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A Performance Measure Approach to composites reliability: a transmission loss application

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1. Abstract
Although the aerospace production process is much better controlled than the process in other industries, it remains true that very small manufacturing variability exists in the geometrical parameters (flange thicknesses, hole diameters …) as well as in material properties. In the current design process, the effect of this manufacturing variability is usually compensated for by applying safety factors. This is not an ideal situation, as it may lead to slightly over-designed structures.
A much more promising approach is to include probabilistic models of design variables into the mechanical simulation process. Then, with a new methodology based on reliability analysis, engineers can obtain a better understanding of the actual effect of the manufacturing tolerances and of variability in material properties. Based on the analysis results, the robustness and reliability of the design can be assessed and improved if needed.
In this paper, the above-mentioned probabilistic approach is demonstrated on a stiffened composite panel of an aircraft and its acoustic performances in terms of transmission loss are assessed. The frequency dependent transmission loss of this composite structure is optimized with respect to two ply thicknesses and four material properties in order to obtain a high transmission loss throughout different frequencies for good acoustic insulation. The objective of this study is to maximize the minimum transmission loss throughout the entire acoustic frequency spectrum.
The structural behavior of the composite panel is calculated with the Finite Element Method, while the acoustic performance is predicted with the Boundary Element method. The combination of both modeling techniques allows accurate prediction of the panel's transmission loss. During the optimization process, the parameters are initially optimized using deterministic approaches: first a global search and then refined by a local search, without any consideration for parameter variability. The solution found by the deterministic approaches is then used as the starting point for the probabilistic approach to improve the reliability and robustness. Therefore, the Performance Measure Approach has been used. The two optimization approaches use surrogate models constructed on the results of a Design of Experiments. In this way, optimal solutions were obtained with a fraction of the otherwise required number of mechanical simulation runs.
The deterministic approaches improved the transmission loss from 8.8 dB of the reference configuration to 13.6 dB. The probabilistic approach improved the reliability from 3.27σ of the benchmark configuration to 4.0σ with a transmission loss of still 13.6 dB.
The probabilistic approach adopted for the current problem has enabled the identification of an optimal material and ply configuration that improves the reliability and robustness of the optimal solution found by a deterministic optimization. Therefore, the solution is less susceptible to variability in material properties and manufacturing tolerances and less likely to violate constraints found by the deterministic optimization, while practically maintaining the transmission loss level found by the deterministic counterpart

2. Keywords: composites, reliability, optimization, performance based approach

3. Introduction
The use of large, long-distance cruise aircraft is increasing the importance of passenger comfort, not only in terms of acceptable noise and vibration levels but also for air quality aspects, addressing the subjective human perception of comfort. As a consequence, aircraft manufacturers are considering the design and production of fuselage structures offering higher standards of comfort, as well as active and passive safety characteristics. Consequently, a big effort is being put in research addressing new materials and manufacturing solutions that could result very attractive not only from the structural point of view (weight and production cost reductions), but also for cabin comfort and health monitoring. In fact, these last aspects are directly related to crewmember compliance issues (due to longer trips) and to passenger expectations, viewing comfort as one of the main aircraft quality indicators. The new material properties and their application in the design of aeronautic structures may lead to a greater

exterior/interior noise transmission. Thus, it is important to have a validated design approach to select and test possible solutions that overcome these noise transmission problems.
Structural analysis techniques are traditionally based on deterministic methods (such as finite element method) and are not able to take into account random characteristics of input parameters like material properties, geometry and applied loads. Thus the uncertainties and the variabilities that characterize composite properties are not properly taken into account. This can lead to oversized structures and damping/soundproofing treatments in order to be able to satisfy the requirement for structural and acoustic performance. This penalizes weight saving advantages that are one of the major reasons in the use of composite materials in aeronautic and aerospace applications.
In last years, many different methods have been developed in order to include uncertain model properties in the FE analysis and to give the opportunity to evaluate the effect of these uncertainties in the analysis results. In the present work, a probabilistic approach has been used to describe the dynamic behavior of a composite ribbed panel subject to incident sound pressure in order to assess the transmitted noise inside the aircraft cabin.
In a probabilistic finite element approach, design parameters are introduced as stochastic variables to model randomness and spatial variability of the geometrical and mechanical properties. The probabilistic characterization of the acoustic response due to the introduced variability is assessed and is used to compute the probability that a value representative of the performance of the system doesn't exceed a selected threshold. In the next sections an overview of the application case will be given, together with a brief overview of the probabilistic techniques used to perform the analysis. Also, details on the analysis implementation will be explained, together with the enabling tools used to integrate a probabilistic approach in the standard composite design process.

4. Test case description

In this paper, the above-mentioned probabilistic approach is demonstrated on a stiffened composite panel of an aircraft and its acoustic performances in terms of transmission loss are assessed (Figure 1). Transmission loss through a panel is a measure of the reduction in sound power (or sound pressure) that occurs from one side to the other side of the panel, and is defined as follows (Figure 2).

Figure 1: View of the composite panel from the Receiver Room in the physical experiment set up.

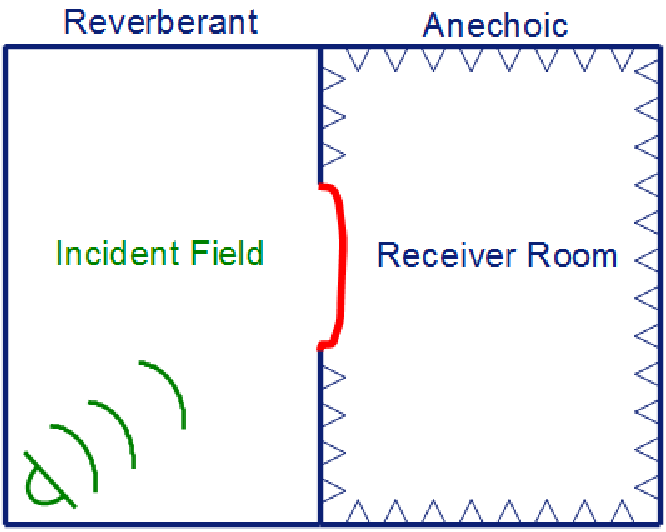

Figure 2: Schematics of physical set up to measure Transmission Loss of the composite panel.

The frequency dependent transmission loss of this composite structure is optimized with respect to two ply thicknesses and four material properties in order to obtain a high transmission loss throughout different frequencies for good acoustic insulation. The objective of this study is to maximize the minimum transmission loss throughout the entire acoustic frequency spectrum.

$$Transmission\ Loss_{dB} = 10 \log \left(\frac{Incident\ Power}{Transmitted\ Power} \right) \tag{1}$$

The structural behavior of the composite panel is calculated with the Finite Element Method, while the acoustic performance is predicted with the Boundary Element method (Figure 3). The combination of both modeling techniques allows accurate prediction of the panel's transmission loss.

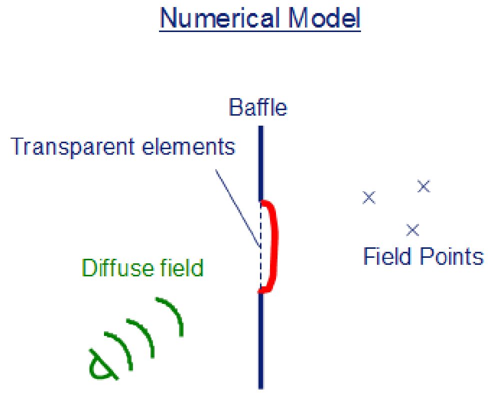

Figure 3: Schematics of numerical set up showing BEM baffled principle

As mentioned before, physical variables have been modeled in the analysis project with probability distribution functions in order to characterize the variabilities concerned with the material properties of the various layers [2] . A total of 8 input parameters have been considered, as reported in Table 1.

Property	Distribution	Mean	Standard Deviation
Ply_thickness_1	Normal	0.21 mm	6.67 10^{-3} mm
Ply_thickness_2	Normal	0.21 mm	6.67 10^{-3} mm
Mat1_YoungModulus_E1	Normal	6.2 E^{10} Pa	2.07 10^{9} Pa
Mat1_YoungModulus_E2	Normal	6.2 E^{10} Pa	2.07 10^{9} Pa
Mat2_YoungModulus_E1	Normal	1.379 10^{11} Pa	4.59 10^{9} Pa
Mat2_YoungModulus_E2	Normal	7.7911 10^{9} Pa	2.59 10^{8} Pa

Table 1: Statistically characterized material properties for the composite panel

5. The Reliability Problem Statement

To better understand the methodology used in the analysis, a brief overview of the reliability problem statement is presented here. In the reliability theory, variabilities of an engineering design can be characterized by the variations of a random system parameter set. These random variables are modeled with probability distribution functions and are representative of the uncertain model parameters. Thus, given a set of random variables $X = [X_i]^T$, with $i = 1 \ldots n$, the probability distribution of each random variable X_i is described either by its cumulative distribution function (CDF, $F_{X_i}(x_i)$) or by its probability density function (PDF, $f_{X_i}(x_i)$) and is often bounded by tolerance limits on the system parameter values.

For each realization of the set of input random variables, the performance of the system has to be evaluated. This performance is generally described by Performance Functions (PF, $G_j(x)$ with $j = 1...m$) that, for a structural design, are usually the selected failure criteria. Thus, if $G_j(x)$ is one of the *m* system PFs, the system is considered to fail if $G_j(x) < 0$ for at least one index *j*. The probability of failure of the system for every j^{th} performance function is then the multi dimensional integral of the joint PDF function of the set of random variables over the failure domain Ω_j . This is expressed by the integral [3] :

$$F_{G_j}(0) = P(G_j(x) < 0) = \int_{\Omega_j} \ldots \int f_X(x) dx_1 \ldots dx_n \qquad \text{where } \Omega_j : \{x \in \Re^n : G_j(x) < 0\} \tag{2}$$

The event space Ω_j is the region of the stochastic space where only failure events occur. Thus, the integral of the joint probability density function $f_X(x)$ over Ω_j yields the probability of failure $P_{f,j}$ of the structure for the j^{th} failure criterion.

$$P_{f,j} = F_{G_j}(0) = P(G_j(x) < 0) \quad \text{with } j = 1...m$$

In general, given a fixed performance index (or measure) g_j of the system, Eq (2) can be generalized by computing the probability of the performance function assuming values smaller than the selected threshold.

$$F_{G_j}(g_j) = P(G_j(x) < g_j) = \int_{\Omega_{G_j}} \ldots \int f_X(x) dx_1 \ldots dx_n \text{ where } \Omega_{G_j} : \{x \in \Re^n : G_j(x) < g_j\} \quad (3)$$

In this case, the event space Ω_{G_j} represents the region of the stochastic space where each performance parameter of the structure is below the prescribed quantity g_j.

The main aim of the reliability analysis is to estimate the probability of failure of a structure, given a set of input random variables and a set of failure criteria defining the safe and fail regions Ω_j or Ω_{G_j}. This estimation is usually carried on by approximating the multi dimensional integral with appropriate techniques. These include sampling methods and limit state approximations, usually computed by transforming the problem definition from the parameter space *X* into the standard normal space *Y*.

The total probability of failure of the structure is given by the integral of the joint PDF $f_X(x)$ over the union of all the failure domains. These failure domains are defined by the corresponding Performance Functions $G_j(x)$ and their threshold values g_j (performance indexes or measures).

In the next sections an overview of the techniques used to approximately estimate the multidimensional integral and used to exploit this estimation for optimization procedures is presented.

5.1. The Reliability-based Design Optimization Model

In engineering design, the traditional deterministic design optimization model has been successfully applied to systematically improve the system design process, yielding a reduction of the costs and an improvement of the final quality of the products. However, uncertainties are present in either engineering simulations and in manufacturing processes. This calls for different optimization models that can yield not only an improvement in the design, but also a higher level of confidence. Thus, a reliability-based design optimization (RBDO) model for robust and cost-effective designs can be defined using mean values of the random system parameters as design variables and optimizing the cost subject to prescribed probabilistic constraints (e.g. a maximum on the allowed probability of failure) by solving a mathematically nonlinear programming problem. As a result, the RBDO solution provides not only an improved design but also a higher level of confidence in the design.

The general RBDO model can be defined as

$$\begin{cases} \min_d \{Cost[d(\mu_X)]\} \\ \text{subject to } P_{f,j} = P(G_j(x) < 0) \leq \overline{P}_{f,j} \text{ with } j = 1 \ldots m \end{cases} \quad (4)$$

where

- the cost function can be any function of the mean values μ_X of the input parameters and
- $\overline{P}_{f,j}$ is the probabilistic constraint that can be defined for each failure mode and needs to be satisfied.

For this optimization problem, the constraint definition is now expressed in terms of probability distributions and thus needs to be evaluated, for each optimization step, within the probability framework. Thus, for each iteration of the optimization loop, an estimation of the probabilistic constraint in terms of its multidimensional integral (see Eq. (2) and (3)) has to be computed. For this purpose, different methods exist. Most of them apply a transformation of the input parameter space *X* to the standard normal space *Y*. In this space, each probability of failure $P_{f,j}$ can be represented in terms of the reliability index β_j [[3]] as

$$P_{f,j} = P(G_j(x) < 0) \Rightarrow P_{f,j} = P(G_j(y) < 0) = \Phi(-\beta_j) \Rightarrow \beta_j = -\Phi^{-1}(P_{f,j}) \quad (5)$$

Where $\Phi(\bullet)$ is the standard normal CDF (zero mean and standard deviation 1). This equation can be generalized as

$$F_{G_j}(g_j) = P(G_j(x) < g_j) = \Phi(-\beta_{G_j}) \quad (6)$$

Where $F_{G_j}(\bullet)$ is the CDF of the j^{th} system response.

The same approach can be used also to express the probabilistic constraint of Eq. (4) in a different notation. In this case, the probability of failure $P_{f,j}$ will be the *target probability of failure* $\overline{P}_{f,j}$ and the reliability index β_j the *target reliability index* $\beta_{t,j}$.

$$\overline{P}_{f,j} = \Phi(-\beta_{t,j}) \Rightarrow \beta_{t,j} = -\Phi^{-1}(\overline{P}_{f,j}) \tag{7}$$

Using Eq. (6), the second condition of Eq. (4) can be rewritten as

$$P_{f,j} = F_{G_j}(0) = \Phi(-\beta_j) \le \Phi(-\beta_{t,j}) = \overline{P}_{f,j} \Rightarrow \beta_j \ge \beta_{t,j} \tag{8}$$

The relation expressed in Eq. (8) in terms of the reliability index, can also be expressed in terms of the performance measure through inverse transformation. In fact, using Eq. (7) and Eq (8), the target probability of failure can be expressed in terms of the target performance measure as

$$\overline{g}_j = F_{G_j}^{-1}(\overline{P}_{f,j}) = F_{G_j}^{-1}[\Phi(-\beta_{t,j})] \tag{9}$$

where $\overline{g}_j$ is named "target probabilistic performance measure". It represents the value of the performance function "equivalent" to the target reliability index $\beta_{t,j}$.

The expression of the probabilistic constraint of Eq. (8) and (9) can be used in the optimization problem of Eq. (4) to equivalently replace the original definition.

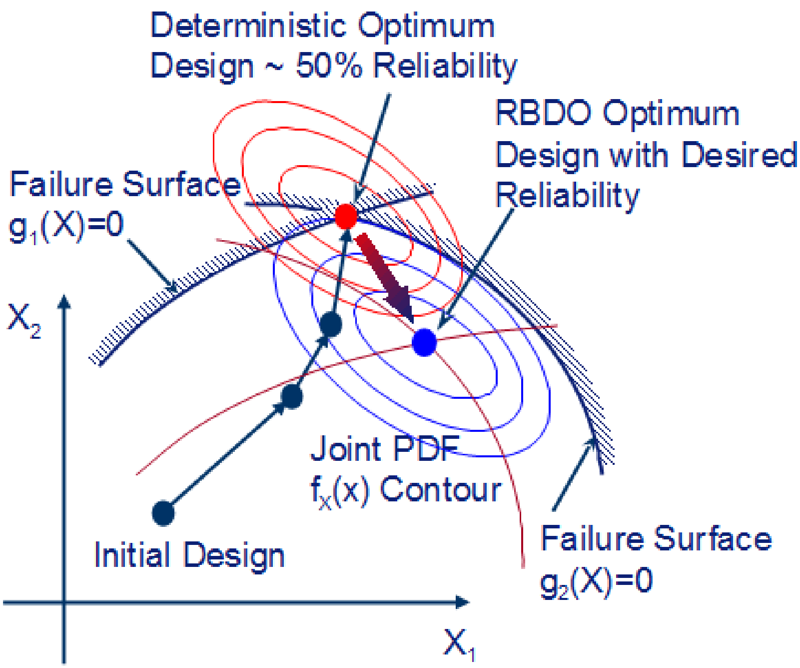

Figure 4: Two step optimization procedure: first optimize without considering variation in design parameters, then optimize for reliability and robustness taking into account the variation in design parameters.

5.2. Performance Measure Approach

The formulation of the general RBDO model of Eq. (4) that uses Eq. (8) to describe the probabilistic constraint is called Reliability Index Approach (RIA) [4] [3] . An alternative and more effective way of formulating the general RBDO model is to use the inverse formulation of the probabilistic constraint of Eq. (9), known as Performance Measure Approach (PMA) [5] [6] . The RBDO model can be re-written as

$$\begin{cases} \min_{d} \{Cost[d(\mu_X)]\} \\ \text{subject to } \bar{g}_j \geq g_j \text{ with } j = 1 \dots m \end{cases} \tag{10}$$

In this case, at a given design $d^{(k)} = \left[d_i^{(k)}\right]^T \equiv \left[\mu_i^{(k)}\right]^T$, the evaluation of the target probabilistic performance measure $\bar{g}_j$ is performed using

$$\bar{g}_j\left(d^{(k)}\right) = F_{G_j}^{-1}\left(\int_{\Omega_{G_j}} \dots \int f_X(x) dx_1 \dots dx_n \right) \tag{11}$$

To facilitate the estimation of the integral in Eq. (11), a transformation from the parameter space *X* to the standard normal uncorrelated space *Y* is generally used [7] . This permits to easily evaluate the integral by transforming the joint PDF function of the input random variables in a multidimensional joint normal PDF, which can be easier evaluated.

Thus, for each iteration of the optimization process the following problem has to be solved

$$\begin{cases} \min_{u} \left[\left\| G_j(y) \right\|\right] \text{ with } j = 1 \dots m \\ \text{s.t. } \|y\| = \beta_{t,j} \end{cases} \tag{12}$$

Where *y* is defined by the transformation *Y=T(X)* from the input parameters space *X* to the standard normal space *Y*. The approximate solution of this problem yields an estimation of the performance measure, given by

$$\bar{g}_j \approx \left\| G\left(\bar{y}_{\beta=\beta_{t,j}}\right) \right\| \tag{13}$$

With the formulation used in Eq. (13), the performance measure g_j^*, which depends on the mean values of the input random parameters (also called *nominal point*), has to be estimated for each iteration. The target of the optimization is to find the nominal point that can satisfy all the probabilistic constraints in terms of the performance measure. This approach yields an improved convergence rate of the PMA methodology with respect to the RIA formulation. In fact, PMA minimizes a complex cost function subject to a simple constraint, which raises less numerical problems and therefore yields a better algorithm performance, while the RIA algorithm minimizes a simple cost function subject to a complex constraint.

Both these methods use a Limit State Approximation to make an approximation of the exact Limit State Function (LSF); refer to [3] [8] for a more elaborate description. In particular, the method considered in this paper is the First Order Reliability Method (FORM) and it has been used for both RIA and PMA. This method has the advantage of being fast and relatively accurate in the analysis for less than 15 input variables [9] . However, it is recognized that such an approach can lead to large approximation errors for a higher number of variables, and also to high computation times when using a gradient-based sensitivity computation [10] . Considering such high-dimensional problems is however outside the scope of this paper.

5.3. Design Of Experiments and Response Surfaces

Design Of Experiments (DOE) [12] is a general approach to investigate complex correlations between input parameters and output response quantities. The experiments are set up in such a way that a maximum amount of information is obtained in a minimum amount of computation time. It is often combined with Response Surface Methodology (RSM) [13] : a meta-model of certain order is estimated from the experimental data, which yields insight in the functional relationship between the input parameters and the true response and output quantities. DOE methodology was introduced in England in the 19^{th} century and first used in agricultural studies. The methodology then disseminated to the rest of the world. Taguchi applied DOE in the automotive industry in the 1980's to improve the product quality and production process. Nowadays, DOE methodology is widely used in all fields of Computer Aided Engineering.

The selection of a DOE method depends on the available computational power and the expected order (linear, quadratic with/without cross terms ...) of the RSM model that is required to accurately represent the actual functional performances. For the present paper, two DOE methods have been used: a Latin Hypercube design of experiment (DOE) method and a full Random one. Latin Hypercube DOE is a method that belongs to the category

of semi-random methods design in which the points are chosen following a semi-random process. However, a method that uses random points throughout the design, could contain many cluster points. This is certainly not an interesting situation for purposes of exploration. It is of greatest interest, however, to get points to fill in as much as possible the area of investigation. In statistical sampling, randomization structure is made through a Latin-Hypercube, which is defined as sample containing a grid positions, if and only if there is only one sample in each row and each column. A Latin-Hypercube DOE generalizes this idea by forcing only a sample size of each axis-aligned plan in more dimensions. Dividing the size of each factor in *n* equidistant levels, one obtains the *nxn* unit with a number of experiments equal to the number of levels. The points of the plan are accomplished by selecting a permutation of levels for each dimension and combining within the plan. In this way the maximum number of levels used because each level is within the plan. Therefore, if one or more factors are less important than others, each point will be able to provide information about the influence of the other factors on the response. In this way none of the time-consuming computer experiments become useless.
In normal random methods, the clustering of experiments in specific parts of the domain provides good information of the specific region but not on the overall design domain. The LHD DOE method is extremely effective if one wants to analyze the domain with a fixed number of experiments, depending on the complexity of the case study, thus having a direct control on the computational cost of the DOE.

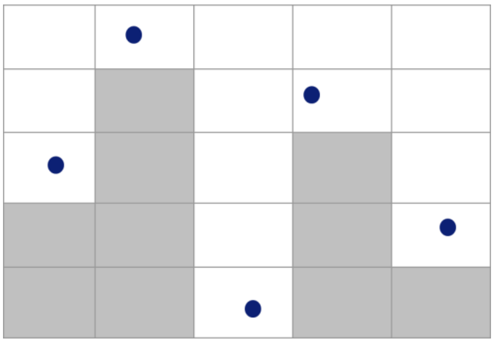

Figure 5: Visualization of a Latin Hypercube typical design for 2 factors and 5 experiments

DOEs are often combined with Response Surface Methodology (RSM) [14] : a meta-model of a certain order is estimated from the DOE data, yielding insight in the functional relationship between the input parameters and the output quantities. For the present case, a response surface model has been created on the basis of the DOE results for the output performances. This response model has then been used to perform a fast optimization procedure and to locate, with sufficient accuracy, a new possible optimal composite panel configuration to improve the transmission loss performance. The main advantage of this approach is that the calculation of the DOE generally has a fixed computational effort that depends on the number of parameters, the type of non linearity expected and few other considerations. On the other side, a global or local optimization process has no fixed computational effort that can be estimated, since this depends on the satisfaction of a convergence criterion for the specified algorithm (e.g. on the dimensionality of the problem for gradient based algorithms like Sequential Quadratic Programming - SQP - or Non Linear Programming by Quadratic Lagrangian - NLPQL). For this paper, given the computational time required for one single execution of the complete analysis, the LH DOE approach limits the total effort that can be spent and relies on the creation of a response model to considerably speed up the optimization process. On the other hand, the results of the optimization process obtained using the response model have to be verified with a single final direct simulation, in order to assess the error between the analytical response model and the simulation analysis only for the optimal point.

6. Analysis Case Implementation

From the CAD model of the composite panel, two kinds of meshes are created. One is the finite element model (FEM) for the structural analysis and the other is the boundary element model (BEM) for the acoustic analysis. The structural analysis was conducted using NASTRAN® to obtain the vibration modes, whereas the sound radiation was computed using the BEM in LMS Virtual.Lab® Acoustics (Figure 6). The FEM and BEM are both deterministic simulations where if the inputs are the same so is the output. The uncertainty is incorporated in the design variables i.e. the two ply thickness and the 4 material properties (Figure 7). The results from the BEM is used to compute the transmission loss in the post-process (Figure 6).
The process integration workflow tools has enabled the automatic execution of the different analysis phases in order to automatically iterate during the optimization process and find the optimal composite panel design. In this

way, the generation of new combinations of input parameters can be fully automated. This integrated process captures the various tasks that are usually performed manually and automates them to reduce user intervention to the minimum. In the current case, Optimus® drives the execution of LMS Virtual.Lab as showed in Figure 8.
In this workflow, Optimus generates new panel configurations (in terms of material properties and ply geometry), changes the geometry (see Figure 6) and updates the acoustic and structural meshes. In this way the analysis files are submitted, first to Nastran to compute a Craig-Bampton modal solution and have an orthogonalized combined mode set between normal modes (related to the natural vibration of the body) and static modes (related to the deformation due to localized loading). The Craig-Bampton Method combines the constraint modes with a limited number of the lowest Eigen modes of the structure, the fixed-interface normal modes, in order to allow the reduced model to correctly represent both static and dynamic behavior. The results of the Craig Bampton analysis are then used as structural modes in the acoustic analysis together with the BEM approach to compute the acoustic solution in terms of Transmission Loss in the range between 100 Hz and 1kHz. The minimum in this frequency interval is used as performance measure of the system and delivered as result of the single analysis run.
These outputs are read and then incorporated in the optimization routines to find the most suitable design among all admissible design. This integrated process includes the batch execution of LMS Virtual.Lab to change the geometry, update the mesh and perform the acoustic analysis. Optimus then extracts the results that are needed for the optimization process.

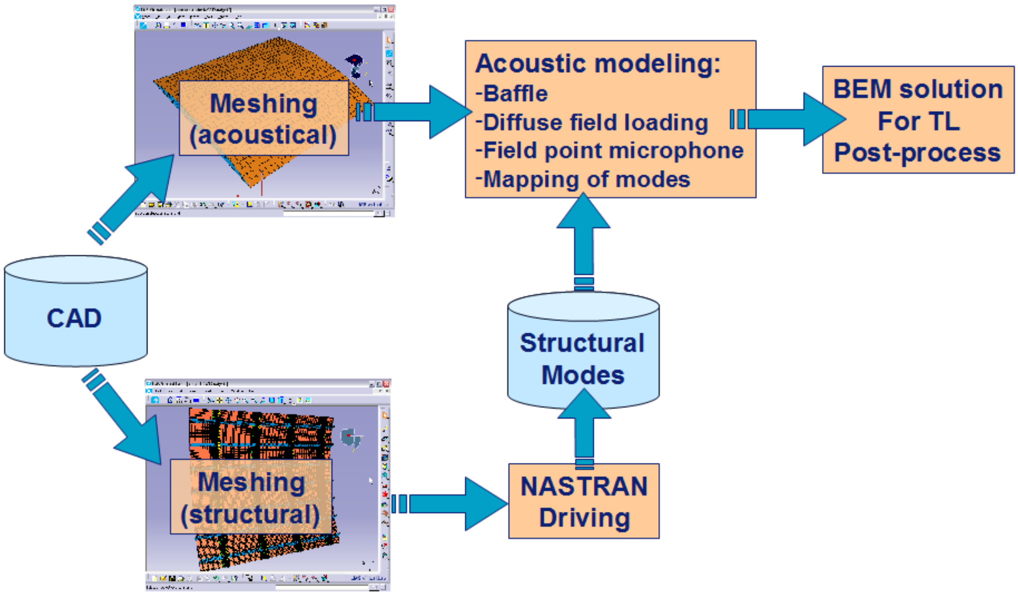

Figure 6: Simulation using acoustic boundary element method and structural finite element method models.

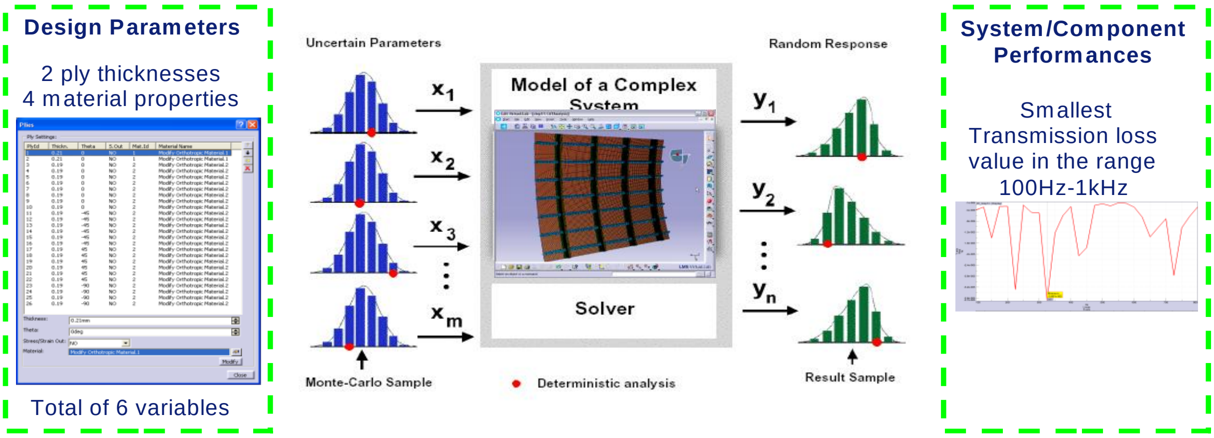

Figure 7: Uncertainties in the analysis process

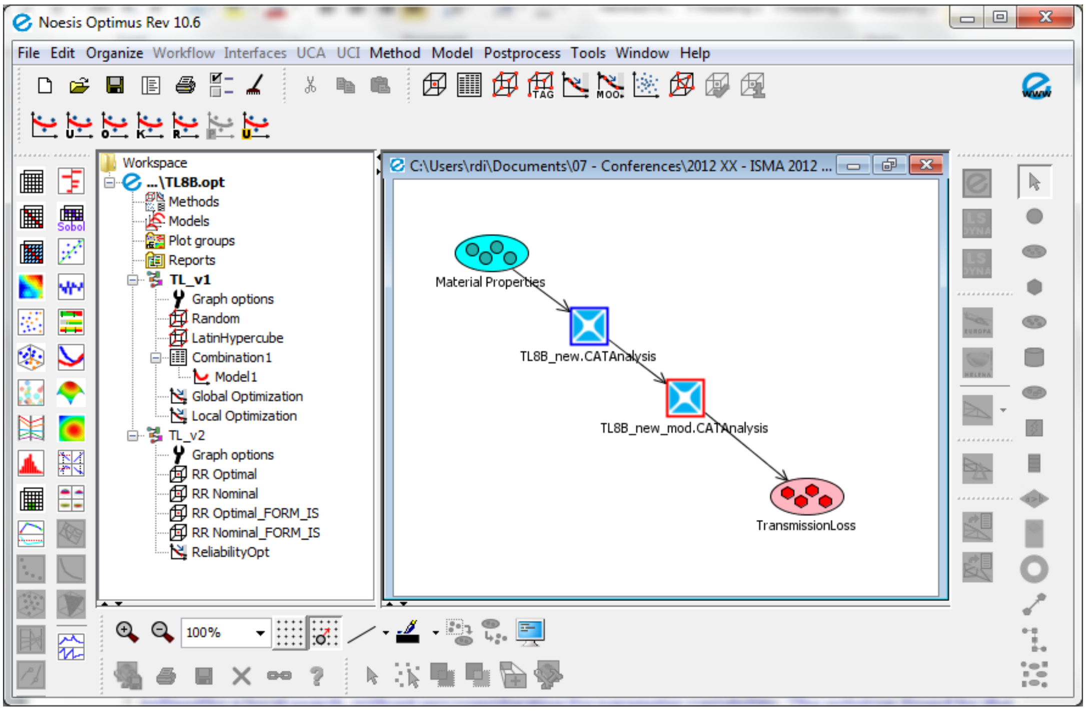

Figure 8: Process integration workflow with Noesis Optimus

5. Results

A two step approach was employed in the determination of an optimal design of the composite rib panel. The parameters were initially optimized using deterministic approaches: first a global search and then refined by a local search, without any consideration for parameter variability. The solution found by the deterministic approaches was then used as the starting point for the probabilistic approach to improve the reliability and robustness (Figure 4). Optimus® developed at Noesis Solutions was used to perform both the deterministic and probabilistic approach of optimization.

The optimization was performed on the Radial Basis Function Network constructed from 31 random samples and 50 Latin Hypercube samples. The global search was performed using Self-Adaptive Evolution which is an implementation of genetic algorithm, and the local search refinement was done using the Nonlinear Programming Quadratic Search (NLPQL) which is an implementation of Sequential Quadratic Programming [14] . Then, in the probabilistic approach, First Order Reliability Method (FORM) was used in Performance Measure Approach (PMA). The maximization of transmission loss was performed using NLPQL with reliability index constraint to be greater than 4.0.

The deterministic approaches improved the transmission loss from 8.8 dB of the reference configuration to 13.6 dB (Table 2). The probabilistic approach improved the reliability from 3.27σ of the benchmark configuration to 4.01σ with a transmission loss of still 13.6 dB (Table 2).

		DETERMINISTIC		PROBABILISTIC	
		Global Optimization	Local Optimization	Start	End
	Start	End [4225] (4225)	End [16] (4)	Start	End [45582] (50)
Inputs					
Ply_thickness_1	0.21	0.21344	0.21341	0.21341	0.2134
Ply_thickness_2	0.21	0.20876	0.20879	0.20879	0.20878
Mat1_YoungModulus_E1	6.2e+10	6.17403e+10	6.17401e+10	6.17401e+10	6.17281e+10
Mat1_YoungModulus_E2	6.2e+10	5.72393e+10	5.72396e+10	5.72396e+10	5.72287e+10
Mat2_YoungModulus_E1	1.379e+11	1.39495e+11	1.39512e+11	1.39512e+11	1.39473e+11
Mat2_YoungModulus_E2	7.7911e+09	7.30972e+09	7.30998e+09	7.30998e+09	7.31057e+09
Outputs					
TL1	8.86443	13.60201	13.60593	13.60594	13.60586
				1.28103	1.20227
Reliability Index				3.27401	4.01341
				-13.60594	-13.60586

Table 2: Results of two step optimization

5. Conclusion

The probabilistic approach has enabled the identification of an optimal material and ply configuration that improves the reliability and robustness of the optimal solution found by a deterministic optimization. Therefore, the solution is less susceptible to variability in material properties and manufacturing tolerances and less likely to violate constraints found by the deterministic optimization, while practically maintaining the transmission loss level found by the deterministic counterpart.

6. References

References should be listed at the end of the paper and consecutively numbered. Refer to references in the text with reference number in brackets as [1]. Style the reference list according to the following examples.

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