Structural Optimization of Permanent Magnet Motor for Resonance Avoidance

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1. Abstract
This paper proposes a new design method for avoiding resonance of the permanent magnet (PM) motor by considering both the electromagnetic sources and mechanical behavior of the motor structure. To avoid the resonance band between the electromagnetic sources and the motor structure, the optimization problem is formulated with the multi-objective function to maximize the lowest eigenfrequency of motor structure and minimize the harmonic component of magnetic force. Since the magnetic and mechanical behaviors of PM motor are highly sensitive to the structural boundaries of PM and ferromagnetic material (FM), the level set method which can handle a detailed change of motor configuration is employed. The optimization is performed by using the normalized design sensitivities which are calculated from both the magnetic and structural analysis. A design example of SPM motor is provided to investigate the usefulness of the proposed method and achieve a novel motor design that promises enhanced performance.
2. Keywords: permanent magnet motor, resonance avoidance, structural optimization, multi-objective function

3. Introduction
Since the resonance of a motor system reduces the machine's durability and causes the driving noise, the avoidance of mechanical resonance is the basic design target in the development of permanent magnet (PM) motor which is widely used in many industrial applications due to high power density [1]. The resonance of motor is generated by the interaction between the harmonic component of electromagnetic force and the natural frequency of the motor structure [2-3]. Hence, many studies have performed design optimization with two objectives: one is the elimination of certain harmonics of the exciting magnetic force [4] and the other is the adjustment of structural frequencies to avoid the resonance band [5]. Unfortunately, by previous design methods using the experimental data and parametric study, the optimal solution for resonance avoidance can bring the great change of the motor structure such as the pole-slot combination and an increase of the motor size.

This paper proposes a new design method which can guarantee localized geometrical change for avoiding resonance of PM motor. Several level set functions are employed as design variables to express boundaries of PM, FM, copper coil and air in PM motor and obtain an innovative optimal design [6]. To calculate the magnetic force and the eigenfrequency which is varied due to the motor configuration, both magnetostatic and modal analysis are performed with material properties such as magnetic reluctivity and elasticity, which are defined by level set functions. The optimization problem is formulated to minimize the harmonic component of magnetic force which creates the resonance band according to the rotating speed, and maximize the natural frequency of motor structure. The optimal motor shape is obtained by implicit boundaries of level set function which is moved by the design sensitivity. The proposed method is applied to the structural design of PM motor with consideration of its speed range and resonance band.

4. Problem Formulation

4.1. Boundary Representation of Motor Structure
To design the whole motor structure that satisfies our design goal in the conceptual step, three different level set functions are employed as design variables for representing the distribution of PM, coil, ferromagnetic material (FM) and air in the motor. Figure 1 depicts how the structural boundary of each material in the motor can be defined by level set functions. The one level set function (ϕ_1) in the stator is used to divide coil and FM region, and additional two level set functions (ϕ_2, ϕ_3) are distributed in the rotor to express the configuration of PM, FM and air.

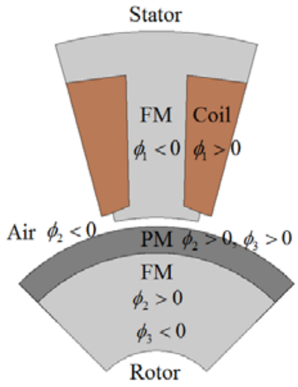

Figure 1: Boundary representation of PM motor

Each level set function has the sign for distinguishing the different material domain (Ω_i) and the zero level of them represents the structural boundaries ($\partial\Omega_i$) as follows:

$$\begin{cases} \phi_i(\mathbf{x}) > 0 & \text{for } \mathbf{x} \in \Omega_i^+ \\ \phi_i(\mathbf{x}) = 0 & \text{for } \mathbf{x} \in \partial\Omega_i \qquad (i = 1, 2, 3) \\ \phi_i(\mathbf{x}) < 0 & \text{for } \mathbf{x} \in \Omega_i^- \end{cases} \tag{1}$$

The material properties for mechanical and magnetic analysis can be defined by the characteristic function (χ_i) of which value is 0 or 1 according to the sign of each level set function. In the stator, magnetic and mechanical properties of coil (p_{coil}) and FM (p_{FM}) such as the relative magnetic reluctivity, the current density, Young's modulus and density are defined as follows:

$$p(\phi_1) = p_{\text{coil}}\chi_1 + p_{\text{FM}}(1 - \chi_1) \tag{2}$$

To represent the magnetic properties of PM (p_{PM}), FM (p_{FM}) and air (p_{air}) in the rotor of PM motor, the combinations of two characteristic functions are used as in the following equation:

$$p(\phi_2, \phi_3) = \chi_2\left[p_{\text{PM}}\chi_3 + p_{\text{FM}}(1 - \chi_3)\right] + p_{\text{air}}(1 - \chi_2) \tag{3}$$

In this case, the relative reluctivity and the remanent flux density are accepted as p to express a characteristic of PM.

4.2. Optimization Problem Formulation

The objective function (F) is composed with two values: one is the first eigenvalue (ω_1^2) with a minus sign and the other is the difference between the radial force (f_r) on the teeth of the stator and the target value ($f_{r,\text{target}}$) from the fundamental waveform that harmonics of radial force up to m^{th} order are eliminated as shown in Figure 2.

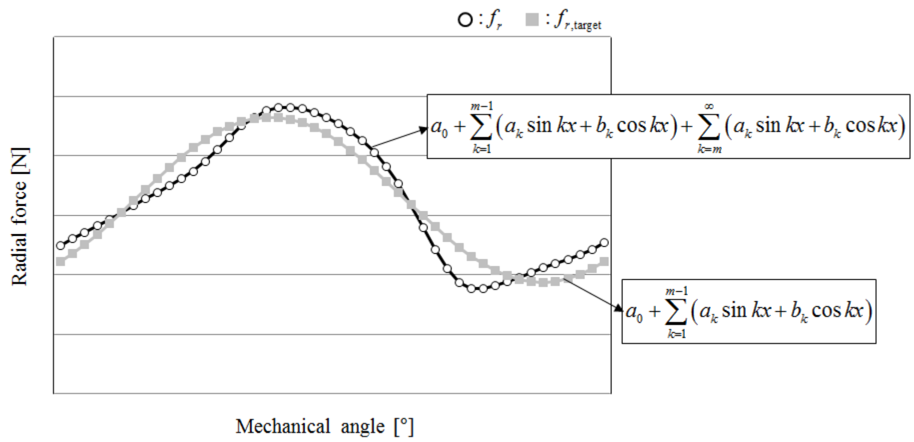

Figure 2: Target radial force

By minimizing F , we can optimize the motor structure with consideration of the eigenfrequency maximization and the harmonic elimination of the radial force which are the important issues for resonance avoidance. The optimization problem is formulated with the volume constraints of each level set function (VF_i) as follows:

$$\text{minimize } F(\phi_1,\phi_2,\phi_3) = -\omega_1^2(\phi_1) + \sum_{j=1}^{n}\left(\frac{f_r^{\,j}(\phi_1,\phi_2,\phi_3) - f_{r,\text{target}}^{\,j}}{f_{r,\text{target}}^{\,j}}\right)^2$$

$$\text{subject to } G_i(\phi_i) = \int_{\Omega_{(\text{stator or rotor})}} \chi_i \, d\Omega_{(\text{stator or rotor})} \Big/ \int_{\Omega_{(\text{stator or rotor})}} d\Omega_{(\text{stator or rotor})} - \mathrm{VF}_i \le 0 \quad (i=1,2,3) \tag{4}$$

$$G_4(\phi_1,\phi_2,\phi_3) = T_{\text{avg,target}} - T_{\text{avg}}(\phi_1,\phi_2,\phi_3) \le 0$$

$$G_5(\phi_1,\phi_2,\phi_3) = T_{\text{ripple}}(\phi_1,\phi_2,\phi_3) - T_{\text{ripple,target}} \le 0$$

where n is the total number of rotating position, Ω_{stator} and Ω_{rotor} mean the design domain in the rotor and stator of the motor. It is noted that two design constraints are added for satisfying the target output torque ($T_{\text{avg, target}}$) and minimizing the torque ripple (T_{ripple}) which also creates the resonance band.

Each level set function is updated by the following equation until the convergence conditions are satisfied.

$$\frac{\partial \phi_i}{\partial t} = -\left[\frac{\delta F}{\delta \phi_i} + \varepsilon_4 \frac{\delta G_4}{\delta \phi_i} + \varepsilon_5 \frac{\delta G_5}{\delta \phi_i}\right] + \lambda_i \qquad (i=1,2,3) \tag{5}$$

where ε_4 and ε_5 are the Lagrange multipliers to satisfy the design constraints for torque performance and λ_i is for the volume fraction of each level set function.

5. Design Example

5.1. Initial Design of SPM Motor

The proposed method is applied to the design optimization of 8-pole 12-slot SPM motor which was developed for an electric power steering (EPS) system of electric vehicle. Figure 3 shows the initial design of SPM motor and the driving frequency of the radial force (ω_r) and the torque (ω_T) which is calculated by the following equations.

$$\omega_r = \frac{N[rpm]}{60} \times \text{pole pair} \times 2 \times h \quad [Hz] \tag{6}$$

$$\omega_T = \frac{N[rpm]}{60} \times \text{pole pair} \times 2 \times 3 \times h \quad [Hz] \tag{7}$$

where N is the rotating speed of the motor and h is the order of the harmonic component.

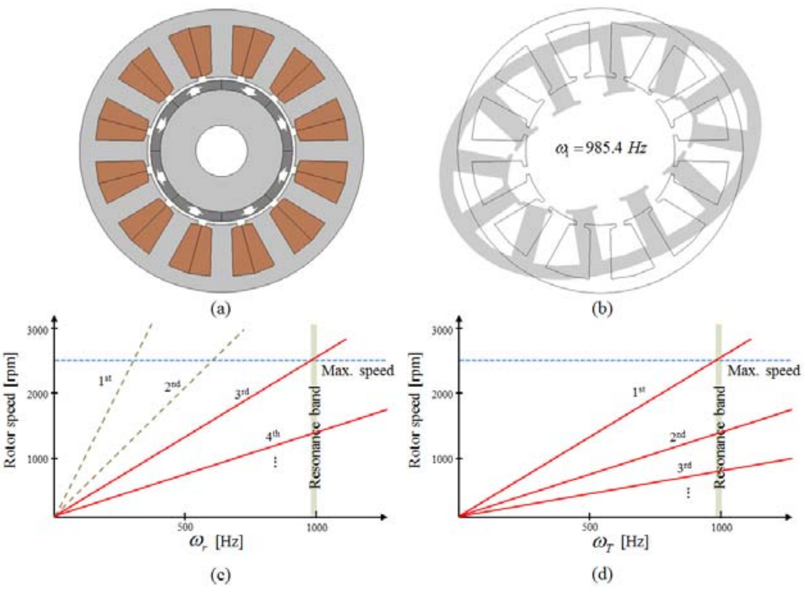

Figure 3: Initial design of SPM motor: (a) configuration (b) 1st eigen mode (c) driving frequency of harmonics of radial force (d) driving frequency of harmonics of torque

Unfortunately, ω_T of all harmonic order and ω_r of more than 3rd order pass through the resonance band which is generated by the lowest eigenfrequency of the stator of the initial design under the maximum rated speed (2500 rpm), as illustrated in Figure 3.

5.2. Design Optimization for Resonance avoidance

To avoid the resonance band between the electromagnetic source and the motor structure, the target wave of the radial force is generated without harmonic components up to 3rd order and the target torque ripple ($T_{\text{ripple,target}}$) is set to 0.5% for reducing the fluctuation of the torque by more than 95% compared with the initial design. The shape of the stator is optimized to maximize the 1st eigenfrequency with satisfaction of the target output torque, 3.2 Nm. The design domain is the entire area of the motor except for the teeth of the stator where the radial magnetic force is calculated and the lower part of the rotor which is bordered with the rotating shaft, as shown in Figure 4(a).

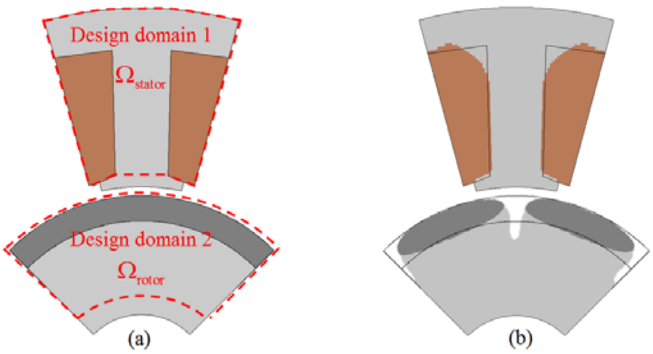

Figure 4: Configuration of SPM motor: (a) design domain (b) optimal design

Figure 4(b) shows the optimal distribution of each material in comparison with the initial design noticed by the solid lines. The boundaries of the PM in the rotor become a rounded shape to eliminate the distortion of magnetic flux waveform in air gap and reduce the magnetic saturation effect in the teeth of the stator for eliminating the harmonic components of the radial force. It is noted that the central PM thickness increases slightly to provide the magnetic flux to the stator sufficiently and FM around the edge of PM is eliminated to reduce the leakage flux. In the stator, FM is distributed to eliminate the sharp corner and concentrated in the upper side of the yoke to maximize the stiffness of the stator within the same volume fraction as the initial design.

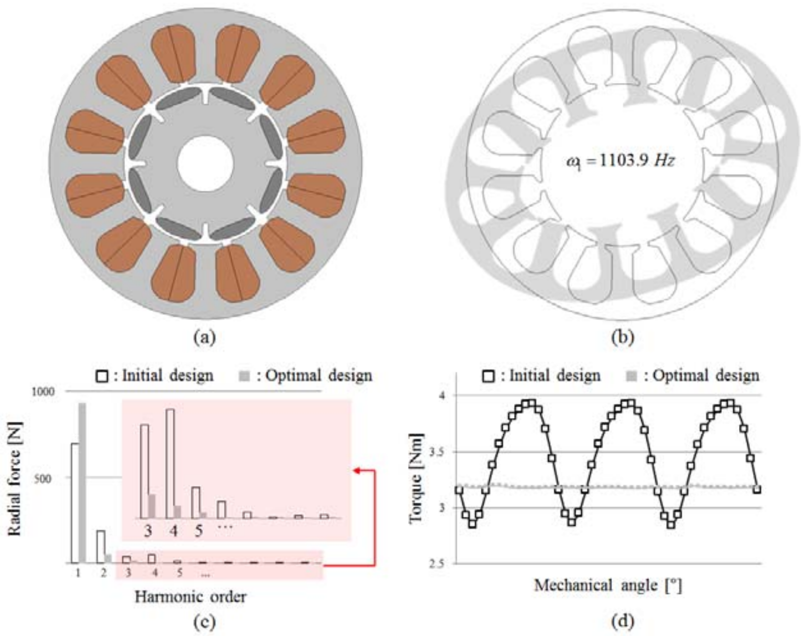

Figure 5: Optimal design of SPM motor: (a) configuration (b) 1st eigen mode (c) amplitudes of harmonics of radial force (d) torque profiles

Figure 5 depicts the magnetic and mechanical behavior of the optimal motor design. It is confirmed that the 1st eigenfrequency of the modified stator shape increases by 12% and the amplitudes of harmonics of the radial force which are overlapped on the resonance band decrease greatly. The fluctuation of the torque, the other magnetic source of resonance, nearly disappeared while maintaining the target average torque as illustrated in Figure 5(d). Table 1 summarizes that the optimal motor design provides a large decrease in the high harmonics generated by the radial force and the design constraints for the torque performance are satisfied.

Table 1: Comparison between initial and optimal design

	Amplitude of radial force [N]			ω_1 [Hz]	T_{avg} [Nm]	T_{ripple} [%]
	3rd	4th	5th			
Initial design	40.4	47.3	13.7	985.4	3.5	16.1
Optimal design	10.5 (74.0% ↓)	5.3 (88.8% ↓)	2.6 (81.0% ↓)	1103.9 (12.0% ↑)	3.2 (8.6% ↓)	0.3 (98.1% ↓)

6. Conclusion

This paper proposes a new design optimization method to avoid a resonance of the PM motor. The design problem is formulated to maximize the eigenfrequency of the motor structure and minimize the electromagnetic source such as the harmonics of radial force and the torque fluctuation which pass through the resonance band. The design example provides the way to avoid the mechanical resonance of an SPM motor and it is expected that the proposed method can provide the novel structural design for avoiding the motor’s resonance in the conceptual design stage.

7. Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) Grant (616-2010-1-D00006).

8. References

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