SHAPE SENSITIVITY ANALYSIS AND OPTIMIZATION FOR A CONTACT PROBLEM IN THE MANUFACTURING PROCESS DESIGN

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OUTLINE

- **DSA for Contact Problem with Frictional Return-Mapping**
  - ✓ Material Derivative of Frictional Return-Mapping Scheme
  - ✓ Die Shape Design Parameters
- **Smooth Contact Surface**
  - ✓ Meshfree Interpolation of C^2-Continuous Contact Surface
- **Numerical Examples**
  - ✓ Gasket Design Problem
  - ✓ Metal Punch DSA
  - ✓ Metal Extrusion Problem

3D CONTACT FORMULATION

- Penalty-Based Contact Formulation
  - Contact Form

    $$b_N(\mathbf{z},\bar{\mathbf{z}}) = \omega_N \int_{\Gamma_X^c} g\mathbf{n}\cdot\hat{\bar{\mathbf{z}}}\,d\Gamma$$

  - Gap Function

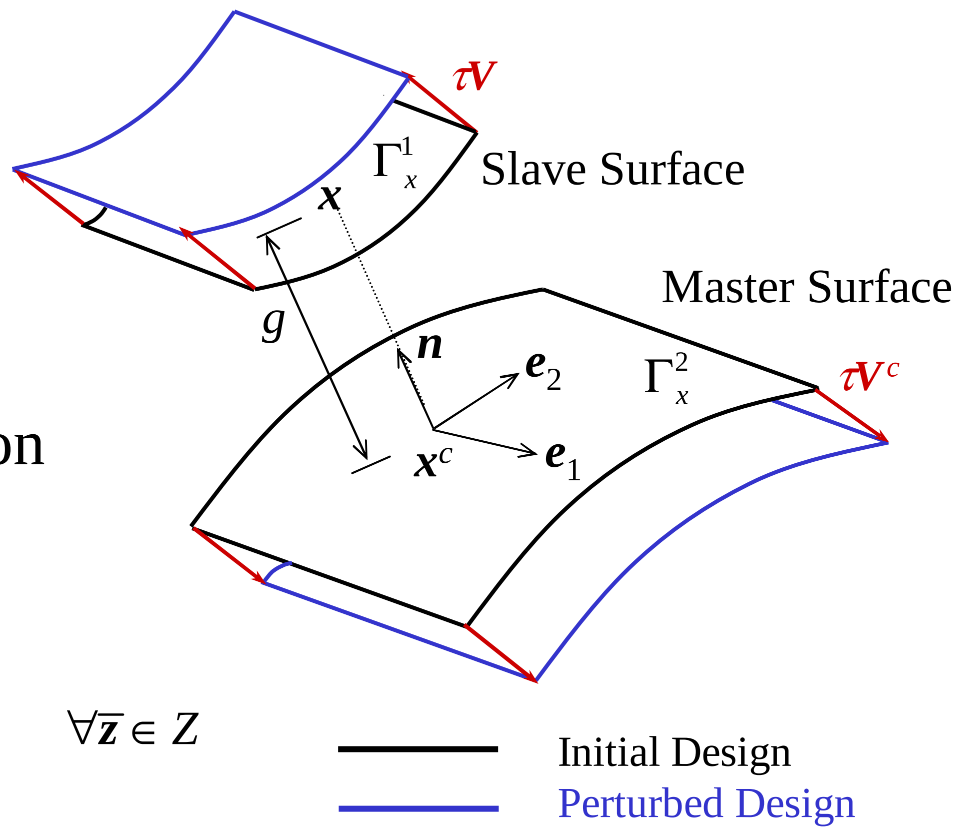
    $$g = \mathbf{n}\cdot(\mathbf{x}-\mathbf{x}^c) \geq 0$$

  - Consistency Condition

    $$\mathbf{e}_\alpha\cdot(\mathbf{x}-\mathbf{x}^c) = 0$$

  - Variational Equation

    $$a_\Omega(\mathbf{z},\bar{\mathbf{z}}) + b_N(\mathbf{z},\bar{\mathbf{z}}) = \ell_\Omega(\bar{\mathbf{z}}), \quad \forall\bar{\mathbf{z}}\in Z$$



3D CONTACT FORMULATION cont.

- Material Derivative Formulas

$$\frac{d}{d\tau}(\boldsymbol{x}_\tau)\Big|_{\tau=0} = \boldsymbol{V}(\boldsymbol{X}) + \dot{\boldsymbol{z}}(\boldsymbol{X})$$: Slave Particle

$$\frac{d}{d\tau}(\boldsymbol{x}_\tau^c)\Big|_{\tau=0} = \boldsymbol{V}^c(\boldsymbol{X}) + \dot{\boldsymbol{z}}^c + \boldsymbol{e}_\alpha \dot{\xi}_\alpha$$: Contact Point on Master Surface

- Material Derivative of the Contact Form

$$\frac{d}{d\tau}[b_N(\boldsymbol{z}_\tau, \overline{\boldsymbol{z}}_\tau)]\Big|_{\tau=0} \equiv b_N^*(\boldsymbol{z}; \dot{\boldsymbol{z}}, \overline{\boldsymbol{z}}) + b_N'(\boldsymbol{z}, \overline{\boldsymbol{z}})$$

$$b_N'(\boldsymbol{z}, \overline{\boldsymbol{z}}) = b_N^*(\boldsymbol{z}; \boldsymbol{V}, \overline{\boldsymbol{z}}) + \omega_N \int_{\Gamma_X^c} \kappa g \hat{\overline{\boldsymbol{z}}} \cdot \boldsymbol{n} V_n \, d\Gamma$$: Contact Fictitious Load

– $b_N^*(\boldsymbol{z}; \bullet, \bullet)$ is same as the tangent stiffness operator that appears in contact analysis. Thus, the contact fictitious load can be calculated readily.

FRICTIONAL CONTACT DSA

- Elastoplasticity-Type Friction Model
  - Trial Frictional Force

$$\boldsymbol{f}^{tr} = f_\alpha^{tr} \boldsymbol{e}^\alpha$$

$$f_\alpha^{tr} = f_\alpha^{n-1} - \omega_T M_{\alpha\gamma}(\xi_\gamma - \xi_\gamma^{n-1})$$

Relative Slip Amount

  - Frictional Consistency Condition

$$h = \left| f_\alpha^{tr} \right| - \left| \mu \omega_N g \right|$$

If $h \le 0$, Stick Condition

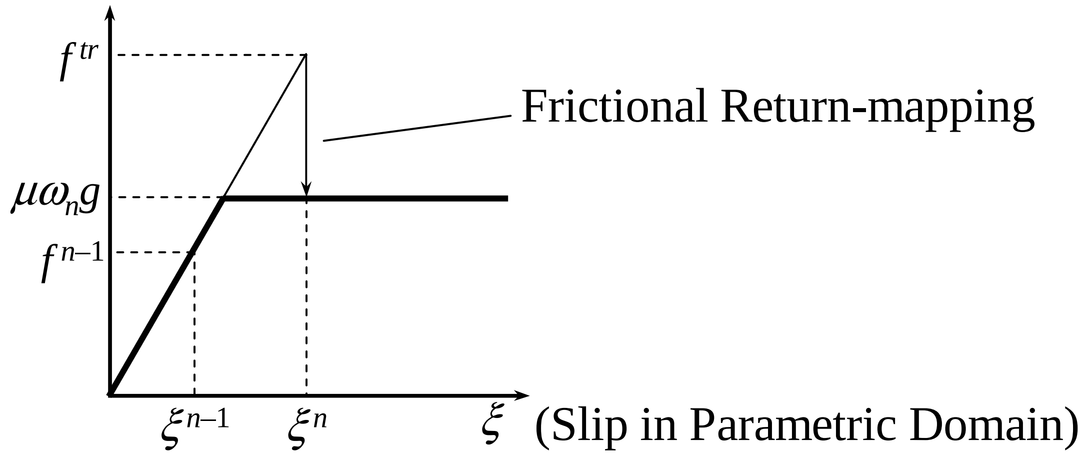
$$f_\alpha = f_\alpha^{tr}$$

Or else, Slip Condition

$$f_\alpha = \mu \omega_N g \, p_\alpha \qquad p_\alpha = f_\alpha^{tr} / \left\| \boldsymbol{f}^{tr} \right\|$$

Return-mapping algorithm in Elastoplasticity

FRICTIONAL CONTACT DSA cont.



– Frictional Form

$$b_T(\mathbf{z}, \overline{\mathbf{z}}) = \int_{\Gamma_X^c} f_\alpha \overline{\xi}_\alpha \, d\Gamma$$

– Material derivative of the frictional form that is consistent with the frictional return-mapping algorithm has to be taken.

FRICTIONAL CONTACT DSA cont.

- Material Derivative of the Stick Condition

$$\dot{f}_\alpha = \omega_T \Phi_{\alpha\beta} \xi_\beta(\dot{\mathbf{z}}) + \omega_T \Phi_{\alpha\beta} \xi_\beta(\mathbf{V}) + \dot{f}_\alpha^{n-1} + \omega_T M_{\alpha\beta} \xi_\beta^{n-1}(\dot{\mathbf{z}})$$

$$\Phi_{\alpha\beta} = M_{\alpha\beta} + M_{\alpha\gamma,\beta}(\xi_\gamma - \xi_\gamma^{n-1})$$

$$\frac{d}{d\tau}[b_T(\mathbf{z}, \overline{\mathbf{z}})]\Big|_{\tau=0} = b_T^*(\mathbf{z}; \dot{\mathbf{z}}, \overline{\mathbf{z}}) \quad \text{: Implicit Term}$$

$$+ b_T^*(\mathbf{z}; \mathbf{V}, \overline{\mathbf{z}}) \quad \text{: Explicit Term}$$

$$+ \int_{\Gamma_X^c} (\dot{f}_\alpha^{n-1} \overline{\xi}_\alpha + \omega_T M_{\alpha\beta} \overline{\xi}_\alpha \dot{\xi}_\beta^{n-1})\, d\Gamma \quad \text{: Path- Depenent Term}$$

– The expressions of implicit and explicit terms are same.

– The contact stiffness matrix from the response analysis can be used for DSA.

FRICTIONAL CONTACT DSA cont.

- Material Derivative of the Slip Condition

$$\dot{f}_\alpha = \mu\omega_N p_\alpha \boldsymbol{n}\cdot(\hat{\dot{\boldsymbol{z}}} + \hat{\boldsymbol{V}}) + \frac{\mu\omega_N g}{\|\boldsymbol{f}^{tr}\|}[\dot{f}_\alpha^{tr} - p_\alpha p^\beta \dot{f}_\beta^{tr} - f_\alpha^{tr} p_\alpha \boldsymbol{p}\cdot\dot{\boldsymbol{e}}^\beta]$$

$$\left.\frac{d}{d\tau}[b_T(\boldsymbol{z},\overline{\boldsymbol{z}})]\right|_{\tau=0} = b_T^*(\boldsymbol{z};\dot{\boldsymbol{z}},\overline{\boldsymbol{z}}) \left. \begin{aligned} &+ b_T^*(\boldsymbol{z};\boldsymbol{V},\overline{\boldsymbol{z}}) \\ &+ \mu\omega_N \int_{\Gamma_X^c} \frac{g\overline{\xi}_\beta}{\|\boldsymbol{f}^{tr}\|}(\delta_\alpha^\beta - p_\alpha p^\beta)(\dot{f}_\alpha^{n-1} + \omega_T M_{\alpha\gamma}\dot{\xi}_\gamma^{n-1})\, d\Gamma \end{aligned} \right\} b_T'(\boldsymbol{z},\overline{\boldsymbol{z}})$$

  - $\dot{f}_\alpha^{tr}$ has the same expression as $\dot{f}_\alpha$ in the stick condition.
  - The sensitivities of the frictional force and displacement contribute to the path-dependency of frictional contact DSA.

FRICTIONAL CONTAT DSA cont.

- Design Sensitivity Equation

$$a^{*}_{\Omega}(\mathbf{z};\dot{\mathbf{z}},\overline{\mathbf{z}}) + b^{*}_{\Gamma}(\mathbf{z};\dot{\mathbf{z}},\overline{\mathbf{z}}) = \ell'_{V}(\overline{\mathbf{z}}) - a'_{V}(\mathbf{z},\overline{\mathbf{z}}) - b'_{V}(\mathbf{z},\overline{\mathbf{z}}), \quad \forall \overline{\mathbf{z}} \in Z$$

  – Contact Fictitious Load

$$b'_{V}(\mathbf{z},\overline{\mathbf{z}}) = b'_{N}(\mathbf{z},\overline{\mathbf{z}}) + b'_{T}(\mathbf{z},\overline{\mathbf{z}})$$

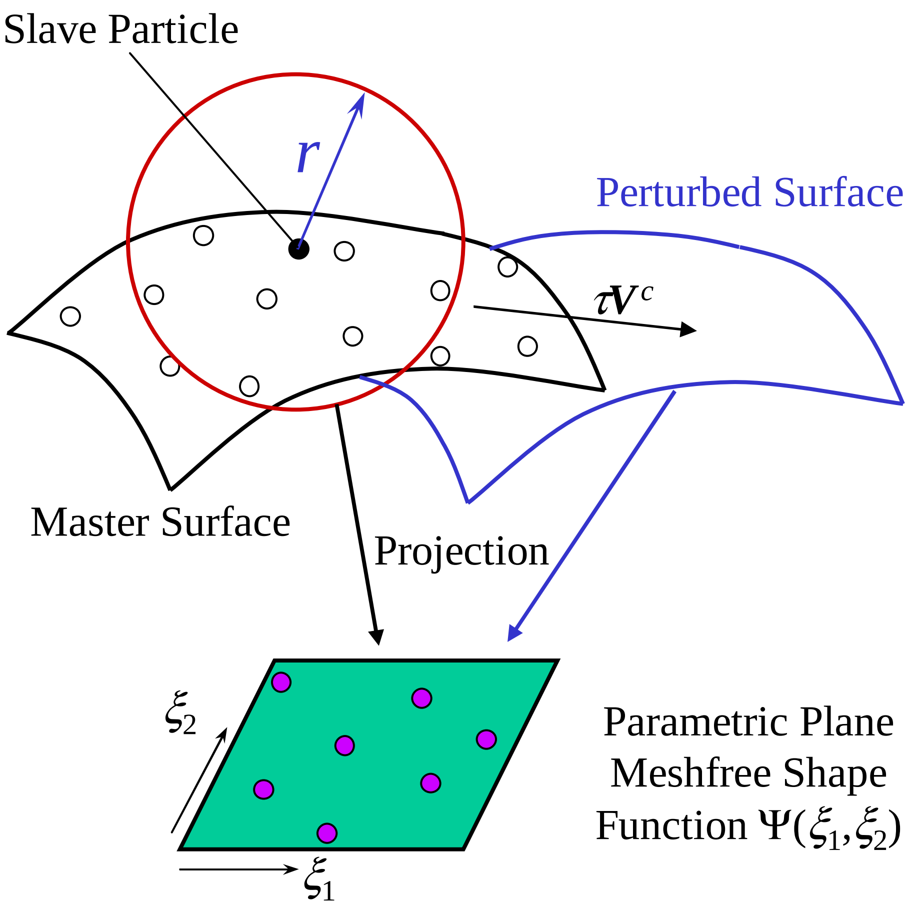
Normal Contact (b'_{N}) — Frictional Slip (b'_{T})

– Path-dependency comes from the tangential friction. (Frictional force and contact particle displacement)

– The same tangent stiffness matrix from response analysis is used for DSA.

SMOOTH CONTACT SURFACE

- Construction of C^2-continuous surface from a scattered set of particles.
- A design independent parametric plane is generated using surrounding particles.
- Meshfree shape function is independent of die shape design parameters.



- Meshfree Interpolation

$$\boldsymbol{x}(\xi_1,\xi_2)=\sum_{I=1}^{NP}\Psi_I(\xi_1,\xi_2)\boldsymbol{x}_I$$

$$\boldsymbol{x}_{,\alpha}(\xi_1,\xi_2)=\sum_{I=1}^{NP}\frac{d\Psi_I(\xi_1,\xi_2)}{d\xi_\alpha}\boldsymbol{x}_I$$

SMOOTH CONTACT SURFACE cont.

- Meshfree shape function in the local parametric domain

$$\Psi_I(\xi_1,\xi_2) = \boldsymbol{H}^T(0,0)\boldsymbol{M}^{-1}(\xi_1,\xi_2)\boldsymbol{H}(\xi_1-\xi_1^I,\xi_2-\xi_2^I)\Phi_a(\xi_1-\xi_1^I,\xi_2-\xi_2^I)$$

Moment Matrix

Kernel Function

- The continuum-based design sensitivity formulation is independent of the surface generation algorithm. Only information that is already available from response analysis is necessary.

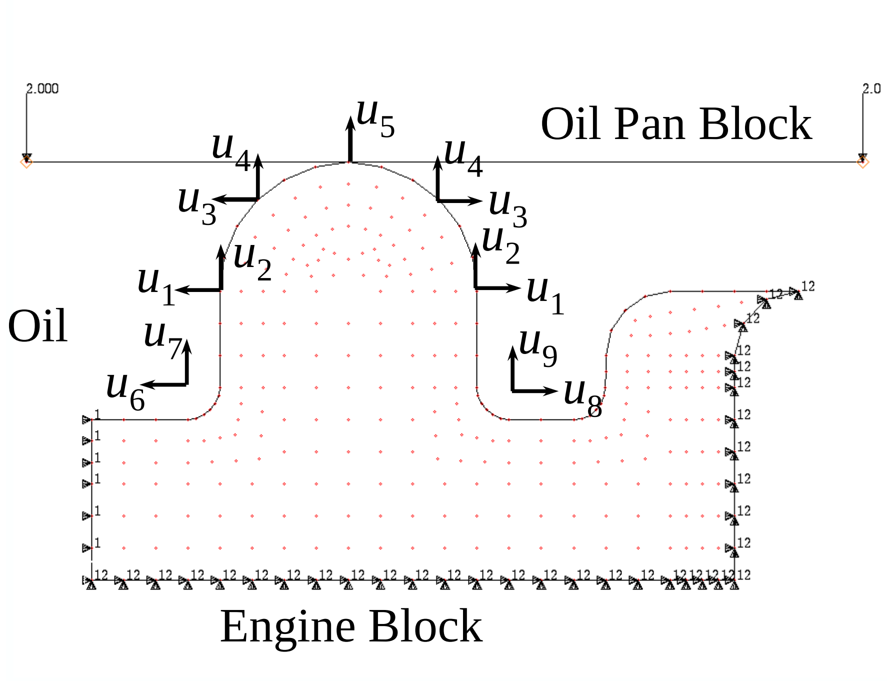
GASKET SHAPE OPTIMIZATION

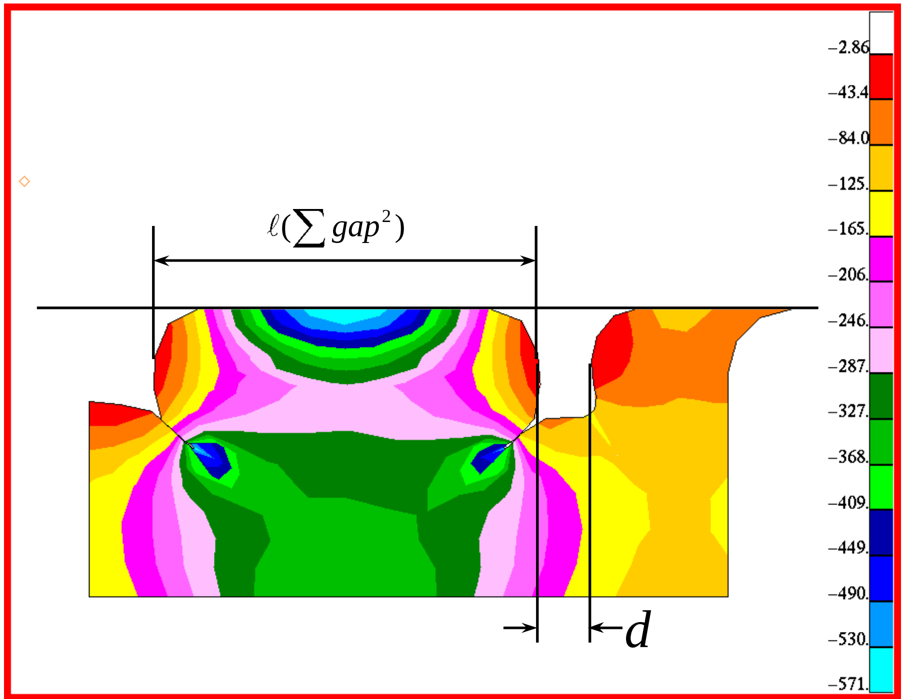
– Oil Pan Gasket to Reduce Leakage

– Mooney-Rivlin Rubber Material

– Flexible-Rigid Body Contact and Self-Contact Conditions

– Significant Distortion in Self-Contact Regions

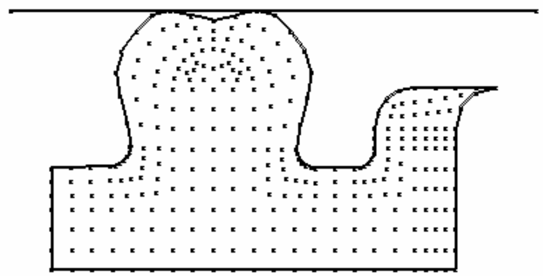


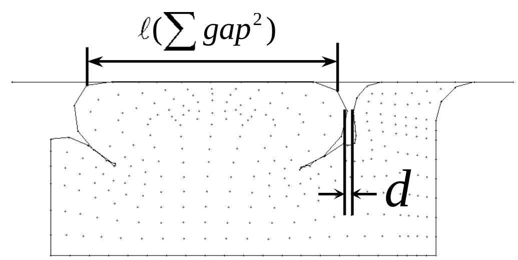


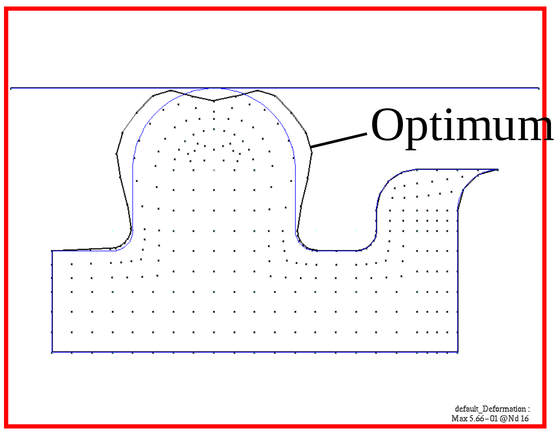
DESIGN OPTIMIZATION

Optimization Problem 1

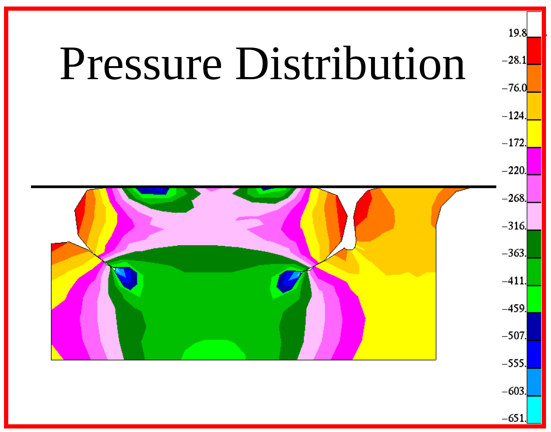
$$\begin{aligned} &\min && d \\ &s.t. && |F_C| \geq 300\ kN \\ & && \sigma \leq 1700\ kPa \\ & && \sum gap^2 \leq 1.0\ mm \\ & && -0.5 \leq u_i \leq 0.5 \end{aligned}$$



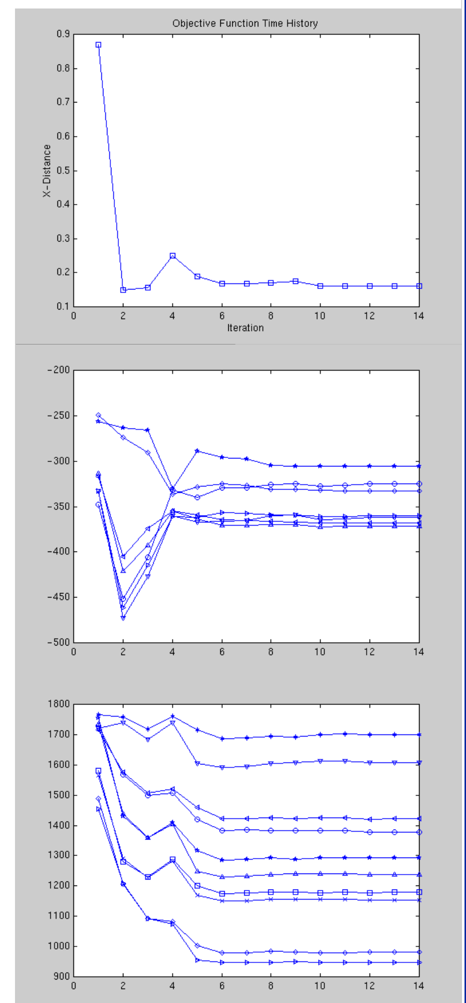




Optimum Design



Optimization History



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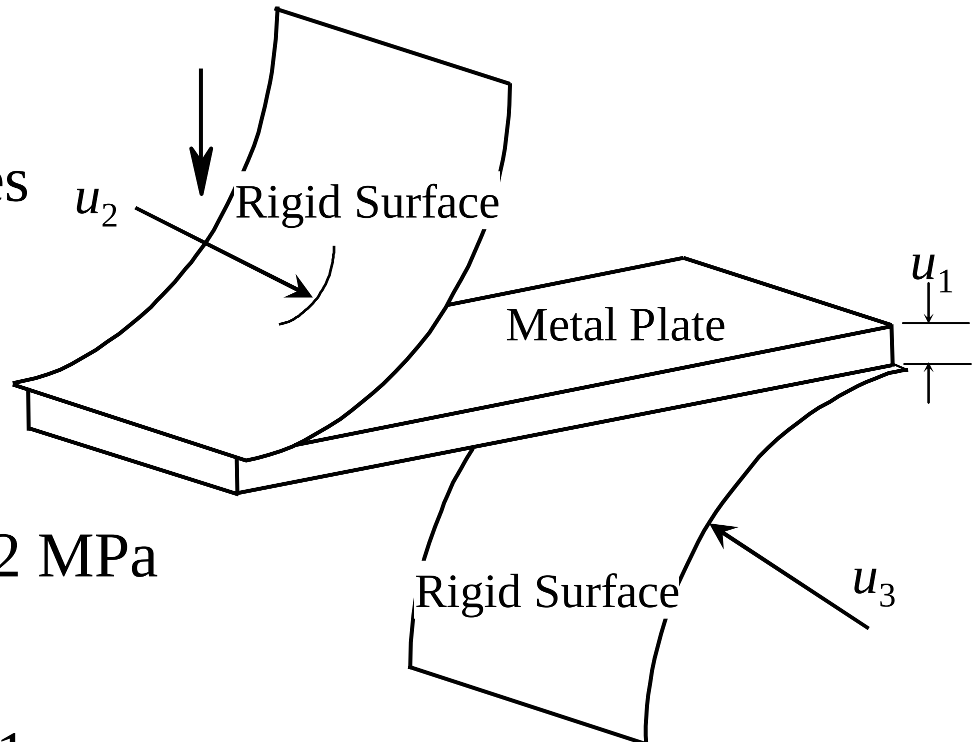
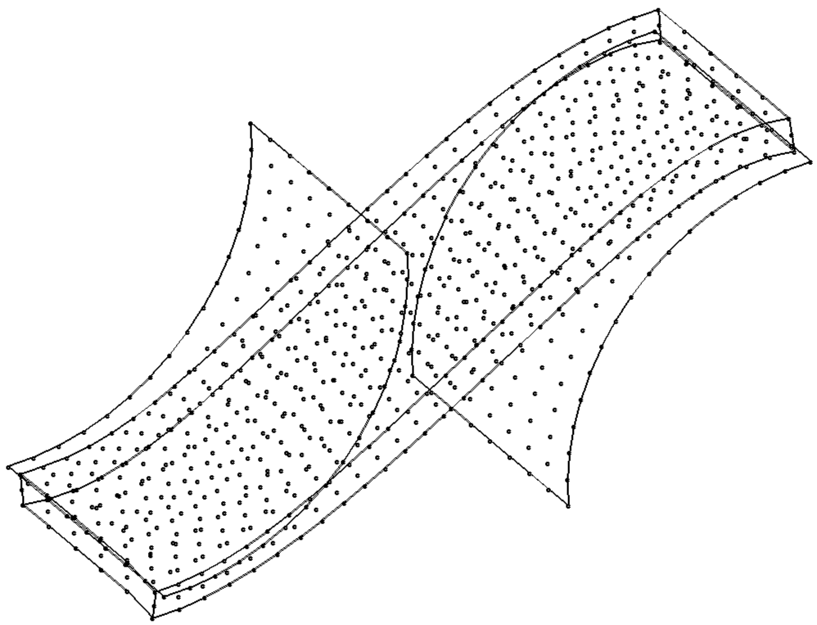
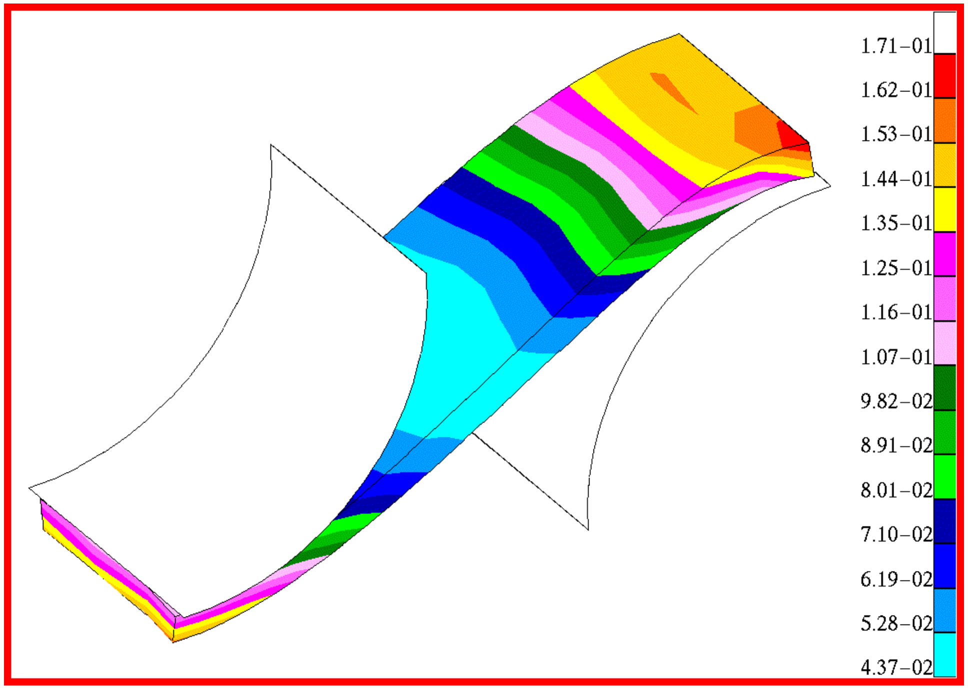
METAL PUNCH PROBLEM

- 558 Meshfree Particles
  (1,674 DOF)
- 306 Rigid Surface Particles
  (Smooth Surface)
- E = 207 GPa, ν = 0.29
- Yield Stress = 167 MPa
- Isotropic Hardening = 77.2 MPa
- Penalty Parameter = 10^{10}
- Frictional Coefficient = 0.1



MESHFREE ANALYSIS RESULT



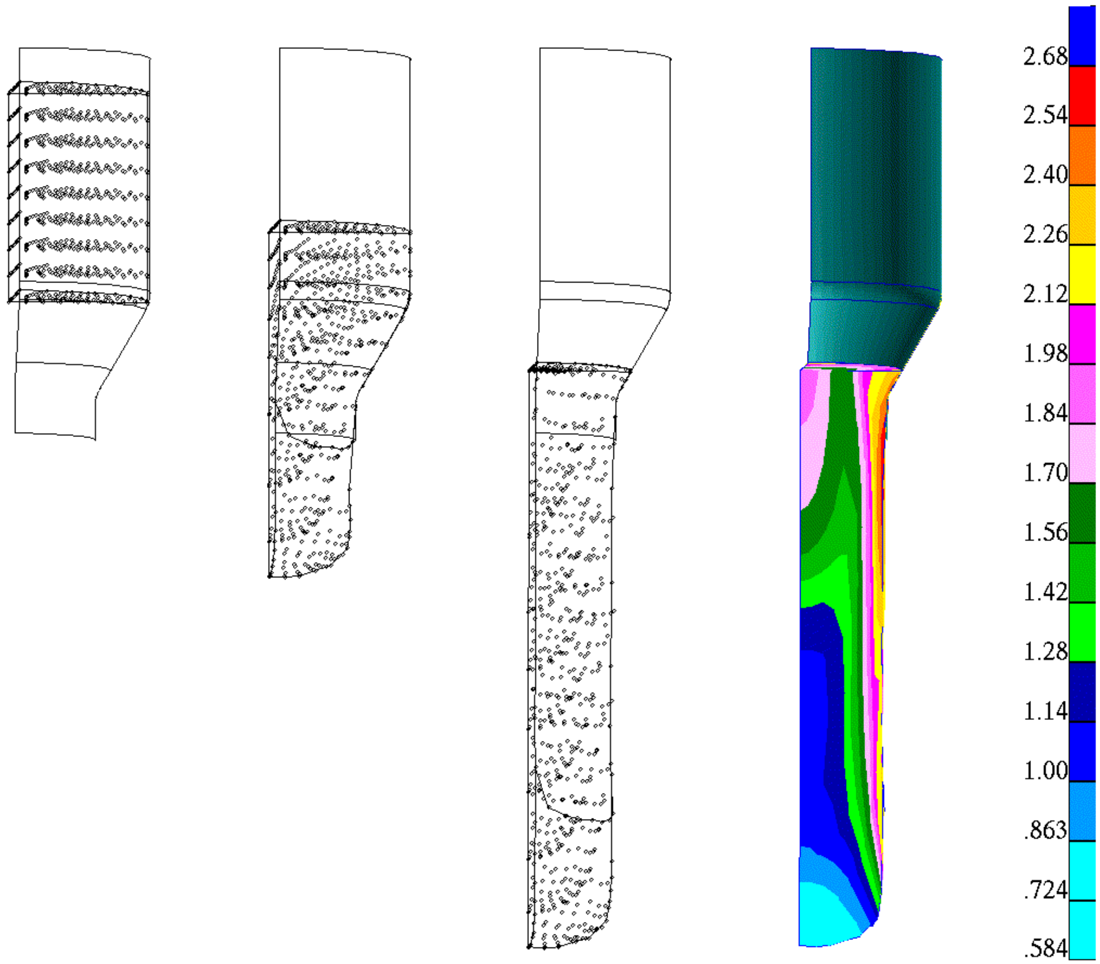


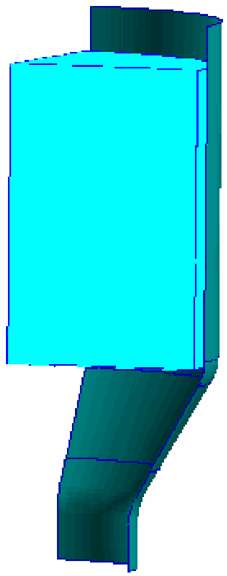
Effective Plastic Strain Plot

DESIGN SENSITIVITY RESULTS

Design	Performance	ψ	$\Delta\psi$	$\psi'\Delta\tau$	$\Delta\psi / \psi'\Delta\tau \times 100$
u_1	z_{344}	-7.66403	-8.74269E-7	-8.74284E-7	100.00
	z_{319}	-6.46946	-8.41994E-7	-8.42008E-7	100.00
	z_{286}	-4.33574	-5.13241E-7	-5.13267E-7	100.00
	z_{283}	-4.36168	-5.64229E-7	-5.64249E-7	100.00
	z_{312}	-5.77780	-7.17279E-7	-7.17305E-7	100.00
u_2	z_{344}	-7.66403	1.01763E-6	1.01722E-6	100.04
	z_{319}	-6.46946	9.34760E-7	9.34128E-7	100.07
	z_{286}	-4.33574	6.24759E-7	6.23955E-7	100.13
	z_{283}	-4.36168	6.36105E-7	6.35389E-7	100.11
	z_{312}	-5.77780	8.39500E-7	8.38630E-7	100.10
u_3	z_{344}	-7.66403	-1.73855E-7	-1.73653E-7	100.12
	z_{319}	-6.46946	-5.02555E-7	-5.02153E-7	100.08
	z_{286}	-4.33574	-1.46946E-6	-1.46873E-6	100.05
	z_{283}	-4.36168	-1.13062E-6	-1.12997E-6	100.06
	z_{312}	-5.77780	-1.28150E-6	-1.28086E-6	100.05

METAL EXTRUSION PROBLEM





METAL EXTRUSION PROBLEM cont.

- Area Reduction Ratio

  2.65

- Maximum Plastic Strain

  2.68

- Length Extension Ratio

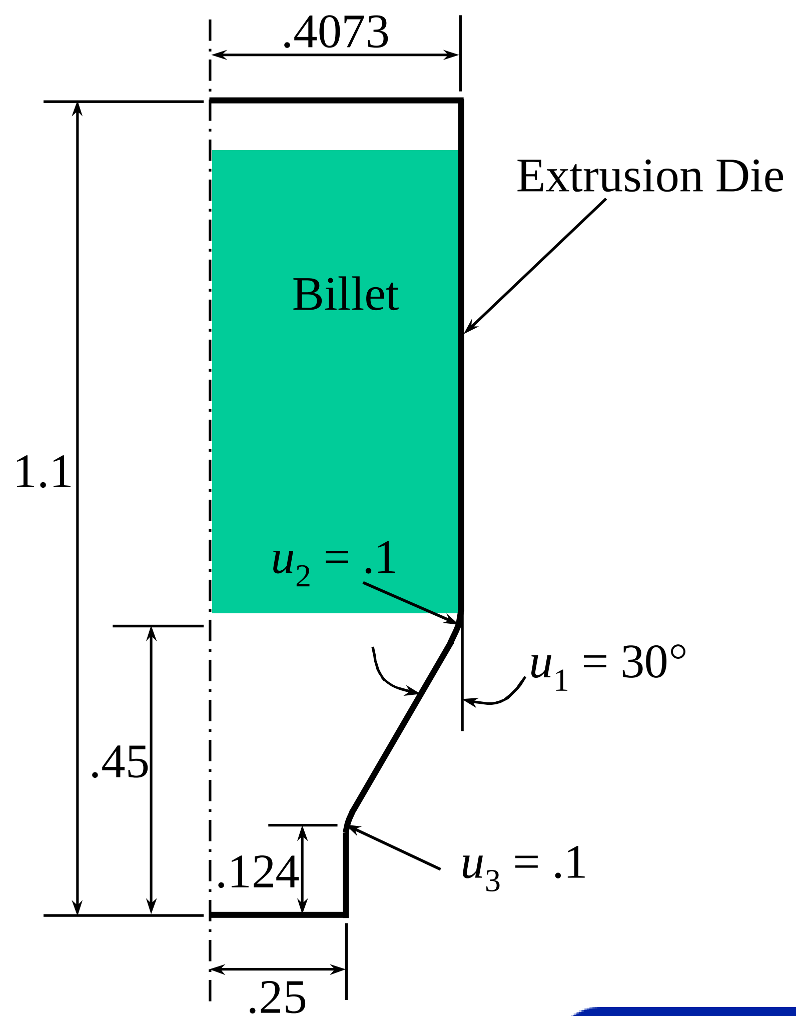
  2.76

DESIGN PARAMETERIZATION

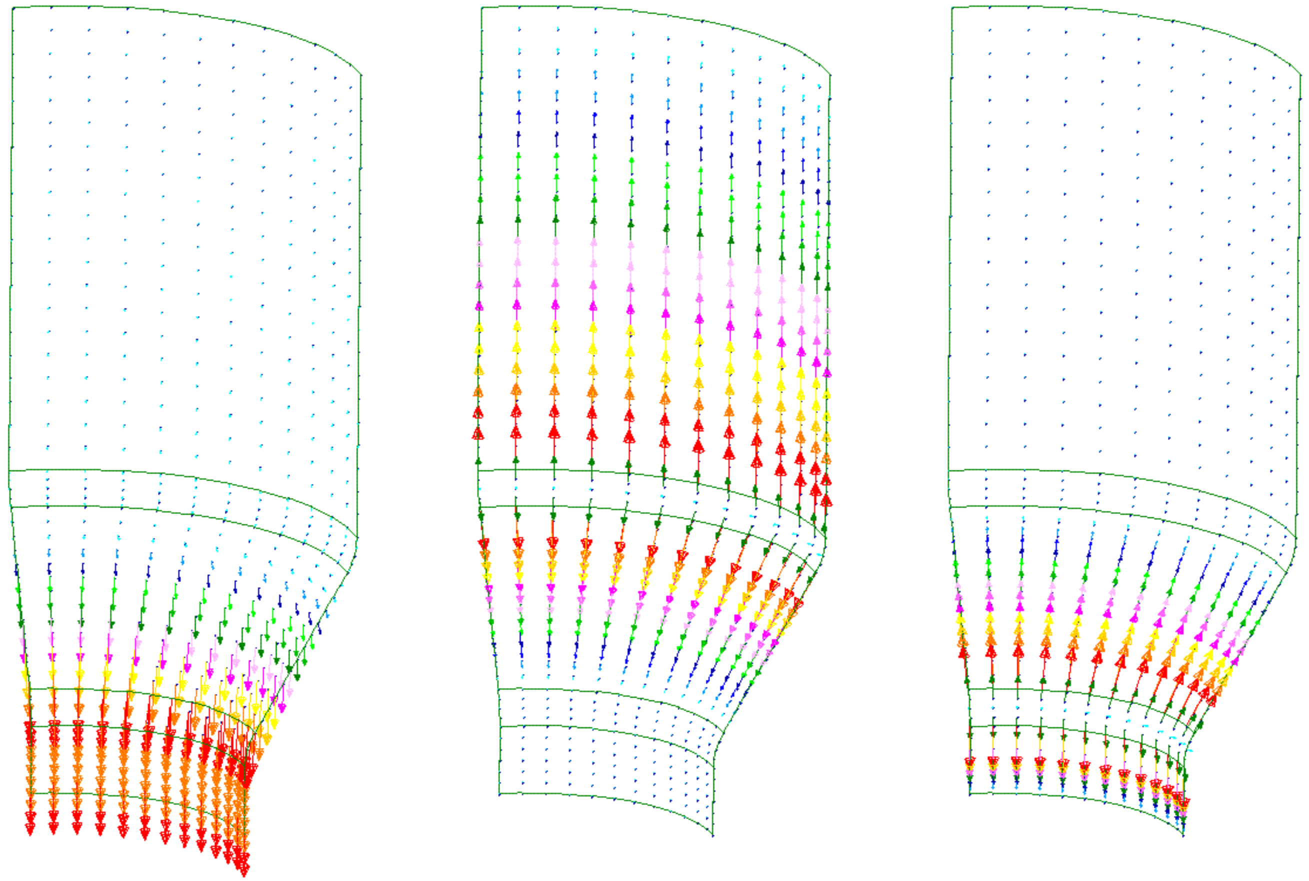
- CAD-Based Design Parameterization
- Die Shape Design Parameters
- Initial Billet Length = 0.6
- Final Billet Length = 1.6583

  → 276% Extension

- u_1: Die Angle
- u_2: Fillet Radius 1
- u_3: Fillet Radius 2



DESIGN VELOCITY FIELDS



Design: u_1 u_2 u_3

DESIGN SENSITIVITY RESULTS

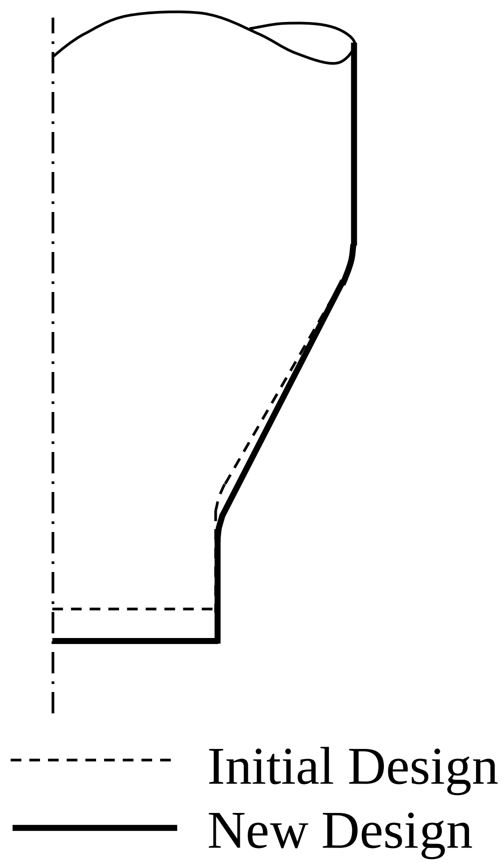
Design Parameter	Process Work Sensitivity	Plastic Strain Sensitivity
u_1	–5.50E–3	–5.32E–2
u_2	6.60E–4	–6.24E–3
u_3	–6.65E–5	–1.10E–4

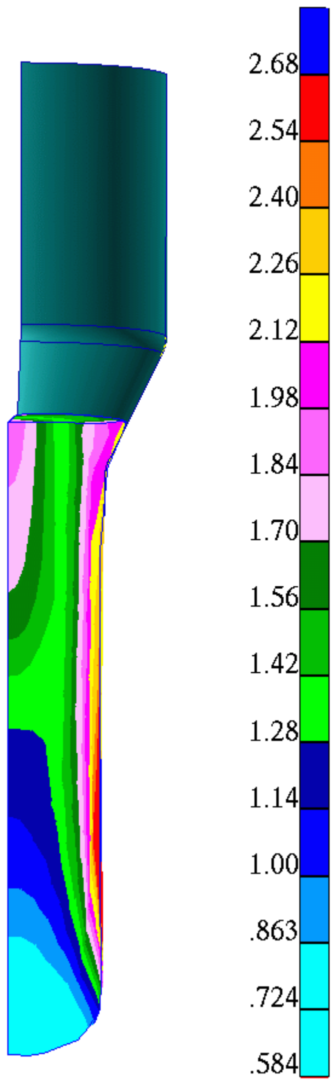
$$\frac{\text{DSA Cost}}{\text{Finite Difference Cost}} = 0.2$$

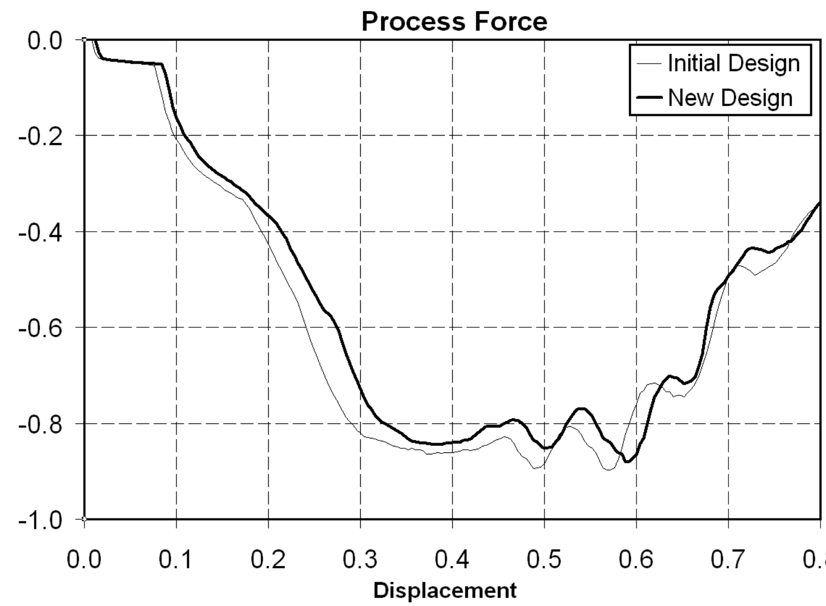
Design Optimization Problem

Minimize	Process Work (W)
Subject to	Effective Plastic Strain (e_p) ≤ 2.68

DESIGN IMPROVEMENT







Process Force Plot

Process Work

$$\frac{\text{New Design}}{\text{Initial Design}} = 0.945$$

CONCLUSIONS

- DSA and optimization of the frictional contact problem is presented by using the continuum approach.
- The material derivative that is consistent with the frictional return mapping algorithm is derived.
- The smooth contact surface is used in the sensitivity formulation with design independent meshfree interpolation function.
- Numerical examples show the efficiency and accuracy of the proposed sensitivity calculation method.
- The current solid-based design approach will be further extended to the nonlinear shell structure.