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MULTI-OBJECTIVE WIND FARM DESIGN: EXPLORING THE TRADE-OFF BETWEEN CAPACITY FACTOR AND LAND USE

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Abstract
The performance of a wind farm is affected by several key factors that can be classified into two categories: the natural factors and the design factors. Hence, the planning of a wind farm requires a clear quantitative understanding of how the balance between the concerned objectives (*e.g.*, socia-economic, engineering, and environmental objectives) is affected by these key factors. This understanding is lacking in the state of the art in wind farm design. The wind farm capacity factor is one of the primary performance criteria of a wind energy project. For a given land (or sea area) and wind resource, the maximum capacity factor of a particular number of wind turbines can be reached by optimally adjusting the layout of turbines. However, this layout adjustment is constrained owing to the limited land resource. This paper proposes a Bi-level Multi-objective Wind Farm Optimization (BMWFO) framework for planning effective wind energy projects. Two important performance objectives considered in this paper are: (i) wind farm Capacity Factor (CF) and (ii) Land Area per MW Installed (LAMI). Turbine locations, land area, and nameplate capacity are treated as design variables in this work. In the proposed framework, *the Capacity Factor - Land Area per MW Installed (CF - LAMI)* trade-off is parametrically represented as a function of the nameplate capacity. Such a helpful parameterization of trade-offs is unique in the wind energy literature. The farm output is computed using the wind farm power generation model adopted from the Unrestricted Wind Farm Layout Optimization (UWFLO) framework. The Smallest Bounding Rectangle (SBR) enclosing all turbines is used to calculate the actual land area occupied by the farm site. The wind farm layout optimization is performed in the lower level using the Mixed-Discrete Particle Swarm Optimization (MDPSO), while the *CF - LAMI* trade-off is parameterized in the upper level. In this work, the *CF - LAMI* trade-off is successfully quantified by nameplate capacity in the 20 MW to 100 MW range. The Pareto curves obtained from the proposed framework provide important insights into the trade-offs between the two performance objectives, which can significantly streamline the decision-making process in wind farm development.

Keywords: Bi-level Multi-objective Wind Farm Optimization (BMWFO), *CF - LAMI* trade-off, land use, wind farm design

1. INTRODUCTION

Wind energy is harvested through wind farms that can consist of hundreds of wind turbines. The planning of wind farms is a comprehensive and complex process where numerous mutually-dependent parameters and design variables need to be appropriately considered. To develop wind farms that are profitable, reliable, and meet community acceptance, it is important to carefully consider the trade-offs between the concerned objectives in the wind farm planning. In this paper, a *Bi-level Multi-objective Wind Farm Optimization* (BMWFO) framework is proposed to quantify and explore the balance between the wind farm Capacity Factor (CF) and the turbines' land use which is represented by the Land Area per MW Installed (LAMI).

Many studies have been done related to the design of wind farms (at the wind farm level). Three research directions are mainly involved in theses studies. The first direction being focused on improving the accuracy of wind farm design by including many sub (cost/performance) models [1, 2, 3]. Algorithms being used to solve the concerned wind farm layout optimization problem is another research direction. Popular algorithms may include Genetic Algorithms (GA) [4], Evolutionary Algorithms (EA) [5], Particle Swarm Optimization (PSO) algorithm [6], and an Extended Pattern Search (EPS) approach [7]. The third research direction puts the emphasis on solving various design objectives [8, 9]. Of all these studies, however, the layout optimization problem is generally project-specific. The farm boundaries and the number of turbines to be installed are normally given or pre-defined. Such restrictions may cause inefficient use of the land resource or limit the wind farm performance; especially when the availability of suitable farm sites has become a limiting factor [10]. It should be emphasized that the land use of a farm site plays an important role in determining the performance of a wind farm design.

This paper presents a bi-level multi-objective wind farm optimization framework aimed to develop first-of-its-kind generalized guidelines for the design of wind farms. It provides an approach to find the best combination of the installed nameplate capacity and the land resource in order to make the best use of the available land resource. In the lower level, a multi-objective wind farm layout optimization is performed, while the trade-off between the design objectives is parameterized in the upper level. Two design objectives that are accounted for include the energy production and the land use. Applying the proposed framework, it can significantly streamline the wind farm planning process (*e.g.*, site selection) and reduce the undesirable delays in large-scale wind farm constructions.

2. DESIGN OBJECTIVES

Two design objectives considered in this work are: (i) the farm output which is represented by the capacity factor (CF), and (ii) the land use of layout which is represented by the Land Area per MW Installed (LAMI).

2.1. Power Generation Model

In this paper, the capacity factor is computed using the power generation model adopted from the Unrestricted Wind Farm Layout Optimization (UWFLO) framework [3]. This power generation model quantifies the wind farm power output as a function of turbine characteristics, turbine locations, and incoming wind speed. The power output of each turbine is evaluated based on its order of encountering the incoming wind flow. A *generalized power curve* is used to evaluate the power output of each turbine. This *generalized power curve* is scaled back to represent the approximate power response of a particular commercial turbine using its specifications. The power generated from a single turbine, P, can be evaluated using the following expression [11]:

$$\frac{P}{P_r} = \begin{cases} P_n\left(\frac{U_0-U_{in}}{U_r-U_{in}}\right), & \text{if } U_{in} < U_0 < U_r \\ 1, & \text{if } U_r < U_0 < U_{out} \\ 0, & \text{if } U_{out} < U_0 \text{ or } U_0 < U_{in} \end{cases} \tag{1}$$

where U_0 is the velocity immediately in front of the turbine; P_r is the turbine rated capacity; U_{in}, U_{out}, and U_r are turbine's cut-in, cut-out, and rated speeds, respectively; and P_n represents the polynomial fit for the *generalized power curve*.

Another feature of this model is that it uses a variable axial induction factor. Hence, the power coefficient, C_P, can be expressed as a function of incoming wind speed and turbine characteristics, as given by

$$C_P = \frac{P}{\frac{1}{2}\rho\pi\frac{D}{4}U^3} \tag{2}$$

where U represents the incoming wind speed; ρ is the air density; and D is the rotor diameter.

This work also accounts for the power reduction caused by the wake effect using the Jensen wake model [12], in which the wake behind the wind turbine is assumed to have a linear expansion, as given by

$$D_{wake} = D(1 + 2ks) \tag{3}$$

where s is the normalized downstream distance (in terms of the rotor diameter) behind the turbine; and k is the wake decay constant. The velocity deficit v in the (fully developed) wake is expressed as

$$v = U\left[\frac{1-\sqrt{1-C_T}}{(1+2ks)^2}\right] \tag{4}$$

where C_T is the turbine thrust coefficient.

Additionally, the Katic model [13] is also used to account for the wake merging and partial wake overlapping. If Turbine-j is in the influence of multiple wakes created by totally K upstream turbines, the corresponding velocity deficit v_j is given by

$$v_j = \sqrt{\sum_{k=1}^{K} \frac{A_{kj}}{A_j} \left(U_j - U_{kj}\right)^2} \tag{5}$$

where U_{kj} represents the wake speed (created by Turbine-k) at the location of Turbine-j; and A_{kj} is the effective influence area of the wake (created by Turbine-k) on Turbine-j. Therefore, the overall energy production of a N-turbine wind farm over a set of randomly distributed N_P wind conditions, E_{farm}, can be calculated as

$$\begin{aligned} & E_{farm} && = (365 \times 24) \textstyle\sum_{j=1}^{N_P} P_{farm}(U^i, \theta^i) p(U^i, \theta^i) \Delta U \Delta\theta, \\ \text{where} \quad & \Delta U \Delta\theta && = U_{max} \times 360^\circ / N_P \\ & P_{farm}(U^i, \theta^i) && = \textstyle\sum_{j=1}^{N} P_j \end{aligned} \tag{6}$$

Here, U_{max} represents the maximum possible wind speed in the current wind distribution; and $p(U^i, \theta^i)$ represents the probability of the occurrence of a wind condition defined by incoming wind speed U and direction θ.

Generally, the capacity factor is used to measure the wind farm performance. The capacity factor of a wind farm is defined as the ratio of the actual energy production over a time period to the potential farm output if the farm was operating at the full nameplate capacity over that time period. The annual wind farm capacity factor can be expressed as

$$CF = \frac{E_{farm}}{(365 \times 24) NC} \tag{7}$$

where NC is the nameplate capacity of the concerned wind farm.

2.2. Land Use of Farm Layout

As each turbine adjusts its position to reach an optimal layout that yields the maximum capacity factor, the total land area used by this optimal layout is measured based on the geometric concept of a "2D convex hull". Given the coordinates of each turbine, the Graham scan algorithm [14] is used to find points (turbines) that comprise the facets of a convex hull. After determining this convex hull, the rotating calipers algorithm [15] is used to find the Smallest Bounding Rectangle (SBR) that encloses all turbines. As shown in Fig. 1, the region enclosed by the solid line box is obtained using the SBR algorithm. However, owing to the turbines' impact on the surroundings, a "buffer area" is considered. Therefore, the actual land use of a certain layout is represented by the buffer area (as illustrated by the dash line box in Fig. 1), which is created by utilizing a $2D$ spacing from the SBR.

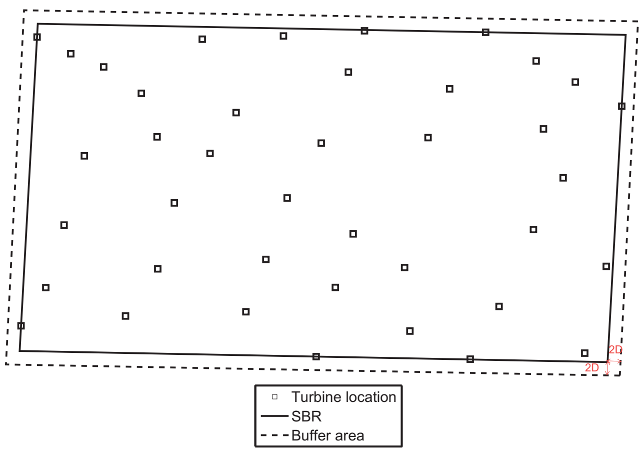

Figure 1: Illustration of land use for a certain layout

3. IMPLEMENTATION OF THE BI-LEVEL MULTI-OBJECTIVE WIND FARM OPTIMIZATION FRAMEWORK

In this work, it is assumed that the wind distribution over the farm site is unique and identical turbines are considered. The GE 1.5 MW turbine is selected and its specifications are listed in Table 1. At first, a series of sample nameplate capacities are generated in a pre-specified range from 20 MW to 100 MW. Since identical turbines are used, the number of turbines to be installed can be then determined based on the nameplate capacity, which is ranged from 13 to 66. For each set of turbines, a wind farm layout optimization is performed at the lower-level of the proposed framework. The Pareto curve obtained from each set of turbines is to be parameterized at the upper-level of this framework. The details of the proposed framework will be introduced as follows.

Table 1: GE 1.5xle Turbine [16]

Specifications	Value
Rated capacity	1500 kW
Cut-in	3.5 m/s
Cut-out	20 m/s
Rated speed	11.5 m/s
Rotor diameter	82.50 m
Swept Area	5346 m^2
Hub height	80 m

3.1. Lower-level: Multi-objective Wind Farm Layout Optimization

In this level, the Mixed-Discrete Particle Swarm Optimization (MDPSO) algorithm developed by Chowdhury et al. [6] is used to perform the wind farm layout optimization. One prominent advantage that the MDPSO algorithm has over a conventional PSO algorithm is that it has the diversity preservation capability to prevent premature stagnation of particles. The basic steps of the advanced algorithm are summarized as

$$\begin{aligned} x_i^{t+1} &= x_i^t + v + i^{t+1} \\ v_i^{t+1} &= \alpha v_i^t + \beta_l r_1(p_i - x_i^t) + \beta_g r_2(p_g - x_i^t) \\ &\quad + \gamma r_3(x_i^t - p_g) \end{aligned} \tag{8}$$

where x_i^t and v_i^t represent the position and the velocity of the i^{th} particle at the t^{th} generation, respectively; r_1, r_2, and r_3 are random numbers between 0 and 1; p_i is the best candidate solution found for the i^{th} particle; p_g is the best candidate solution for the entire population (swarm); α, β_l, and β_g are user defined constants that are respectively associated with inertial weight, exploitation, and exploration; γ is the diversity preservation coefficient that is evaluated adaptively as a function of the prevailing diversity in the population at the concerned iteration. This diversity preservation coefficient is scaled using an user defined parameter, γ_0. In Eq.(8), the last term in the velocity update equation decelerates the motion of particles towards p_g, i.e., the global best, thereby maintaining diversity and preventing premature convergence.

Such an explicit diversity preservation operator is often necessary for solving complex optimization problems that involve multimodal criterion functions and a large number of design variables, as is the case with the maximization of the capacity factor presented in this paper. More details of the population diversity and the formulation of the diversity coefficient γ can be found in Ref. [6, 17].

Initially, all turbines are distributed within a square region, of which size is larger than the allowable land area. The actual land use will be obtained based on the optimal layout. Given that the farm capacity factor is a monotonically increasing function of the land area used by turbines, this bi-objective optimization problem can be solved by performing a constrained single objective optimization, which is formulated as:

$$
\begin{array}{ll}
 & \max\ CF \\
\textit{subject to} & \\
 & g_1(\vec{x},\vec{y}) \geq 2D \\
 & g_2(\vec{x},\vec{y}) \leq A^*(\vec{x},\vec{y}) \\
 & x_{min} \leq x_i \leq x_{max} \\
 & y_{min} \leq y_i \leq x_{max} \\
\text{where} & \\
 & i = 1, 2, \ldots, N
\end{array}
\tag{9}
$$

where CF is the wind farm capacity factor given by Eq.(7); N is the number of turbines to be installed; g_1 represents the constraint that the minimum spacing between turbines (distance measured from hub to hub) should not be less than two rotor diameters; g_2 gives the constraint of the area of layout, A^*, calculated based on the SBR determined by the layout; $\vec{x}$ and $\vec{y}$ are design vectors that represent the turbine coordinates as given by

$$
\begin{array}{rcl}
\vec{x} & = & x_1, x_2, \ldots, x_N \\
\vec{y} & = & y_1, y_2, \ldots, y_N
\end{array}
\tag{10}
$$

The parameter setup of this layout optimization problem is shown in Table 2.

Table 2: User-defined parameters in MDPSO

Parameter	Value
W	0.5
β_g	1.4
β_l	1.4
γ	1.0
γ_0	$1e-10$
Population size	$10 \times N$
Max. allowable function calls	$500,000$

3.2. Upper-level: Parameterization of the Capacity Factor - Land Area per MW Installed Trade-off

After performing the layout optimization for all sample nameplate capacities, a set of Pareto curves can be obtained, from which the trend of the trade-off between the capacity factor and the LAMI can be observed (as shown in Fig. 2). Based on this observation, the power function is selected to fit each of these *CF-LAMI* curves. In this case, the relationship between the capacity factor and the LAMI can be expressed as

$$CF = aLAMI^b + c \tag{11}$$

Figure 2 also shows the curves fitted by using power functions. The coefficients for the power functions are listed in Table 3.

Table 3: Parameterization of *CF-LAMI* Trade-off

Sample	Coef. a	Coef. b	Coef. c
1	-0.2415	-0.9424	0.5076
2	-0.2453	-0.8423	0.4981
3	-0.3027	-0.7985	0.4976
4	-0.9575	-1.133	0.4937
5	-1.107	-1.170	0.4884

Here, we can observe that, for a certain allowable land area, the predicted capacity factor decreases as the installed nameplate capacity increases. This trend is also similar to that presented by Chowdhury et al. [18]. Another observation is that the predicted capacity factor becomes less sensitive to the LAMI as the LAMI increases. By following the above two rules, each coefficient shown in Eq.(11) can be fitted as a

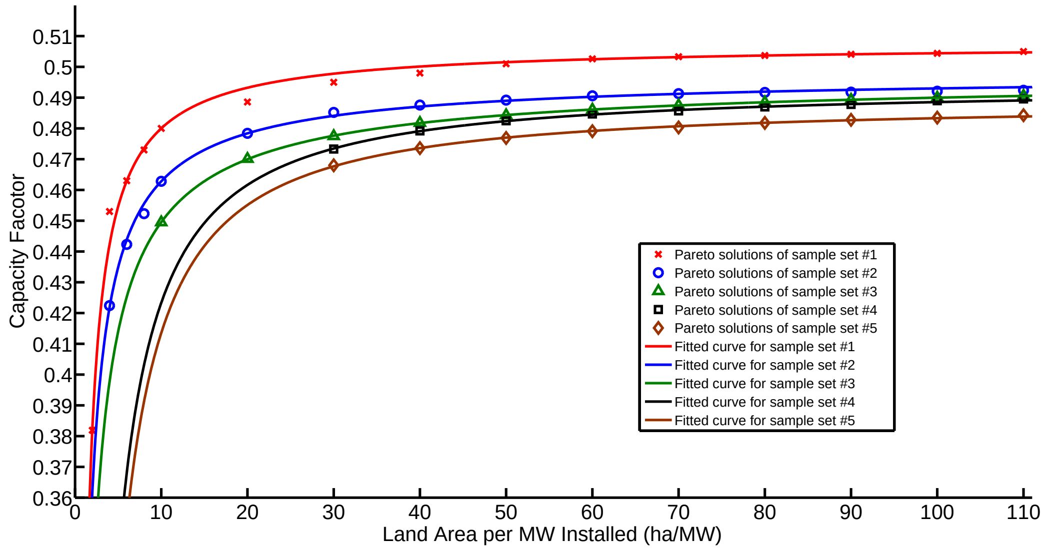

Figure 2: *CF-LAMI* Trade-off Curves

function of the nameplate capacity, which provides an approach to parameterize the *CF-LAMI* trade-off by nameplate capacity. Hence, Eq.(11) can be modified by

$$CF = a(NC)LAMI^{b}(NC) + c(NC) \tag{12}$$

The equations for the three coefficients expressed in the power function are given by:

$$\begin{aligned} a &= -6\times10^{-5}NC^2 - 3.7\times10^{-3}NC - 0.1432 \\ b &= -1\times10^{-5}NC^2 - 1.4\times10^{-3}NC - 0.9099 \\ c &= 5\times10^{-7}NC^2 - 2\times10^{-4}NC + 0.5091 \end{aligned} \tag{13}$$

Therefore, the *CF-LAMI* trade-off can be quantified by the nameplate capacity, as given by

$$\begin{aligned} CF = \; & (-6\times10^{-5}NC^2 - 3.7\times10^{-3}NC - 0.1432)LAMI^{-1\times10^{-5}NC^2 - 1.4\times10^{-3}NC - 0.9099} \\ & +5\times10^{-7}NC^2 - 2\times10^{-4}NC + 0.5091 \end{aligned} \tag{14}$$

3.3. Results and Discussion

Figure 3 shows the optimal layout obtained from the case of 40 turbines. It is observed that turbines tend to be placed very close to each other when the allowable area of layout is small. Subsequently, the capacity factor predicted is relatively low due to the power reduction caused by the wake effect. When turbines have more space, i.e., a larger allowable area, a better capacity factor can be predicted. However, the capacity factor becomes less sensitive to the land area when the LAMI exceeds 30 ha/MW. It is also interesting that the actual land use of the optimal layouts shown in Fig. 3 have similar geometric shapes, which indicates that such layouts can best capture the wind energy over the particular wind distribution assumed in this paper.

Eq.(14) can be helpful for wind farm developers to explore the *CF-LAMI* trade-off by selecting a certain value of nameplate capacity in the 20 MW to 100 MW range. For a particular land resource, $LAMI^*$, an optimal nameplate capacity/number of turbines can be decided to reach the maximum capacity factor.

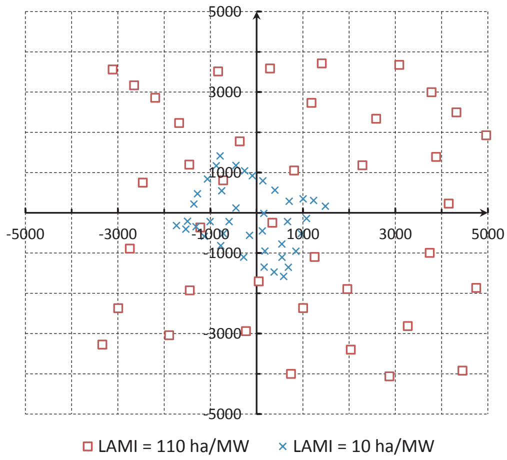

Figure 3: Optimal Layouts of 40 turbines with different allowable areas

Therefore, the optimal layout can be efficiently solved by using the following formulation:

$$\begin{aligned}
&\max\ CF \\
\textit{subject to}\quad & \\
& g_1(\vec{x},\vec{y}) \geq 2D \\
& g_2(\vec{x},\vec{y}) \geq CF_{max} \\
& g_3(\vec{x},\vec{y}) \leq LAMI^* \\
& x_{min} \leq x_i \leq x_{max} \\
& y_{min} \leq y_i \leq x_{max} \\
\text{where}\quad & \\
& i = 1, 2, \ldots, N^*
\end{aligned} \tag{15}$$

where CF_{max} is the maximum capacity factor obtained by optimizing Eq.(14) with given LAMI; and N^* represents the optimal number of turbines to be installed, which can be determined by solving the optimization problem as addressed in Eq.(14).

4. CONCLUDING REMARKS

This paper proposed a bi-level multi-objective wind farm optimization framework that provides an understanding of how the trade-off between the capacity factor and the land use is influenced by the nameplate capacity. General layout optimization methods/wind farm design frameworks require given farm boundaries and/or a given number of turbines to be installed. However, the framework presented in this paper provides a way to implement wind farm designs without those restrictions. For a certain wind distribution, wind farm developers are able to explore the balance between the capacity factor and the land use by setting different values of the nameplate capacity. Moreover, an optimal nameplate capacity can be selected based on the *CF-LAMI* trade-off , in order to make the best use of the land resource.

Since identical turbines were assumed, the nameplate capacity can also be represented in terms of the number of turbines to be installed. To this end, with the proposed framework, the optimal combination of the number of turbines and the actual land use can be determined as well. Future work should focus on accounting for the use of multiple turbines, such that the *CF-LAMI* trade-off can be parametrically represented as a function of both the nameplate capacity and the turbine selection. The impact of land configuration (shape, aspect ratio of the land use, *etc.*) should be also considered.

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