DESIGN SENSITIVITY ANALYSIS for MESHFREE SHELL STRUCTURE

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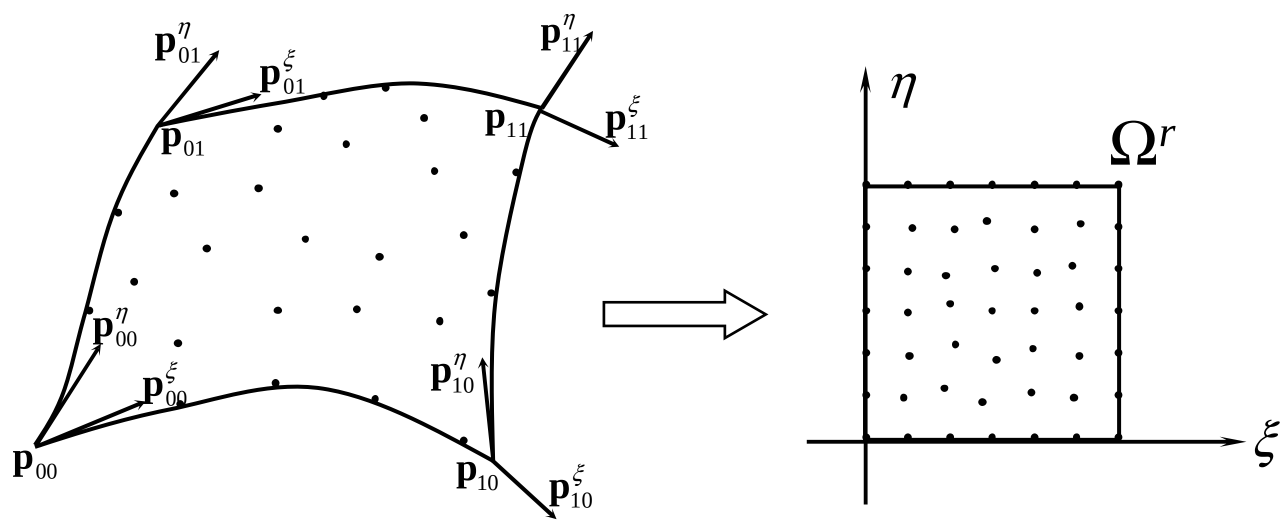
PARAMETERIZATION OF SHELL SURFACE



Continuous Surface Geometry

Parametric Domain

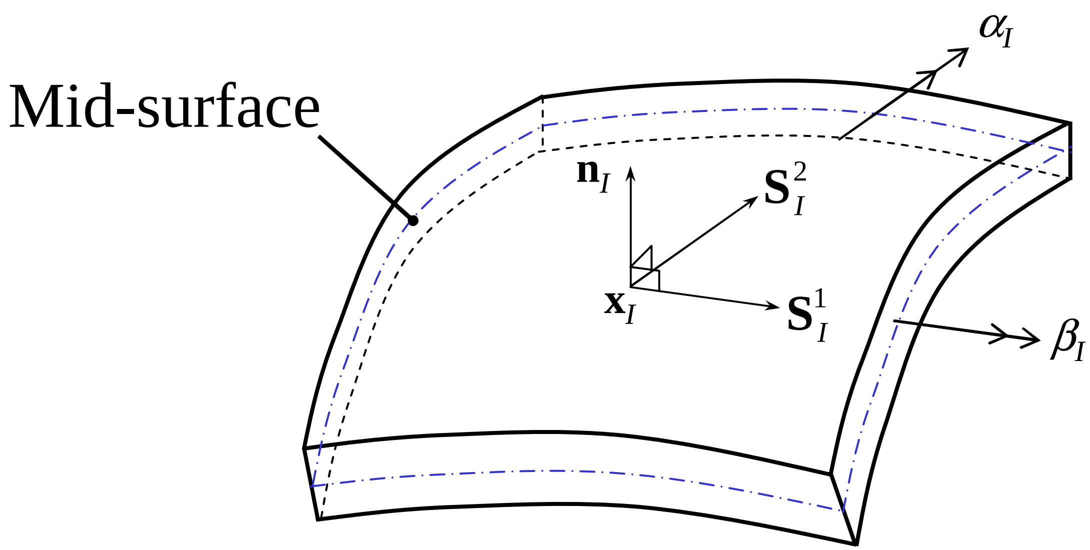
$$\mathbf{x}^n(\xi,\eta) = \mathbf{U}(\xi)^T \mathbf{MGM}^T \mathbf{W}(\eta)$$

$$\mathbf{U}(\xi) = [\xi^3, \xi^2, \xi, 1]^T$$

$$\mathbf{W}(\eta) = [\eta^3, \eta^2, \eta, 1]^T$$

$$\mathbf{G}(u) = \begin{bmatrix} \mathbf{p}_{00}(u) & \mathbf{p}_{01}(u) & \mathbf{p}_{00}^{\eta}(u) & \mathbf{p}_{01}^{\eta}(u) \\ \mathbf{p}_{10}(u) & \mathbf{p}_{11}(u) & \mathbf{p}_{10}^{\eta}(u) & \mathbf{p}_{11}^{\eta}(u) \\ \mathbf{p}_{00}^{\xi}(u) & \mathbf{p}_{01}^{\xi}(u) & \mathbf{p}_{00}^{\xi\eta}(u) & \mathbf{p}_{01}^{\xi\eta}(u) \\ \mathbf{p}_{10}^{\xi}(u) & \mathbf{p}_{11}^{\xi}(u) & \mathbf{p}_{10}^{\xi\eta}(u) & \mathbf{p}_{11}^{\xi\eta}(u) \end{bmatrix}_{4\times4\times3}$$

SHEAR-DEFORMABLE SHELL



$$\mathbf{S}_I^1 = \mathbf{n}_I \times \mathbf{e}_1$$

$$\mathbf{S}_I^2 = \mathbf{n}_I \times \mathbf{S}_I^1$$

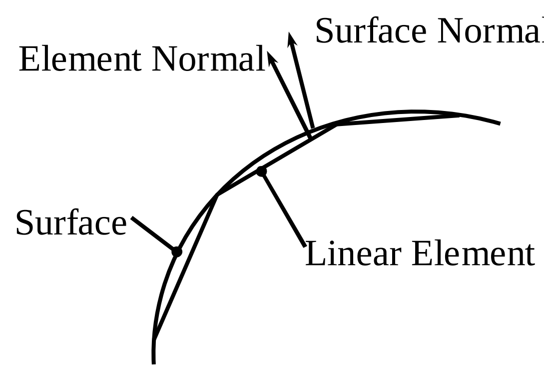
– Accurate Geometry Representation

$$\mathbf{x}(\xi,\eta,\zeta) = \mathbf{U}(\xi)^T \mathbf{MGM}^T \mathbf{W}(\eta) + \zeta \frac{t}{2} \mathbf{n}(\xi,\eta)$$

$$\mathbf{x}_{,\xi}^n = \mathbf{U}_{,\xi}(\xi)^T \mathbf{MGM}^T \mathbf{W}(\eta)$$

$$\mathbf{x}_{,\eta}^n = \mathbf{U}(\xi)^T \mathbf{MGM}^T \mathbf{W}_{,\eta}(\eta)$$

$$\mathbf{n}(\xi,\eta) = \frac{\mathbf{x}_{,\xi}^n \times \mathbf{x}_{,\eta}^n}{\left\| \mathbf{x}_{,\xi}^n \times \mathbf{x}_{,\eta}^n \right\|}$$



MESHFREE SHELL FORMULATION

❑ Continuum Form

– Variational Equation

$$a(\mathbf{z},\overline{\mathbf{z}}) = \ell(\overline{\mathbf{z}}), \qquad \forall \overline{\mathbf{z}} \in Z$$

– Structural Energy Form

$$a(\mathbf{z},\overline{\mathbf{z}}) = 2\iint_{\Omega^r} \varepsilon_{ij}^1(\overline{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^1(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r + \tfrac{2}{3}\iint_{\Omega^r} \varepsilon_{ij}^2(\overline{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^2(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r$$

– Load Linear Form

$$\ell(\overline{\mathbf{z}}) = 2\iint_{\Omega^r} \overline{\mathbf{z}}^T \mathbf{f}^B \left|\mathbf{J}\right| d\Omega^r$$

$$\mathbf{z}(\xi,\eta,\zeta) = \mathbf{z}^1(\xi,\eta) + \zeta \mathbf{z}^2(\xi,\eta)$$

Displacement Function

$$\varepsilon_{ij}^1(\mathbf{z}) = sym\left(\frac{\partial z_i^1}{\partial \xi_m} J_{mj}^{-1} + z_i^2 J_{3j}^{-1}\right)$$

Membrane–Shear Strain

$$\varepsilon_{ij}^2(\mathbf{z}) = sym\left(\frac{\partial z_i^2}{\partial \xi_m} J_{mj}^{-1}\right)$$

Bending Strain

- Analytically Integrated over Thickness Direction
- Shell Surface Is Transformed into the Parametric Plane Ω^r

SHELL FORMULATION (cont')

❑ Meshfree Discretization

$$\mathbf{z}(\xi,\eta,\zeta)=\sum_{I=1}^{IP}\Psi_I(\xi,\eta)\mathbf{d}_I+\sum_{I=1}^{IP}\Psi_I(\xi,\eta)\frac{t_I}{2}\zeta[\mathbf{S}_I^{1^T},-\mathbf{S}_I^{2^T}]\begin{bmatrix}\alpha_I\\ \beta_I\end{bmatrix}$$

$$\boldsymbol{\varepsilon}^1(\mathbf{z})=\mathbf{S}_q\mathbf{T}\sum_{I=1}^{IP}\mathbf{G}_I^1\mathbf{r}_I\equiv\sum_{I=1}^{IP}\mathbf{B}_I^1\mathbf{r}_I$$

$$\boldsymbol{\varepsilon}^2(\mathbf{z})=\mathbf{S}_q\mathbf{T}\sum_{I=1}^{IP}\mathbf{G}_I^2\mathbf{r}_I\equiv\sum_{I=1}^{IP}\mathbf{B}_I^2\mathbf{r}_I$$

$$\mathbf{r}_I=\begin{bmatrix}d_{I1}\\ d_{I2}\\ d_{I3}\\ \alpha_I\\ \beta_I\end{bmatrix}_I$$

$$\mathbf{G}_I^1=\begin{bmatrix}\Psi_{I,\xi}&0&0&0&0\\ \Psi_{I,\eta}&0&0&0&0\\ 0&0&0&\frac{1}{2}\Psi_I t_I S_{Ix}^1&-\frac{1}{2}\Psi_I t_I S_{Ix}^2\\ 0&\Psi_{I,\xi}&0&0&0\\ 0&\Psi_{I,\eta}&0&0&0\\ 0&0&0&\frac{1}{2}\Psi_I t_I S_{Iy}^1&-\frac{1}{2}\Psi_I t_I S_{Iy}^2\\ 0&0&\Psi_{I,\xi}&0&0\\ 0&0&\Psi_{I,\eta}&0&0\\ 0&0&0&\frac{1}{2}\Psi_I t_I S_{Iz}^1&-\frac{1}{2}\Psi_I t_I S_{Iz}^2\end{bmatrix}$$

$$\mathbf{G}_I^2=\begin{bmatrix}0&0&0&\frac{1}{2}\Psi_{I,\xi}t_I S_{Ix}^1&-\frac{1}{2}\Psi_{I,\xi}t_I S_{Ix}^2\\ 0&0&0&\frac{1}{2}\Psi_{I,\eta}t_I S_{Ix}^1&-\frac{1}{2}\Psi_{I,\eta}t_I S_{Ix}^2\\ 0&0&0&0&0\\ 0&0&0&\frac{1}{2}\Psi_{I,\xi}t_I S_{Iy}^1&-\frac{1}{2}\Psi_{I,\xi}t_I S_{Iy}^2\\ 0&0&0&\frac{1}{2}\Psi_{I,\eta}t_I S_{Iy}^1&-\frac{1}{2}\Psi_{I,\eta}t_I S_{Iy}^2\\ 0&0&0&0&0\\ 0&0&0&\frac{1}{2}\Psi_{I,\xi}t_I S_{Iz}^1&-\frac{1}{2}\Psi_{I,\xi}t_I S_{Iz}^2\\ 0&0&0&\frac{1}{2}\Psi_{I,\eta}t_I S_{Iz}^1&-\frac{1}{2}\Psi_{I,\eta}t_I S_{Iz}^2\\ 0&0&0&0&0\end{bmatrix}$$

SHELL FORMULATION (cont')

- Global Stiffness Matrix and Load Vector

$$[\mathbf{K}] = \sum_{L=1}^{NP} (\mathbf{K}_L^1 + \mathbf{K}_L^2)$$

$$\left[\mathbf{K}_L^1\right] = \sum_{I=1}^{IP} \sum_{J=1}^{IP} 2\mathbf{B}_I^1(\mathbf{x}_L)^T \mathbf{C}\mathbf{B}_J^1(\mathbf{x}_L)\left|\mathbf{J}\right| A_L$$

$$\left[\mathbf{K}_L^2\right] = \sum_{I=1}^{IP} \sum_{J=1}^{IP} \tfrac{2}{3}\mathbf{B}_I^2(\mathbf{x}_L)^T \mathbf{C}\mathbf{B}_J^2(\mathbf{x}_L)\left|\mathbf{J}\right| A_L$$

$$\{\mathbf{F}\} = \sum_{L=1}^{NP} \mathbf{F}_L$$

$$\left[\mathbf{F}_L\right] = \sum_{I=1}^{IP} 2\Psi_I(\mathbf{x}_L)\mathbf{f}^B(\mathbf{x}_L)\left|\mathbf{J}\right| A_L$$

$$[\mathbf{K}]\{\mathbf{r}\} = \{\mathbf{F}\}$$

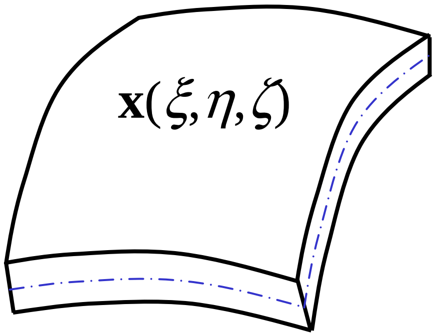
- Meshfree Shape Function Is Constructed on Parametric Plane
- Direct Nodal Integration for Membrane–Shear Part and SC Nodal Integration for Bending Part

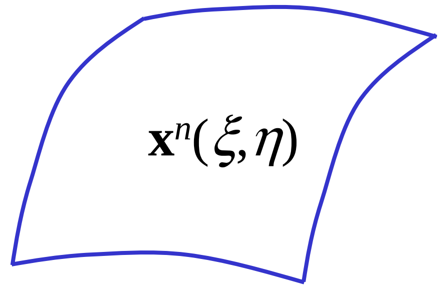
UNIFIED DESIGN SENSITIVITY FORMULATION

- A Unified Design Sensitivity Formulation Is Obtained for Sizing, Shape, and Configuration Design Variables
- Design Velocity Field Is Defined in the Continuum Domain and Then Degenerated Using Shell Kinematics
- Parametric Domain Ω^r Is Independent of Design Perturbation
- Local to Global Coord. Transformation Depends on Design Explicitly
- Design Velocity of Neutral Surface Is Obtained by Perturbing CAD Geometric Matrix

$$\mathbf{V}_u^n(\xi,\eta) = \left.\frac{d\mathbf{x}^n(u+\tau\delta u)}{d\tau}\right|_{\tau=0} = \mathbf{U}(\xi)^T\mathbf{M}(\frac{\partial \mathbf{G}}{\partial u}\delta u)\mathbf{M}^T\mathbf{W}(\eta)$$

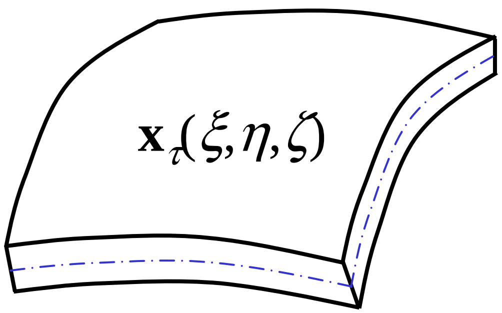
UNIFIED DESIGN VELOCITY FIELD

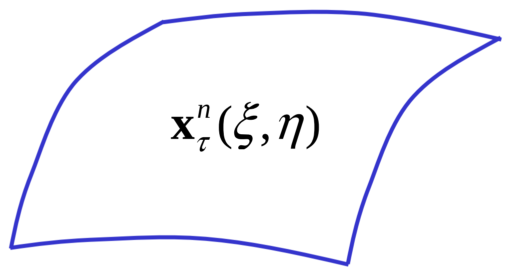




$$\mathbf{x}(\xi,\eta,\zeta) = \mathbf{x}^n(\xi,\eta) + \zeta\frac{t(\xi,\eta)}{2}$$

Design Change ⇓ ⇓ ⇓





$$\mathbf{x}_\tau(\xi,\eta,\zeta) = \mathbf{x}_\tau^n(\xi,\eta) + \zeta\frac{t(\xi,\eta)+\tau\delta t(\xi,\eta)}{2}$$

$$\mathbf{V}_u(\xi,\eta,\zeta) = \underbrace{\mathbf{V}_u^n(\xi,\eta)}_{\text{Shape and Configuration}} + \underbrace{\zeta\frac{\delta t}{2}\mathbf{n}(\xi,\eta)}_{\text{Size}}$$

Design Velocity

Shape and Configuration

Size

DIRECT DIFFERENTIATION METHOD

- Structural Performance Measure

$$\psi = \iint_{\Omega^r} g(\mathbf{z}, u)\,|\mathbf{J}|\, d\Omega^r$$

$$\psi' = \iint_{\Omega^r} (g_{,\mathbf{z}}^T \dot{\mathbf{z}} + g\,div\mathbf{V} + g_{,u}\delta u)\,|\mathbf{J}|\, d\Omega^r$$

- Design Sensitivity Equation

$$[a(\mathbf{z}, \overline{\mathbf{z}}) = \ell(\overline{\mathbf{z}})]'$$

$$\Longrightarrow \quad a(\dot{\mathbf{z}}, \overline{\mathbf{z}}) = \ell'_V(\overline{\mathbf{z}}) - a'_V(\mathbf{z}, \overline{\mathbf{z}}), \qquad \forall \overline{\mathbf{z}} \in Z$$

– External Fictitious Load

$$\left. \frac{d}{d\tau} \ell(\overline{\mathbf{z}}) \right|_{\tau=0} = 2 \iint_{\Omega^r} \overline{\mathbf{z}}^T \mathbf{f}^B div\mathbf{V}\,|\mathbf{J}|\, d\Omega^r$$

$$\equiv \ell'_V(\overline{\mathbf{z}})$$

DIRECT DIFFERENTIATION METHOD (cont')

– Structural Fictitious Load

$$
\begin{aligned}
a'_V(\mathbf{z},\bar{\mathbf{z}}) = {} & 2\iint_{\Omega^r} \varepsilon_{ij}^{V1}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{1}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +2\iint_{\Omega^r} \varepsilon_{ij}^{1}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{V1}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +2\iint_{\Omega^r} \varepsilon_{ij}^{1}(\bar{\mathbf{z}}) C_{ijkl}^{V} \varepsilon_{kl}^{1}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +2\iint_{\Omega^r} \varepsilon_{ij}^{1}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{1}(\mathbf{z})\, div\mathbf{V} \left|\mathbf{J}\right| d\Omega^r \\
& +\tfrac{2}{3}\iint_{\Omega^r} \varepsilon_{ij}^{V2}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{2}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +\tfrac{2}{3}\iint_{\Omega^r} \varepsilon_{ij}^{2}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{V2}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +\tfrac{2}{3}\iint_{\Omega^r} \varepsilon_{ij}^{2}(\bar{\mathbf{z}}) C_{ijkl}^{V} \varepsilon_{kl}^{2}(\mathbf{z}) \left|\mathbf{J}\right| d\Omega^r \\
& +\tfrac{2}{3}\iint_{\Omega^r} \varepsilon_{ij}^{2}(\bar{\mathbf{z}}) C_{ijkl} \varepsilon_{kl}^{2}(\mathbf{z})\, div\mathbf{V} \left|\mathbf{J}\right| d\Omega^r
\end{aligned}
$$

$$\varepsilon_{ij}^{V1}(\mathbf{z}) = -sym(z_{i,m}^{1} V_{m,j} + z_i^2 V_{3,j})$$

$$\varepsilon_{ij}^{V2}(\mathbf{z}) = -sym(z_{i,m}^{2} V_{m,j})$$

Explicitly Dependent Terms of Strains

$$div\mathbf{V} = \frac{\partial V_i}{\partial \xi_i}$$

Variation of |**J**|

$$\mathbf{C}^V(u) = \dot{\mathbf{Q}}^T \mathbf{D}\mathbf{Q} + \mathbf{Q}^T \mathbf{D}\dot{\mathbf{Q}}$$

Local to Global Coord. Transformation

ADJOINT VARIABLE METHOD

- Adjoint Equation

$$a(\boldsymbol{\lambda}, \bar{\boldsymbol{\lambda}}) = \iint_{\Omega^r} g_{,\mathbf{z}}^T \bar{\boldsymbol{\lambda}} \left| \mathbf{J} \right| d\Omega^r, \qquad \forall \bar{\boldsymbol{\lambda}} \in Z$$

- Performance Measure Sensitivity

$$\psi' = \ell'_V(\boldsymbol{\lambda}) - a'_V(\mathbf{z}, \boldsymbol{\lambda}) + \iint_{\Omega^r} (g\,div\mathbf{V} + g_{,u}\delta u) \left| \mathbf{J} \right| d\Omega^r$$

- Efficient for Small Number of Performance Measures and Large Number of Design Parameters

MESHFREE DISCRETIZATION OF DESIGN SENSITIVITY EQUATION

- Explicitly Dependent Displacement

$$\mathbf{z}'_{\delta u} = \sum_{I=1}^{IP} \Psi_I(\xi,\eta)\frac{\delta t}{2}\zeta[\mathbf{S}_I^{1^T}, -\mathbf{S}_I^{2^T}]\begin{bmatrix}\alpha_I \\ \beta_I\end{bmatrix}$$

$$+\sum_{I=1}^{IP} \Psi_I(\xi,\eta)\frac{t}{2}\zeta[\dot{\mathbf{S}}_I^{1^T}, -\dot{\mathbf{S}}_I^{2^T}]\begin{bmatrix}\alpha_I \\ \beta_I\end{bmatrix}$$

- Explicitly Dependent Strain

$$\boldsymbol{\varepsilon}^{V1}(\mathbf{z}) = \mathbf{S}_q \sum_{I=1}^{IP} \mathbf{T}_I^V \mathbf{G}_I^1 \mathbf{r}_I + \mathbf{S}_q \sum_{I=1}^{IP} \mathbf{T}_I \mathbf{G}_I^{V1} \mathbf{r}_I \equiv \sum_{I=1}^{IP} \mathbf{B}_I^{V1} \mathbf{r}_I$$

$$\boldsymbol{\varepsilon}^{V2}(\mathbf{z}) = \mathbf{S}_q \sum_{I=1}^{IP} \mathbf{T}_I^V \mathbf{G}_I^2 \mathbf{r}_I + \mathbf{S}_q \sum_{I=1}^{IP} \mathbf{T}_I \mathbf{G}_I^{V2} \mathbf{r}_I \equiv \sum_{I=1}^{IP} \mathbf{B}_I^{V2} \mathbf{r}_I$$

$$\mathbf{T}_I^V = \begin{bmatrix} \mathbf{J}^{-1}\dfrac{\partial \mathbf{V}}{\partial \mathbf{x}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}^{-1}\dfrac{\partial \mathbf{V}}{\partial \mathbf{x}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}^{-1}\dfrac{\partial \mathbf{V}}{\partial \mathbf{x}} \end{bmatrix}_I$$

DISCRETE
DESIGN SENSITIVITY EQUATION

- Direct Differentiation Method

$$\mathbf{F}_L^a = \boldsymbol{L}\{\sum_{I=1}^{IP} 2[\mathbf{B}_I^{V1^T}\mathbf{C}\boldsymbol{\varepsilon}^1 + \mathbf{B}_I^{1^T}\mathbf{C}\boldsymbol{\varepsilon}^{V1} + \mathbf{B}_I^{1^T}(\mathbf{C}^V + \mathbf{C}\,div\mathbf{V})\boldsymbol{\varepsilon}^1]\left|\mathbf{J}\right| A_L\}$$

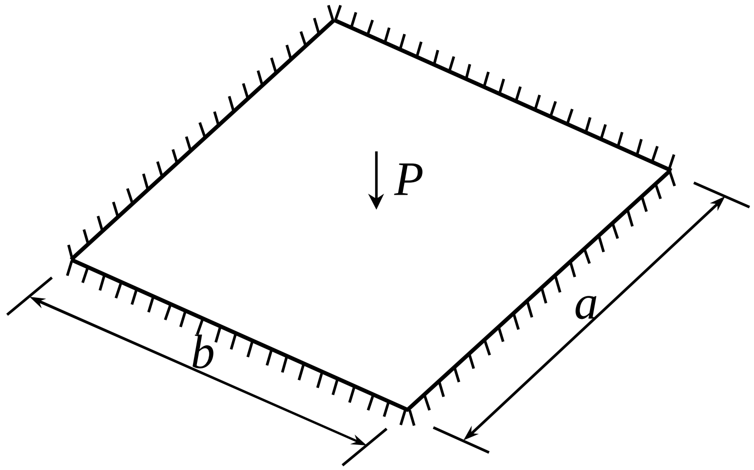
$$+\boldsymbol{L}\{\sum_{I=1}^{IP} \tfrac{2}{3}[\mathbf{B}_I^{V2^T}\mathbf{C}\boldsymbol{\varepsilon}^2 + \mathbf{B}_I^{2^T}\mathbf{C}\boldsymbol{\varepsilon}^{V2} + \mathbf{B}_I^{2^T}(\mathbf{C}^V + \mathbf{C}\,div\mathbf{V})\boldsymbol{\varepsilon}^2]\left|\mathbf{J}\right| A_L\}$$

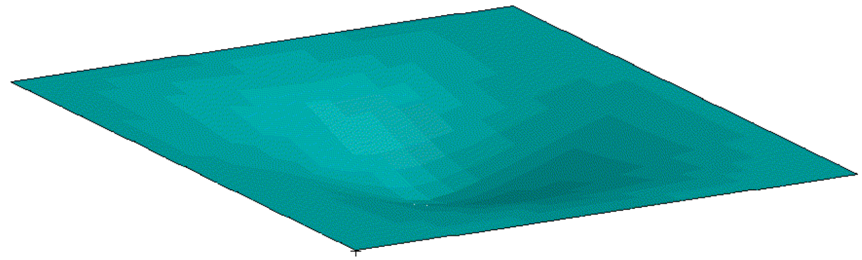
$$\mathbf{F}_L^\ell = \boldsymbol{L}\{\sum_{I=1}^{IP} 2(\Psi_I \mathbf{f}^B div\mathbf{V})\left|\mathbf{J}\right| A_L\}$$

$$[\mathbf{K}]\{\dot{\mathbf{r}}\} = \{\mathbf{F}^\ell\} - \{\mathbf{F}^a\}$$

- Chain Rule of Differentiation Can Be Used To Calculate Sensitivity of Performance Measure

PLATE BENDING EXAMPLE





Deformed Shape of Plate

Design Parameter: Plate Thickness

Parameters	**Value**
Young's modulus E	3×10^{6}
Poisson's ratio ν	0.3
Plate dimension	80×80
Plate thickness t	0.8
Boundary condition	clamped
Applied force P	1.0 (vertical)
Kernel function	cubic spline
Normalized dilation parameter (a , a)	(2.0, 2.0)
Completeness condition	1-st order

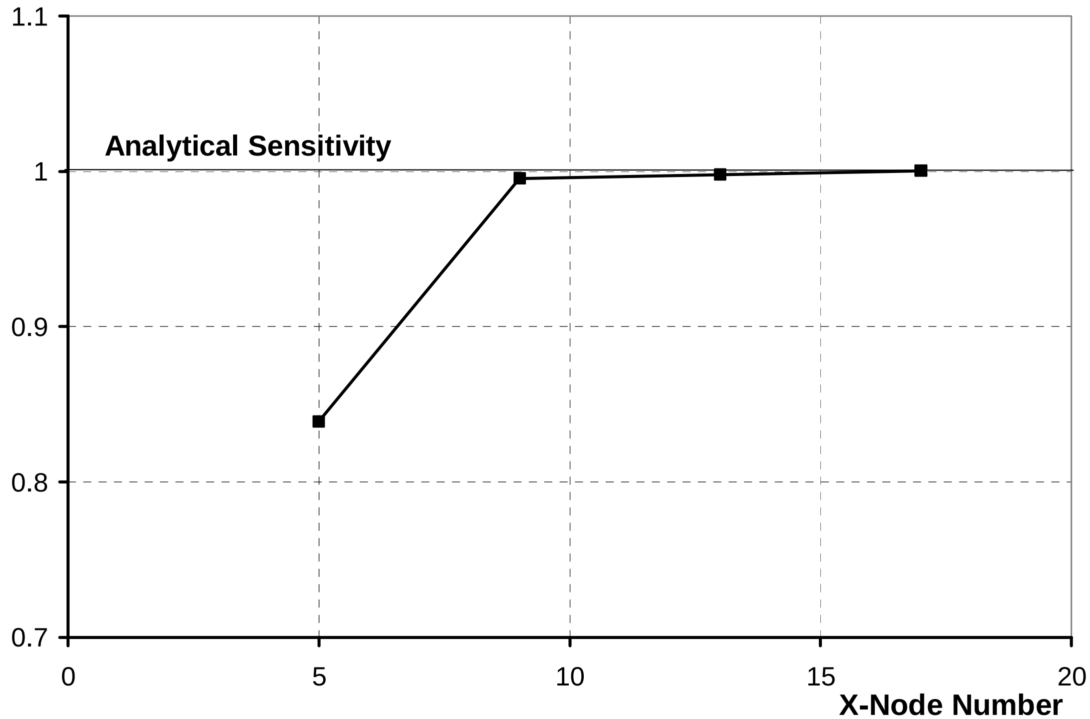
PLATE BENDING EXAMPLE (cont')

Analytical Displacement

$$z_{\max} = \alpha \frac{Pa^2}{D} \qquad D = \frac{Et^3}{12(1-\nu^2)}$$

Analytical Sensitivity

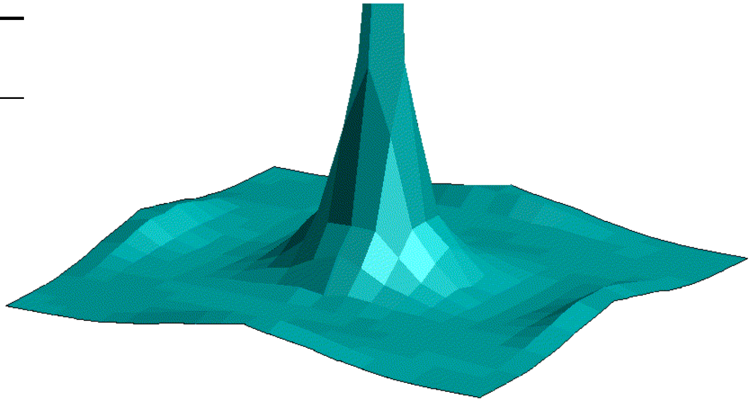
$$\dot{z}_{\max} = -\alpha \frac{3Pa^2}{Dt}$$



DESIGN SENSITIVITY RESULTS

Accuracy of Sensitivity Analysis

Design	Δz	$z'\tau\Delta u_i$	$(\Delta z / z'\tau\Delta u_i) \times 100\%$
11	0.2667110678E–06	0.2681489993E–06	99.46
32	0.2722314095E–06	0.2689011302E–06	101.24
53	0.1464059417E–06	0.1430103828E–06	102.37
74	0.6533997742E–07	0.6578831239E–07	99.32
95	0.5806800927E–07	0.5945678529E–07	97.66
116	0.9747570553E–07	0.9537862770E–07	102.20
137	0.1450910631E–06	0.1409421884E–06	102.94
158	0.4739573975E–06	0.4702366147E–06	100.79
179	0.6748613380E–06	0.6736937651E–06	100.17
200	0.2170313115E–05	0.2150760629E–05	100.91
221	0.6155689150E–05	0.6151250631E–05	100.07



Displacement Sensitivity at Center w.r.t. Plate Thickness

Comparison of Sensitivity Methods

Design	Direct differentiation method	Adjoint variable method
11	0.536297998631E–03	0.536305808329E–03
32	0.537802260496E–03	0.537801119644E–03
53	0.286020765581E–03	0.286020204320E–03
74	0.131576624781E–03	0.131576397114E–03
95	0.118913570576E–03	0.118913690400E–03
116	0.190757255398E–03	0.190758979318E–03
137	0.281884376712E–03	0.281882711447E–03
158	0.940473229450E–03	0.940474560876E–03
179	0.134738753020E–02	0.134738839984E–02
200	0.430152125821E–02	0.430151666983E–02
221	0.123025012625E–01	0.123025063345E–01

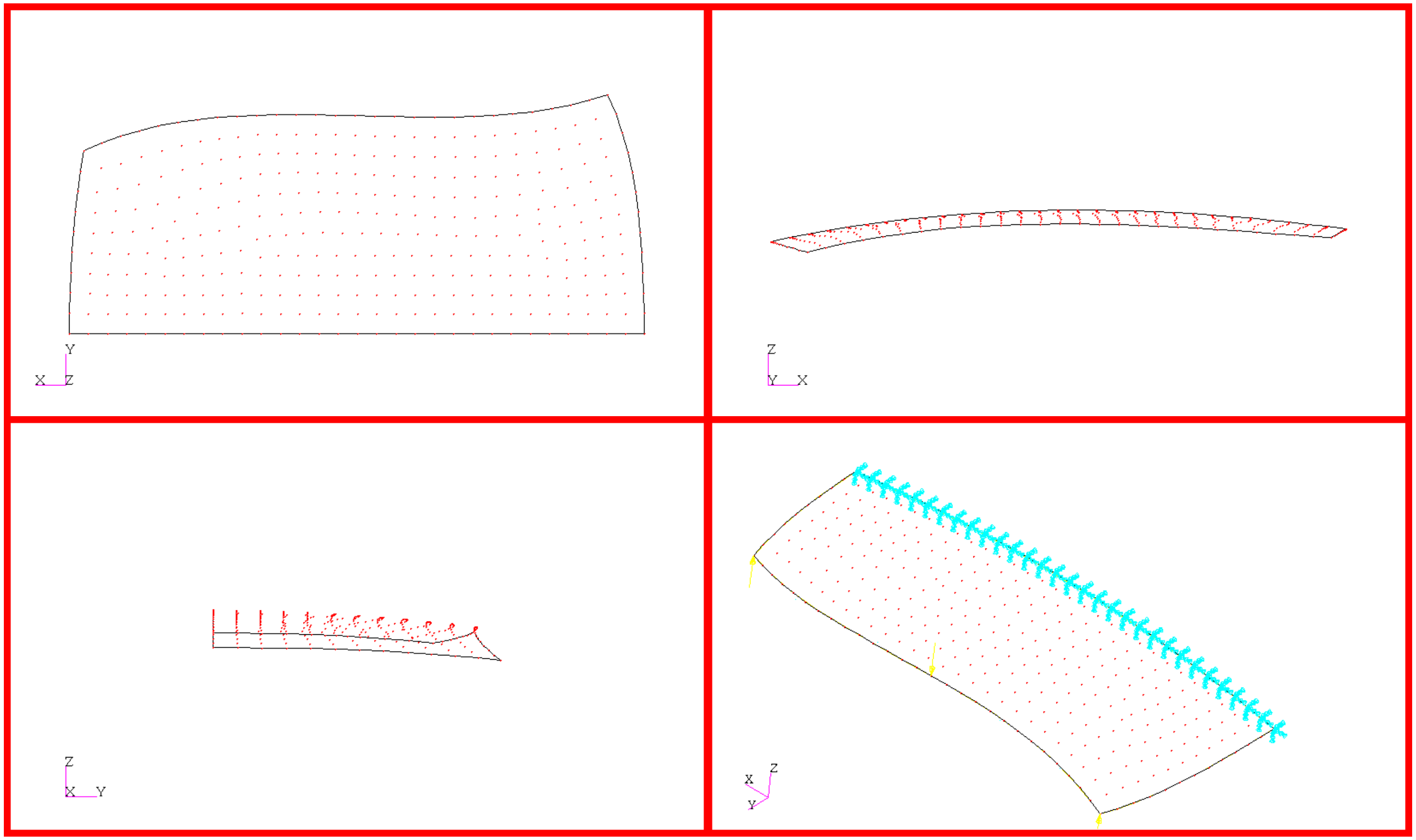
Adjoint Variable Method
0.26 sec

Direct Differentiation Method
0.174 × 441 = 76.75 sec

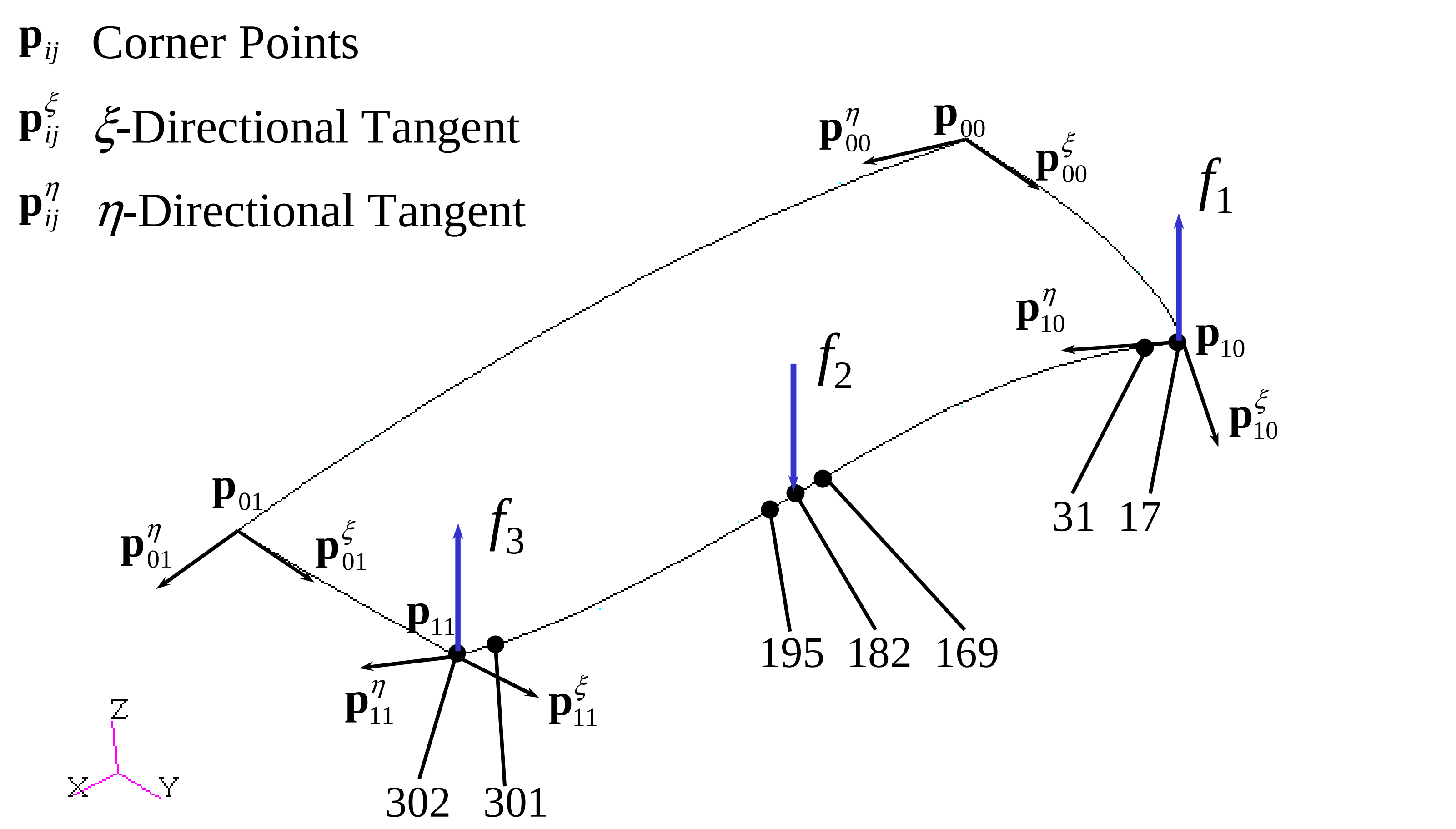
Cost of Sensitivity Analysis

VEHICLE ROOF MODEL

Half Model

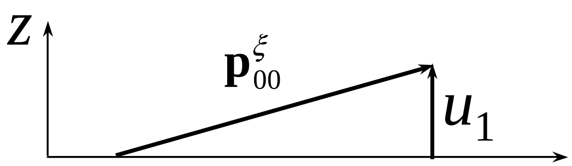


DESIGN PARAMETERIZATION



DESIGN PARAMETERIZATION (cont')

- Design parameters are chosen from the vertical movement of the tangent vector

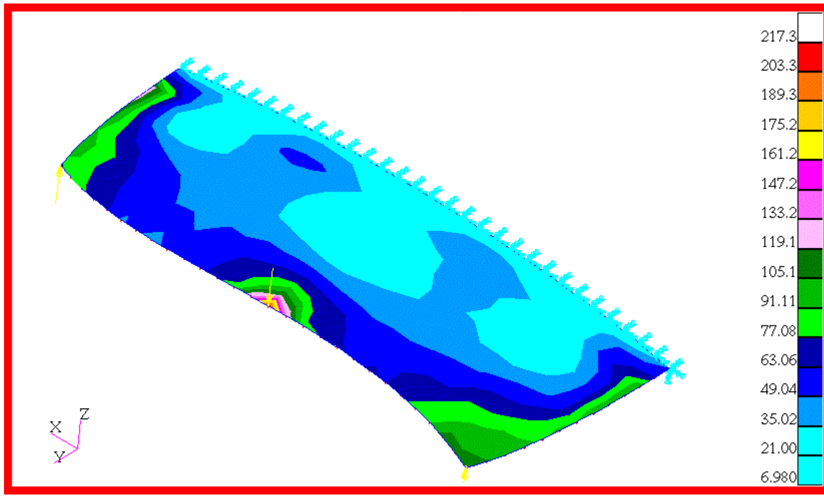


- Three nodal forces and the material property E and ν are chosen as design variables

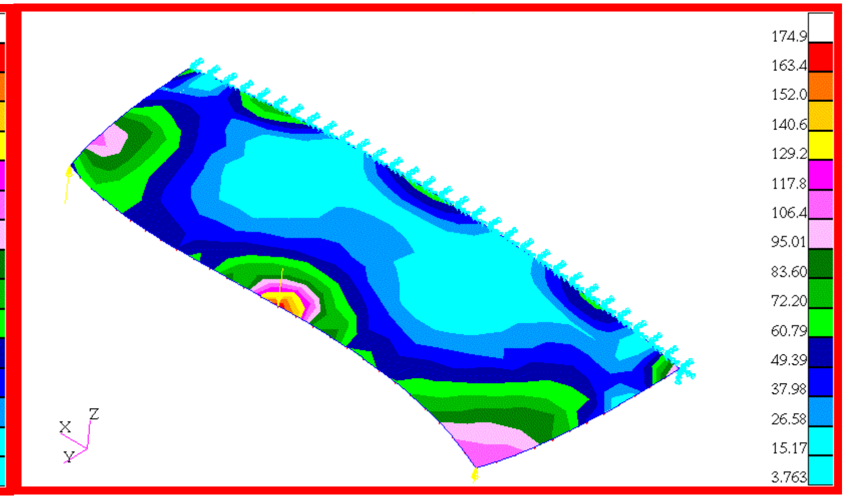
$u_1 : \mathbf{p}_{00}^{\xi}$
$u_2 : \mathbf{p}_{10}^{\xi}$
$u_3 : \mathbf{p}_{01}^{\xi}$
$u_4 : \mathbf{p}_{11}^{\xi}$
$u_5 : \mathbf{p}_{00}^{\eta}$
$u_6 : \mathbf{p}_{10}^{\eta}$
$u_7 : \mathbf{p}_{01}^{\eta}$
$u_8 : \mathbf{p}_{11}^{\eta}$

$u_9 : f_1$
$u_{10} : f_2$
$u_{11} : f_3$
$u_{12} : E$
$u_{13} : \nu$

MESHFREE ANALYSIS RESULTS



Bottom Surface (Max = 217.3 *MPa*)



Top Surface (Max = 174.9 *MPa*)

- 347 nodes (1735 DOF)
- Force = 100 kN @ node 17
  –200 kN @ node 182
  100 kN @ node 302
- CPU Time (HP Workstation)
  Meshfree Analysis : 37.12 sec
  Sensitivity Analysis : 0.94 sec

Material Property

$E(MPa)$	26,000
ν	0.34
$G(MPa)$	9,701
$t(mm)$	2.0

DESIGN SENSITIVITY RESULTS

Design = u_1 : Vertical Movement of $\mathbf{p}_{00}^{\xi}$
Perturbation Size = 1.602E–02

Type	Ψ	$\Delta\Psi$	$\Psi'\Delta\tau$	$(\Delta\Psi/\Psi')\times100$
Volume	1.51033E+06	1.95212E-01	1.95180E-01	100.02
σ_{169}	1.34898E+02	9.43339E-03	9.43429E-03	99.99
σ_{182}	2.17291E+02	8.96848E-03	8.96976E-03	99.99
σ_{195}	1.35578E+02	7.27985E-03	7.28063E-03	99.99
σ_{17}	1.21676E+02	1.32524E-03	1.32552E-03	99.98
σ_{31}	1.09005E+02	2.88250E-03	2.88289E-03	99.99
σ_{301}	8.66971E+01	2.66191E-03	2.66156E-03	100.01
σ_{302}	7.34885E+01	1.72660E-03	1.72636E-03	100.01

Design = u_3 : Vertical Movement of $\mathbf{p}_{01}^{\xi}$
Perturbation Size = 1.354E–02

Type	Ψ	$\Delta\Psi$	$\Psi'\Delta\tau$	$(\Delta\Psi/\Psi')\times100$
Volume	1.51033E+06	2.08691E-01	2.08665E-01	100.01
σ_{169}	1.34898E+02	6.58233E-03	6.58289E-03	99.99
σ_{182}	2.17291E+02	7.80610E-03	7.80689E-03	99.99
σ_{195}	1.35578E+02	7.50906E-03	7.50947E-03	99.99
σ_{17}	1.21676E+02	2.64271E-04	2.64279E-04	100.00
σ_{31}	1.09005E+02	7.37644E-04	7.37657E-04	100.00
σ_{301}	8.66971E+01	4.77743E-03	4.77783E-03	99.99
σ_{302}	7.34885E+01	1.06057E-03	1.06138E-03	99.92

DESIGN SENSITIVITY RESULTS (cont')

Design = u_{10} Force f_2 at the Edge

Perturbation Size = 3.793E–03

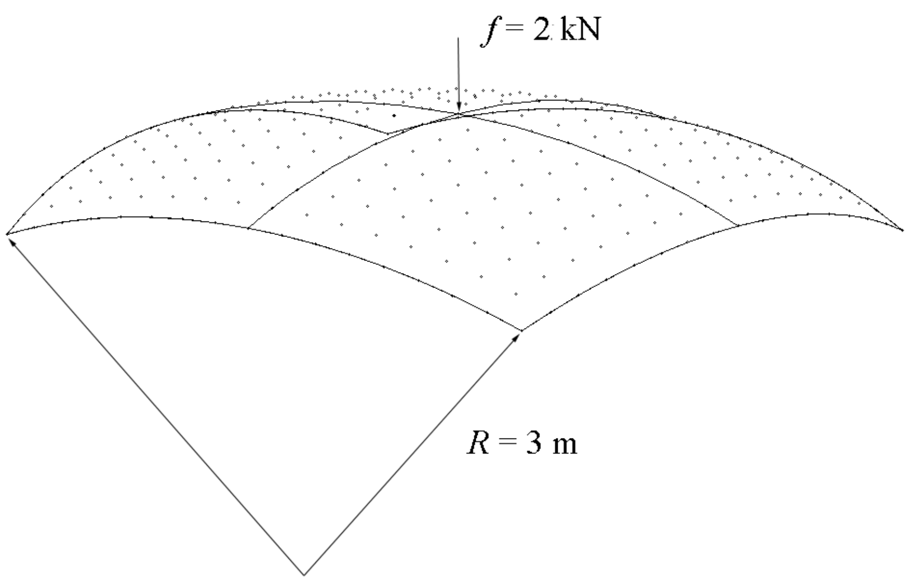
Type	Ψ	$\Delta\Psi$	$\Psi'\Delta\tau$	$(\Delta\Psi/\Psi')\times 100$
Volume	1.51033E+06	0.00000E+00	0.00000E+00	0.00
σ_{169}	1.34898E+02	-2.12311E-03	-2.12310E-03	100.00
σ_{182}	2.17291E+02	-3.52794E-03	-3.52793E-03	100.00
σ_{195}	1.35578E+02	-2.11374E-03	-2.11373E-03	100.00
σ_{17}	1.21676E+02	-1.37942E-04	-1.37941E-04	100.00
σ_{31}	1.09005E+02	1.94032E-05	1.94029E-05	100.00
σ_{301}	8.66971E+01	-1.72318E-04	-1.72319E-04	100.00
σ_{302}	7.34885E+01	-9.10049E-05	-9.10035E-05	100.00

Design = u_{13} Poisson's Ratio

Perturbation Size = 1.853E–05

Type	Ψ	$\Delta\Psi$	Ψ'	$(\Delta\Psi/\Psi')\times 100$
Volume	1.51033E+06	0.00000E+00	0.00000E+00	0.00
σ_{169}	1.34898E+02	3.98915E-04	3.98709E-04	100.05
σ_{182}	2.17291E+02	6.15776E-04	6.15426E-04	100.06
σ_{195}	1.35578E+02	3.67303E-04	3.66965E-04	100.09
σ_{17}	1.21676E+02	-4.93757E-05	-4.93850E-05	99.98
σ_{31}	1.09005E+02	-2.60711E-05	-2.60801E-05	99.97
σ_{301}	8.66971E+01	3.88180E-04	3.87581E-04	100.15
σ_{302}	7.34885E+01	1.02153E-03	1.02096E-03	100.06

ELASTOPLASTIC PROBLEM

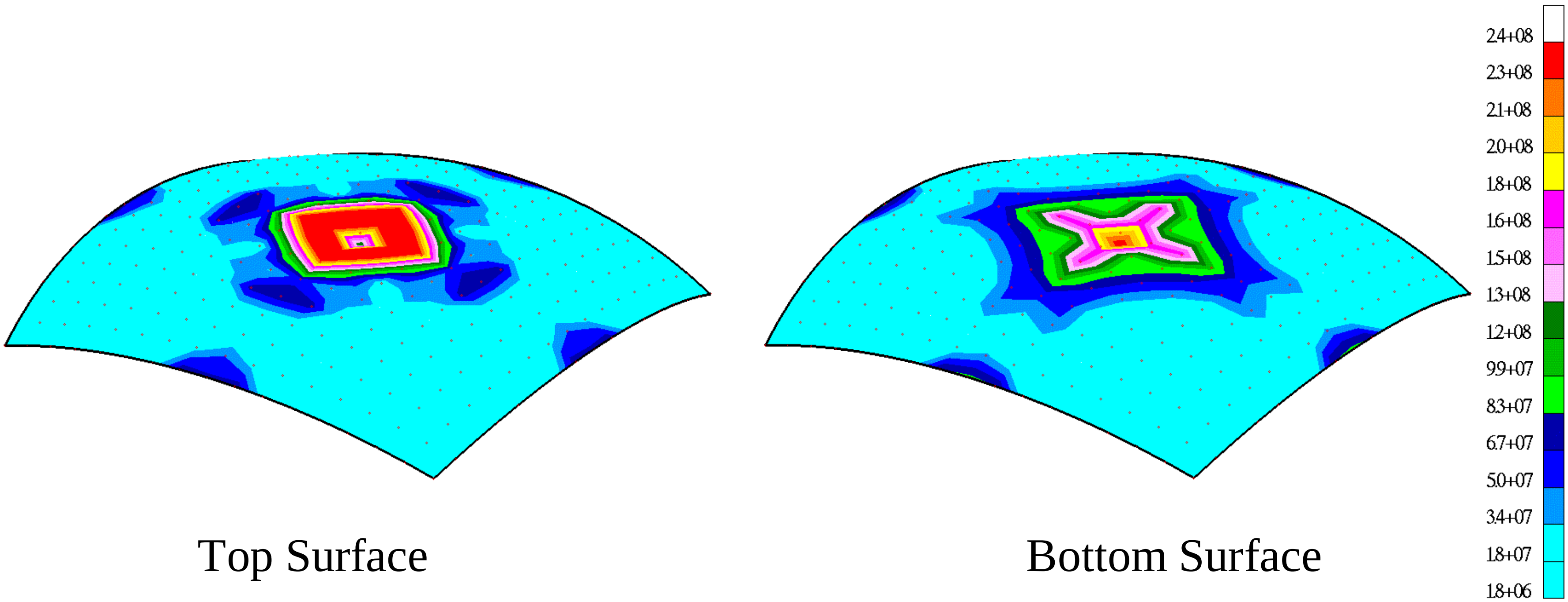


Spherical Shell (1,445 DOF)

Properties	***Value***
Young's modulus E	70 GPa
Poisson's ratio ν	0.35135
Yielding strength σ_Y	241 MPa
Hardening slope H	241 MPa

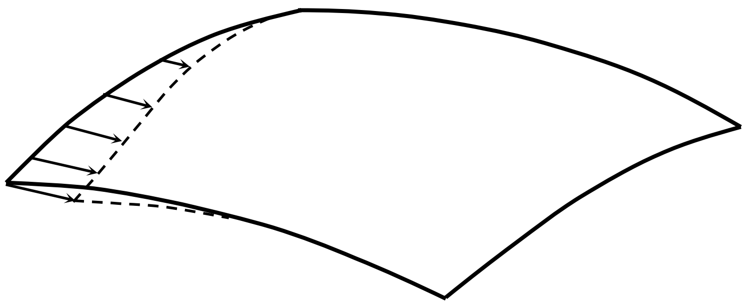
- J_2 Plasticity Theory with Isotropic/Kinematic Hardening
- Return-Mapping That Satisfies Plane-Stress Condition
- Bending: SC Nodal Integration
- Membrane/Shear: Direct Nodal Integration

STRESS CONTOUR PLOTS

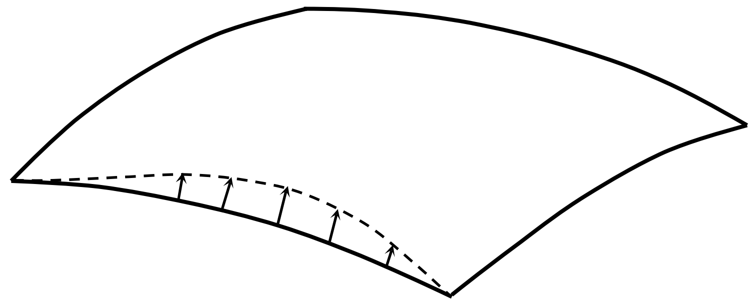


Iteration	Residual Norm
1	6.50061E+02
2	7.41409E+01
3	1.82775E–01
4	4.26552E–07

DESIGN SENSITIVITY RESULTS



(a) $u_1 = \mathbf{p}_{00}$



(b) $u_2 = \mathbf{p}_{01}^{\eta}$

Node	$\Delta\psi$	$\psi'\Delta\tau$	Ratio
Z_{54}	3.31830E–7	3.31520E–7	100.09
Z_{145}	1.75315E–6	1.79128E–6	97.87
Z_{164}	7.40097E–6	7.40136E–6	99.99
Z_{175}	–1.85079E–6	–1.84984E–6	100.05
Z_{199}	2.10897E–6	2.10980E–6	99.96
Z_{250}	3.40716E–7	3.39494E–7	100.36
Z_{261}	1.99752E–7	1.99943E–7	99.99

Node	$\Delta\psi$	$\psi'\Delta\tau$	Ratio
Z_{54}	–3.09593E–8	–3.09622E–8	99.99
Z_{145}	1.67601E–6	1.67607E–6	100.00
Z_{164}	–1.84748E–7	–1.84758E–7	99.99
Z_{175}	2.96181E–8	2.96222E–8	99.99
Z_{199}	–2.66952E–8	–2.66960E–8	100.00
Z_{250}	–1.77193E–8	–1.77203E–8	99.99
Z_{261}	3.27533E–9	3.27513E–9	100.01

- Forward Finite Difference Method
- Perturbation Size = 10^{-4}

SUMMARY

- A Unified DSA and Parameterization Methods for Meshfree Shell Structure Is Developed That Can Handle Sizing/Shape/Configuration Design Parameters Simultaneously.
- A Design Velocity Field Is Defined in the Continuum Domain and Then Degenerated Using Shell Kinematics
- A Connection with CAD Tool Provides Useful Information for Meshfree Shell Analysis (Surface Normal, Jacobian, Coordinate Transformation)
- SC Nodal Integration Method of Meshfree Analysis Is Used in the Parametric Domain
- Accuracy of the Proposed Sensitivity Calculation Method Is Compared with the Analytical Solution and with the Finite Difference Results for Various Examples.
- Current Development Will Be Extended to the Die Shape Design Capability to Remove the Springback Problem in Stamping Process.