Constructing Probabilistic Safety Envelope for Load Tolerance
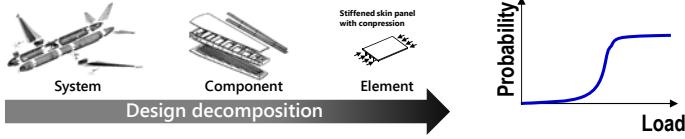## WCSMO 10 Poster Exhibition

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Introduction

- **Complex system design needs to be decomposed into various components/elements**complexity
- **Load tolerance of structural element is important information for evaluating design**
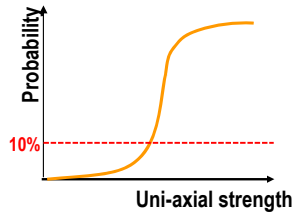- **Due to various uncertainties, load tolerance has to be defined probabilistically**

Objectives

(1) Constructing safety envelope for load tolerance with considering aleatory and epistemic uncertainties with respect to applied loads
(2) Incorporating structural test results for reducing uncertainty in constructing safety envelope
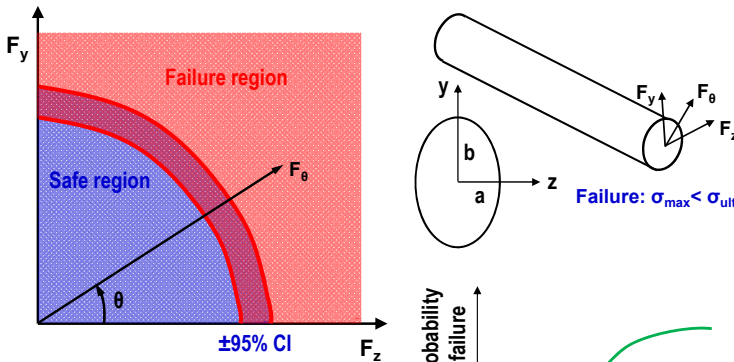
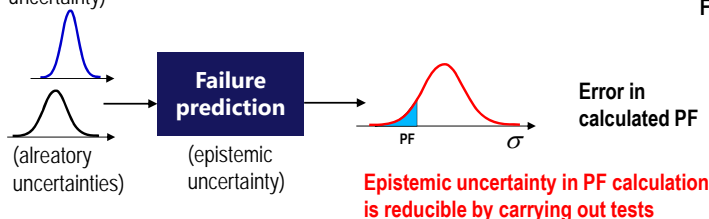## Design allowable of material strength (B-basis)

Design allowable
(1) Conservative estimate of uni-axial strength
(2) **Aleatory uncertainty** due to natural variability
(3) **Epistemic uncertainty** due to finite sample size
(4) Estimating 10th percentile (aleatory uncertainty) of material strength distribution with 95% confidence (epistemic uncertainty)

Probabilistic safety envelope for load tolerance

Failure: $\sigma_{max} < \sigma_{ult}$

(1) Due to aleatory uncertainty (variability in **a**, **b** and σ_{ult}, there is no clear border line
(2) Probability of failure for given load is estimated
(3) Numerical simulation to estimate the envelope causes error, numerical simulation has to be used (epistemic uncertainty)
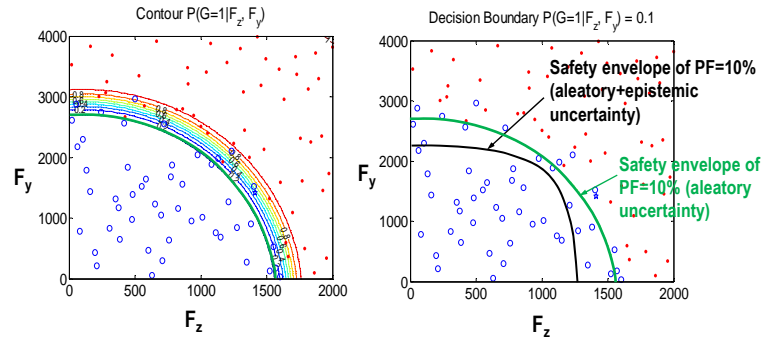

(aleatory uncertainties) → **Failure prediction** (epistemic uncertainty) → Error in calculated PF

Epistemic uncertainty in PF calculation is reducible by carrying out tests

Estimating probabilistic safety envelope with respect to aleatory uncertainty

Probabilistic classification
(1) Estimating safety envelope involving **aleatory uncertainty**
(2) $P(G=1|F_z,F_y)$ is the probability of failure for given load F_z and F_y
(3) Discriminated approach to construct the envelope

$$P\left(G = 1 \mid F_z, F_y\right) = \frac{1}{1 + \exp\left(-w^T \phi(F_z, F_y)\right)}$$

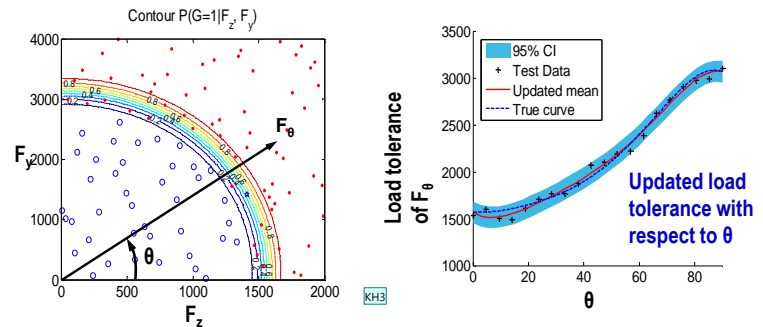G=0: Safe,
G=1: Failed;

However, the safety envelope has error due to epistemic uncertainty

Error in prediction with respect to epistemic uncertainty

Estimating error in failure prediction
(1) Estimating safety envelope involving error in failure prediction (normal distr., COV=5%)
(2) The band represents error in predicting the safety envelope

Updating the error in predicting the safety envelope with test data
(1) Extracting F_θ versus θ from the predicted failure envelope
(2) Incorporating test results to reduce the error in the predicted safety envelope

Conclusions

Safety envelope for load tolerance can be estimated
(1) Probabilistic classification method is used to estimate uncertainty in estimating safety envelope for tolerance of combined loads
(2) Structural test result can be incorporated by separating the contribution of aleatory and epistemic uncertainty

Future works

Updating the error distribution with test results
(1) Gaussian process will be used to update the error distribution
(2) Efficient sampling method for estimating probabilistic safety envelope regarding aleatory uncertainty will be investigated

KH1 I don't know why we need this

KH2 Right now, it is confusing because the plot on the right has epistemic uncertainty curve (black), and yet, you said "however, the safety envelope has error due to epistemic uncertainty"

KH3 Show how you included epistemic uncertainty in equation. Is left graph before or after test? It is important to show (at least one theta), what's the uncertainty before, after tests.