Addressing Risk in Design Through Decision Analysis in MDO/VDD Frameworks

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1. Abstract

Large-scale complex engineered systems are products of large organizations with hundreds, if not thousands, of individuals making design decisions. Due to the uncertain nature of complex engineered products, decision makers are often confronted with choosing the best design according to both value and risk preferences. The current methods of preference communication from stakeholder to decision maker primarily focus on value preferences. Without proper communication of risk preferences the designs can be risk biased by the decision makers, resulting in non-optimal designs with respect to the stakeholder’s preferences. This paper proposes the incorporation of decision analysis techniques into value-driven design and multidisciplinary design optimization frameworks to properly disseminate the value and risk preferences of the stakeholder to the decision maker. Two examples involving a continuous design variable system and a system composed of sub-systems are examined to illustrate the impact that risk biases have on the design and how an objective function incorporating both value and risk preferences can eliminate the biases.

2. Keywords: Risk, Utility Theory, Multidisciplinary Design Optimization, Value-Driven Design, Decision-making

3. Introduction

Highly complex systems, such as aircraft, are generally products of large organizations with hundreds, if not thousands, of individuals making decisions that will impact the final design. The Boeing Company alone has over 170,000 employees[1]. The design of the Boeing 787 Dreamliner, a large-scale complex engineered system (LSCES), involved not only Boeing employees, but also more than 50 suppliers scattered across the globe[2]. The individuals involved in LSCES design occupy various levels of the hierarchical design structure, from conceptual system design to physical component characteristic determination. These individuals come from various cultural backgrounds, educational backgrounds and work at different tiers and groups of the organizational hierarchy. These differences result in many different preferences on risk aversion in design.

The goal of the design process is to determine the design alternative which is most preferred by product’s stakeholder, the individual at the top of the product’s design hierarchy. George Hazelrigg has stated that, “Engineering design is increasingly recognized as a decision-making process.”[3] In LSCES these decision-making processes are happening throughout the levels of the hierarchical design structure and by all of the individuals within those levels. With so many decisions that are being made, it is important for the stakeholder to properly communicate their preferences in order to result in decisions which are in line with his/her preferences. These preferences which impact the choices being made are not simply the value preferences of the stakeholder, but also their degree of risk aversion.

System design frameworks and tools, such as multidisciplinary design optimization (MDO) [4] [5] and value-driven design (VDD) [6] are currently being used to communicate the preferences of the stakeholder. VDD properly captures the value preferences of the stakeholder but does not capture the degree of risk aversion of the stakeholder when faced with uncertain design outcomes. Without a mathematical representation of the stakeholder’s desires concerning risk the decision makers will be forced to speculate on a risk preference, resulting in risk biases by the decision makers. The impact of risk biases will impact the final design, likely resulting in a design that is not optimal in the stakeholder’s eyes. Decision analysis offers mathematical methods to determine decisions that individuals will make under uncertainty. This paper will discuss these topics and, through examples, demonstrate the impact of risk biases and how to properly communicate risk preferences through use of utility functions.

The notion of uncertainty and the associated risk of designs is a topic that will need to be addressed when designing for LSCES. Research has been conducted in the use of utility functions to capture risk preferences in design[7] and how utility theory can be incorporated in the preference models of various design methods such as Reliability Based Design, Robust Design and Risk-Based Design[8]. This paper differentiates from the past research by examining the incorporation of utility functions, through decision analysis, into multidisciplinary design optimization and value-driven design frameworks. A method involving MDO, VDD and decision analysis allows for both the value and the risk preferences of the stakeholder to be properly communicated to throughout an organization hierarchy, eliminating the effects of risk biases in the design process.
4. Current System Design Tools

LSCESs can be decomposed based on disciplines, components, organizations, and many other methods. When decomposed the components of the decomposition are rarely independent from the other components. Couplings connect the components together through both physical means, such as a shaft, and non-physical means, such as information distribution. When couplings between components exist then a framework is needed to allow for proper optimization of the system as a whole. In the 1980s, a concept known as multidisciplinary design optimization (MDO) was created to address the challenge of designing a system consisting of multiple disciplines. MDO provides frameworks for designers to determine the optimal system design knowing that the system is composed of multiple coupled disciplines. Some frameworks that have been produced are Concurrent Subspace Optimization (CSSO) [9-11], Collaborative Optimization (CO) [12], Individual Discipline Feasible (IDF) approaches [13], and All-At-Once (AAO) approaches [14].

The various MDO frameworks differ in the way they handle the system decomposition and couplings; however, all of the frameworks have a system level objective function that is being used to determine the optimum. Traditionally MDO researchers have assumed that the objective function is an exogenous input, the work of another group, commonly Systems Engineering. The current Systems Engineering waterfall process[15] does not provide for the formation of an objective function. The waterfall process flows down the preferences of the stakeholders through the use of requirements. Requirements do not inform decision makers of what the stakeholders want, only what they do not want. The requirements themselves are generally arbitrarily determined, leading to ambiguous stakeholder preferences for the decision makers to act upon.

Value-driven design (VDD)[6] was developed in the early 2000s to provide an alternative Systems Engineering approach that addresses the concerns of requirement-driven design. VDD provides a framework for establishing an objective function which focuses on the true value preference of the stakeholder[16]. In terms of engineering design, value preferences are the desires of an individual with regard to a measurement, such as preferring the most profit. Typically, objective functions would focus on performance attributes of the system to minimize or maximize, such as weight or lift. VDD instead focuses on the worth of the system. For example, objective functions formed using VDD for commercial systems are commonly in terms of profit. VDD also provides methods to decompose the stakeholder’s value functions to be used by each of the sub-systems of a system[17, 18]. In certain environments, VDD value functions result in each design having a singular value, easily comparable with other designs. VDD does not provide a framework for determining the design which would result in the best value.

The two LSCES tools of MDO and VDD complement each other. MDO provides a framework for determining the best design, but lacks the ability to form a meaningful objective function. VDD provides the framework for determining a value function that captures the stakeholder’s value preference, but lacks the ability to determine the best design. Recent research has begun examining these two related tools in frameworks leveraging the benefits of each[19]. The use of MDO and VDD together provides a useful framework for determining the best design according to a stakeholder’s preferences in situations void of uncertainty.

5. Comparison of Uncertain Designs

In the design of LSCES uncertainty is present in many aspects, including but not limited to physical measurements, model assumptions, manufacturing tolerances and communication within organizations. Figure 1 depicts the values of two certain design alternatives. In a certain design the choice is fairly simple; choose the design alternative associated with the highest value, in this case design alternative 2 (assuming more value is preferred). Figure 2 depicts the probability distributions of the values of two uncertain design alternatives. A design choice is not as straightforward in an uncertain design, requiring the decision maker to express their risk preference to enable the best choice to be made. A person’s risk preference expresses their desires on uncertainties. To determine the alternative that a person should choose only the value preferences of the person is needed for certain alternatives, for uncertain alternatives both the person’s value preference and risk preference are needed.

Decision analysis is a set of mathematical methods to determine the alternative a person or group of people should selected given a set of alternatives. Decision analysis’s core is found in utility theory[20, 21]. Utility theory is a mathematical method to collapse a probability distribution of outcome utilities to a single expected utility that is consistent with the risk preferences of a specific individual. The expected utilities of the alternatives can then be compared directly to determine the most preferred design. Utility theory describes the risk preferences of an individual through a utility function, which, for VDD contexts, is a manipulation of the value function. A utility function describes the degree of risk aversion of an individual. For example, individuals with identical value functions may choose different alternatives in Figure 2. These differences are found in their risk preferences, and hence these individuals with identical value functions would have different utility functions.
A person may be placed in three risk categories for a specific decision: risk averse, risk neutral and risk proverse. A risk averse individual may prefer a certain alternative’s outcome over an uncertain alternative’s higher expected outcome. A risk neutral individual is indifferent between an uncertain alternative’s expected outcome and a certain alternative with the same outcome. A risk proverse individual would never choose a certain alternative whose outcome is less than an uncertain alternative’s expected outcome. A more continuous description of an individual’s risk preference is through their degree of risk aversion, the greater an individual’s degree of risk aversion, the more risk averse they are.

In utility theory the worth of an alternative to an individual is determined by calculating the alternative’s expected utility[21]. The expected utility of one alternative is then compared to the expected utility of another alternative to determine the preferred alternative under uncertainty. In Figure 2 an individual would choose alternative 1 if alternative 1’s expected utility were higher than alternative 2, and vice versa. Decision analysis is further expanded by examining how interacting individuals make decisions using game theory[21-23] and Bayesian game theory[24]. In this paper the decision making process that will be explored will focus on single individuals, with the acknowledgment that the design process for LSCES is inherently a group endeavor and will be examined as such in future work. Research has been conducted in this area[25, 26] but not in the examination of incorporating a utility function in a MDO/VDD framework and the impact of risk biases on decision making.

A MDO-VDD design framework can incorporate the principles of decision analysis by first capturing the risk preferences of the stakeholder through a utility function. This utility function would make use of the previously created value function, as the value function captures the value preferences the utility function would capture both the value and risk preferences. The expected utility can then be used as the objective function, where the expected utility is maximized to produce the most preferred design alternative in terms of the stakeholder’s preferences. The utility function can be decomposed and distributed to the decision makers to enable decisions consistent with not only the stakeholder’s value preference but also their risk preference.

The formation of the utility functions used to capture the decision makers’ value and risk preference and decompose the function will not be examined in this paper. This paper will assume that techniques are used to...
determine the functions appropriately, and are exogenous variables to the method. This paper will focus on the impact of various degrees of risk aversion on design, represented through simple utility functions rather than deriving complex but more accurate utility functions. Hyperbolic absolute risk aversion (HARA) utility functions will be used in this paper due to their simplicity. The standard formulation of a HARA function\[27\] is seen in Eq.(1). The measurement variable that is found in all utility functions will be represented by the value function determined from VDD. Hence if the value function results in a net profit then the utility function will be a function of the net profit. In a HARA function a risk averse individual has a positive coefficient and a risk proverse individual has a negative coefficient. The greater the magnitude of the coefficient the more risk averse or risk proverse the individual is. A risk neutral individual will be represented by a utility function equivalent to the VDD derived value function. By capturing the VDD value as the measurement in the utility function both the decision maker’s risk and value preferences are captured.

\[
u(m) = -\frac{1}{a} e^{-am}
\]

where:

\[a: \text{coefficient of absolute risk aversion}
\]

\[m: \text{measurement/value}
\]

6. Communication of preferences

The communication of stakeholder preferences is a significant attribute of any design method, and is a key research area of LSCES\[28\]. The design methods of requirement-driven design, MDO/VDD and MDO/VDD incorporating DA have various effectiveness in communicating the preferences of the stakeholders to the designers (decision makers) to enable consistent decision making. Requirement-driven design captures the preferences of the stakeholders through requirements, stating what the stakeholder does not want through generally arbitrary design space constraints. The MDO/VDD method captures the stakeholder’s value preference through the distributed value functions, but does not capture the risk preferences of the stakeholders. Distribution of only value functions results in designs which are impacted by the risk biases of the designers, whom will likely use a different risk preference than that of the stakeholder. The MDO/VDD/DA method will create a utility function based off of the value function, resulting in the stakeholder’s value and risk preferences being distributed to the designers. Assuming the designers make decisions based solely off of the utility function then the decisions will be consistent with that of the stakeholder’s.

Preferences often are communicated in organizations by means other than requirements, value functions or utility functions. Communication by stakeholders of their desires, and how stakeholders can influence people to perform as desired, is vastly complex, with its own research community (i.e. IEEE Professional Communication Society, Journal of Organizational Behavior) and numerous papers\[29, 30\] and books\[31\]. Such tactics as statements from stakeholders to appease certain groups may misrepresent the stakeholder’s true preferences to the designers. For example, a company may make a public statement that they are constantly working to produce ever more environmental friendlier products. This statement is interpreted by designers as the stakeholder’s preference, where in actuality the preference is likely a combination of system attributes, one bring environmental impact, which results in a profit value function. It is also possible for preferences of stakeholders to be distorted as it is passed along the organization hierarchy. This loss of information due to communication between individuals or groups is seen in the child game of telephone, where the statement passed along a group becomes increasingly misinterpreted, resulting in beginning and end statements that are rarely the same. The use of utility functions in a system design MDO/VDD framework will enable a simple function to be passed along the organization hierarchy with minimal information loss.

7. Impact of Risk Biases Examples

In order to investigate the impact of risk biases in a MDO/VDD method and the benefits of a MDO/VDD/DA method, two example problems will be explored. These examples will consist of a stakeholder and a single designer trying to determine the optimal design in the eyes of the stakeholder. These examples assume that the value function and utility functions have been previously decomposed if the optimization is within a multiple system/designer context.

\[V_L = -(X^2 - 10)^2 + 1000 - 10 \times X\]

\[V_M = -(X^2 - 10)^2 + 1000\]

\[V_U = -(X^2 - 10)^2 + 1000 + 10 \times X\]

where:

\[V_L: \text{value lower bound}\]

\[V_M: \text{value most likely value}\]

\[V_U: \text{value upper bound}\]

\[X: \text{design variable (Measurement)}\]
7.1. Impact of Risk in Continuous Value Function

The first example to demonstrate the impact of risk biases in design involves a stakeholder asking a designer to determine an optimal continuous design variable (X) that will maximize the value function (Y). This is a common request which can be easily determined when the value function is certain. When the value function is uncertain then risk preferences of the designer will impact the determination of the optimal value, since the stakeholder did not communicate a risk preference. For this continuous problem the value function will be represented by a triangular distribution[32] for each design variable. Triangular distributions are commonly used to represent the more complex normal distributions due to their ease of mathematical formation. The probability distribution of the value function for each design variable is characterized by a lower bound, an upper bound and a most likely value seen in Eq.(2). The value function is depicted graphically in Figure 3. The design variable X has a lower bound of 0 and an upper bound of 30.

![Figure 3: Continuous Value Function Example](image)

In this scenario, the only information distributed to the designer is that the stakeholder prefers more measurement value. In a certain environment, where the most likely value would occur 100% of the time when choosing its respective design variable, the optimal measurement would be X=10 resulting in a value of 1000. With the inclusion of uncertainty in the value and with only a value preference to work with, the true optimum cannot be determined. For example, while a measurement of X=15 results in a lower most likely value than a measurement of X=10, it also has a possibility of resulting in a much higher value than measurement X=10 ever could. Measurement X=5 has a lower most likely value than measurement X=10; however, it also has a high lower bound, guaranteeing that the lowest possible value is higher than the lowest possible value from measurement X=10. A risk averse individual may choose measurement X=5 due to its higher certainty and a risk proverse person may choose measurement X=15 due to its higher possible upper value.

In order for the designer to determine an optimal design in an uncertain environment he must employ a risk preference, and since he has not been distributed one from the stakeholder he must create one to use. This risk preference may be his own, a speculation on the stakeholder’s risk preference or a combination of his and a speculation. The risk preference that the designer chooses to implement has a low probability of matching the stakeholder’s true risk preference due to the continuous nature of the degree of risk aversion. Suppose that the stakeholder’s true risk preference is captured in a HARA function with a coefficient of absolute risk aversion of \( a=-0.01 \), hence he is risk proverse. The stakeholder’s optimal measurement, associated with the highest expected utility or worth according to the stakeholder’s preferences, is X=10.89. If the designer is risk averse and has no information on the stakeholder’s risk preferences then he may assume a risk averse coefficient of risk aversion, say \( a=0.1 \), resulting in an optimal measurement of X=6.521, seen graphically in Figure 4. Perhaps the designer has heard a statement by the stakeholder that they are willing to take risks in their design, driving the designer to make a decision using a coefficient of risk aversion of \( a=-0.1 \), resulting in an optimal design of measurement X=14.296, seen in Figure 5. This is significantly different from the preferred design of the stakeholder. With such a generic statement containing only a broad statement on the degree of risk aversion the development of a utility function to properly characterize the stakeholder’s risk preference is highly unlikely.
This example has shown how a design, characterized by a continuous design variable, can be biased by designer chosen risk preferences. Without proper dissemination of the stakeholder’s risk preference to the designers, along with their value preference, the designer is left to hypothesize on the stakeholder’s degree of risk aversion. This will result in designs that are highly likely to not be the true preferred design of the stakeholder.

7.2. Impact of Risk in Coupled System Design

A simple system composed of three coupled sub-systems will next examined to demonstrate how improper dissemination of risk preferences can impact the final design of a system. Each of the three sub-systems will have five possible outcomes. Each outcome will be uncertain, characterized by a triangular distribution value output. The parameters of the triangular distribution will be partially dependent on the outcomes of the two other sub-systems, hence the sub-systems are coupled. For example, sub-system A’s (SSA) five possible outcomes are seen in Figure 6, given that sub-system B’s (SSB) chosen outcome is alternative 4 and sub-system C’s (SSC) chosen outcome is alternative 4. If SSB’s or SSC’s alternatives were different then SSA’s probability distributions of the alternatives would be different. The lower bound, upper bound, and most-likely value of each outcome of each sub-system are defined in Appendix A, as well as the outcome couplings from the other sub-systems.

In this system a stakeholder is tasking an MDO designer with determining the optimal system, which consists of one alternative from each sub-system. The stakeholder has distributed a system value function to the

![Figure 4: Continuous Value Function Example a=0.1](image.png)

![Figure 5: Continuous Value Function Example a=-0.1](image.png)
designer which is seen in Eq.(3). The value function is a simple summation equation of the value of each sub-
system. In an industry setting this value will likely be in monetary units. For this example it is assumed that the
measurement values are in millions of dollars of profit. The stakeholder fails to properly communicate to the
designer a mathematically based risk preference.

\[ V = V_A + V_B + V_C \] (3)

Due to the non-representation of a usable risk preference to the designer from the stakeholder, and the
uncertainty present in the system, the designer is forced to create a risk preference in order to determine an
optimal system design. To illustrate the impact of risk preferences on design, three risk preferences
characterized by three HARA function’s coefficient of absolute risk aversions will be used. The three risk
preferences are: risk averse \((a=2)\), relatively risk neutral \((a=0.00000001)\) and risk proverse \((a=-2)\). These three
risk preferences are possible assumptions the designer is forced to make in order to proceed with the
optimization of the uncertain system. An extensive search is conducted examining all possible configurations of
the sub-system alternatives to determine the optimal expected utility for each of the risk preferences. Due to the
uncertainty of the system the objective function value being maximized is the system’s expected utility,
determined from Equation 4, for each of the system configurations. Note that in order to mathematically handle
the uncertainty the continuous probability distributions representing each of the sub-system outcomes are
segmented into 100 equally spaced intervals. In Eq.(4) the summations are stepping through each of those
segments, each segment having an associated probability and value.

\[ E_u = \sum_{p_a=1}^{100} \left( \sum_{p_b=1}^{100} \left( \sum_{p_c=1}^{100} p_{a,i} p_{b,j} p_{c,k} \cdot U[V_{a,i} + V_{b,j} + V_{c,k}] \right) \right) \] (4)

where:
\[ E_u : \text{expected utility} \]
\[ p = \text{probability of SSA, B or C alternative i, j or k, for each segment} \]
\[ V = \text{value of SSA, B or C alternative i, j or k, for each segment} \]

The optimal system configuration for each of the three risk preferences that the designer in this scenario
may use is seen in Table 1. As can be seen in the table each of these three risk preferences result in a different
system configuration, displaying the impact that risk assumptions can have on MDO. Let’s assume that the
stakeholder’s actual risk preference is associated with a HARA coefficient of absolute risk aversion of \((a=1)\).
This risk preference, if distributed to the designer, would result in an optimal system configuration of SSA
alternative 2, SSA alternative 1 and SSA alternative 4. The associated expected utility of the stakeholder’s
optimal design is -0.0572. The least preferred design associated with this risk preference is SSA alternative 5,
SSB alternative 4 and SSC alternative 1, which results in an expected utility of -0.1209. The expected utilities
and rank out of 125 possible configurations for the designer’s three risk coefficient designs, according to the stakeholder’s true risk preference, are seen in Table 2. This table shows that the designer would have been closest to the stakeholder’s true risk preference if he had unknowingly assumed the risk preference of ($a=2$). This random assumption would only get the designer in the top 8 of the stakeholder’s actual preferred design rank ordering. If the designer had assumed the risk preference of the stakeholder would have been selected, illuminating the need for stakeholders to properly disseminate their risk preferences through a mathematical formulation.

<table>
<thead>
<tr>
<th>Coefficient of Absolute Risk Aversion</th>
<th>Sub-System A Alternative</th>
<th>Sub-System B Alternative</th>
<th>Sub-System C Alternative</th>
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</thead>
<tbody>
<tr>
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<td>1</td>
<td>3</td>
<td>4</td>
</tr>
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<td>1</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>2</td>
<td>3</td>
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</table>

Table 2: Design Characteristics with $a=.1$

<table>
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<tr>
<th>Design [SSA SSB SSC]</th>
<th>Expected Utility</th>
<th>Rank</th>
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<tbody>
<tr>
<td>[2 1 4]</td>
<td>-.0572</td>
<td>1</td>
</tr>
<tr>
<td>[1 3 4]</td>
<td>-.0590</td>
<td>8</td>
</tr>
<tr>
<td>[2 1 3]</td>
<td>-.0603</td>
<td>9</td>
</tr>
<tr>
<td>[5 2 3]</td>
<td>-.0782</td>
<td>68</td>
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</table>

This example has displayed the importance of proper risk preferences dissemination through utility functions in MDO problems where one designer is optimizing a set of sub-systems simultaneously. In many MDO problems sub-systems consists of a set of sub-systems which may consist of further sub-systems with designers at the different system tiers. MDO/VDD method promotes decomposition of value functions to enable these sub-system designers to make decisions independently, within larger system loops to guarantee system consistency. When uncertainty is present in the system then the utility function would replace the value function as the function to be decomposed and distributed to sub-systems, enabling the sub-systems to make decisions which would be consistent with the stakeholder’s risk and value preferences.

8. Conclusion

The MDO/VDD method improves upon the traditional MDO objective function by creating a value function converting system attributes into a meaningful measurement, such as profit. In uncertain environments both traditional objective functions and value functions can be impacted by risk biases of the decision makers, resulting in designs that may not be most preferred by the stakeholder. The examples in this paper have shown situations where decision makers, forced to make assumptions on the stakeholder’s risk preference due to improper communication, have designed products which were not preferred by the stakeholder. In any design process that may involve uncertainty and where the stakeholder is not the designer, a mathematical representation of the stakeholder’s risk preference is necessary. Utility theory, using the VDD value function as the measurement value, provides a meaningful, mathematical sound representation of risk preferences for easy dissemination to decision makers in the system hierarchy using the MDO/VDD framework.

9. Appendix A

Table A.1: Sub-system A Alternative Triangular Distribution Characteristics

<table>
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<tr>
<th>Alt</th>
<th>Lower Bound Value</th>
<th>Most Likely Value</th>
<th>Upper Bound Value</th>
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### Table A.2: Sub-system B Alternative Triangular Distribution Characteristics

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<th>Alt</th>
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### Table A.3: Sub-system C Alternative Triangular Distribution Characteristics

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<th>Alt</th>
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### Table A.4: Modifiers passed to Sub-system B and C from sub-system A

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### Table A.5: Modifiers passed to Sub-system A and C from sub-system B

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### Table A.6: Modifiers passed to Sub-system A and B from sub-system C

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### 10. References


