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Topology Optimization Method for Dynamic Fatigue Constraints Problem

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1. Abstract

In this research, a new topology optimization (TO) method was proposed to consider dynamic failure criteria (fatigue) under constant and proportional loading. Despite the great development of the topology optimization, the TO method considering the static or dynamic failure constraints has been regarded as one of the difficult problems. Although the TO method for the static failure has been studied extensively nowadays, the TO method considering the dynamic fatigue constraints is remained as an unexplored field. In order to address the dynamic failure in TO, this research develops a new fatigue-constrained topology optimization procedure. Because the dynamic responses as well as the static responses should be considered, it is more difficult than the stress-based topology optimization due to the non-differentiable fatigue criteria of the modified Goodman, the Soderberg and the Gerber theories. By addressing these issues numerically, this research can solve the topology optimization problem considering the fatigue constraint successfully.

2. Keywords: topology optimization, dynamic fatigue constraints

3. Introduction

This research develops a novel fatigue constraint topology optimization (FCTO) method to consider dynamic fatigue failure criteria based on the stress-life approach under constant and proportional loading as shown in Figure 1. In the initial phase of topology optimization development, the stress-based topology optimization is considered as one of difficult problems due to several numerical difficulties like the singularity [1-9], local behavior [7-11], and highly nonlinear behavior of stress constraint [9, 12]. These difficulties are successfully resolved by previous elaborated studies therefore various kinds of stress-based topology optimization methods are developed and applied for structural optimization problem nowadays. Despite great development of stress-based topology optimization methods, the TO method considering dynamic fatigue failure criteria remains an unexplored research field. Because our FCTO method is based on the stress-life approach under constant and proportional load, three difficulties of stress-based topology optimization are basically inherent as well as newly observed numerical difficulties. In this study, a stable FCTO method is developed by resolving aforementioned numerical difficulties successfully. In short, this research contributes one of important research topic of TO considering dynamic fatigue failure by developing a new FCTO method.

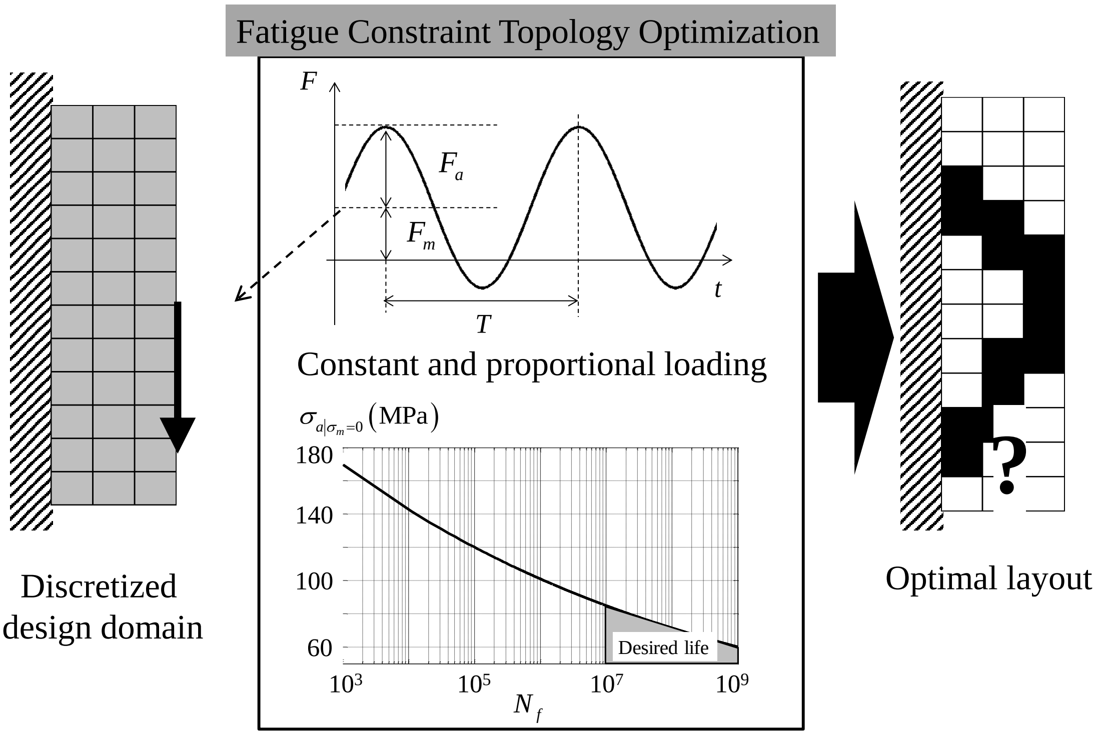


Figure 1: The application of topology optimization with the fatigue constraint.

4. Formulation of Fatigue Constraint Topology Optimization

The FCTO method considers a static failure as well as dynamic fatigue failure. Also the TO problem can be formulated to minimize material usage subject to static failure and dynamic fatigue constraint as follows.

$$\begin{aligned}
&\underset{\gamma}{\text{Minimize}} && V(\gamma)=\sum_{e=1}^{NE}\tilde{\gamma}_e v_e && (\tilde{\gamma}:\text{Filtered density})\\
&\text{subject to} && \left\langle f_{1,\max}\right\rangle_k \le 1, && \\
& && \left\langle f_{2,\max}\right\rangle_k \le 1, && \\
& && \left\langle f_{3,\max}\right\rangle_k \le 1, && k=1,2,\cdots,RN\\
& && \tilde{\gamma}=\Xi(\gamma)\text{ with the denstiy filter }\Xi &&
\end{aligned} \tag{1}$$

where the objective function is formulated to minimize the sum of multiplication of filtered design variable, $\tilde{\gamma}_e$ and volume of *e*th element v_e. The first constraint of TO problem (1), $\left\langle f_{1,\max}\right\rangle_k$ represents the regional dynamic fatigue constraint formulated by using the *p*-norm constraint approach. Also, the first constraint is formulated by using the stress life approach considering mean stress effect such as the modified Goodman, Soderberg, and Gerber criterion [13-16]. The second and third constraints, $\left\langle f_{2,\max}\right\rangle_k$ and $\left\langle f_{3,\max}\right\rangle_k$ represent the regional static failure constraints which are formulated to prevent one time static failure. These constraints are formulated by using the alternating and mean stress to constrain the maximum or minimum stress to be smaller than the yield strength. Because the design domain is divided into *RN* to consider local failure constraint and three kinds of constraints are considered, the number of constraints in the problem (1) is $3\times RN$. Because the failure constraints are local constraint, the regional *p*-norm approach is applied for computational efficiency as follows.

$$\left\langle f_{i,\max}\right\rangle_k = c_{i,k}^{iter}\left\langle f_{i,PN}\right\rangle_k,\ \left(e\in\Omega_k\right) \tag{2}$$

$$\left\langle f_{i,PN} \right\rangle_k \equiv \left(\sum_e (f_{i,e})^p \tilde{\gamma}_e \right)^{1/p} \ , \ f_{i,e} \geq 0, \ (e \in \Omega_k) \tag{3}$$

$$c_{i,k}^{iter} = \alpha \frac{f_{i,k,\max}^{iter-1}}{\left\langle f_{i,PN} \right\rangle_k^{iter-1}} + (1-\alpha) c_{i,k}^{iter-1} \quad 0<\alpha<1 \tag{4}$$

In the above equations, the *p*-norm criterion is multiplied by the correction parameter, $c_{i,k}^{iter}$, determined as the ratio between the value of the *p*-norm criterion and the value of the maximum constraint function of the previous iteration with the damping parameter, α, to avoid oscillations of the constraints.

5. Numerical Example

To demonstrate usefulness of the present FCTO method, cantilever beam topology optimization problem is solved. The material properties of plain carbon steel 1020 were used. Young's modulus and Poisson's ratio are E=200 GPa and v=0.3, respectively. The ultimate tensile strength, Yield strength, and density were set to 300 MPa, 165 MPa, and 7860 kg/m^3, respectively. To determine the alternating stress value without mean stress, $\sigma_{a|\sigma_m=0}$, for a minimum desired number of loading cycles in the S-N diagram, the *Basquin* equation was employed as follows.

$$\sigma_{a|\sigma_m=0} = \sigma_f \left(2N_f\right)^{b_f} \tag{5}$$

where the fatigue strength coefficient and the exponent of the *Basquin* equation are denoted by σ_f and b_f, respectively. The value of σ_f and b_f are set as 300 MPa and -0.075, respectively. The minimum desired loading cycles, N_f, was set at 10^7, therefore the feasible region of S-N diagram can be represented as shown in Figure 2.

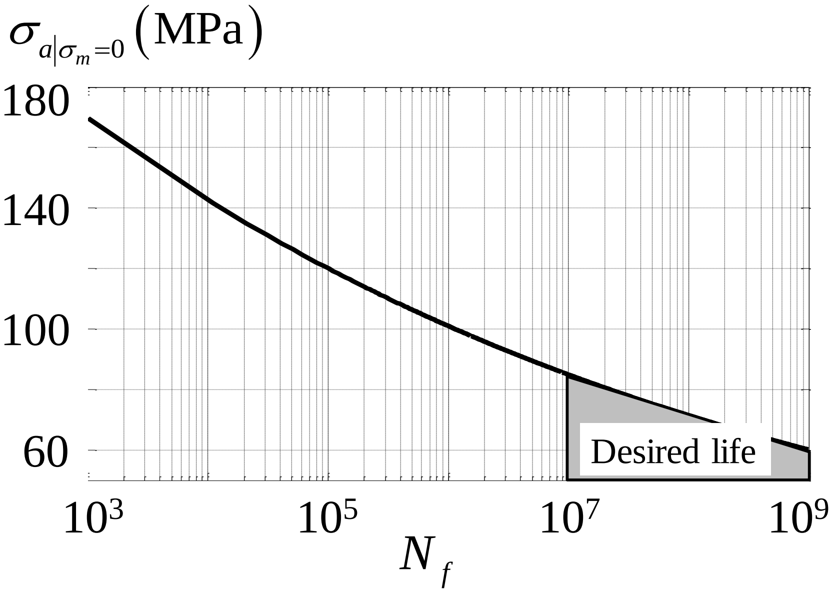
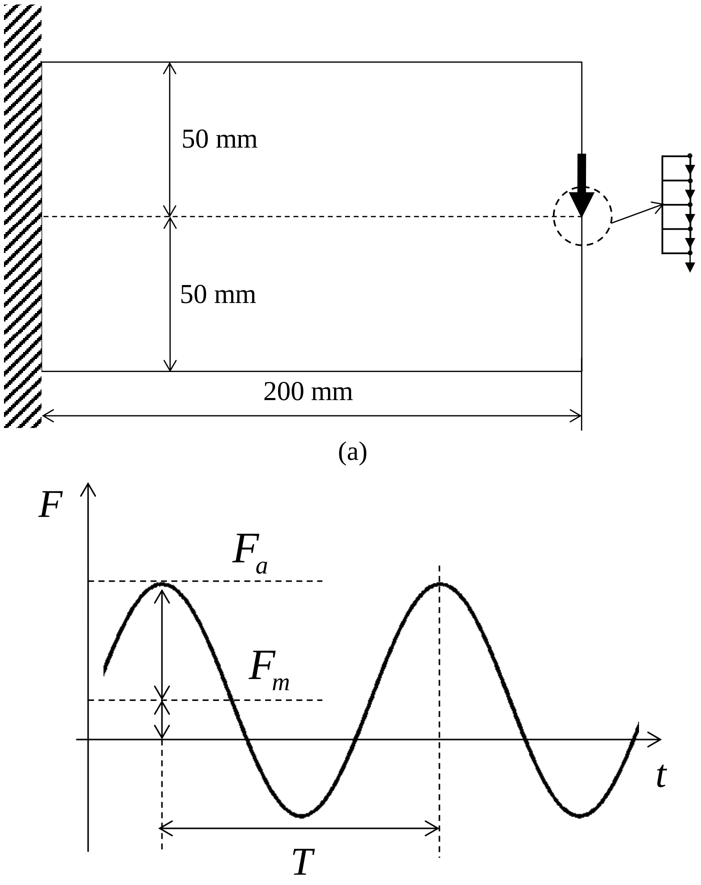


Figure 2: Feasible region of considered problem in the S-N diagram.

Figure 3(a) represents the design domain and loading condition of cantilever beam problem. The fixed boundary condition is applied to the left edge of design domain and the downward distributed proportional loading is applied at the center of right side edge. In Figure 3(b), F_m and F_a represent the mean and alternating force. The magnitudes of mean and 1Hz alternating force are 100 N and 400 N, respectively. The design domain was discretized by 5,000 2mm by 2mm plane stress elements. The design domain is divided into 10 region for the calculation of constraint functions thus the number of constraints is 30.

Figure 4(a) and Figure 4(b) represent the optimized layouts of cantilever beam considering the modified Goodman and Gerber criterion, respectively. It is interesting that the asymmetric layouts are obtained although the symmetric boundary condition is applied. This phenomenon appeared as the modified Goodman or Gerber criterion ignores the compressive effect. Therefore, the optimized layouts tend to endure tensile stress by distributing more material at the upper part. Also, more material is used for the layout considering the modified Goodman criterion which is more conservative than the Gerber criterion.



(a)

(b)

Figure 3: The design domain of cantilever beam with mean and alternating forces: (a) The geometry and boundary condition and (b) the mean and alternating forces.

(a)

(b)

Figure 4: The optimized layouts of the cantilever beam problem: (a) an optimized layout considering the modified Goodman ($V(\gamma)/V_0 = 0.345$) and (b) Gerber criterion ($V(\gamma)/V_0 = 0.333$)

6. Conclusions

In this research, a new FCTO method was developed which is one of the important structural topology optimization problems. Because the stress-life approach is used to estimate fatigue life, three well known difficulties of stress-based topology optimization are basically inherent as well as the newly found difficulties. These difficulties are successfully solved by using the proposed FCTO approach and the usefulness of the proposed TO method is verified by solving the cantilever beam design problem. Also, an interesting phenomenon appeared in the cantilever beam design problem because an asymmetric layout is obtained even though symmetric boundary condition is applied. In short, this research proposes a new topology optimization method considering dynamic fatigue constraint under constant and proportional loading.

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8. References

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