SHAPE DESIGN SENSITIVITY ANALYSIS OF CONTACT PROBLEM WITH FRICTION

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APPROACH

- Analysis
  - Reproducing Kernel Particle Method (RKPM)
  - Contact Problem - Variational Inequality
  - Penalty Method for Frictional Contact
- DSA of Variational Inequality
  - Material Derivative of Contact Variational Forms
  - Shape Design Sensitivity Analysis
- Numerical Examples of Shape Design Optimization
  - Door Seal Problem
  - Windshield Blade Problem

REPRODUCING KERNEL PARTICLE METHOD (RKPM)

Reproduced Displacement Function

$$z^R(x) = \int_\Omega C(x; y-x)\phi_a(y-x)z(y)\,dy$$

$z^R(x) \to z(x)$ as $a \to 0$ Dirac Delta Measure

Correction Function

$$C(x; y-x) = \mathbf{q}(x)^T \mathbf{H}(y-x)$$

$$\mathbf{H}(y-x)^T = [1, (y-x), (y-x)^2, \cdots, (y-x)^n]$$

$$\mathbf{q}(x)^T = [q_0(x), q_1(x), \cdots, q_n(x)]$$

N-th Order Completeness Requirement (Reproducing Condition)

$$z^R(x) = \int_\Omega C(x; y-x)\phi_a(y-x)z(y)\,dy$$

$$= \overline{m}_0(x)z(x) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\overline{m}_n(x)\frac{d^n z(x)}{dx^n}$$

$\overline{m}_0(x) = 1$ $\overline{m}_k(x) = 0$ $k = 1, \ldots, n$

RKPM (cont.)

Reproducing Condition

$$\mathbf{M}(x)\mathbf{q}(x) = \mathbf{H}(0) \qquad \mathbf{H}(0)^T = [1,0,\ldots,0]$$

$$\mathbf{M}(x) = \begin{bmatrix} m_0(x) & m_1(x) & \ldots & m_n(x) \\ m_1(x) & m_2(x) & \ldots & m_{n+1}(x) \\ . & . & \ldots & . \\ m_n(x) & m_{n+1}(x) & \ldots & m_{2n}(x) \end{bmatrix}$$

$$C(x; y-x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \mathbf{H}(y-x)$$

$$z^R(x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \int_\Omega \mathbf{H}(y-x)\phi_a(y-x)z(y)\,dy$$

$$z^R(x) = \sum_{I=1}^{NP} C(x; x_I - x)\phi_a(x_I - x)z_I \Delta x_I = \sum_{I=1}^{NP} \Phi_I(x) d_I$$

RKPM (cont.)

- Shape Function $\Phi_I(x_I)$ Depends on Current Coordinate Whereas FEA Shape Functions Depend on Coordinate of the Reference Geometry
- Does Not Satisfy Kronecker Delta Property: $\Phi_I(x_J) \neq \delta_{IJ}$
- Lagrange Multiplier Method for Essential B.C.

$$\Pi = U - \int_{\Gamma_D} \lambda^T (z - \zeta)\, d\Gamma$$

  - First-order variation is

$$\overline{\Pi} = \overline{U} - \int_{\Gamma_D} \lambda^T \overline{z}\, d\Gamma - \int_{\Gamma_D} \overline{\lambda}^T (z - \zeta)\, d\Gamma$$

- For Contact Problem, a Direct Transformation Method Is Used (Chen *et al*.-1996)

CONTACT PROBLEM

- Variational Inequality

$$a(\mathbf{z}, \mathbf{w}-\mathbf{z}) \geq \ell(\mathbf{w}-\mathbf{z}) \quad \forall \mathbf{w} \in V$$

Solution is obtained using a projection $\mathbf{z} = P_v(\mathbf{F})$ where

$$V = \left\{ \mathbf{w} \in H^1(\Omega) : \mathbf{w} = 0 \text{ on } \Gamma_D \text{ and } \mathbf{w} \cdot \mathbf{n} \leq g \ \text{ a.e. on } \Gamma_C \right\}$$

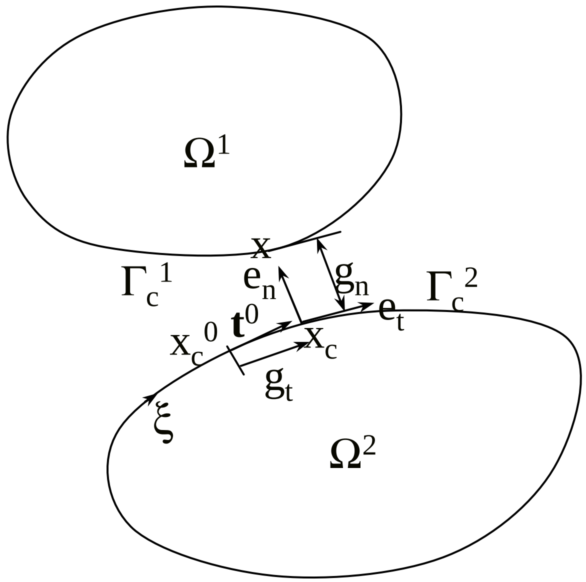
$a(\mathbf{z}, \mathbf{w})$: Nonlinear strain energy form

$\ell(\mathbf{w})$: Load liner form for conservative load

- Equivalent Constrained Minimization Problem

$$\frac{1}{2} a(\mathbf{z}, \mathbf{z}) - \ell(\mathbf{z}) = \min_{\mathbf{w} \in V} \left[\frac{1}{2} a(\mathbf{w}, \mathbf{w}) - \ell(\mathbf{w}) \right]$$

PENALTY METHOD FOR FRICTIONAL CONTACT



Impenetration Condition

$$g_n \equiv (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_n(\xi_c) \geq 0, \quad \mathbf{x} \in \Gamma_c^1, \mathbf{x}_c \in \Gamma_c^2$$

Tangential Slip Function

$$g_t \equiv \left\| \mathbf{t}^0 \right\| (\xi_c - \xi_c^0)$$

Contact Consistency Condition

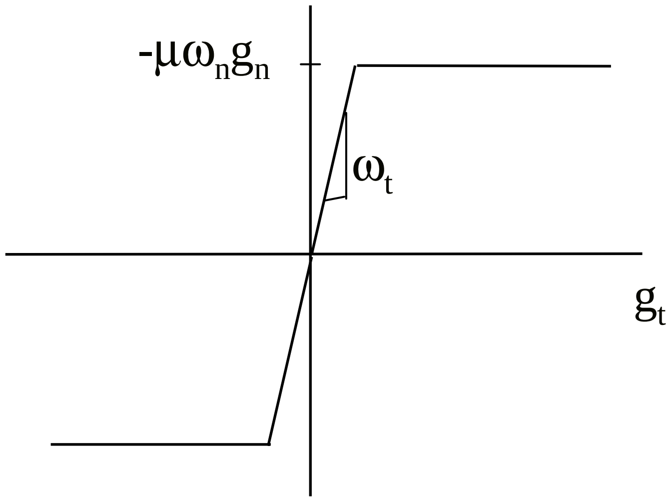
$$\varphi(\xi_c) = (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_t(\xi_c) = 0$$

Contact Penalty Function

$$P = \frac{1}{2}\omega_n \int_{\Gamma_C} g_n^{\,2} d\Gamma + \frac{1}{2}\omega_t \int_{\Gamma_C} g_t^{\,2} d\Gamma$$

where integration is performed only on the region where $g_n < 0$ on Γ_C

PENALTY METHOD (cont.)



Contact Variational Form

$$b(\mathbf{z}, \bar{\mathbf{z}}) \equiv \bar{P} = \omega_n \int_{\Gamma_C} g_n \bar{g}_n \, d\Gamma + \omega_t \int_{\Gamma_C} g_t \bar{g}_t \, d\Gamma$$

CONTACT VARIATIONAL FORMS

- Stick Condition: $|\omega_t g_t| \le |\mu\omega_n g_n|$

$$b(\mathbf{z},\bar{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_n d\Gamma$$

$$+ \omega_t \int_{\Gamma_C} g_t \left[\nu (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_t + (g_n \|\mathbf{t}^0\| / c) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \right] d\Gamma$$

- Slip Condition : $|\omega_t g_t| > |\mu\omega_n g_n|$

$$b(\mathbf{z},\bar{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_n d\Gamma$$

$$- \mu\omega_n \,\mathrm{sgn}(g_t) \int_{\Gamma_C} g_n \left[\nu (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_t + (g_n \|\mathbf{t}^0\| / c) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \right] d\Gamma$$

where

$$\alpha \equiv \mathbf{e}_n{}^T \mathbf{x}_{c,\xi\xi} \qquad \beta \equiv \mathbf{e}_t{}^T \mathbf{x}_{c,\xi\xi} \qquad \gamma \equiv \mathbf{e}_n{}^T \mathbf{x}_{c,\xi\xi\xi}$$

$$c \equiv \|\mathbf{t}\|^2 - g_n \alpha \qquad \nu \equiv \frac{\|\mathbf{t}\| \|\mathbf{t}^0\|}{c}$$

DSA OF VARIATIONAL INEQUALITY

- Contact Is a Variational Inequality (VI) Problem
- Penalty Method (Wriggers et al.-1990 and Kikuchi & Oden-1988) Can Be Used for an Approximate Solution of VI
- Shape Design Sensitivity of VI Has Been Obtained by Sokolowski and Zolesio (1991) for Linear Problem
- Design Sensitivity Equation is Another VI
- For Linear Problems, Application of the Penalty Method to Solve the Sensitivity Equation (VI) Yields the Same Result As the Sensitivity Equation Obtained From the Penalty Equation

DSA FORMULATION FOR CONTACT PROBLEM

- Material Derivative of Structural Point

$$\frac{d}{d\tau}({}^{n}\mathbf{x}_{\tau})\Big|_{\tau=0} \equiv \mathbf{V}({}^{n}\mathbf{x}) = \mathbf{V}({}^{0}\mathbf{x}) + {}^{n}\dot{\mathbf{z}}({}^{0}\mathbf{x}), \qquad {}^{0}\mathbf{x} \in \Omega^{1} \cup \Omega^{2}$$

- Material Derivative of Contact Point

$$\frac{d}{d\tau}({}^{n}\mathbf{x}_{c\tau})\Big|_{\tau=0} = \mathbf{V}_{c}({}^{0}\mathbf{x}_{c}) + {}^{n}\dot{\mathbf{z}}_{c}({}^{0}\mathbf{x}_{c}) + {}^{n}\mathbf{t}\,{}^{n}\dot{\xi}_{c}\,, \qquad {}^{0}\mathbf{x}_{c} \in \Gamma_{c}^{2}$$

- Material Derivative of Natural Coordinate at Contact

$$\dot{\xi}_{c} = \left(\|\mathbf{t}\|/c\right)\mathbf{e}_{t}^{T}(\mathbf{V} + \dot{\mathbf{z}} - \mathbf{V}_{c} - \dot{\mathbf{z}}_{c}) + (g_{n}/c)\mathbf{e}_{n}^{T}\left(\mathbf{V}_{c,\xi} + \dot{\mathbf{z}}_{c,\xi}\right)$$

MATERIAL DERIVATIVE OF CONTACT VARIATIONAL FORMS

$$\frac{d}{d\tau}\left[b_{\Gamma_\tau}({}^{n}\mathbf{z}_\tau, \bar{\mathbf{z}}_\tau)\right] = b^*({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}}) + b'_V({}^{n}\mathbf{z}, \bar{\mathbf{z}})$$

Linearized Contact Variational Form

$$b^*({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}}) = b^*_N({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}}) + b^*_T({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}})$$

$$b'_V({}^{n}\mathbf{z}, \bar{\mathbf{z}}) = b'_N({}^{n}\mathbf{z}, \bar{\mathbf{z}}) + b'_T({}^{n}\mathbf{z}, \bar{\mathbf{z}})$$

- Normal Contact Fictitious Load Form for DSA

$$\begin{aligned}
b'_N(\mathbf{z}, \bar{\mathbf{z}}) = {} & \omega_n \int_{\Gamma_C} (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_n \mathbf{e}_n{}^T (\mathbf{V} - \mathbf{V}_c)\, d\Gamma - \omega_n \int_{\Gamma_C} (\alpha g_n / c)(\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_t \mathbf{e}_t{}^T (\mathbf{V} - \mathbf{V}_c)\, d\Gamma \\
& - \omega_n \int_{\Gamma_C} \left(g_n \|\mathbf{t}\| / c\right)(\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_t \mathbf{e}_n{}^T \mathbf{V}_{c,\xi}\, d\Gamma - \omega_n \int_{\Gamma_C} \left(g_n \|\mathbf{t}\| / c\right) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \mathbf{e}_t{}^T (\mathbf{V} - \mathbf{V}_c)\, d\Gamma \\
& - \omega_n \int_{\Gamma_C} \left(g_n{}^2 / c\right) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \mathbf{e}_n{}^T \mathbf{V}_{c,\xi}\, d\Gamma + \omega_n \int_{\Gamma_C} \kappa g_n (\bar{\mathbf{z}} - \bar{\mathbf{z}}_c)^T \mathbf{e}_n (\mathbf{V}^T \mathbf{n})\, d\Gamma
\end{aligned}$$

MATERIAL DERIVATIVE OF CONTACT VARIATIONAL FORMS (cont.)

- Tangential Stick Fictitious Load Form for DSA

$$
\begin{aligned}
b'_T({}^n\mathbf{z},\bar{\mathbf{z}}) &= b^*_T({}^n\mathbf{z};\mathbf{V},\bar{\mathbf{z}}) \\
&+ \omega_t \int_{\Gamma_C} {}^n\left(2g_t\|\mathbf{t}\|/c\right)(\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^T(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
&+ \omega_t \int_{\Gamma_C} \left({}^n\nu(2\,{}^{n-1}\beta\,{}^n g_t - \|{}^{n-1}\mathbf{t}\|^2\right)(\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^T(\mathbf{V}+{}^{n-1}\dot{\mathbf{z}}-\mathbf{V}_c-{}^{n-1}\dot{\mathbf{z}}_c)\,d\Gamma \\
&+ \omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta\,{}^n g_n\,{}^n g_t\left(\|{}^{n-1}\mathbf{t}\|+\|{}^n\mathbf{t}\|\right)/\,{}^n c\,{}^{n-1}c\right](\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_n^T(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
&- \omega_t \int_{\Gamma_C} \left[{}^n g_n\|{}^n\mathbf{t}\|\|{}^{n-1}\mathbf{t}\|^2/\,{}^n c\,{}^{n-1}c\right](\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{t-\Delta t}\mathbf{e}_n^T(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
&+ \omega_t \int_{\Gamma_C} \left[{}^n g_n\|{}^{n-1}\mathbf{t}\|\left(2\,{}^{n-1}\beta\,{}^n g_t - \|{}^{n-1}\mathbf{t}\|^2\right)/\,{}^n c\,{}^{n-1}c\right]\bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_t^T(\mathbf{V}+{}^{n-1}\dot{\mathbf{z}}-\mathbf{V}_c-{}^{n-1}\dot{\mathbf{z}}_c)\,d\Gamma \\
&+ \omega_t \int_{\Gamma_C} \left[{}^n g_n\,{}^{n-1}g_n\left(2\,{}^{n-1}\beta\,{}^n g_t - \|{}^{n-1}\mathbf{t}\|^2\right)/\,{}^n c\,{}^{n-1}c\right]\bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_n^T(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
&+ \omega_t \int_{\Gamma_C} \kappa\left\{{}^n\nu\,{}^n g_t(\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t + \left({}^n g_n\,{}^n g_t\|{}^{n-1}\mathbf{t}\|/c\right)(\bar{\mathbf{z}}-\bar{\mathbf{z}})^T\, {}^n\mathbf{e}_t\right\}(\mathbf{V}^T\mathbf{n})\,d\Gamma
\end{aligned}
$$

MATERIAL DERIVATIVE OF CONTACT VARIATIONAL FORMS (cont.)

- Tangential Slip Fictitious Load Form for DSA

$$
\begin{aligned}
& b'_T({}^n\mathbf{z},\bar{\mathbf{z}}) \equiv b^*_T({}^n\mathbf{z};\mathbf{V},\bar{\mathbf{z}}) \\
& + \omega_t \int_{\Gamma_C} \left({}^n g_n \left\|{}^n\mathbf{t}\right\| / {}^n c\right)(\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^T (\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \left({}^n\nu\, {}^{n-1}\beta\, {}^n g_n / {}^{n-1}c\right)(\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^T (\mathbf{V} + {}^{n-1}\dot{\mathbf{z}} - \mathbf{V}_c - {}^{n-1}\dot{\mathbf{z}}_c)\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta\, {}^n g_n\, {}^{n-1}g_n \left\|{}^{n-1}\mathbf{t}\right\| / {}^n c\, {}^{n-1}c\right](\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_n^T (\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \left[{}^n g_n^2 / {}^n c\right] \bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_t^T (\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta \left\|{}^{n-1}\mathbf{t}\right\| {}^n g_n^2 / {}^n c\, {}^{n-1}c\right] \bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_t^T (\mathbf{V} + {}^{n-1}\dot{\mathbf{z}} - \mathbf{V}_c - {}^{n-1}\dot{\mathbf{z}}_c)\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta\, {}^n g_n^2\, {}^{n-1}g_n / {}^n c\, {}^{n-1}c\right] \bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_n^T (\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& + \omega_t \int_{\Gamma_C} \kappa\left[{}^n\nu\, {}^n g_n (\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T\, {}^n\mathbf{e}_t + \left({}^n g_n^2 \left\|{}^{n-1}\mathbf{t}\right\| / {}^n c\right) \bar{\mathbf{z}}_{c,\xi}^T\, {}^n\mathbf{e}_n\right](\mathbf{V}^T\mathbf{n})\, d\Gamma
\end{aligned}
$$

NONLINEAR CONTACT ANALYSIS

- Variational Equation for Mixed Formulation of Hyperelastic Solid

$$a({}^{n+1}\mathbf{r},\bar{\mathbf{r}}) + b({}^{n+1}\mathbf{r},\bar{\mathbf{r}}) = \ell(\bar{\mathbf{r}}) \quad \forall \bar{\mathbf{r}} \in Z \qquad \mathbf{r}^T \equiv [\mathbf{z}^T, p, \lambda^T]$$

- Linearized Incremental Equation

$$a^*({}^{n+1}\mathbf{r}^k;\Delta\mathbf{r}^{k+1},\bar{\mathbf{r}}) + b^*({}^{n+1}\mathbf{r}^k;\Delta\mathbf{r}^{k+1},\bar{\mathbf{r}}) = \ell(\bar{\mathbf{r}}) - a({}^{n+1}\mathbf{r}^k,\bar{\mathbf{r}}) - b({}^{n+1}\mathbf{r}^k,\bar{\mathbf{r}})$$

where linearized strain energy form is

$$a^*({}^{n}\mathbf{r};\Delta\mathbf{r},\bar{\mathbf{r}}) \equiv \int_\Omega \left[\bar{\mathbf{E}} : (\mathbf{C} : \Delta\mathbf{E} + J_{3,\mathbf{E}}\Delta p) + \mathbf{S} : \Delta\bar{\mathbf{E}}\right] d\Omega$$

$$+ \int_\Omega \bar{p}\left(J_{3,\mathbf{E}} : \Delta\mathbf{E} - \frac{\Delta p}{K}\right) d\Omega - \int_{\Gamma_D} \bar{\mathbf{z}}^T \Delta\lambda \, d\Gamma - \int_{\Gamma_D} \Delta\mathbf{z}^T \bar{\lambda} \, d\Gamma$$

SHAPE DESIGN SENSITIVITY ANALYSIS

- Variational Equation for Perturbed Shape Design

$$a_{\Omega_\tau}({}^n\mathbf{r}_\tau, \bar{\mathbf{r}}_\tau) + b_{\Gamma_\tau}({}^n\mathbf{r}_\tau, \bar{\mathbf{r}}_\tau) = \ell_{\Omega_\tau}(\bar{\mathbf{r}}_\tau) \qquad \forall \bar{\mathbf{r}}_\tau \in Z_\tau$$

- Shape Sensitivity Equation - 1st Order Variation

$$a^*_\Omega(\mathbf{r}; \dot{\mathbf{r}}, \bar{\mathbf{r}}) + b^*_\Gamma(\mathbf{r}; \dot{\mathbf{r}}, \bar{\mathbf{r}}) = \ell'_V(\bar{\mathbf{r}}) - a'_V(\mathbf{r}, \bar{\mathbf{r}}) - b'_V(\mathbf{r}, \bar{\mathbf{r}})$$

where fictitious load forms for DSA are

$$a'_V(\mathbf{r}, \bar{\mathbf{r}}) = \int_\Omega \left[\bar{\mathbf{E}} : \mathbf{C} : \mathbf{E}_V(\mathbf{z}) + \mathbf{S} : \bar{\mathbf{E}}_V(\mathbf{z}, \bar{\mathbf{z}}) + \mathbf{S} : \bar{\mathbf{E}}\,\mathrm{div}\mathbf{V}\right] d\Omega$$
$$- \int_{\Gamma_D} \kappa \bar{\mathbf{z}}^T \lambda (\mathbf{V}^T \mathbf{n})\, d\Gamma - \int_{\Gamma_D} \bar{\lambda}^T (\mathbf{z} - \zeta)(\mathbf{V}^T \mathbf{n})\, d\Gamma$$

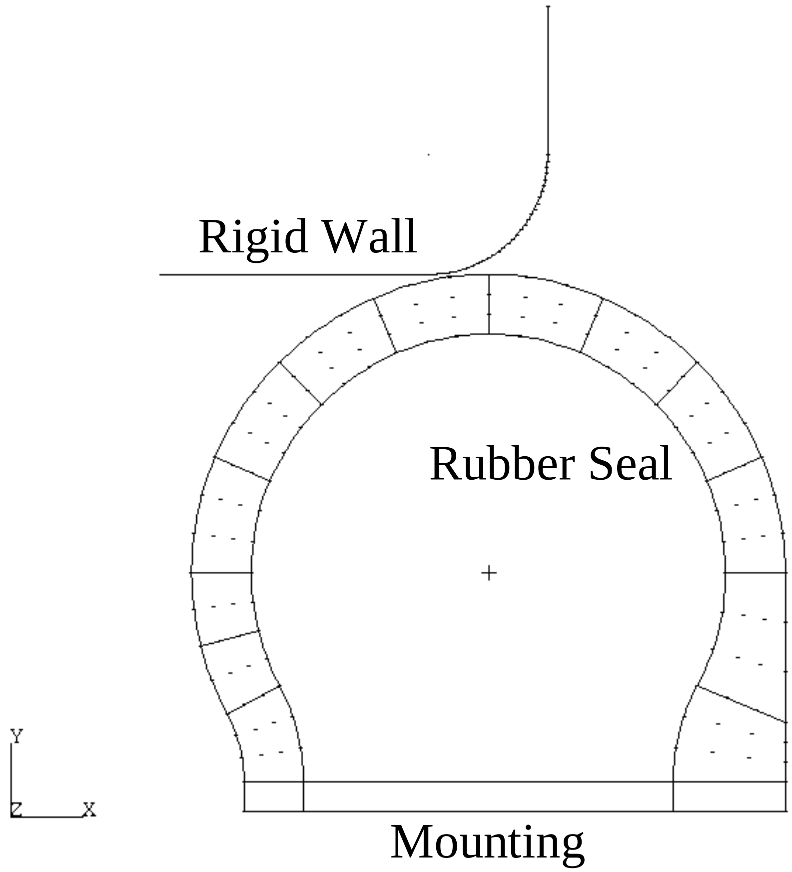
SHAPE DESIGN SENSITIVITY ANALYSIS (cont.)

$$\ell'_{\mathrm{V}}(\bar{\mathbf{r}}) = \iint_{\Omega} \left[\bar{\mathbf{z}}^{\mathrm{T}} (\nabla \mathbf{f}^{\mathrm{B}^{\mathrm{T}}} \mathbf{V}) + \bar{\mathbf{z}}^{\mathrm{T}} \mathbf{f}^{\mathrm{B}} \,\mathrm{div}\mathbf{V} \right] \mathrm{d}\Omega$$

$$+ \int_{\Gamma_{\mathrm{T}}} \left[\bar{\mathbf{z}}^{\mathrm{T}} (\nabla \mathbf{f}^{\mathrm{T}^{\mathrm{T}}} \mathbf{V}) + \kappa \bar{\mathbf{z}}^{\mathrm{T}} \mathbf{f}^{\mathrm{T}} (\mathbf{V}^{\mathrm{T}} \mathbf{n}) \right] \mathrm{d}\Gamma$$

- Remarks
  - Total form of sensitivity equation
  - No iteration is required
  - Frictional contact is path-dependent
  - DSA needs to be carried out at each converged load step
    - Direct Differentiation Method is used
  - Material derivatives of displacement at contact nodes are used for DSA at next time step

DOOR SEAL CONTACT MODEL



Material Constant D_{10} = 80 KPa
D_{01} = 20 KPa

Bulk Modulus K = 80 MPa

Frictional Coefficient μ = 0.25

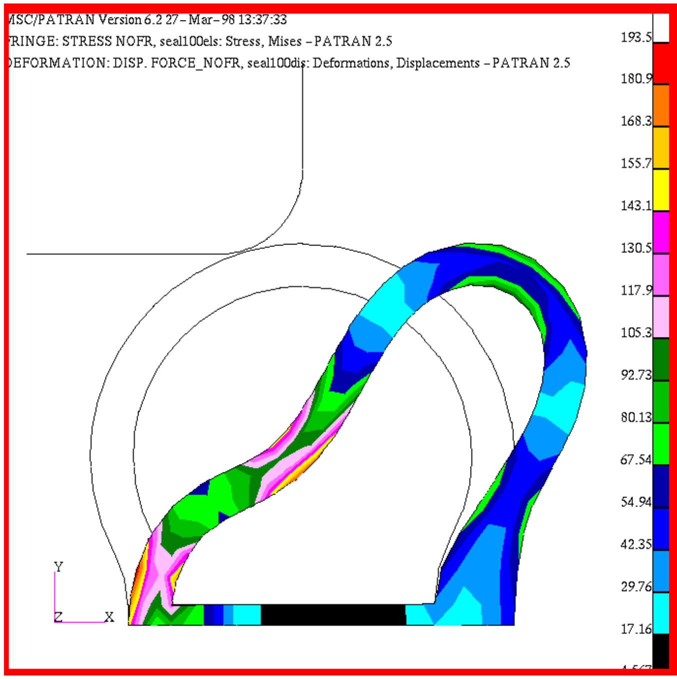
Rubber Seal = 174 Particle Points

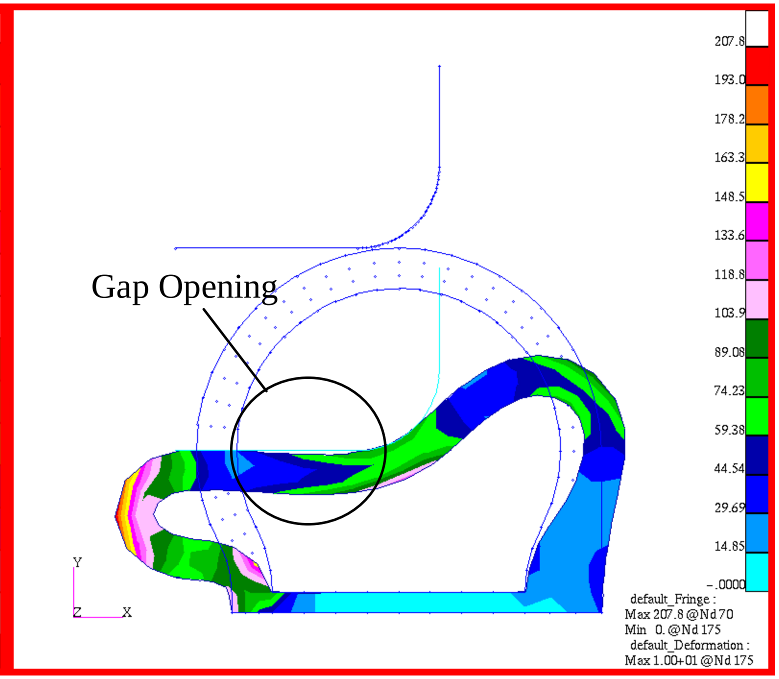
Rigid Wall = 32 Piecewise Linear Master Segments

100 Load Steps for Analysis with Displacement Driven Procedure

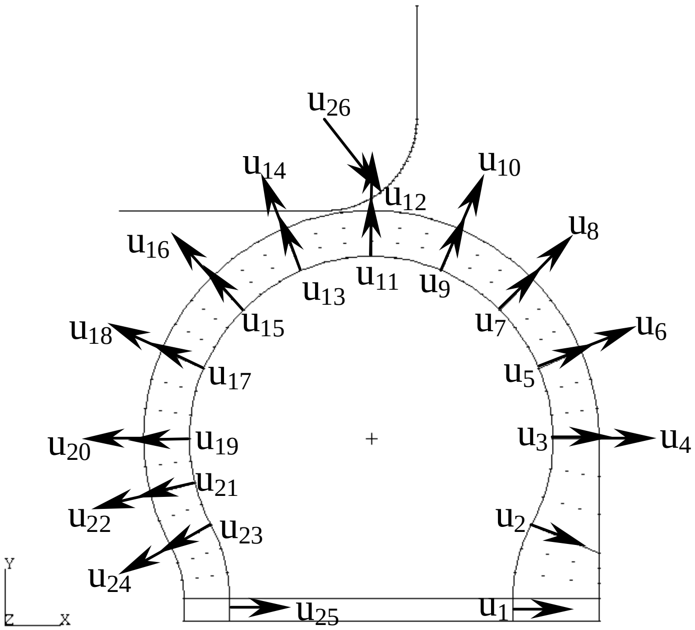
MESHLESS ANALYSIS OF DOOR SEAL

Frictionless Contact | Frictional Contact





DOOR SEAL CONTACT DESIGN



26 Shape Design Parameters

$$
\begin{aligned}
&\text{Min} && \sum g_n{}^2 \\
&\text{s.t.} && \text{Area}(109) \le 110 \\
& && \sigma_{75}(130) \le 100 \\
& && \sigma_{86}(129) \le 100 \\
& && \sigma_{44}(209) \le 160 \\
& && \sigma_{114}(207) \le 160 \\
& && \sigma_{31}(98) \le 100 \\
& && \sigma_{38}(191) \le 160 \\
& && \sigma_{115}(148) \le 160 \\
& && -0.5 \le u_i \le 0.5 \qquad i = 0, 26
\end{aligned}
$$

Optimizer: DOT-SQP

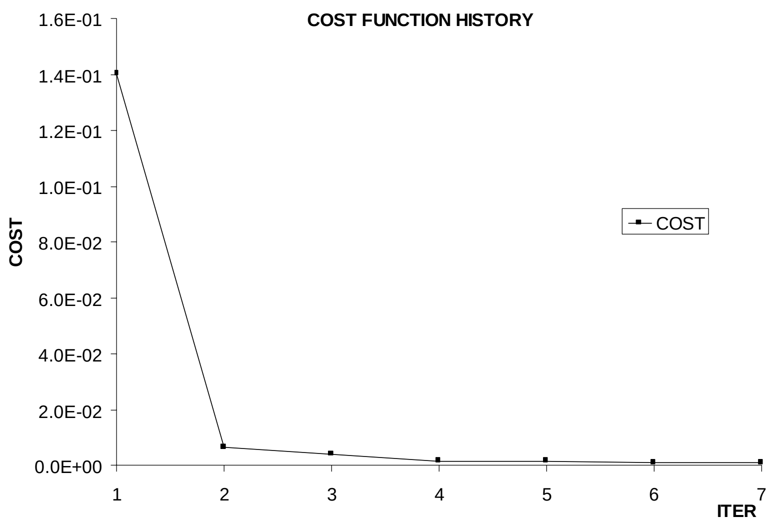
Initial Design is Infeasible

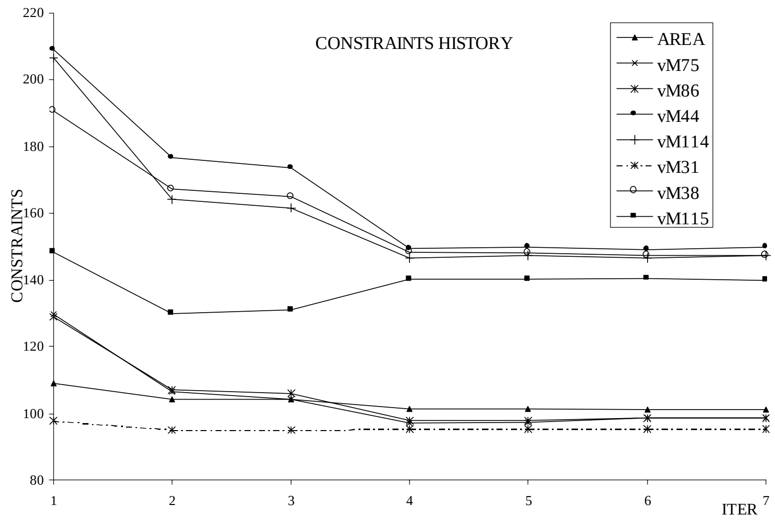
ACCURACY OF SHAPE DSA RESULTS

Analysis: 1968 Sec, Sensitivity: 1084 Sec for 26 DV = 41.7 Sec

Ψ	$\Delta\Psi$ (FDM)	Ψ' (PROPOSED)	$(\Delta\Psi / \Psi')\times 100$
u_1			
Area	-.163895E-5	-.163895E-5	100.00
σ_{75}	-.501565E-6	-.501563E-6	100.00
σ_{86}	-.255777E-5	-.255775E-5	100.00
σ_{44}	-.247860E-6	-.247893E-6	99.99
σ_{114}	.525571E-6	.525554E-6	100.00
σ_{31}	-.149431E-6	-.149300E-6	100.09
Σg_n^2	-.114879E-8	-.114878E-8	100.00
u_2			
Area	.163894E-5	.163895E-5	100.00
σ_{75}	.514388E-6	.514395E-6	100.00
σ_{86}	-.268130E-5	-.268129E-5	100.00
σ_{44}	.292610E-4	.292609E-4	100.00
σ_{114}	.126237E-4	.126237E-4	100.00
σ_{31}	.947482E-6	.947679E-6	99.98
Σg_n^2	.223116E-7	.223116E-7	100.00
u_3			
Area	-.405671E-5	-.405671E-5	100.00
σ_{75}	-.858554E-7	-.858371E-7	100.02
σ_{86}	-.361270E-5	-.361266E-5	100.00
σ_{44}	-.481447E-6	-.481452E-6	100.00
σ_{114}	.143501E-5	.143499E-5	100.00
σ_{31}	.869462E-7	.868284E-7	100.14
Σg_n^2	-.311421E-9	-.311370E-9	100.02
u_4			
Area	-.351300E-5	-.351300E-5	100.00
σ_{75}	-.105863E-4	-.105864E-4	100.00
σ_{86}	-.130646E-4	-.130647E-4	100.00
σ_{44}	.122614E-5	.122615E-5	100.00
σ_{114}	-.329776E-5	-.329777E-5	100.00
σ_{31}	-.243310E-6	-.243378E-6	99.97
Σg_n^2	-.210381E-8	-.210383E-8	100.00
u_5			
Area	.447486E-5	.447486E-5	100.00
σ_{75}	.629273E-5	.629276E-5	100.00
σ_{86}	.835460E-5	.835467E-5	100.00
σ_{44}	.122614E-5	.122615E-5	100.00
σ_{114}	.143353E-5	.143352E-5	100.00
σ_{31}	.116624E-6	.116676E-6	99.96
Σg_n^2	-.430791E-9	-.430725E-9	100.02

SHAPE OPTIMIZATION HISTORY

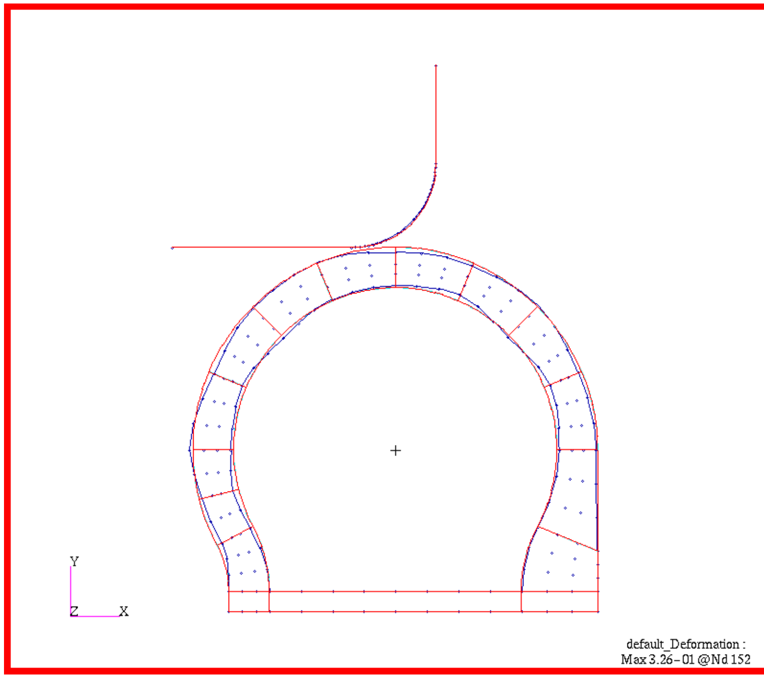




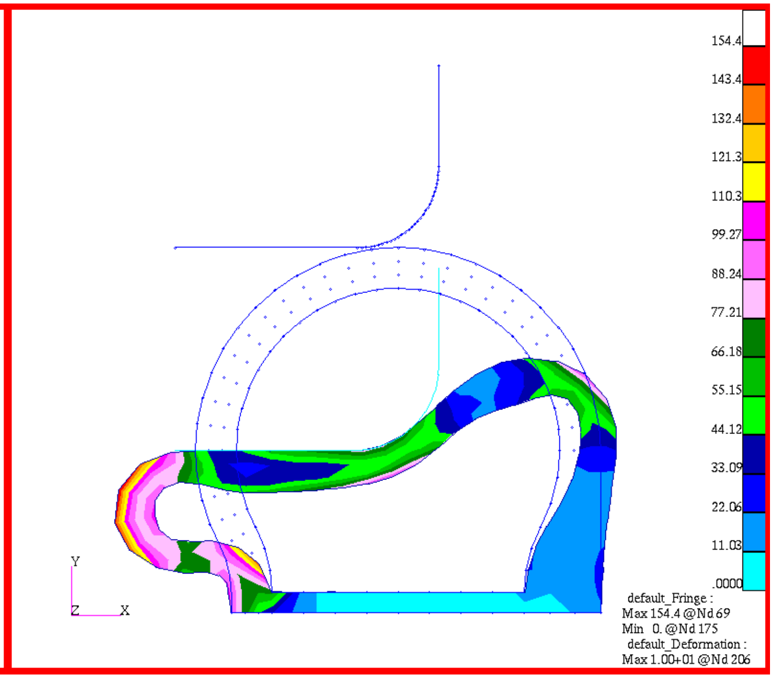
The University of Florida
College of Engineering

OPTIMIZED DOOR SEAL

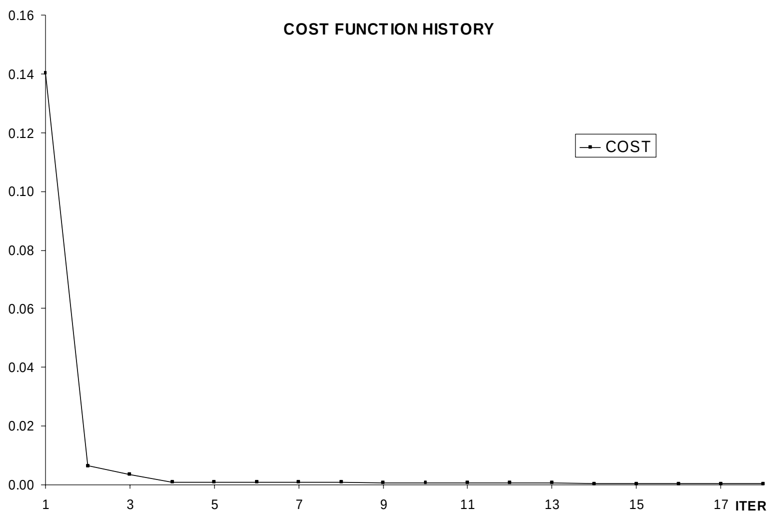
Optimum Shape

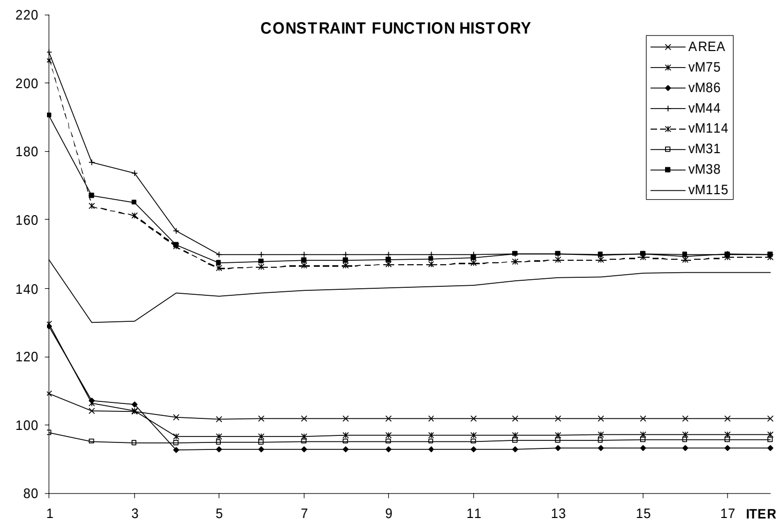


Analysis of Optimum Design

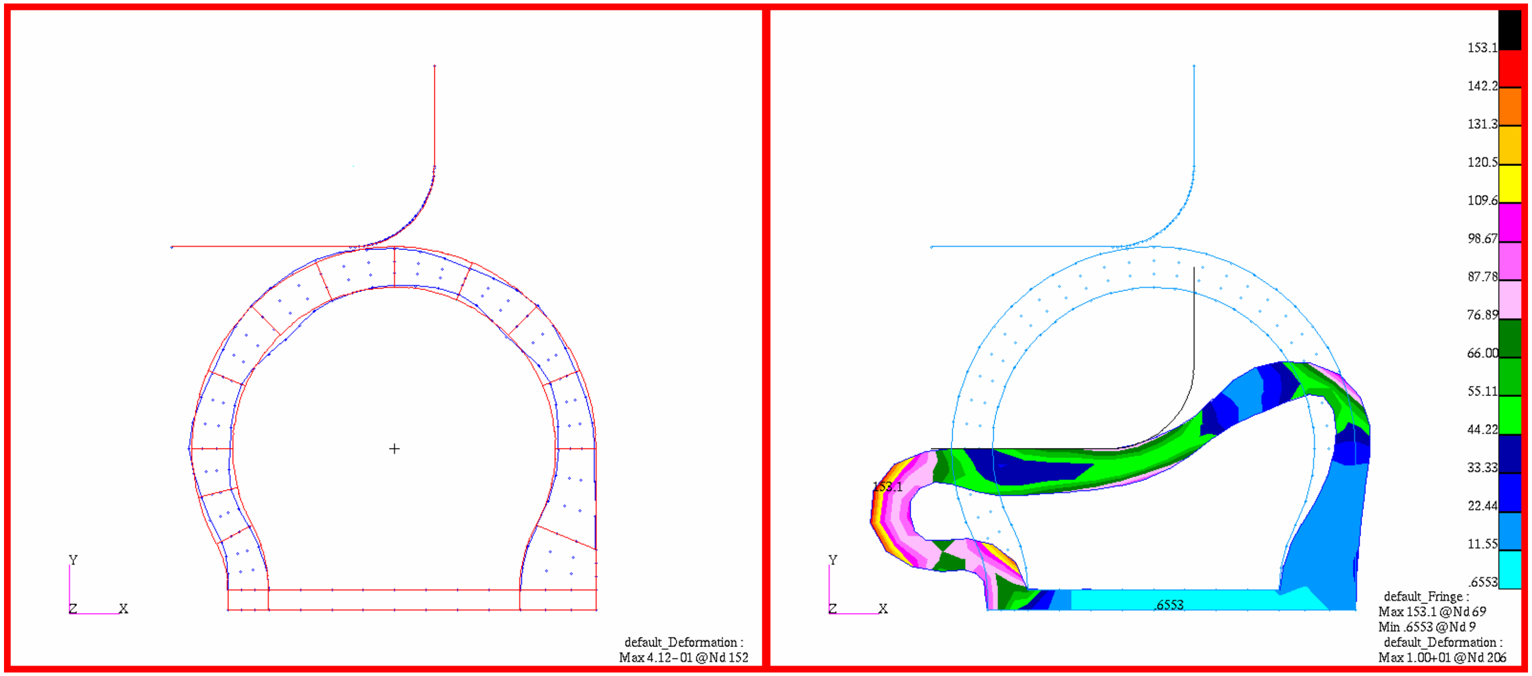


SHAPE OPTIMIZATION HISTORY (FDM)



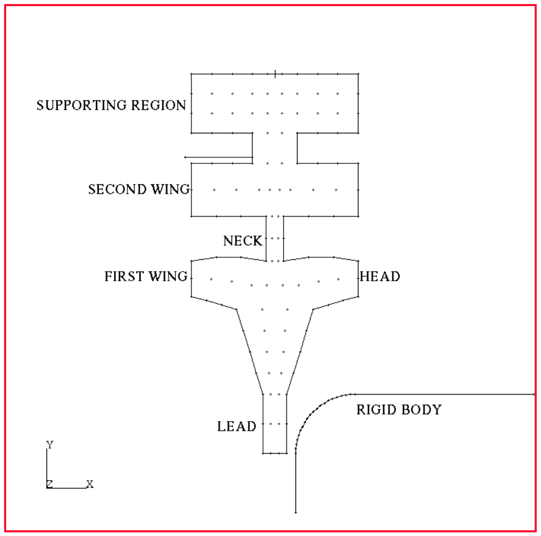


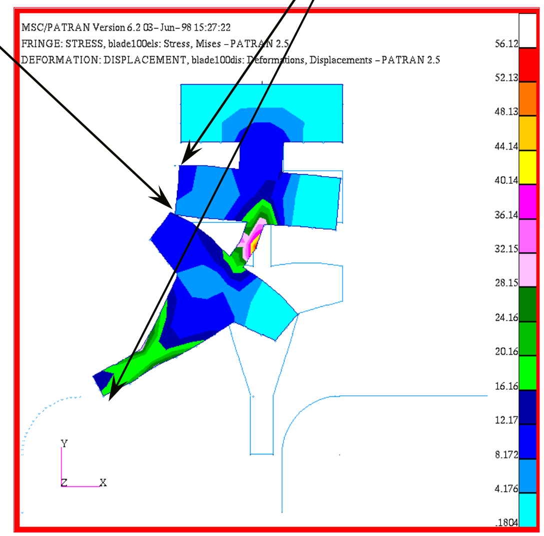
OPTIMIZED DOOR SEAL (FDM)



WINDSHIELD BLADE MODEL

Multi-body Contact   Flexible-rigid Body Contact





Material Constant D_{10} = 80 Kpa,   Bulk Modulus K = 80 MPa

D_{01} = 20 Kpa

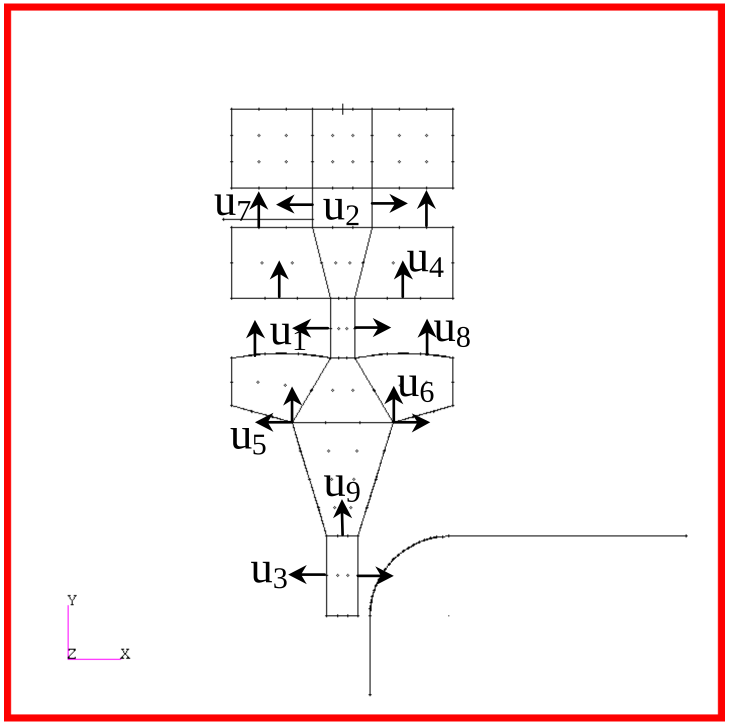
Frictional Coefficient $\mu = 0.15$

Windshield Blade = 128 Particle Points

Rigid Wall = 26 Piecewise Linear Master Segments

100 Load Steps for Analysis with Displacement Driven Procedure

SHAPE DESIGN PARAMETERS OF BLADE



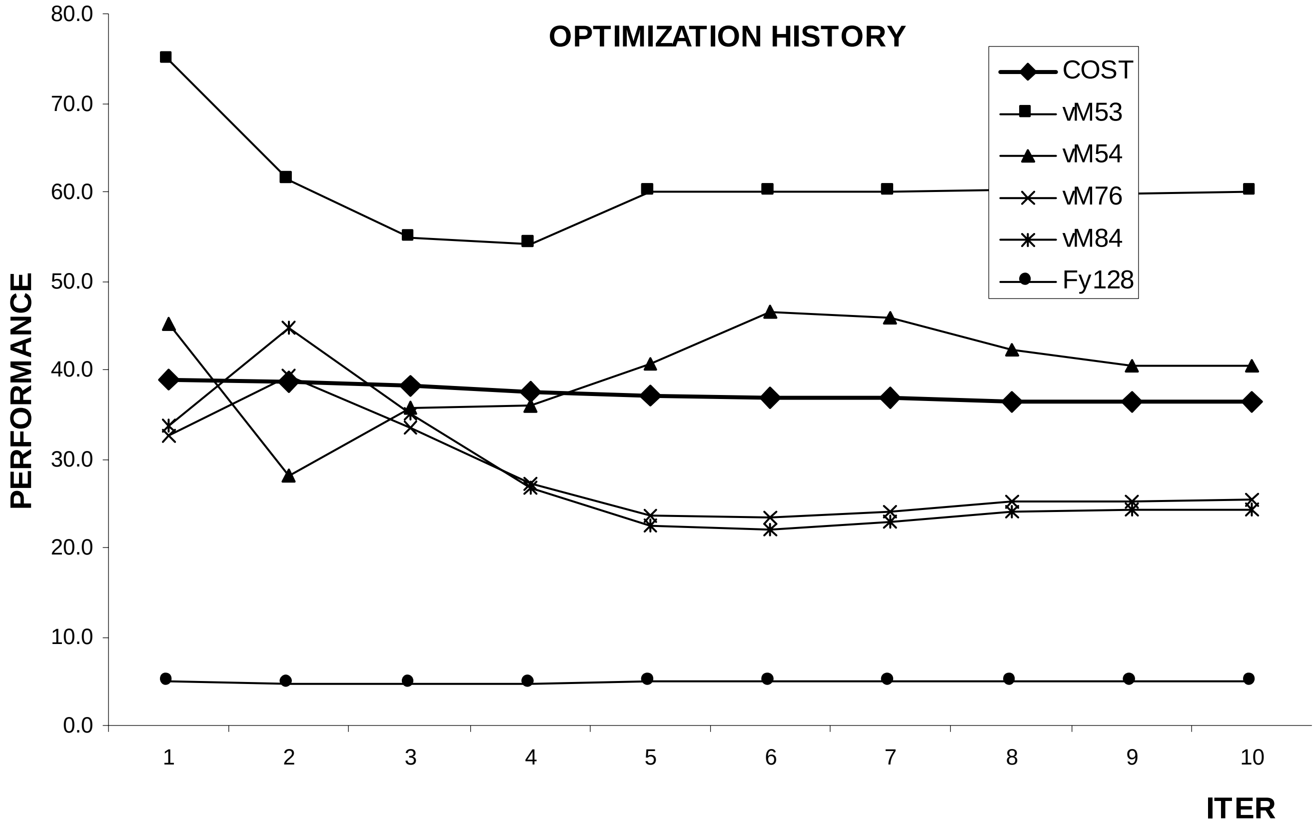
$$
\begin{array}{llllll}
\text{Min} & \text{Area}(39) & & & & \\
\text{s.t.} & \sigma_{53}(75) & \le & 55 & & \\
 & \sigma_{54}(45) & \le & 55 & & \\
 & \sigma_{76}(32) & \le & 55 & & \\
 & \sigma_{84}(34) & \le & 55 & & \\
 & F_{y128}(5) & \ge & 5.5 & & \\
 & -0.2 & \le u_i \le & 0.2 & & i = 1,3,7,8 \\
 & -0.3 & \le u_i \le & 0.3 & & i = 2,4 \\
 & -0.6 & \le u_i \le & 0.6 & & i = 5,6 \\
 & -0.1 & \le u_i \le & 0.1 & & i = 9
\end{array}
$$

ACCURACY OF SHAPE DSA RESULTS

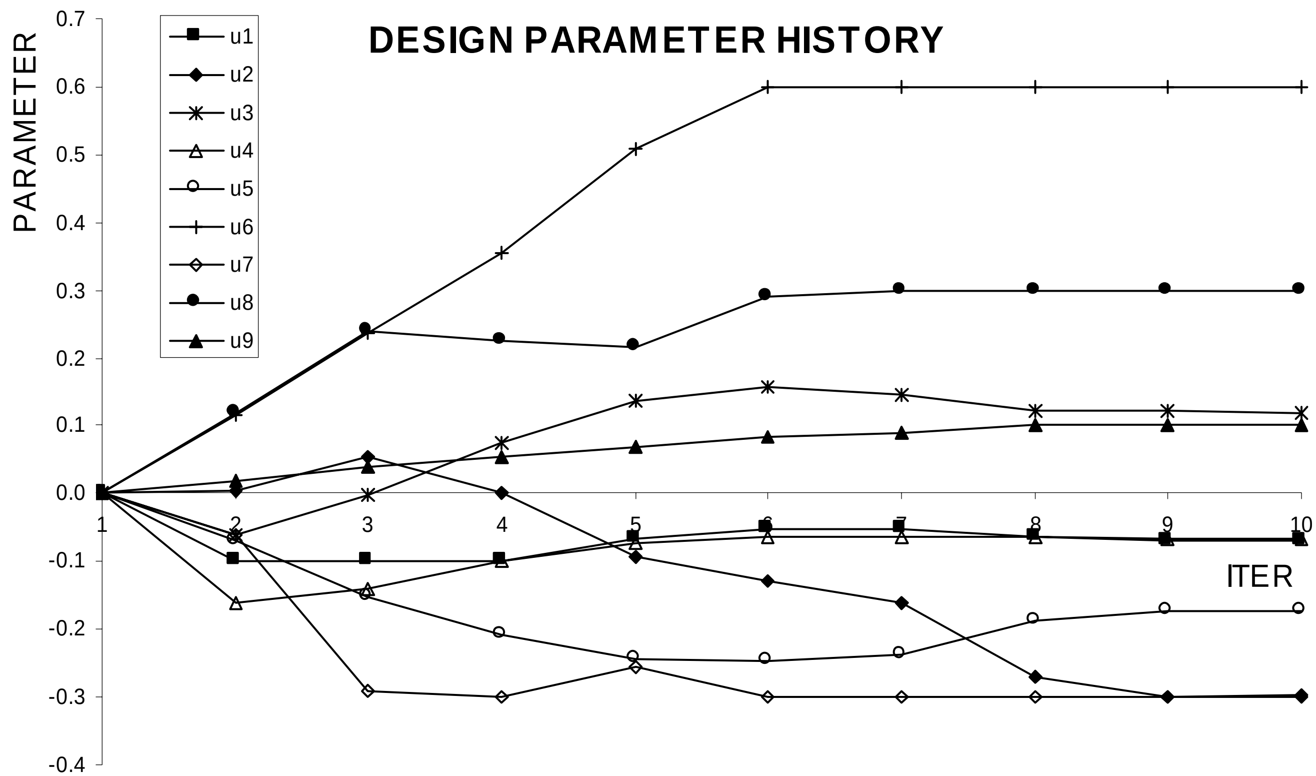
Analysis: 633 Sec, Sensitivity: 133 Sec for 6 DV = 22.2 Sec

Ψ	$\Delta\Psi$(FDM)	Ψ'(PROPOSED)	$\Delta\Psi/\Psi'\times100$	Ψ	$\Delta\Psi$(FDM)	Ψ'(PROPOSED)	$\Delta\Psi/\Psi'\times100$
DV = 1				DV = 3			
Area	.284063E-05	.284063E-05	100.00	Area	.686628E-05	.686628E-05	100.00
σ_{53}	.804656E-04	.804720E-04	99.99	σ_{53}	.875426E-04	.875487E-04	99.99
σ_{54}	.155088E-03	.155089E-03	100.00	σ_{54}	.215552E-04	.215548E-04	100.00
σ_{76}	.168538E-04	.168559E-04	99.99	σ_{76}	.751887E-07	.765740E-07	98.19
σ_{84}	.223094E-04	.223117E-04	99.99	σ_{84}	-.158343E-04	-.158331E-04	100.01
F_{x128}	-.406709E-06	-.406753E-06	99.99	F_{x128}	-.244081E-05	-.244084E-05	100.00
F_{y128}	.271140E-05	.271169E-05	99.99	F_{y128}	.162721E-04	.162723E-04	100.00
DV = 2				DV = 4			
Area	.200000E-05	.200000E-05	100.00	Area	-.500000E-05	-.500000E-05	100.00
σ_{53}	-.458509E-05	-.457972E-05	100.12	σ_{53}	-.757898E-05	-.757548E-05	100.05
σ_{54}	.927484E-05	.927534E-05	99.99	σ_{54}	-.884933E-05	-.884885E-05	100.01
σ_{76}	.458977E-05	.459232E-05	99.94	σ_{76}	-.513851E-04	-.513846E-04	100.00
σ_{84}	.597551E-05	.597838E-05	99.95	σ_{84}	-.646750E-04	-.646747E-04	100.00
F_{x128}	-.102118E-06	-.102180E-06	99.94	F_{x128}	.112802E-05	.112801E-05	100.00
F_{y128}	.680789E-06	.681198E-06	99.94	F_{y128}	-.752015E-05	-.752009E-05	100.00

PERFORMANCE MEASURE HISTORY

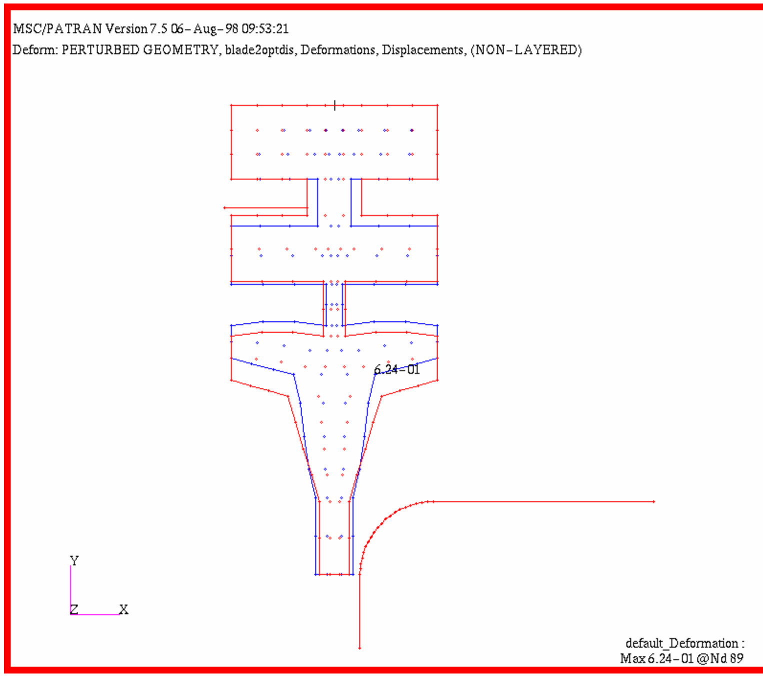


DESIGN PARAMETER HISTORY



OPTIMIZED WINDSHIELD BLADE

Optimum Shape



Analysis of Optimum Design