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Multi-Objective Structural Optimization of Wind Turbine Tower and Foundation Systems using Isight: A Process Automation and Design Exploration Software

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1. Abstract

Currently, wind turbine towers are being built to increasing heights in order to tap into higher and more consistent winds available at these heights. Additionally, increasingly large wind turbines are being deployed atop these towers in order to capture more energy. This trend of larger turbines being deployed at greater heights makes obtaining the most efficient and safe, or optimal, designs of the structures that support them ever more important. Towards this goal, the present work formulates the design of an integral wind turbine tower and foundation system as a multi-objective optimization problem using the process automation and design exploration software Isight. Specifically, a new general metamodeling and optimization methodology for highly non-linear multi-objective problems is applied to the design of a 130-m hybrid pre-cast concrete and tubular steel tower supported by a gravity based foundation by: 1) applying Design of Experiments (DOE) techniques to identify response drivers and collect experimental data 2) developing metamodels of responses from this experimental data using Elliptical Basis Functions (EBF) 3) performing Multi-Objective Genetic Algorithm (MOGA) optimization to generate a Pareto Front of Pareto-Optimal designs 4) performing Multi-Objective Gradient Based (MOGB) optimization from select Pareto-Optimal designs 5) validating the resulting optimal designs by full finite element simulations 6) performing Reliability Based Design Optimization (RBDO) to ensure reliability of the final design. To analyze the support structure and obtain response information, a geometrically non-linear transient finite element analysis is performed at each DOE sample point using Abaqus. This problem is multi-objective in nature with an ideal design being one that both minimizes costs and maximizes structural stiffness to reduce vibrational wear on turbine components. Therefore, a composite objective function is developed that minimizes raw material costs and deflections. The physical dimensions of the wind turbine support structure are taken as design variables. Design requirements such as yielding or fracture of the tower wall, limits on tower top deflection and rotation, limits on the natural frequency of the tower and foundation system, bearing capacity of the foundation, stiffness of the foundation, and foundation overturning moment are evaluated using Isight's calculator component and converted to optimization constraints by Isight. Results obtained show that the proposed methodology yields significant value to the design engineer in terms of improved designs and enhanced understanding of the problem. Additionally, this work shows how Isight can be used to overcome many of the practical barriers associated with applying advanced optimization methods at the detailed design level.

2. Keywords: Multi-Objective Optimization, Metamodeling, Reliability Based Design Optimization (RBDO), Wind Turbine Tower and Foundation System, Isight.

3. Introduction

A recent study estimates world wind power potential to be 40 times greater than total current power consumption [1]. This large increase over previous studies, which found this multiple to be closer to 7, is primarily due to the deployment of larger turbines that rise to greater heights where winds are higher and more consistent. As larger turbines are deployed at greater heights, obtaining the most efficient and safe, or optimal, design of the tower and foundation systems that support them is critical to their successful proliferation.

The success of wind energy relies heavily on wind power's levelized energy cost being lower than that of other energy sources. While current total system levelized energy cost projections (in 2010 $/megawatthour) for 2017 predict that wind energy will be more cost effective than coal, nuclear, geothermal, biomass, and solar, much work still needs to be done to make wind competitive with natural gas and hydro: wind, 96.0; coal, 97.7; nuclear, 111.4; geothermal, 98.2; biomass, 115.4; solar, 152.7; natural gas, 63.1; hydro, 88.9 [2]. The key factors in calculating the levelized energy cost for wind energy, which can be directly affected by better engineering design, include: initial capital investment cost, unscheduled maintenance costs, levelized replacement cost. The first factor can be reduced by decreasing the cost of the installed system. The last two factors can be reduced by improving system reliability and extending the lifespan of the system, respectively, which can both be improved by minimizing tower vibrations. Therefore, we define an optimal wind turbine tower and foundation system as one that costs as little as possible to build and is as stiff as possible in order to reduce vibrational wear on the turbine components.

Much work has been done over the past 20 years to optimize various pole and tower structures. Kocer and Arora 1996a [3] formulated the design of a dodecagonal steel transmission pole as a mixed continuous-discrete variable optimization problem and showed that the optimal design process can yield more efficient and safe designs than the conventional design process. This work also demonstrated how the optimal design process allows for various design options to be trialed relatively quickly through the example of substituting a circular cross section for the dodecagonal one and re-solving, which resulted in a 2.4% cost savings in material. In this work, various structural mechanics equations and numerical methods were implemented to complete the analysis of the structure. Kocer and Arora 1997 [4] extended the work of Kocer and Arora 1996a. It was especially interesting in that it used discrete optimization to select the optimal design from a set of prefabricated pole sections in a catalog. This work also incorporated cross-sectional shape and steel grade as additional design variables, which expanded the feasible design space and allowed for further reductions in cost. Kocer and Arora 1996b [5] formulated the design of a prestressed concrete transmission pole as a mixed discrete-integer-continuous variable optimization problem. This work discussed many of the practical aspects of converting precast concrete design requirements into optimization constraints and showed that the inclusion of second order moments due to deflections (i.e. nonlinear geometric effects) in the analysis leads to substantially different final designs. Here, an iterative numerical method was implemented to calculate second order moments. Negm and Maalawi 1999 [6] formulated the design of a wind turbine tower, which was made up of multiple uniform segments, as a continuous variable optimization problem and tested various optimization objectives. Additionally, a variety of design criteria were converted into optimization constraints. Using a simplified model of the tower allowed for an analytical analysis of the structure, which made use of Euler-Bernoulli beam theory, to be performed. Silva, Arora, and Brasil 2008 [7] formulated the design of a reinforced concrete wind turbine tower as a continuous variable optimization problem that minimized material cost and incorporated a numerical procedure for performing a non-linear dynamic analysis. This work highlighted the fact that the foundation is not infinitely rigid and suggested incorporating the foundation into tower dynamic analyses. Nicholson 2011 [8], which the present work is an extension of, formulated the optimal design of a steel wind turbine tower and shallow concrete foundation as an integral system where the foundation is considered in tower natural frequency calculations. A combination of structural mechanics equations, numerical methods, and structural design calculations were implemented in Excel to analyze the structure and an equivalent lumped mass method was employed to estimate the system natural frequency.

While these works are quite commendable, many areas for further innovation still exist: the incorporation of a full transient finite element analysis into the optimization process, optimization of new space frame and hybrid concrete-steel towers that rise to extreme heights, application of discrete multi-objective and other advanced optimization techniques, etc. The challenge that arises as more complex analyses and designs are incorporated into the optimization process is that the problem becomes highly non-linear and often discontinuous. This limits the choice of optimization strategies to discrete methods that can be prohibitively computationally expensive if the analysis is complex. Furthermore, advanced optimization methods, such as RBDO, are almost always too cumbersome when expensive analyses are used. The present work overcomes these challenges and addresses some of the areas for further innovation listed above by suggesting and applying a general metamodeling and optimization methodology for highly non-linear multi-objective problems. Section 4 summarizes the wind turbine tower and foundation system optimization problem and the analysis methodology. Section 5 introduces the general metamodeling and optimization methodology. Section 6 shows the results of applying this methodology to the problem at hand and section 7 summarizes the conclusions of this work and areas for further study.

4. Wind Turbine Tower and Foundation System Design Optimization Problem

The conversion of an engineering design problem into a design optimization problem involves three primary steps: choosing an analysis methodology, identification of objective(s) and design variables to be optimized, and transformation of design requirements into optimization constraints [9]. In this section, we describe how this general methodology can be used to convert the design of a wind turbine tower and foundation system into a design optimization problem. For implementation details beyond those given in this section, the reader is directed to the work of Nicholson [8] of which the present work is an extension.

4.1. Analysis Methodology

The wind turbine tower and foundation system optimization problem presented here is based on a full-scale transient Finite Element Analysis (FEA) of a hybrid concrete-steel 130-m tower and a spreadsheet analysis of a reinforced concrete shallow foundation. The overall Isight [11] analysis process flow is outlined in Figure 1, which shows the steps required to analyze the structure and calculate the outputs to be used in optimization. As can be seen, the foundation design is completed as a sub-optimization problem, at each tower analysis task iteration, since we're analyzing a structural system where the tower analysis depends on the foundation design.

The foundation spreadsheet analysis, which is run at each foundation design optimization iteration, calculates the ultimate soil bearing capacity, maximum allowable pressure on soil, soil rotational stiffness, and the overturning and resisting moments for the assumed soil parameters and the current foundation geometry.
The tower transient FE analysis, which accounts for geometric nonlinearity, is analyzed using Abaqus Standard 6.12-2 [12] after the foundation design optimization loop completes. It consists of 130 quadratic beam (B32) elements as well as 130 material, section, and pipe profile definitions (one of each for each element) that are dynamically updated from an Excel spreadsheet by Isight at each analysis iteration. Additionally, 130 horizontal distributed line loads resulting from wind action on the tower are calculated in Excel and dynamically updated by Isight for the current geometry. Vertical distributed loads due to tower self-weight and internal fixture weight are also considered. All degrees of freedom at the tower base are assumed fixed except those for the rotational and horizontal-translation degrees of freedom orthogonal to the plane of loading, which are dynamically set by Isight to the calculated foundation rotation and translation. The lumped masses of the foundation and turbine are also automatically input into the FEA by Isight.

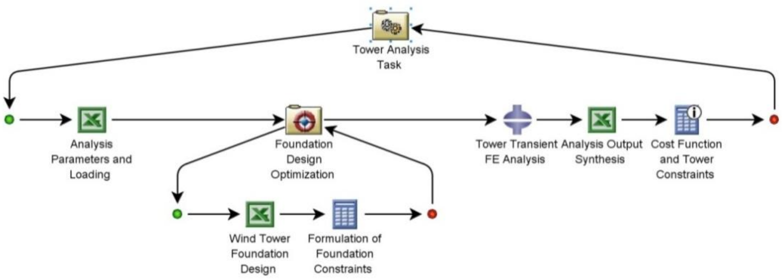

Figure 1: Isight tower analysis task process flow starting at the green dot and finishing at the red dot

4.2. Design Objectives and Variables

As we discussed earlier, an optimal wind turbine tower and foundation system design is best described as one that costs as little as possible to build and is as stiff as possible in order to reduce vibrational wear on the turbine components. Therefore, this problem is multi-objective in nature with an ideal design being one that both minimizes cost and maximizes stiffness. In order to formulate the cost objective, $f_1(\boldsymbol{x})$, the volumes of tower steel, tower concrete, and foundation concrete are multiplied by their respective cost per volume values, which were obtained by averaging values from multiple completed projects [13], and summed. In formulating the stiffness objective, tower top deflection is used as a rough indicator of stiffness since stiffer structures will deflect less. Therefore, in order to maximize stiffness, the second objective, $f_2(\boldsymbol{x})$, is to minimize tower top deflection. The physical dimensions of the tower and foundation shown in Figure 2 and listed in Table 1 make up the design variable vector, $\boldsymbol{x}$.

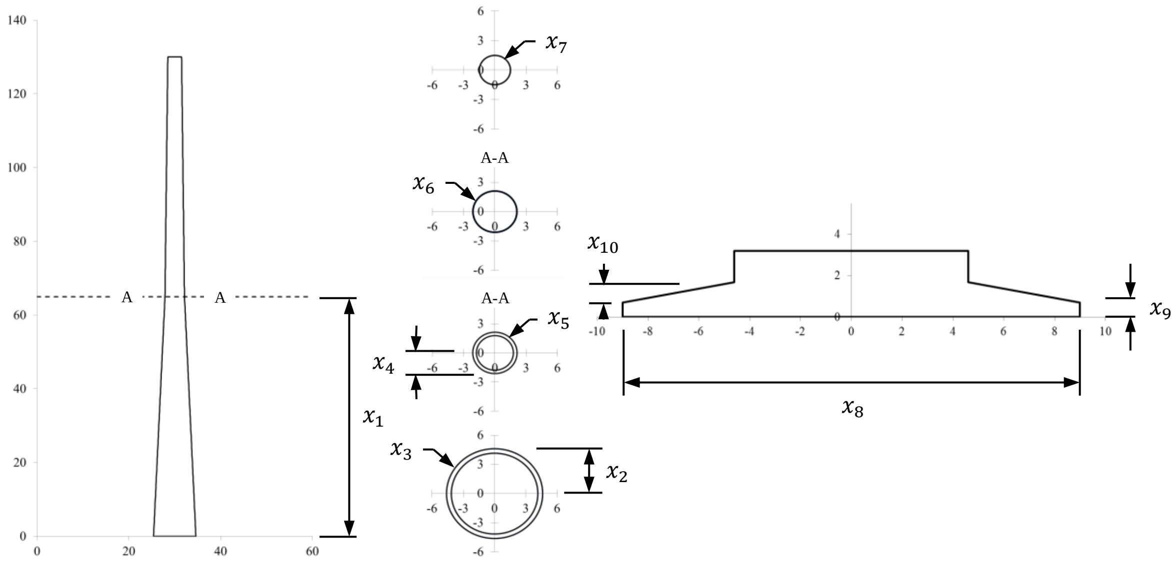

Figure 2: Tower and foundation cross-sectional dimensions taken as design variables

4.3. Transformation of Design Requirements into Constraints

An optimal wind turbine tower and foundation system design must satisfy certain design requirements. These design requirements ensure the safety and functionality of a design and protect against: yielding or fracture of the tower wall, buckling of the tower wall, excessive tower top deflection and rotation, natural frequency of the tower and foundation system coinciding with that of the turbine, bearing capacity failure of the soil being exceeded, excessive foundation rotation, foundation overturning, and others. Transforming these design requirements into optimization constraints allows the optimization algorithm to automatically handle the laborious task of enforcing these design requirements as it minimizes the chosen objectives. The present work converts twelve such design requirements into constraints. Table 1 lists these constraints and whether they are greater-than-or-equal-to type or less-than-or-equal-to type by giving their lower or upper bound, respectively. Here, constraints with very large values have been normalized by dividing them by their limiting values to avoid ill-conditioning of the optimization problem.

Table 1: Design variable, objective, and constraint values at initial design

Variable	Description	Lower Bound	Initial Design	Upper Bound	Unit
	Tower Design Variables				
x_1	Concrete height	60.000	65.000	70.000	m
x_2	Concrete base outer radius	3.3500	4.6125	5.8750	m
x_3	Concrete thickness at base	0.4250	0.4625	0.5000	m
x_4	Concrete-steel interface outer radius	2.0000	2.1250	2.2500	m
x_5	Concrete thickness at interface	0.3000	0.3500	0.4000	m
x_6	Steel thickness at interface	0.0300	0.0350	0.0400	m
x_7	Steel thickness at top	0.0100	0.0150	0.0200	m
	Foundation Design Variables				
x_8	Concrete base diameter	10.000	13.029	30.000	m
x_9	Concrete outer edge thickness	0.7000	0.7000	1.5000	m
x_{10}	Concrete top taper height	1.0000	1.0000	4.0000	m
	Objectives				
$f_1(\boldsymbol{x})$	Tower and foundation cost	-	1.86e+6	-	$
$f_2(\boldsymbol{x})$	Tower top deflection	-	1.1815	-	m
	Tower Constraints				
$c_1(\boldsymbol{x})$	Min. allowable natural frequency	0.2400	**0.2317**	-	Hz
$c_2(\boldsymbol{x})$	Von Mises yield criterion at A-A	-	0.0270	1	
$c_3(\boldsymbol{x})$	Von Mises yield criterion top	-	3.90e-4	1	
$c_4(\boldsymbol{x})$	Max. principal stress failure criterion bottom	-	0.1734	1	
$c_5(\boldsymbol{x})$	Max. principal stress failure criterion at A-A	-	0.8118	1	
$c_6(\boldsymbol{x})$	Max. tower top rotation	-	0.0254	0.0873	rad
	Foundation Constraints				
$c_7(\boldsymbol{x})$	Reqd. rotational stiffness	-	0.1052	1	
$c_8(\boldsymbol{x})$	Reqd. horizontal stiffness	-	0.0390	1	
$c_9(\boldsymbol{x})$	Reqd. factor of safety bearing capacity failure	-	**1.0000**	1	
$c_{10}(\boldsymbol{x})$	Reqd. factor of safety overturning	-	0.7457	1	
$c_{11}(\boldsymbol{x})$	Reqd. factor of safety max. pressure on soil	-	0.9372	1	

5. General Metamodeling and Optimization Methodology

This section describes a metamodeling and optimization methodology suitable for a large class of highly non-linear multi-objective engineering design problems. The overall strategy is to develop accurate metamodels of the highly non-linear objectives and constraints and use them, in place of the original objectives and constraints, to efficiently apply advanced optimization methods that thoroughly search the design space.

5.1. Design of Experiments to Collect Experimental Data and Identify Response Drivers

Design of experiments not only provides us with a means to systematically select experimental or full-analyses points from the design space, which will be used to train the metamodels, but also allows us to better understand the relationship between inputs and responses. Understanding this relationship permits us to select appropriate response drivers for the various objective and constraint metamodels created in the next step.

5.2. Metamodel Generation from Experimental Data and Error Analysis
After implementing a design of experiments technique to collect experimental training data and identify the response drivers for each objective and constraint response, various metamodels for each objective and constraint should be created and the metamodel that provides the best fit for an individual objective or constraint selected. Metamodel types that typically give the best fit for highly non-linear problems with experimental data that may be unevenly spaced include: Elliptical Basis Functions (EBF), Radial Basis Functions (RBF), Quadratic Response Surface Methodology (RSM). The coefficient of determination or R-squared, which varies between 0 and 1 with higher values representing a better fit, is taken as the primary error analysis measure when comparing fitness of metamodels. A higher order metamodel than 2 should only be selected if both the maximum and average errors do not increase; since an increase would indicate that the higher order model may not fit interpolated points located between the experimental points well. Last, the residual plot of the selected metamodel should not contain a discernible pattern (e.g. curved, upward/downward trend, increasing/decreasing trend, etc.) as this is an indication that there is a problem with using the selected model to approximate the experimental data [10].

5.3. Multi-Objective Genetic Algorithm (MOGA) Optimization to Generate Pareto Front
Performing MOGA optimization using the Neighborhood Cultivated Genetic Algorithm (NCGA) technique [11] ensures that the entire design space is searched and reduces the likelihood of converging to a local minimum. Additionally, it provides the engineer with a set of Pareto-Optimal designs to choose from and optimize further using gradient based optimization techniques. The Pareto Front of Pareto-Optimal designs is generated by plotting each objective vs. another objective and shows the best that could be achieved without disadvantage to at least one objective [11]. Therefore, it is a useful tool in understanding the trade-off between objectives as well.

5.4. Multi-Objective Gradient Based (MOGB) Optimization from Pareto-Optimal Designs
Multi-Objective Genetic Algorithms will terminate after a predetermined number of evaluations based on the genetic algorithm population size and number of generations parameter values chosen. The Pareto-Optimal design set found at termination may be near optimal but often not sufficiently so. Therefore, gradient based optimization techniques such as Nonlinear Programming by Quadratic Lagrangian (NLPQL) [11] can be performed starting from the Pareto-Optimal designs identified by the engineer as best (using his or her intuition and judgment) in order to fine tune the designs to the desired level. It is important to note that the chosen Pareto-Optimal designs were obtained by considering each objective separately and hence no scaling factors were employed. In contrast, gradient based optimization will require us to combine the objectives into one scalar function. Therefore, to preserve the trade-off identified by the engineer as best, it is recommended that each objective be scaled by its current Pareto-Optimal value before performing gradient based optimization. This will help ensure that the chosen trade-off is maintained and one objective does not dominate the optimization as much as possible.

5.5. Validation of Optimal Design by Full Finite Element Simulation
Once a deterministic optimal design has been obtained using the process outlined above, a full analysis can be carried out and the error between the predicted and actual objectives and constraints at the optimal design calculated. If a large error exists between the predicted and actual values then take steps to refine the metamodels and re-optimize before moving on; since RBDO results will only be meaningful if accurate metamodels are used.

5.6. Reliability Based Design Optimization (RBDO)
Previously, we used MOGB optimization to obtain a deterministic optimum that assumes no variability in the design variables and exactly satisfies the binding constraints. Now we wish to consider variability in the design variables and ensure that all of the constraints are satisfied 99% of the time by performing RBDO.

6. Results
In this section, the general metamodeling and optimization methodology described above is applied to the problem of wind turbine tower and foundation system design. Implementation details for each step applied to the specific problem at hand are given and results of each step are discussed.

6.1. Design of Experiments Results
To select experimental or full-analyses points from the design space, the Optimal Latin Hypercube DOE technique in Isight was used with 50 sample points, initially. As we'll discuss in section 6.6, however, a 3-level full factorial technique was eventually required to achieve sufficiently accurate metamodels. Once design of experiments has been run, various plots can be created to assist the engineer in visualizing DOE results. Engineering data mining plots can be created that clearly illustrate interactions among design variables and trade-offs among design responses. Figure 3 clearly shows how, in general, any reduction in cost requires an increase in deflection and a decrease in natural frequency. For instance, the green line shows how choosing the lowest cost design increased

top deflection and lowered natural frequency. Conversely, the blue line shows how choosing the highest cost design decreased top deflection and increased natural frequency. Another type of plot useful in visualizing DOE results is the Pareto plot. Pareto plots illustrate how various design variables, or interactions between design variables, affect design responses. Figures 4, 5, and 6 are examples of Pareto plots and show concrete height to be one of the leading factors affecting cost, deflection, and natural frequency, respectively.

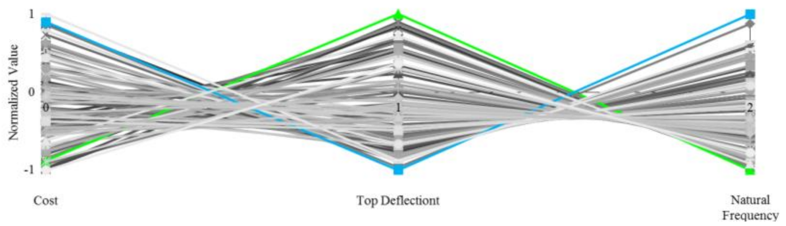

Figure 3: Engineering data mining plot showing trade-offs in cost, top deflection, and natural frequency.

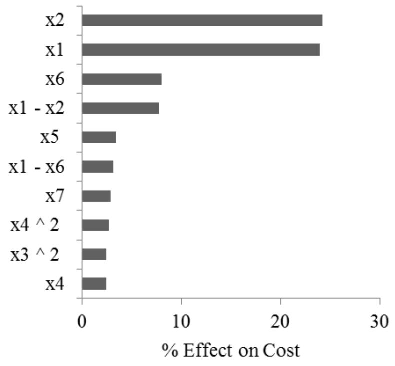

Figure 4: Pareto plot: design variable effect on cost.

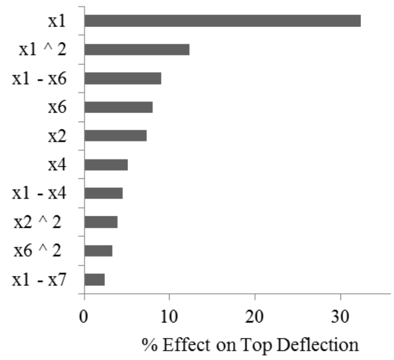

Figure 5: Pareto plot: design variable effect on top deflection.

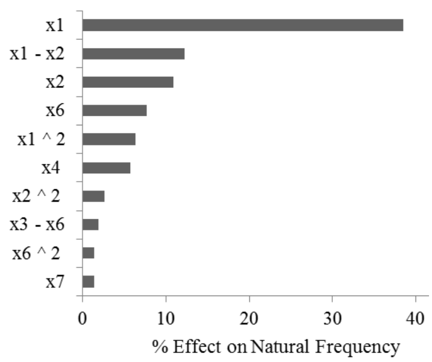

Figure 6: Pareto plot: design variable effect on natural frequency.

6.2. Metamodel Results

All of the metamodel types recommended in section 5.2 were trialed and RBF and EBF methods provided the best approximations of the objective and constraint responses for the DOE data. EBF was chosen over RBF for its property that it ranks input variables in the order of influence on output variables [11]. Quadratic RSM appeared to provide a good fit for the data as well; however, some plots of the residuals of the responses exhibited curved or sine wave behavior suggesting that quadratic RSM is not well suited for approximating those responses. Figures 7, 8, and 9 show that responses predicted by EBF (red dots) match actual response (black line) very well for cost, top deflection, and natural frequency, respectively. Figures 10, 11, and 12 show how increasing concrete base outer radius increases cost linearly at a high rate, but decreases top deflections and increases natural frequency nearly quadratically. Figures 13, 14, and 15 show how increasing concrete base outer radius increases cost linearly at a low rate and decreases top deflections and increases natural frequency linearly at a medium rate.

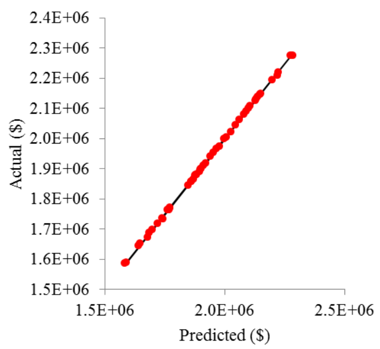

Figure 7: Predicted vs. actual cost response ($R^2 = 0.99$).

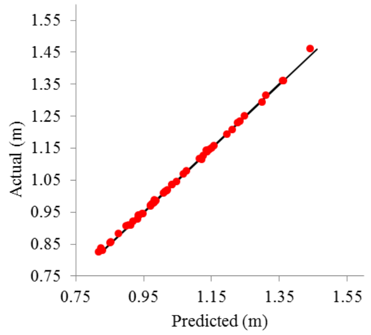

Figure 8: Predicted vs. actual top deflection response ($R^2 = 0.99$).

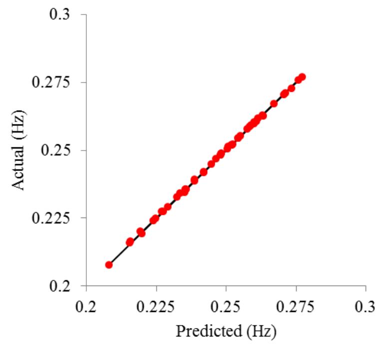

Figure 9: Predicted vs. actual natural frequency response ($R^2 = 0.99$).

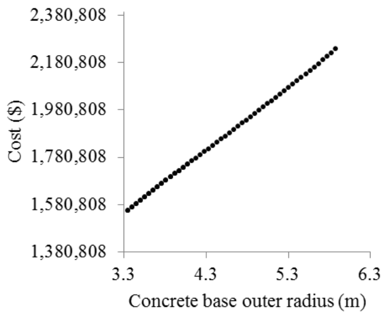

Figure 10: Concrete base outer radius effect on cost.

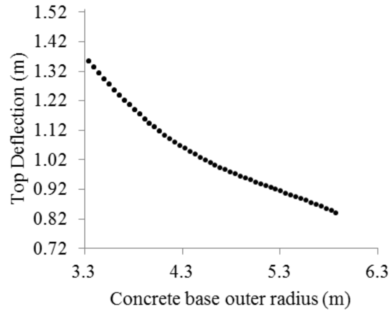

Figure 11: Concrete base outer radius effect on top deflection.

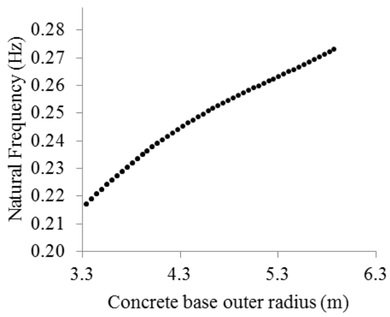

Figure 12: Concrete base outer radius effect on natural frequency.

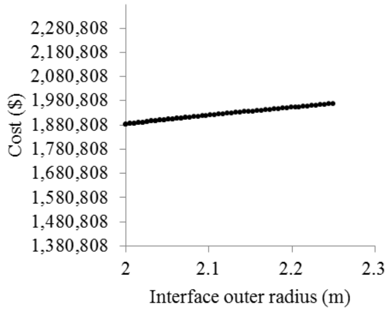

Figure 13: Concrete-steel interface outer radius effect on cost.

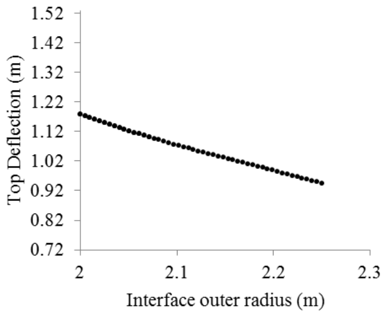

Figure 14: Concrete-steel interface outer radius effect on top deflection.

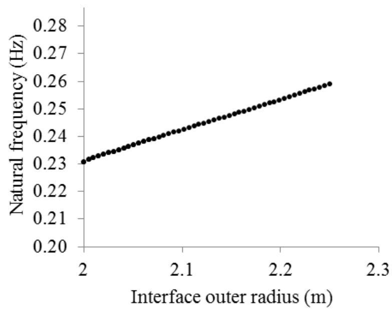

Figure 15: Concrete-steel interface outer radius effect on natural frequency.

6.3. MOGA Optimization Results

To identify a set of Pareto-Optimal designs, the NCGA optimization technique in Isight was used with genetic algorithm population size and number of generations parameter values set to 10 and 20, respectively, which resulted in 200 NCGA iterations. Figure 16 shows the Pareto Front of Pareto-Optimal designs (blue dots) generated by plotting the cost objective vs. the top deflection objective. From Figure 16, we can see that the cost-deflection trade-off is nearly linear with every cm reduction in deflection costing roughly $7,000. In our case, minimizing cost is more important than minimizing deflection so we select the Pareto-Optimal design represented by the green diamond in Figure 16 to optimize further using MOGB and RBDO techniques.

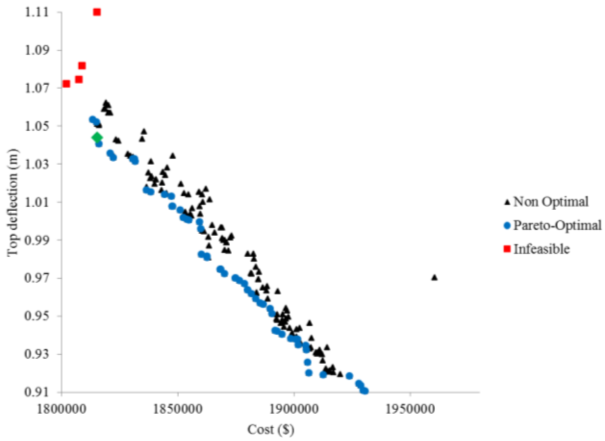

Figure 16: Pareto plot of cost vs. deflection from MOGA optimization.

6.4. MOGB Optimization Results

To further optimize the chosen Pareto-Optimal design, a gradient based NLPQL technique in Isight is used to minimize the composite objective function, (1). Scaling factors s_1 and s_2 are taken as the values of $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$, respectively, at the Pareto-Optimal design selected in the previous step. Weights w_1 and w_2 are taken as 0.8 and 0.2, respectively, in order to preserve the desired trade-off of minimizing cost over deflection. In Table 2, it can be seen that performing MOGB optimization resulted in an additional \$110,000 reduction in cost with only a slight increase in deflection. Figures 17 and 18 show how the composite objective function varies with some of the design variables. Figure 17 suggests that some combinations of concrete base outer radius and thickness may greatly reduce the chosen objective and some may greatly increase it and shows us where those occur. Figure 18 suggests that the chosen objective can be decreased by reducing steel interface thickness and reducing or possibly maximizing outer radius.

$$Min: \quad f = \frac{f_1(x)}{s_1} \times w_1 + \frac{f_2(x)}{s_2} \times w_2 \tag{1}$$

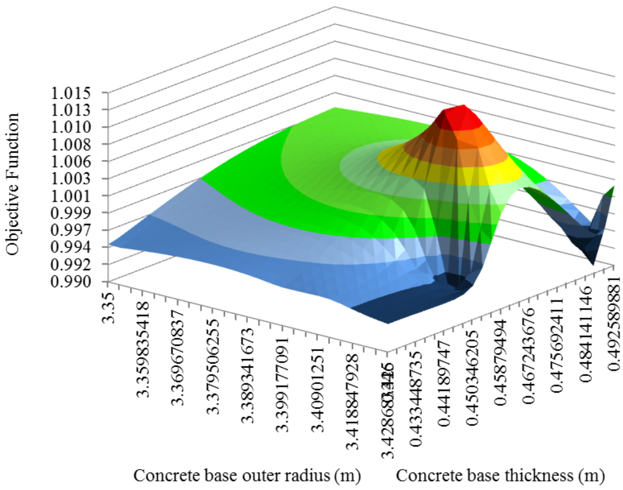

Figure 17: Concrete base outer radius and thickness effect on objective function

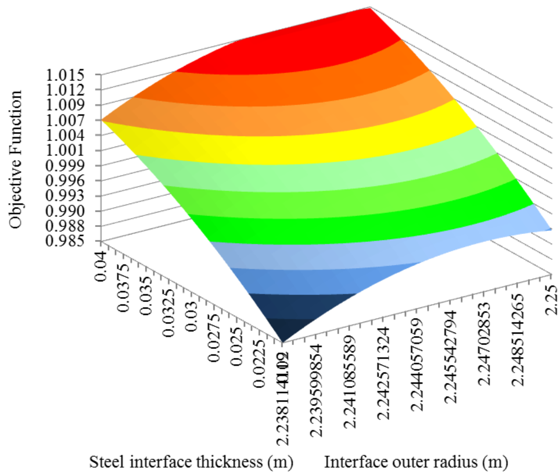

Figure 18: Concrete-steel interface outer radius and steel thickness effect on objective function

6.5. Validation Results

The second to last column in Table 2 shows the percent error between the objective and constraint metamodel values at the deterministic optimal design and the actual objective and constraint values at the deterministic optimal design, which were obtained by performing a full FEA. This percent error represents the error in the underlying objective and constraint metamodels, which have been used so far during optimization, near the solution. While the percent errors shown are relatively small, many DOE points had to be added to achieve this. In all, a 3-level full factorial requiring $3^7 = 2187$ design points for 7-variables was executed in parallel in Isight. Another strategy for refining metamodels, which doesn't involve adding additional DOE points, is to scale the DOE sample point variables [9]. This strategy may be helpful if additional analyses are prohibitively expensive or unavailable. Isight does this automatically for all metamodel types.

6.6. RBDO Results

To ensure that all of the constraints are satisfied 99% of the time, when some variability is present in concrete and steel wall thicknesses, the Six Sigma Optimization or RBDO component is invoked in Isight. A Monte Carlo analysis is used and a descriptive sampling technique with 1000 simulation points is implemented in order to ensure a broad spread of data across the distributions of each random variable while still following the shape of the individual distributions. In addition to the standard design constraints and objectives, we create lower bound constraints on the probability of success of each standard constraint and incorporate additional terms in the objective to maximize the probability of success of each standard constraint. Since the optimization problem now depends on randomness, we have a very discontinuous and expensive problem requiring an optimization technique that doesn't rely on gradients but is still efficient. Therefore, a Hooke-Jeeves technique is chosen and is modified by setting the relative-step-size and the step-size-reduction-factor to 0.8 to handle the highly discontinuous design space and reduce the likelihood of convergence to a local minimum.

Once RBDO has been run, we obtain a design that is more conservative, but satisfies the constraints 99% of the time. Table 2 shows how cost has increased, but natural frequency is no longer close to falling below its lower bound. Figure 19 further illustrates this by showing how, for the assumed probability distributions of the design variables, the probability of natural frequency falling below its lower bound of 0.24 is extremely small.

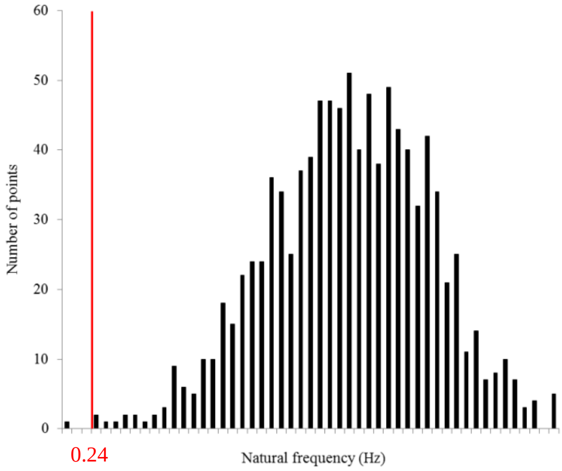

Figure 19: Natural frequency probability distribution histogram.

Table 2: Design variable, objective, and constraint values at MOGA, MOGB, and RBDO optimal designs

Var.	Final Lower Bound	Final Upper Bound	Initial Design	MOGA	MOGB	FEA at MOGB	% Error	RBDO
x_1	60.000	70.000	65.000	69.544	**70.000**	**70.000**	-	**70.000**
x_2	3.3500	5.8750	4.6125	4.1950	3.9300	3.9300	-	4.5512
x_3	0.4250	0.5000	0.4625	0.4954	**0.4250**	**0.4250**	-	0.4525
x_4	2.0000	2.2500	2.1250	2.2471	**2.2500**	**2.2500**	-	**2.2500**
x_5	0.3000	0.4000	0.3500	0.3090	**0.3000**	**0.3000**	-	**0.3000**
x_6	0.0300	0.0400	0.0350	0.0318	0.0363	0.0363	-	0.0358
x_7	0.0100	0.0200	0.0150	0.0190	**0.0100**	**0.0100**	-	**0.0100**
x_8	10.000	30.000	13.029	13.133	13.004	13.004	-	13.385
x_9	0.7000	1.5000	**0.7000**	**0.7000**	**0.7000**	**0.7000**	-	**0.7000**
x_{10}	1.0000	4.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	-	**1.0000**
$f_1(\boldsymbol{x})$	-	-	1.86e+6	1.81e+6	1.70e+6	1.70e+6	0.00%	1.85e+6
$f_2(\boldsymbol{x})$	-	-	1.1815	1.0420	1.1285	1.1088	1.75%	0.9858
$c_1(\boldsymbol{x})$	0.2400	-	**0.2317**	0.2433	**0.2400**	0.2404	0.17%	0.2548
$c_2(\boldsymbol{x})$	-	1	0.0270	0.0226	0.0221	0.0226	2.26%	0.0222
$c_3(\boldsymbol{x})$	-	1	3.90e-4	7.78e-4	0.0011	0.0009	18.18%	0.0005
$c_4(\boldsymbol{x})$	-	1	0.1734	0.2287	0.2731	0.2624	3.92%	0.1809
$c_5(\boldsymbol{x})$	-	1	0.8118	0.7386	0.7494	0.7492	0.03%	0.7437
$c_6(\boldsymbol{x})$	-	0.0873	0.0254	0.0213	0.0237	0.0237	0.00%	0.0225
$c_7(\boldsymbol{x})$	-	1	0.1052	0.1031	0.1057	0.1057	-	0.0982
$c_8(\boldsymbol{x})$	-	1	0.0390	0.0388	0.0391	0.0391	-	0.0381
$c_9(\boldsymbol{x})$	-	1	**1.0000**	**1.0000**	**1.0000**	**1.0000**	-	**1.0000**
$c_{10}(\boldsymbol{x})$	-	1	0.7457	0.8352	0.8978	0.8978	-	0.7598
$c_{11}(\boldsymbol{x})$	-	1	0.9372	0.9246	0.9135	0.9135	-	0.9343

7. Conclusions

In this paper, we proposed a general metamodeling and optimization methodology and applied this methodology to the problem of wind turbine tower and foundation system design. It was found that the proposed methodology yields significant value to the design engineer in terms of improved designs and enhanced understanding of the problem. Implementing DOE not only allowed us to systematically select design points to be used in generating the metamodels, but clearly showed how cost and natural frequency are inversely related to top deflection and how individual design variables affect the responses. Using the DOE sample points to generate accurate metamodels provided us with computationally inexpensive continuous representations of the objectives and constraints that allowed us to apply advanced optimization methods. These metamodels also gave us further insights into how the design variables influence responses. For instance, it was shown that both concrete base outer radius and concrete-steel interface outer radius could be increased to decrease top deflection and increase natural frequency. Performing MOGA optimization provided us with a set of Pareto-Optimal designs to choose from that clearly illustrated the trade-off between minimizing deflection and cost. Performing MOGB optimization on the chosen Pareto-Optimal design and using appropriate scale factors and weightings allowed us to further optimize the design while maintaining the chosen trade-off in objectives. We were also able to develop 3D contour plots that allowed us to better understand how individual design variables affect the composite objective function. Finally, performing RBDO allowed us to ensure that the constraints will be satisfied 99% of the time when small uncertainties in tower wall thickness are present. Performing a full FEA at the final MOGB and RBDO designs confirmed that the error in the objective and constraint metamodels near the optimized designs is small and the MOGB and RBDO designs are valid.

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9. References

[1] X. Lu, M.B. McElroy and J. Kiviluoma, Global potential for wind-generated electricity, *Proceedings of the National Academy of Sciences of the United States*, 106 (27), 10933-10938, 2009.

[2] U.S. Energy Information Admin., Levelized Cost of New Generation Resources in the Annual Energy Outlook 2012, *http://www.eia.gov/forecasts/aeo/electricity_generation.cfm*, Accessed March 20, 2013.

[3] F.Y. Kocer and J.S. Arora, Optimal Design of Steel Transmission Poles, *Journal of Structural Engineering*, 122 (11), 1347-1356, 1996a.

[4] F.Y. Kocer and J.S. Arora, Standardization of Steel Pole Design Using Discrete Optimization, *Journal of Structural Engineering*, 123 (3), 345-349, 1997.

[5] F.Y. Kocer and J.S. Arora, Design of Prestressed Concrete Transmission Poles: Optimization Approach, *Journal of Structural Engineering*, 122 (7), 804-814, 1996b.

[6] H.M. Negm and K.Y. Maalawi, Structural Design Optimization of Wind Turbine Towers, *Computers and Structures*, 74, 649-666, 1999.

[7] M.A. Silva, J.S. Arora and R.M.L.R.F. Brasil, Formulations for the Optimal Design of RC Wind Turbine Towers, *International Conference on Engineering Optimization*, Rio de Janeiro, Brazil, 2008.

[8] J.C. Nicholson, Design of wind turbine tower and foundation systems: optimization approach, *Iowa Research Online: Theses and Dissertations (http://ir.uiowa.edu/etd/1042)*, University of Iowa, 2011.

[9] J.S. Arora, *Introduction to Optimum Design: Third Edition*, Elsevier, Boston, 2012.

[10] Origin Lab, Graphic Residual Analysis, *http://www.originlab.com/www/helponline/Origin/en/UserGuide/Graphic_Residual_Analysis.html*, Accessed March 20, 2013.

[11] Dassault Systèmes SIMULIA, *Isight 5.7 User's Guide*, Providence, Rhode Island, 2012.

[12] Dassault Systèmes SIMULIA, *Abaqus/CAE User's Manual*, Providence, Rhode Island, 2012.

[13] S. Engström, T. Lyrner, M. Hassanzadeh, et al., Tall towers for large wind turbines, *http://www.natverketforvindbruk.se/Global/Affarsutveckling/Dokument/10_48_report_high_towers.pdf*, Accessed March 3, 2013.