h}_i(\mathbf{x}_i, \mathbf{y}_i) \le \mathbf{0}, \textit{for } i = 1, \ldots, m$$

where $\boldsymbol{v}_i$ is the vector of Lagrangian multipliers of subproblem i, ρ is the weight for the quadratic penalty term .
Problem (3) can be solved using a Gauss-Seidel iterative method: the primal variables $\mathbf{x}_i$, $\mathbf{y}_i$ ad $\mathbf{z}_i$ are collected in two blocks **[**$\mathbf{x}_i$, $\mathbf{y}_i$**]** and **[**$\mathbf{z}_i$**]** that are updated sequentially [9][11]. Block **[**$\mathbf{x}_i$, $\mathbf{y}_i$**]** (**[**$\mathbf{z}_i$**]**) is obtained solving (3) while considering **[**$\mathbf{z}_i$**]** (**[**$\mathbf{x}_i$, $\mathbf{y}_i$**]**) constant and equal to the most recent update. The decomposition paradigm is exploited in the first step when **[**$\mathbf{x}_i$, $\mathbf{y}_i$**]** can be computed optimizing each subproblem independently (Step 1 in Figure 2 (right)). Note that the consensus variables $\mathbf{z}_i$ are obtained in a closed form since (3) is an unconstrained quadratic problem with respect to $\mathbf{z}_i$ (Step 2 in Figure 2 (right)) Once the primal variables are obtained, the dual variables must be calibrated in order to achieve the feasibility of the consistency constraints. These steps read

$$\textit{step } 1: \; \mathbf{y}_i = \arg \min_{\mathbf{x}_i, \mathbf{y}_i} (f_i(\mathbf{x}_i, \mathbf{y}_i) + (\mathbf{v}^k{}_i)^{\mathrm{T}}(\mathbf{y}_i - \mathbf{z}_i) + \frac{\rho}{2}\|\mathbf{y}_i - \mathbf{z}_i\|^2)$$
$$\textit{step } 2: \; \mathbf{z}_{ij} = \frac{1}{2}(\mathbf{y}_{ij} + \mathbf{y}_{ji} + \frac{1}{\rho}\mathbf{v}_{ij}{}^k + \frac{1}{\rho}\mathbf{v}_{ji}{}^k), \qquad j \in N(i) \tag{4}$$
$$\mathbf{v}_{ij}{}^{k+1} = \mathbf{v}_{ij}{}^k + \rho(\mathbf{y}_{ij} - \mathbf{z}_{ij}), \qquad j \in N(i)$$

N(i) is the index of subproblems that share the same coupling variables with the subproblem i. $\boldsymbol{y}_{ij}$ are the coupling variables shared by subproblems i and j. $\boldsymbol{v}_{ij}$ are the Lagrangian multipliers associated with $\boldsymbol{y}_{ij}$.
The optimization terminates when the consistency error between the copies of coupling variables and the consensus are satisfied within the desired tolerance and no improvement on the objective functions is possible. The consistency error is calculated as following:

$$\mathbf{c}_{ij} = z_{ij} - y_{ij}, \qquad \textit{for } i = 1, \ldots, m, \; j \in N(i) \tag{5}$$

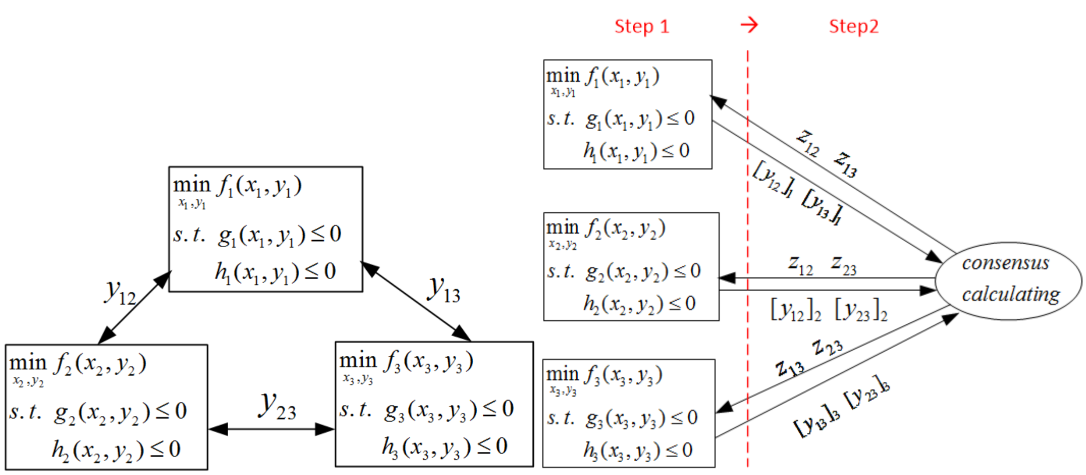


Figure 2: Original problem (left) and decoupled problem (right) for CADMM

3. Micro-accelerometer Benchmark Problem and its Hybrid Decomposition

The micro-accelerometer problem is a non-convex, nonlinear engineering test problem which is proposed as a benchmark problem for testing and comparing different multidisciplinary optimization methods. It involves four disciplines: Structures, Electrostatics, Dynamics, and Circuits. The four disciplines may depend on the same design variables or one discipline can depend on the output of another discipline [10]. There are 22 design variables at most. The design objective is to minimize the footprint area A (which is proportional to fabrication cost). The design constraints are requirements that make sure the performance with respect to sensitivity, noise, and range is at least as good as the baseline design.
In [10], this problem is decomposed in four ways and solved using the Augmented Lagrangian Method (ALC)

Details about this problem can be found in reference [10].

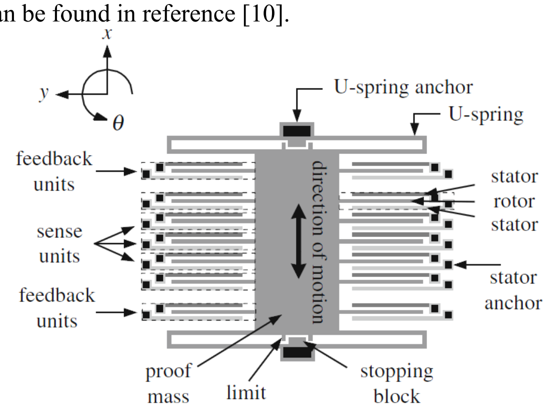


Figure 3: Schematic illustration of micro-accelerometer design [10]

In this paper, by adding and removing disciplines or components, the flexibility of the hybrid network (non-hierarchical) is explored. We propose two hybrid decompositions for solving this problem. In decomposition 1, a new discipline (Dynamics) is added into the original component decomposed problem, resulting in a hybrid problem. In decomposition 2, a new component (Springs) is added into the original discipline decomposed problem. Both decompositions do not include design variables and constraints in the Circuit discipline, which is one aspect considered in [10]. The number of design variables is 16.

3.1 Hybrid Decomposition 1

The original problem is decomposed into two components: Springs and Mass-Fingers. These two components share five design variables. Mass-Fingers also needs the output of Springs ($k_{x,m}$) as its parameter. The two components only involve two disciplines: Mechanics and Electrostatics. The constraints g_s and g_e correspond to the two disciplines and g_s is distributed among the different components.

When we add Dynamics as a new discipline to the original problem, the new subproblem needs the outputs of Springs and Mass-Fingers. As shown in figure 4, for this decomposition, each subproblem is coupled with the other two subproblems. It is a typical network optimization problem [11].

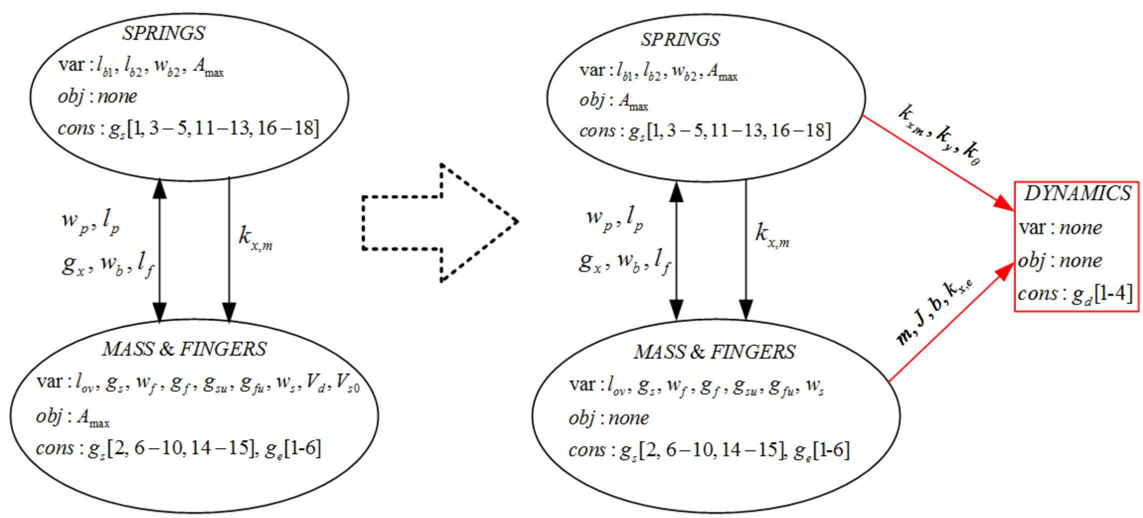


Figure 4: Hybrid decomposition 1 for the micro-accelerometer problem

3.2 Hybrid Decomposition 2

The original problem is decomposed into three disciplines: Mechanics, Electrostatics and Dynamics. They couple with each other either through sharing the same design variable or linking functions. In addition to g_s and g_e, g_d is the constraint corresponding to the dynamics discipline.

When we add springs as a new component to this original problem, this new subproblem shares 6 design variables with the mechanics subproblem, and generates stiffness for the dynamics and electrostatics subproblems. The resulting hybrid decomposition is shown in figure 5. Each subproblem in this decomposition network problem is coupled with the other three subproblems.

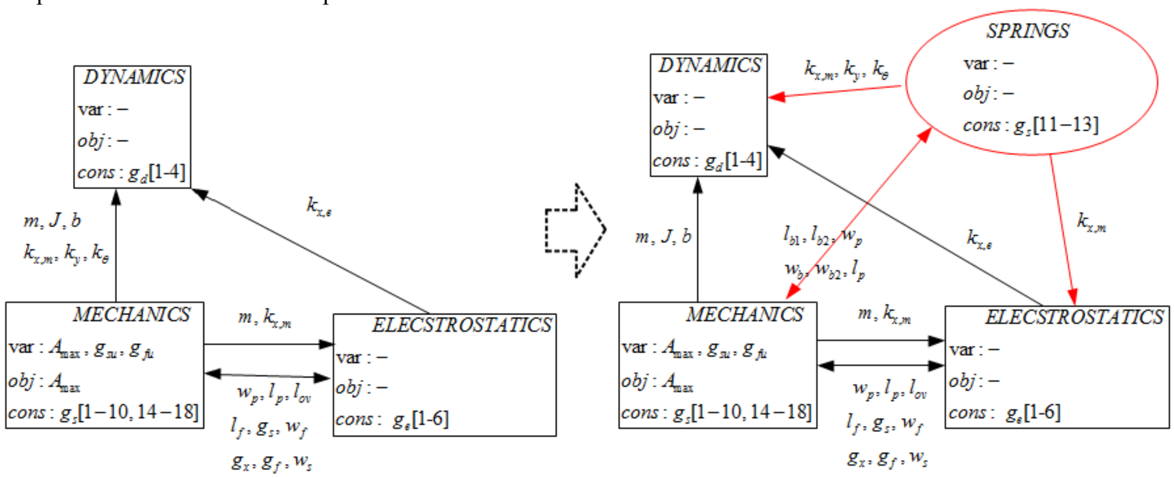


Figure 5: Hybrid decomposition 1 for the micro-accelerometer problem

4. Application of CADMM to hybrid Micro-accelerometer problems and Results

4.1 Experimental Setup

In this paper, numerical experiments are conducted in Matlab R2012a. The constrained minimization function "*fmincon*" is used.

The limit for the number of iterations of the optimization loop is 1e3. The program converges if the consistency error (equation 5) is less than the termination tolerance ε = 1e-3. The initial starting points for each subproblem are randomly selected between 50% - 150% of the baseline design value. At each iteration, the exitflag output of *fmincon* of each subproblem is checked to ensure they converge. If one subproblem does not converge within 10 *fmincon* tries, its optimal value from last iteration is used for the optimization of the other subproblems and parameter updating.

The initial value of ρ is 0.03. Instead of using a fixed penalty parameter ρ, we update ρ based on the inconsistency error as follows:

$$\begin{cases} if \quad \max(\mathbf{c}^k)/\max(\mathbf{c}^{k-1}) > \xi,\ \rho^{k+1} = \lambda * \rho^k \\ if \quad \max(\mathbf{c}^k)/\max(\mathbf{c}^{k-1}) \le \xi,\ \rho^{k+1} = \rho^k \end{cases} \tag{6}$$

In this way, the penalty term is small at the beginning of optimization process. In this way, during the first iterations the objective function is minimized and then as the penalty weight increases the consistency error becomes more important and the consistency error is reduced..

The optimal objective value of All-in-one (AIO) formulation is 4.5322e-08 m^2, which is used as the reference solution for both problems.

4.2 Results and Discussion

First, different ξ and λ are tried to find the appropriate penalty weight update parameter. The results are shown in Table 1. The 4th column is the final objective value, which is the footprint area for the micro-accelerometer. The 5th column is the number of iterations it takes for the optimization process to converge. The 6th column is the maximum design variable error compared to the All–In-One reference solution.

It takes less iteration for large ρ and λ to converge compared with small ρ and λ. However, the cost of this is the optimal solutions are farther from the reference solutions. On the base of this simple consideration and some trials, the following parameter setting is chosen: *initial* ρ = 0.03, ζ = 0.99, and λ = 1.01.

Table 1: Optimization results of CADMM applied to the hybrid decomposition2 using different penalty weight update parameter

Initial ρ	ζ	λ	*Area*(mm^2)	*Iter*	*Max_vari _error*
0.1	0.8	1.2	0.0750	50	5.5923
.05	0.9	1.1	0.0618	111	3.2623
0.03	0.99	1.01	0.0460	434	0.9589

Table 2: Optimization results of CADMM applied to the two kind of hybrid decompositions

		Area(mm^2)	*Iter*	*Max_vari _error*
Hybrid 1	Min	0.0458	459	0.7549
90%	Mean	0.0467	538	1.1135
	Max	0.0533	769	2.4103
Hybrid 2	Min	0.0457	367	0.9253
90%	Mean	0.0460	434	0.9589
	Max	0.0475	589	1.1281

Table 2 summarizes the numerical results of both problems. For each hybrid problem, ten runs are attempted starting from different random initial points. The percentage in table 1 indicates how many runs have converged out of all test runs. All three metrics are statistically categorized into min, mean and max.

The high number of iterations may be due to the small initial value for the penalty term ρ and update parameter λ for ρ. One explanation for the high design variable error is that the problems have many local optimal solutions which are far from the reference solution. Another is that the objective value is not sensitive to some design variables. So although the values for these design variables differ a lot from the reference solution, the objective does not. It can be seen that the results for both hybrid decompositions are very close to the reference optimal solution (with mean value 0.0467 for hybrid 1and 0.0460 for hybrid 2).

5. Conclusions

The hybrid component-discipline based decomposition is common in the engineering field. The subproblems in this decomposition are coupled in a network way, thus each subproblem may interact with any of the other subproblems. This kind of structure is difficult for multilevel hierarchical decomposition methods to handle. This paper proposes to apply a systematic mathematical method - Consensus optimization via Alternating Direction Method of Multipliers (CADMM) to the hybrid problem.

By fully exploring the flexibility of decomposition of the micro-accelerometer benchmark problem, two kinds of hybrid network decompositions are proposed. One is adding a new discipline to a component-decomposed problem; the other one is adding a new component to a discipline-decomposed problem. The CADMM is employed to solve these two decompositions. Numerical experiments show that the optimization results of the CADMM are very close to the reference optimal solution (with an error less than 3%). This demonstrates that the CADMM is able to deal with the hybrid network decomposition problem and supports component-discipline decomposition and subsystem optimization to solve the overall problem. Additional implementations are planned to further refine the method and show its flexibility and robustness.

6. Acknowledgements

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