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Improved Surrogate Model Assisted Differential Evolution with an Infill Criterion

Eduardo Krempser[1,2], Douglas A. Augusto[1] and Helio J.C. Barbosa[1,3]

[1]Laboratório Nacional de Computação Científica, Petrópolis - RJ, Brazil
[2]Faculdade de Educação Tecnológica do Estado do Rio de Janeiro, Petrópolis - RJ, Brazil
[3]Universidade Federal de Juiz de Fora, Juiz de Fora - MG, Brazil

1. Abstract
Simulation models in engineering as well as other sciences have become increasingly costly from the computational viewpoint, motivating the development of the approximated but less expensive models known as surrogate models or metamodels. The combination of surrogate models and Differential Evolution (DE), a metaheuristic known for performing well in many applications, works satisfactorily under limited budgets, but the approximation accuracy strongly depends on the exact points used to build the metamodel. Hence, a suitable infill criterion for selecting the most appropriate candidate solutions to be exactly evaluated is highly desirable. This work proposes the application of two infill criteria to a surrogate model assisted DE through a Radial Basis Function neural network (RBF). The proposals are investigated and assessed on a set of problems of weight minimization of trusses.

2. Keywords: Differential Evolution, Surrogate Model, Infill Criterion.

3. Introduction
The increasing complexity of simulation models in engineering as well as other sciences has pushed forward the development and management of less computationally expensive models, known as surrogate models or metamodels, specially when nature-inspired metaheuristics (NIMH), which require a large number of objective function and constraint evaluations, are used to tackle complex optimization problems.

Metamodels can be based on phenomenological simplifications or on data-driven approximation, *i.e.*, an approximated model is formulated based on expensive simulations already performed.

Differential Evolution (DE) is an NIMH that has shown promising results in diverse applications[1], specially when continuous design variables are considered. DE's main movement operation is based on differences among its candidate solutions (vectors). Although in general good solutions can be obtained, DE requires many calls to the objective function evaluator. The Surrogate Model Assisted Differential Evolution (SMDE) proposed in [2] generates the offspring using four DE variants from the literature and the survivor is the one with the best (approximate) fitness evaluated via a metamodel.

The combination between DE and surrogate models is able to find better solutions when a limited budget is considered, but the approximation quality is strongly influenced by the exact points used to build the metamodel. As a result, a suitable infill criterion for selecting the most appropriate candidate solutions to be exactly evaluated by the expensive simulator is desirable. Two well-known infill criteria in the metamodel literature, but still unexplored within DE, are the Probability of Improvement[3] and the Expected Improvement[4]. Basically, they treat the uncertainty in the surrogate model application at any point as the realization of a normally distributed random variable. The distribution of the fitness function value at any untested point can be estimated from data collected during the search. Here, a Radial Basis Function Neural Network (RBF) is used as a surrogate model in the DE scheme proposed in [2], and we study the application of those infill criteria in the selection of new candidate solutions.

The weight minimization of trusses with continuous as well as discrete design variables is used to assess the performance of the proposed procedure. The infill criteria coupled with SMDE is shown to provide better results thus opening new perspectives on the application of DE to expensive optimization problems.

4. Differential Evolution
The class of constrained optimization problems considered here can be written as

$$\begin{aligned}&\text{minimize } f(x)\\&\text{subject to: } g(x) \leq 0 \text{ and } x_i^L \leq x_i \leq x_i^U, \quad \forall i = 1, \ldots, n\end{aligned} \tag{1}$$

where $f : \Re^n \to \Re$, $g : \Re^n \to \Re^m$ is a vector of constraints, and x_i^L and x_i^U contain lower and upper bounds, respectively, for x. This problem will be tackled by the Differential Evolution technique, originally proposed by Storm and Price [5], where the basic operation performed is the addition to each design variable of a given candidate solution of a term which is the scaled difference between the values of such variable in other candidate solutions in the population. Also, a recombination operator is applied between the current/target individual ($x_{i,G}$) and the generated individual ($u_{i,G}$) in a generation G. The number of differences applied, the way in which the individuals are selected, and the recombination operator determine the DE variant, which can strongly influence DE's performance. The DE variants considered here are described below:

- **DE/rand/1/bin:** $u_{i,j,G+1} = x_{r_3,j,G} + F.(x_{r_1,j,G} - x_{r_2,j,G})$, where r_1, r_2 and r_3 are randomly selected individuals in the population.
- **DE/best/1/bin:** Uses the best individual in the population $x_{best,j,G}$ as base vector leading to $u_{i,j,G+1} = x_{best,j,G} + F.(x_{r_1,j,G} - x_{r_2,j,G})$, where r_1 and r_2 are randomly selected individuals.
- **DE/target-to-best/1/bin:** This variant uses the best individual of the population and the target individual (the one that will be used in the comparison after the mutation, also called current individual), leading to $u_{i,j,G+1} = x_{i,j,G} + F.(x_{best,j,G} - x_{i,j,G}) + F.(x_{r_1,j,G} - x_{r_2,j,G})$
- **DE/target-to-rand/1/bin:** This one modifies the previous variant by using a randomly selected individual (r_3) instead of the best one, $u_{i,j,G+1} = x_{i,j,G} + F.(x_{r_3,j,G} - x_{i,j,G}) + F.(x_{r_1,j,G} - x_{r_2,j,G})$

5. Adaptive Penalty Method

The application of nature inspired metaheuristics to constrained optimization problems usually requires a constraint handling technique. Due to their conceptual simplicity, penalty techniques are widely used. However, they require adequate values for penalty parameters so that a good performance is attained.

The constraint handling technique considered here was an adaptive penalty method (APM), proposed in [6]. The APM shown to be quite effective within Genetic Algorithms [7] and DE [8], aims at relieving the user from the task of defining good values for the penalty coefficients by automatically setting those values using feedback from the search process. The idea is to observe how each constraint is being violated by the individuals of a given population and then set a higher penalty coefficient to those constraints which seem to be harder to satisfy. More detailed information about APM can be found in [6] and [7].

6. Surrogate Model

Replacing the original evaluation function (a complex computer simulation) by a substantially less expensive approximation is known as surrogate modeling, or metamodeling. This idea appeared early in the evolutionary computation literature [9] and many possibilities are available today [10].

Herein, a Radial Basis Function Network (RBF) [11] was considered as surrogate model. The RBF has shown a good approximation capability with a simple structure and fast learning algorithms. It is a special neural network architecture with three layers (input, hidden, and output layers) where the units of the hidden layer are called neurons and each neuron implements a radial basis function applied over the input signal and weighted at the output layer. The model can be formulated as:

$$\widehat{y}(x) = \sum_{i=0}^{k} \theta_i \phi(||x - c_i||) \tag{2}$$

where $c_i, i = 1, \ldots, k$ are the centers previously selected, $\theta_i, i = 1, \ldots, k$ are weights connecting the hidden and output layers, and the radial basis function (ϕ) herein considered was the Gaussian function $\phi(r) = \exp(-\frac{r^2}{2\sigma^2})$. The main steps of the RBF learning process are the selection of the centers, the definition of the radius (σ), and the computation of the weights connecting the hidden and output layers.

Here an RBF was applied locally, *i.e.*, for each new candidate solution a local approximation was performed selecting as centers the nearest solutions previously calculated.

The number of centers was considered a parameter in the metamodel and, after an initial set of computational experiments, was fixed to three and the radius were defined as the mean pairwise distance between each center. To compute the connection weights was used a Singular Value Decomposition as used.

7. Infill Criterion

For the optimization problems addressed here, the evaluation of a candidate solution requires a computationally expensive simulation. Moreover, each new evaluated solution will be added to the archive $\mathcal{D}$, and will affect the accuracy of the surrogate model approximations produced subsequently.

Therefore, by selecting new candidate solutions to be evaluated we are in fact selecting new samples that will enrich our model. This selection may be guided by information obtained during the approximation, according to an infill criterion.

The criteria assessed in this work use the concept of improvement of the solution evaluated by the surrogate model when compared with the current best solution, which is defined by $I(y) = f_{min} - \widehat{y}(x)$, where $\widehat{y}(x)$ is the surrogate model evaluation and f_{min} the objective value of the best candidate solution.

However, the main characteristic of the criteria assessed here lies in considering each given solution as being the realization of a random variable $Y(x)$ with mean $\widehat{y}(x)$ and standard deviation $s(x)$, hence modeling the uncertainty of the value generated by the surrogate model.

The first criterion considered here was originally proposed by Ulmer *et al.*[15] and suggests the use of the Probability of Improvement (PoI). Designating by ψ the probability density function and by Ψ the cumulative distribution, PoI is defined by:

$$PoI(x) = \int_{y=-\infty}^{f_{min}} \psi(y)dy = \Psi\left(\frac{f_{min} - \widehat{y}(x)}{s(x)}\right) \tag{3}$$

Another criterion investigated is known as Expected Improvement (EI), and was proposed by Schonolau *et al.*[16]. It takes into account both the variance of the approximated model and the expected improvement amount. The value of EI is obtained by:

$$EI(x) = \int_{y=-\infty}^{f_{min}} \left(\frac{f_{min} - y}{s(x)}\right) \psi(y)dy = (f_{min} - \widehat{y}(x))\Psi\left(\frac{f_{min} - \widehat{y}(x)}{s(x)}\right) + s\psi\left(\frac{f_{min} - \widehat{y}(x)}{s(x)}\right) \tag{4}$$

In our proposal, the comparison among individuals is performed considering the value obtained by the applied criterion.

For such, considering the value $\widehat{y}(x)$ as a realization of a stochastic process, as described earlier, and assuming here a Gaussian random variable, we follow [17] in obtaining $s(x)$ from the application of a RBF by

$$s^2(x) = 1 - \boldsymbol{\phi}\boldsymbol{\Phi}^{-1}\boldsymbol{\phi}^T, \tag{5}$$

where

$$\boldsymbol{\phi} = [\phi(||x - c_1||), \phi(||x - c_2||), \ldots, \phi(||x - c_k||)] \quad \text{and} \quad \boldsymbol{\Phi}_{i,j} = \phi(||c_i - c_j||) \tag{6}$$

with $i = 1, \ldots, k$ and $j = 1, \ldots, k$.

8. Proposed Algorithm

The proposal is based on the SMDE algorithm (Surrogate Model Assisted Differential Evolution), described in [2], which employs surrogate models in a simple scheme and aims at reducing the number of expensive objective function evaluations. In SMDE, all candidate solutions from the initial population are evaluated exactly, and used to initialize the archive $\mathcal{D}$ which will support the surrogate model. The main difference from the standard DE is that in SMDE each individual generates one offspring for each variant considered (four are adopted). Each new individual is then evaluated by the surrogate model but only the best one is exactly evaluated by the simulator. If the exact evaluation of the offspring turns out to be better than that of the parent, the parent is replaced by the offspring. It is important to note that every individual exactly evaluated is then stored in the archive $\mathcal{D}$ used by the surrogate model. The SMDE scheme is depicted in the figure 1. In this paper SMDE is modified in such a way that each generated offspring is selected according to an infill criterion, *i.e.* the $EI(x)$ or $PoI(x)$ values are considered to compare the individuals. Another difference is the use of the current individual's fitness (target) as the f_{min} value, considering the direct comparison between the target individual and the best one generated.

9. Computational Experiments

The performance of the proposed procedure is assessed in the weight minimization of five structures from the literature, for discrete and continuous design variables.

The proposed surrogate model assisted DEs with infill criteria are labeled as SMDE followed by the infill criterion applied: Probability of Improvement (PoI) and Expected Improvement (EI). The SMDE

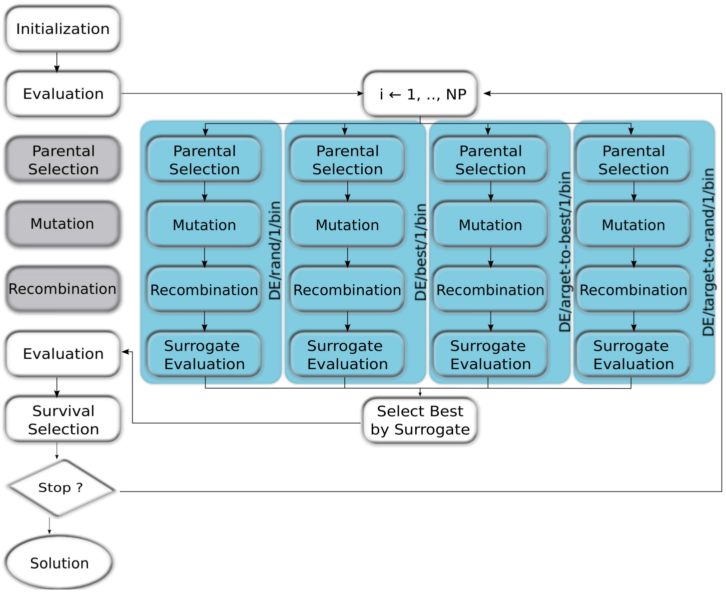

Figure 1: Surrogate Model Assisted Differential Evolution Scheme

technique with plain RBF prediction is indicated simply as SMDE. For all cases, a maximum of 15,000 objective function evaluations were allowed, and 30 independent runs were performed.

The population size and recombination rate were fixed as 30 and 0.9, respectively. For each variant considered an F parameter value was used according to previous evaluations: (i) DE/rand/1/bin: $F = 0.5$, (ii) DE/best/1/bin: $F = 0.7$, (iii) DE/target-to-best/1/bin: $F = 0.6$ and (iv) DE/target-to-rand/1/bin: $F = 0.7$.

9.1. Problems' Description

In the following, a brief description of the five structural design optimization problems considered is presented. The complete data can be found in [6, 7, 18].

The 10-bar Truss Design

In this test problem the stress in each member is limited to ± 25ksi, and the displacements at the nodes are limited to 2 in., in the x and y directions. The design variables are the cross-sectional areas of the bars. The material has $\gamma = 0.1$ lb/in^3, and $E = 10^4$ ksi. Vertical downward loads of 100 kips are applied at nodes 2 and 4. For the discrete case the values of the cross-sectional areas (in^2) are chosen from the set $\mathcal{S}$: 1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50. For the continuous case the cross sectional areas are in the range $[0.1, 40]$ in^2.

The 25-bar Truss Design

In this test-problem, the constraints require that the maximum stresses in the members remain in the interval $[-40, 40]$ ksi and that the maximum displacements at the nodes 1 and 2 be limited to 0.35 in, in both the x and y directions. For the discrete case the cross-sectional areas of the bars are to be chosen from the set (in square inches): 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, and 3.4. For the continuous case the cross-sectional areas are assumed to be in the range $[0.1, 3.4]$ in^2. The design variables are linked in eight groups, and the material has $\gamma = 0.1$ lb/in^3 and $E = 10^4$ ksi.

The 60-bar Trussed Ring

This test-problem includes three load cases.The cross-sectional areas were grouped in 25 design variables. The outer radius of the ring is 100 in and the inner radius is 90 in. The material has $E = 10^4$ ksi and $\gamma = 0.1$ lb/in^3. There are 198 constraints where 180 refer to allowable stress ($\sigma_i = 60$ ksi, $i = 1$ to 60), and 18 refer to displacement constraints along both the x and y directions with magnitude: 1.75 in at node 4, 2.25 in at node 13, and 2.75 in at node 19. For the continuous case the cross-sectional areas of the bars are assumed to be in the range $[0.5, 5]$ in^2. For the discrete case the values of the cross-sectional areas (in^2) are chosen from the set $\mathcal{S}$: $0.5, 0.6, \ldots, 5.0$ in a total of 46 options.

The 72-bar Truss

For this structure the cross-sectional areas of the bars are linked in sixteen groups. For the discrete case the values (in^2) are chosen from the set $\mathcal{S}$: $0.1, 0.2, \ldots, 2.5$ in a total of 25 options.

The material has $\gamma = 0.1$ lb/in^3 and $E = 10^4$ ksi. Two load cases are considered.Displacements at the nodes 1 to 16 along the x and y directions are constrained to a maximum of 0.25 in, and the stress in each bar is restricted to the range $[-25, 25]$ ksi.

The 942-bar Truss Design

In this truss tower [18], the symmetry of the tower around the x and y axes is employed to group the 942 truss members into 59 independent size variables. A single loading condition is considered (see [18]).

For the continuous case the cross-sectional areas are takenin the range $[1, 200]$ in^2 and for the discrete case the values (in^2) are chosen from the set $\mathcal{S}: 1, 2, \ldots, 200$ in a total of 200 options. The constraints for this problem include a maximum stress of 25.0 ksi (170 MPa) both in tension and compression for all members, and a limit of 15 in for the displacements of the top nodes in any global direction. This problem has been previously studied in [18, 19], where more detailed information can be found.

9.2. Results

The results for all the investigated problems are presented in tables 1, 2, 3, 4 and 5, where the proposals are compared to different budgets. Of course, the application of the surrogate model aims to reduce the number of function evaluations, thus the main objective is to obtain the best solutions with few evaluations. To observe this behavior, the average weight curve of each technique is presented in figures 2a, 2c and 3. In order to highlight the results for the 60- and 72-bar problems, two additional plots showing a closer view of the curves are presented in figures 2b and 2d.

Table 1: Weights found for the 10-bar truss problem. The best results for each case are in boldface.

	Best	Median	Average	std	Worst	NEvals
			Discrete case			
SMDE	**5490.74**	5498.28	5516.41	$3.65e+01$	5614.22	3000
SMDE-PoI	**5490.74**	**5494.47**	5510.67	$3.23e+01$	5614.22	3000
SMDE-EI	**5490.74**	5498.20	**5508.41**	$2.27e+01$	**5577.04**	3000
SMDE	**5490.74**	**5490.74**	5506.92	$3.19e+01$	5614.22	6000
SMDE-PoI	**5490.74**	**5490.74**	5505.00	$2.96e+02$	5614.22	6000
SMDE-EI	**5490.74**	**5490.74**	**5497.35**	$1.86e+01$	**5558.48**	6000
SMDE	**5490.74**	**5490.74**	5497.20	$1.66e+01$	5563.72	15000
SMDE-PoI	**5490.74**	**5490.74**	5498.71	$1.84e+01$	5568.40	15000
SMDE-EI	**5490.74**	**5490.74**	**5493.49**	$8.74e+00$	**5525.53**	15000
DUVDE[8]	5562.35	–	5564.90	$6.00e-01$	5565.04	24000
APM[7]	5490.74	–	5545.48	–	5567.84	90000
			Continuous case			
SMDE	5062.77	5076.97	5076.5743	$7.14e+00$	5090.87	3000
SMDE-PoI	**5062.6**	5069.55	**5072.12**	$6.33E+00$	**5082.38**	3000
SMDE-EI	5064.49	**5071.01**	5073.8213	$7.09E+00$	5090.49	3000
SMDE	5060.95	5061.52	5064.60	$6.24e+00$	5077.00	6000
SMDE-PoI	5060.91	5061.56	5064.70	$6.20e+00$	**5076.86**	6000
SMDE-EI	**5060.90**	**5061.29**	**5062.52**	$3.96e+00$	5076.98	6000
SMDE	5060.90	5061.13	5064.31	$6.32e+00$	**5076.79**	15000
SMDE-PoI	5060.90	**5061.05**	5064.19	$6.38e+00$	5076.82	15000
SMDE-EI	**5060.89**	5061.08	**5062.29**	$6.98e+00$	5076.94	15000
DUVDE[8]	5060.85	–	5067.18	$7.94e+00$	5076.66	280000
APM[7]	5069.08	–	5091.43	–	5117.39	280000

The results in tables 1 and 2 show that the techniques proposed in this work can obtain better solutions than when applying directly a RBF. Our results are also better than the ones taken as reference, even using a very small number of function evaluations.

However, the assessment of the performances on another set of problems give us a better idea of the differences among the techniques. In table 3 one can observe a clear improvement of the solutions for the 60-bar truss which involves 25 design variables, where the proposal SMDE-PoI achieved the best results for all the considered budgets. The technique was also able to outperform the reference results yet spending up to 10 times less function evaluations. In order to better assess this behavior regarding the reduction of the number of evaluations, it is shown in figure 2a the decrease of the obtained average weight

Table 2: Weights found for the 25-bar truss problem. The best results for each case are in boldface.

	Best	Median	Average	std	Worst	NEvals
			Discrete case			
SMDE	**484.85**	**484.85**	485.20	$9.89e-01$	490.19	3000
SMDE-PoI	**484.85**	**484.85**	485.31	$1.97e+00$	495.54	3000
SMDE-EI	**484.85**	**484.85**	**484.87**	$1.01e-01$	**485.38**	3000
SMDE	**484.85**	**484.85**	485.17	$9.94e-01$	490.19	6000
SMDE-PoI	**484.85**	**484.85**	484.95	$4.16e-01$	487.13	6000
SMDE-EI	**484.85**	**484.85**	**484.90**	$1.62e-01$	**485.57**	6000
SMDE	**484.85**	**484.85**	485.12	$7.37e-01$	488.69	15000
SMDE-PoI	**484.85**	**484.85**	484.95	$4.16e-01$	487.13	15000
SMDE-EI	**484.85**	**484.85**	**484.90**	$1.62e-01$	**485.57**	15000
DUVDE[8]	485.90	–	498.44	$7.66e+00$	507.77	20000
APM[7]	484.85	–	485.96	–	490.74	20000
			Continuous case			
SMDE	**484.05**	484.08	**484.09**	$6.29e-02$	484.38	3000
SMDE-PoI	**484.05**	**484.07**	484.11	$9.64e-02$	484.43	3000
SMDE-EI	**484.05**	484.08	484.10	$5.77e-02$	**484.24**	3000
SMDE	**484.05**	**484.06**	**484.07**	$1.70e-02$	**484.11**	6000
SMDE-PoI	**484.05**	**484.06**	**484.07**	$1.78e-02$	484.13	6000
SMDE-EI	**484.05**	**484.06**	**484.07**	$1.49e-02$	**484.11**	6000
SMDE	**484.05**	**484.06**	**484.07**	$1.70e-02$	**484.11**	15000
SMDE-PoI	**484.05**	**484.06**	**484.07**	$1.78e-02$	484.13	15000
SMDE-EI	**484.05**	**484.06**	**484.07**	$1.49e-02$	**484.11**	15000
DUVDE[8]	484.05	–	484.05	$1.00e-05$	484.05	240000
APM[20]	484.73	–	487.66	–	493.03	240000

Table 3: Weights found for the 60-bar truss problem. The best results for each case are in boldface.

	Best	Median	Average	std	Worst	NEvals
			Discrete case			
SMDE	**328.69**	**358.55**	**359.32**	$1.77e+01$	393.60	3000
SMDE-PoI	331.91	366.03	362.20	$1.68e+01$	**384.87**	3000
SMDE-EI	337.46	365.78	370.20	$2.11e+01$	418.33	3000
SMDE	315.71	**319.24**	**319.27**	$3.01e+00$	**328.37**	6000
SMDE-PoI	315.69	319.59	322.27	$8.28e+00$	350.95	6000
SMDE-EI	**314.71**	319.78	323.12	$1.07e+01$	362.03	6000
SMDE	**312.76**	**314.73**	**315.74**	$2.28e+00$	**320.81**	9000
SMDE-PoI	313.71	316.73	318.42	$6.35e+00$	347.77	9000
SMDE-EI	313.71	315.68	318.32	$9.53e+00$	352.14	9000
SMDE	**312.71**	**314.68**	**315.26**	$2.06e+00$	**320.77**	15000
SMDE-PoI	313.69	315.69	317.09	$6.18e+00$	347.75	15000
SMDE-EI	312.73	314.69	317.28	$9.73e+00$	352.10	15000
			Continuous case			
SMDE	327.57	357.18	358.15	$1.70e+01$	402.06	3000
SMDE-PoI	**325.86**	**344.36**	**351.14**	$1.80e+01$	394.3	3000
SMDE-EI	336.91	359.62	361.25	$1.40e+01$	**385.33**	3000
SMDE	313.67	316.84	319.04	$8.20e+00$	356.60	6000
SMDE-PoI	**311.95**	**314.05**	**317.98**	$9.43e+00$	346.97	6000
SMDE-EI	314.57	316.95	318.19	$4.56e+00$	**333.29**	6000
SMDE	309.28	310.26	310.39	$7.14e-01$	313.11	15000
SMDE-PoI	**308.85**	**309.54**	**309.58**	$4.86e-1$	**310.83**	15000
SMDE-EI	309.21	310.27	310.91	$3.26e+00$	327.76	15000
DUVDE[8]	309.44	–	311.54	$1.46e+00$	314.70	150000
APM[6]	311.87	–	333.01	–	384.19	800000

(out of 30 runs) for each algorithm. As one could expect, as the optimization process progresses and thus there is more information accumulated about the search space, the different techniques start producing very similar solutions due to the increasing quality of the surrogate models. Figure 2b highlights the average weight curve of each proposal by presenting only the first 200 generations of the optimization processes, *i.e.* the first 6000 function evaluations.

Although the 72-bar problem has a smaller number of design variables (16 variables), a similar behavior with respect to the reduction in the number of evaluations is observed, both for SMDE-PoI and SMDE-EI. This behavior is also shown in figures 2c and 2d, but this time for the first 4500 function evaluations.

The change concerning the quality of the obtained solutions as larger budgets are considered is clearly seen for the 942-bar problem (which has 59 design variables), as shown in table 5 and figure 3.

Table 4: Weights found for the 72-bar truss problem. The best results for each case are in boldface.

	Best	Median	Average	std	Worst	NEvals
			Discrete case			
SMDE	**385.54**	**389.70**	**390.39**	$4.28e+00$	**402.65**	3000
SMDE-PoI	386.53	391.33	392.15	$6.29e+00$	410.13	3000
SMDE-EI	386.81	390.61	397.13	$1.95e+01$	477.86	3000
SMDE	**385.54**	386.81	**387.71**	$2.90e+00$	**397.81**	6000
SMDE-PoI	**385.54**	387.37	389.53	$5.49e+00$	409.13	6000
SMDE-EI	**385.54**	**386.53**	391.05	$1.63e+01$	473.06	6000
SMDE	**385.54**	386.81	**387.27**	$2.45e+00$	**397.81**	9000
SMDE-PoI	**385.54**	386.81	388.98	$5.56e+00$	409.13	9000
SMDE-EI	**385.54**	**385.54**	390.54	$1.55e+00$	468.40	9000
SMDE	**385.54**	386.81	**387.05**	$2.48e+00$	**397.81**	12000
SMDE-PoI	**385.54**	386.81	388.67	$5.41e+00$	409.13	12000
SMDE-EI	**385.54**	**385.54**	390.34	$1.51e+01$	466.00	12000
SMDE	**385.54**	386.81	**386.89**	$2.32e+00$	**397.81**	15000
SMDE-PoI	**385.54**	386.81	388.46	$5.35e+00$	409.13	15000
SMDE-EI	**385.54**	**385.54**	390.22	$1.50e+01$	466.00	15000
			Continuous case			
SMDE	**381.46**	385.96	386.63	$3.78e+00$	398.37	3000
SMDE-PoI	382.16	**384.49**	**385.55**	$3.07e+00$	**393.31**	3000
SMDE-EI	382.99	386.25	387.02	$3.34e+00$	400.65	3000
SMDE	379.81	380.25	380.49	$7.92e-01$	384.03	6000
SMDE-PoI	**379.70**	**380.13**	**380.36**	$9.56e-01$	383.89	6000
SMDE-EI	379.89	380.36	380.42	$4.22e-01$	**381.60**	6000
SMDE	379.66	379.77	379.90	$0.693e-01$	383.53	9000
SMDE-PoI	**379.64**	**379.73**	379.91	$7.11e-01$	383.52	9000
SMDE-EI	379.65	379.79	**379.79**	$1.03e-01$	**380.14**	9000
SMDE	379.63	379.68	379.69	$5.09e-02$	379.84	12000
SMDE-PoI	**379.62**	**379.67**	379.82	$6.90e-01$	383.46	12000
SMDE-EI	379.63	**379.67**	**379.68**	$4.79e-02$	**379.83**	12000
SMDE	**379.62**	379.66	379.67	$3.67e-02$	379.78	15000
SMDE-PoI	**379.62**	379.66	379.78	$6.18e-01$	383.04	15000
SMDE-EI	**379.62**	**379.65**	**379.65**	$2.66e-02$	**379.73**	15000
DUVDE[8]	379.66	–	380.42	$5.72e-01$	381.37	35000
APM[7]	387.04	–	402.59	–	432.95	35000

Summarizing, all the SMDE proposals are able to obtain results similar to the ones taken as reference, but with up to tenfold decrease in the number of function evaluations. Moreover, the observed behaviors of the SMDE variations match what one could expect for them. The application of SMDE-EI results in a balance between the optimization of the surrogate model and the infilling of regions where the surrogate model has high variance; therefore, the improvement of the solutions achieved by the technique after the first function evaluations is compatible with the improvement of the surrogate model's approximation. On the other hand, the SMDE-PoI proposal has shown to be more efficient when the budget is limited, being thus suitable for problems in which the cost of computer simulations is very expensive.

10. Conclusions

This work has presented two proposals regarding the application of infill criteria using Differential Evolution (DE) in which the surrogate model is given by a RBF constructed locally. A surrogate model assisted DE from the literature was taken as a baseline to assess the application of the RBF-based surrogate model as well as two different infill criteria: PoI and EI. Those criteria consider the approximation obtained by the surrogate model as the realization of a random variable and, therefore, they take into account not only the predicted fitness value, but also the surrogate model's variance, which allows us to evaluate the quality of the surrogate model at the point of interest. The application of both those criteria to SMDE has shown to be efficient for the investigated structural design optimization problems. In particular, this becomes more evident for those situations involving few function evaluations, which allows one to tackle problems where the budget for carrying out computer simulations is limited.

Finally, our promising results suggest the need of a more comprehensive assessment of the proposed techniques, be it considering a broader spectrum of problems or evaluating other infill criteria.

11. Acknowledgements

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Table 5: Weights found for the 942-bar truss problem. The best results for each case are in boldface.

	Best	Median	Average	std	Worst	NEvals
			Discrete case			
SMDE	372448.00	**462879.50**	**469870.10**	$5.49e+04$	596806.00	3000
SMDE-PoI	416086.00	471761.00	476339.90	$3.78e+04$	**564646.00**	3000
SMDE-EI	**359858.00**	482884.00	487222.43	$5.41e+04$	595379.00	3000
SMDE	220910.00	264826.00	**269946.87**	$3.49e+04$	**356870.00**	6000
SMDE-PoI	**198411.00**	267410.50	276069.77	$3.38e+04$	380593.00	6000
SMDE-EI	210332.00	**260523.00**	272621.20	$4.51e+04$	372116.00	6000
SMDE	**165219.00**	215633.50	217172.50	$2.93e+04$	**300795.00**	9000
SMDE-PoI	169117.00	216454.50	221090.43	$3.32e+04$	324984.00	9000
SMDE-EI	169279.00	**208072.00**	**214282.17**	$3.19e+04$	304948.00	9000
SMDE-Y	**153279.00**	195953.00	194640.27	$2.42e+04$	**264961.00**	12000
SMDE-PoI	160213.00	193115.50	198704.90	$3.19e+04$	288750.00	12000
SMDE-EI	153698.00	**187648.50**	**192618.67**	$2.95e+04$	284556.00	12000
SMDE	148937.00	176808.50	**179788.90**	$2.05e+04$	**231617.00**	15000
SMDE-PoI	156066.00	174041.00	184189.63	$3.00e+04$	276194.00	15000
SMDE-EI	**147575.00**	**172827.00**	180234.93	$2.70e+04$	260288.00	15000
			Continuous case			
SMDE	354562.00	477906.00	476183.1	$4.60e+04$	**589946.00**	3000
SMDE-PoI	**344174.00**	**448316.50**	**457227.63**	$6.89e+04$	653040.00	3000
SMDE-EI	393242.00	479279.50	489369.87	$5.50e+04$	611609.00	3000
SMDE	**195811.00**	286905.50	285392.43	$3.73e+04$	370493.00	6000
SMDE-PoI	206364.00	**253530.50**	**257771.73**	$3.40e+04$	**348402.00**	6000
SMDE-EI	212637.00	263988.50	279692.50	$4.30e+04$	380294.00	6000
SMDE	167960.00	218842.00	**220691.10**	$2.84e+04$	**284895.00**	9000
SMDE-PoI	**163637.00**	216811.00	214696.70	$3.07e+04$	321454.00	9000
SMDE-EI	176085.00	**213713.00**	222917.07	$4.02e+04$	340877.00	9000
SMDE	157826.00	194090.00	195169.90	$2.91e+04$	**259526.00**	12000
SMDE-PoI	**150113.00**	189546.50	195400.53	$2.79e+04$	291725.00	12000
SMDE-EI	155339.00	**186141.00**	**190121.47**	$2.57e+04$	264176.00	12000
SMDE	151887.00	**176476.00**	182112.30	$2.75e+04$	254958.00	15000
SMDE-PoI	**147781.00**	180084.50	182910.80	$2.11e+04$	230107.00	15000
SMDE-EI	152349.00	178381.50	**178537.53**	$1.87e+04$	**209901.00**	15000
SA [19]	143436.02	–	–	–	–	39834
ES [18]	141241.00	–	–	–	–	150000

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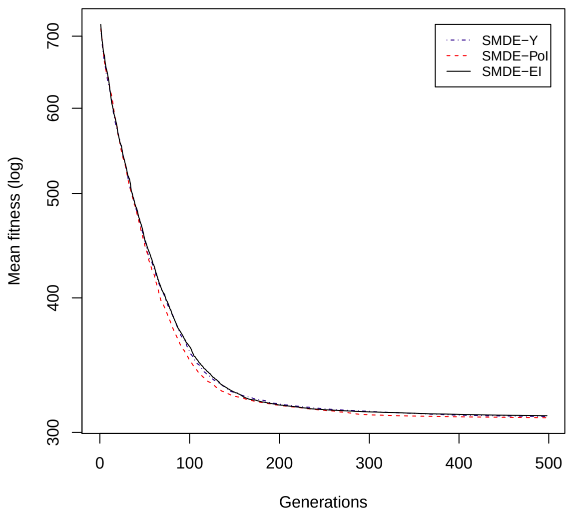
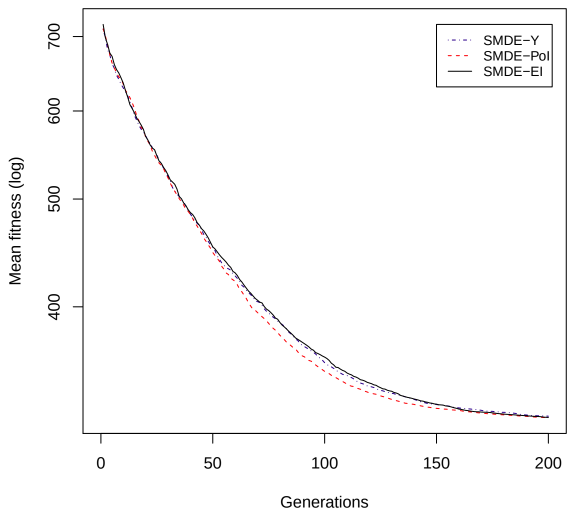
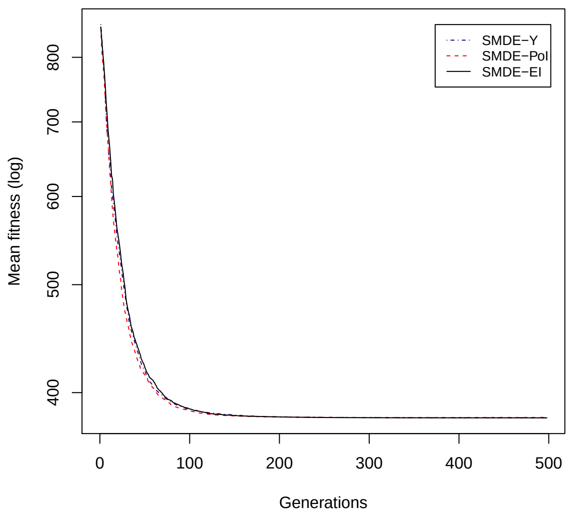


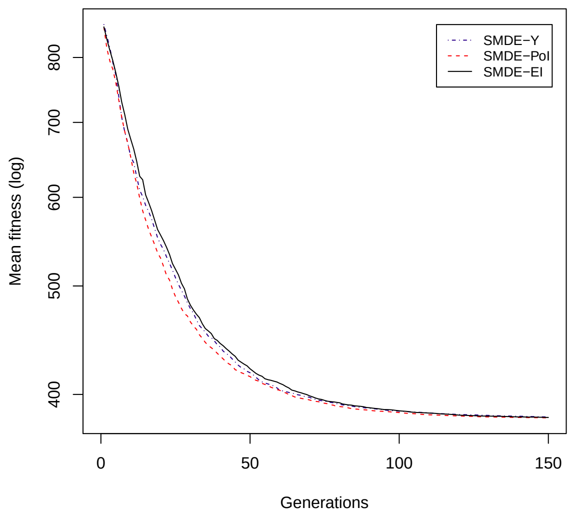
(a) The 60-bar truss



(b) The 60-bar truss for 200 generations



(c) The 72-bar truss



(d) The 72-bar truss for 150 generations

Figure 2: Results for each generation for the 60- and 72-bar trusses (continuous case)

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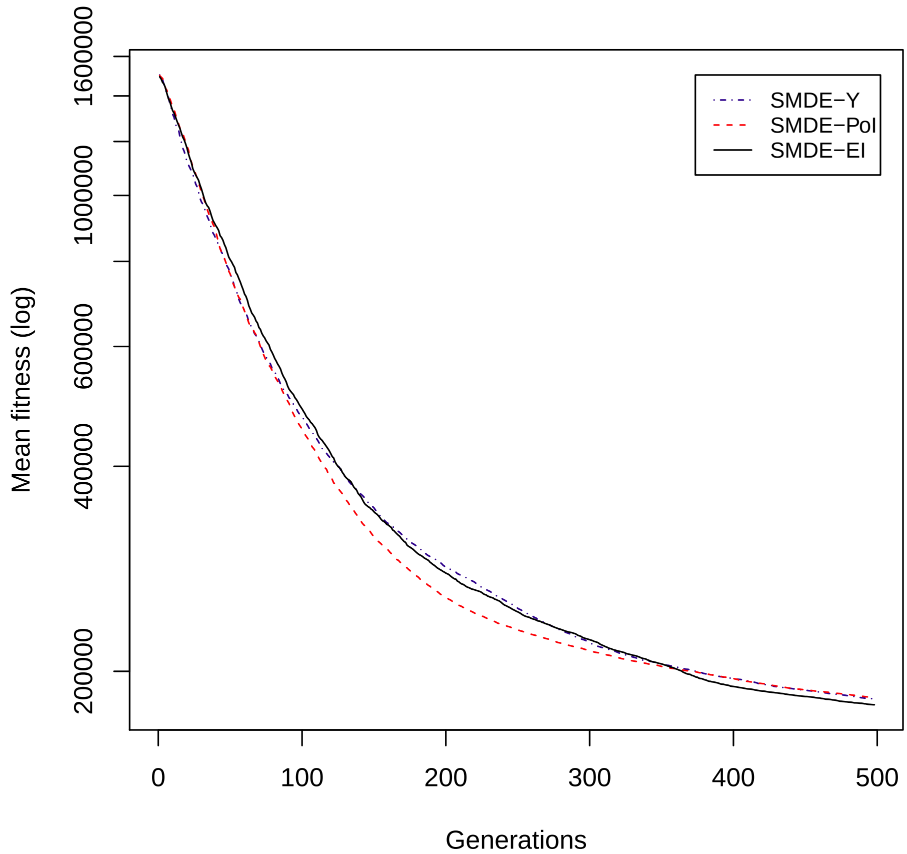

Figure 3: Average of best solutions for each generation for the 942-bar problem (continuous case).

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