

# Conservativeness in Failure Probability Estimate: Redesign Risk vs. Performance

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Being conservative is in the nature of engineering design in order to compensate for uncertainty. Traditionally in the deterministic design approach, the level of conservativeness has not been a major concern for manufacturers in aerospace since it is defined and strictly required by the regulators as factors of safety. However, once a probabilistic design approach comes into play, the selection of conservativeness of probability of failure estimate considering epistemic uncertainty is an inevitable issue because conservativeness penalizes system performance. This paper investigates the tradeoff between performance and risk in development (redesign after certification test), to which regulators do not pay much attention. We conduct the study in the context of risk aversion of expected utility theory. With a practical design problem, a thermal protection system, it is shown that the level of risk aversion of the decision maker is not the key driver of the conservativeness selection, but the severity of redesign cost is. On top of that, the study reveals that expected utility theory may give unreasonable solutions if the level of risk aversion is set too high.

## I. Introduction

Risk mitigation is a crucial part of engineering design since engineering systems are exposed to various uncertainties that can potentially cause critical hazards throughout their lifecycles. For example, unexpected operational conditions may cause a fatal accident. Large errors in analytical predictions may result in failing a certification test leading to costly redesign. To compensate for such uncertainties and avoid undesirable consequences, designing a system conservatively is a common approach, relying on using factors of safety or conservative limits on probability of failure.

A factor of safety [1], which is commonly used in deterministic design, measures the margin between the failure state (so-called capacity, e.g., ultimate strength) and the state in operation (so-called response, e.g., stress). Alternatively, probabilistic design [2] has been gaining popularity in the literature. Probabilistic design uses probability of failure, the likelihood that the response will exceed the capacity, by modeling uncertainties as probability distributions. Due to limited data available to model probability distributions, confidence levels in probability of failure estimate are used to represent its conservativeness, e.g., 95<sup>th</sup> percentile value [3] and maximum value [4]. A conservative estimate of the probability of failure, however, is likely to lead to performance loss. Then, there arises the issue of tradeoff between design conservativeness and performance, which few studies have tackled [5].

Since factors of safety have been well-established over decades and are strictly required by regulatory agencies, which mainly care about safety, the level of design conservativeness has not been a major concern in the community. However, from the standpoint of manufacturers, design conservativeness is associated not only with safety, but also with risk of delays in development process, such as failing the certification test and requiring costly redesign. For effective transition to probabilistic design, an appropriate selection of the conservativeness of probability of failure estimate is imperative.

This paper investigates the tradeoff between risk of redesign after certification test and performance with respect to design conservativeness in the context of decision-based design. Decision-based design is a method of decision-making for engineering design under uncertainty and risk [6, 7] and deploys expected utility theory. Expected utility theory bases its decision on decision maker's utility related to attributes of system rather than attributes themselves,

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such as performance and safety. In addition, the decision maker's risk aversion is modeled by utility functions. A key question is how the design conservativeness is interpreted in risk aversion of the utility.

We conduct the study with a practical structural design problem, such as a thermal protection system for space vehicles. This paper is organized as follows. Sections II and III describe the procedure of probability of failure estimate and the expected utility theory accounting for risk aversion, respectively. The design problem of the thermal protection system is described in Section IV. Section X discusses the tradeoff problem and Section V concludes the study.

## II. Conservativeness in Probability of Failure Estimate

In probability of failure calculation, uncertainties are usually categorized according to their nature into two classes: (1) aleatory uncertainty that stems from physical randomness, e.g., variability in material properties and geometry, and (2) epistemic uncertainty due to lack of knowledge, e.g., errors in computational models [8]. There is a consensus that aleatory uncertainty is modeled by a probability density function (PDF) because it is actually distributed. The treatment of epistemic uncertainty, on the other hand, is debatable [9] because it is not distributed, but just unknown. Various techniques to model epistemic uncertainty have been studied, such as probability theory [2, 10], Dempster-Shafer evidence theory [11, 12], and possibility theory [13, 14]. In this paper, to model epistemic uncertainty, we use probability theory, which is widely accepted in the literature.

Figure 1 illustrates how to calculate a probability of failure considering only aleatory uncertainty. The procedure starts with a calculated system response of interest,  $R_{cal}$ , e.g., predicted stress from a finite element model. After introducing variability in the response due to, for example, variability in geometry and loading conditions, the response forms a distribution. The area beyond the capacity (e.g., strength), the integration of probability density, represents the probability of failure. For the sake of simplicity, the capacity is assumed deterministic. It is noted that a single value of failure probability is calculated from the given aleatory uncertainty.

Next, we introduce epistemic uncertainty; that is, an error in the response prediction. Figure 2 shows how the probability of failure deviates when the error is taken into account. If the error is conservative, the true response is smaller than  $R_{cal}$ , and the distribution of response shifts left resulting in a smaller probability of failure (Fig. 2.(a)). On the other hand, an unsafe error leads to a higher probability of failure (Fig. 2.(b)). Thus, the probability of failure estimate will have a variation according to the level of error.

If the error is modeled by a PDF, we can randomly generate a set of possible errors and calculate the corresponding probabilities of failure by Monte Carlo simulation, forming a distribution of the probability of failure. If the designer wants to be conservative, he may take the maximum value or a high percentile value of the distribution. Using the mean of the distribution has been often used in the literature in part because of its easiness of implementation<sup>4</sup>. Note that a distribution of probability of failure is skewed and heavily right-tailed, even if the distribution of epistemic uncertainty is symmetric. This means that the mean value of the distribution is higher than the median. However, the percentile value corresponding to the mean is not consistent over the design space and varies depending on design point.

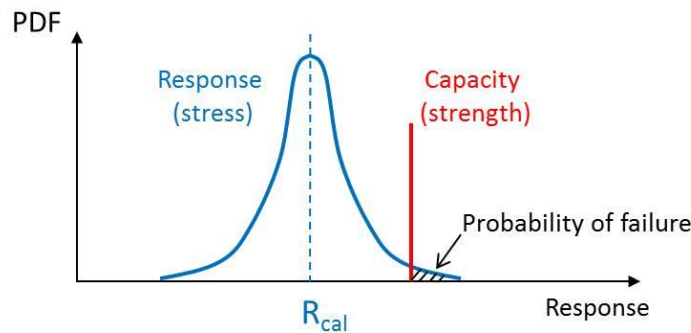
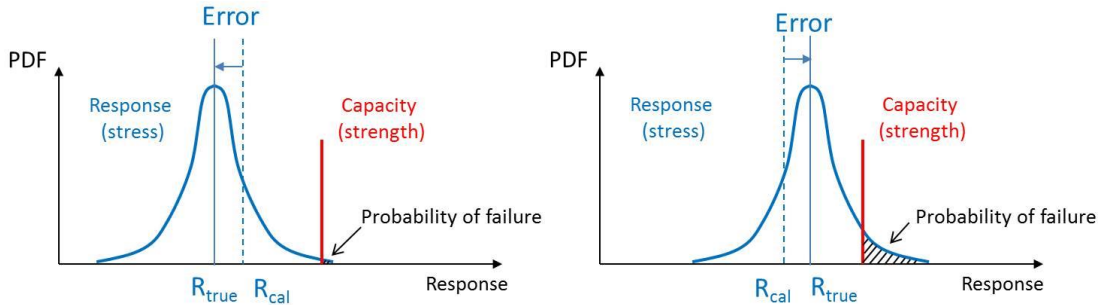


Figure 1. Probability of failure estimate when only aleatory uncertainty is considered.

<sup>4</sup>Using the mean value of the distribution of probability of failure is essentially equivalent to the case that aleatory uncertainty and epistemic uncertainties are treated equally and combined together via convolution as a wider distribution of response. This offers a single number of probability of failure.



(a) Conservative error

(b) Unconservative error

Figure 2. Probability of failure estimate when both aleatory and epistemic uncertainties are considered.

### III. Expected Utility Theory and Risk Aversion

Decision-making theory under risk and uncertainty originated in the 18<sup>th</sup> century (Bernoulli), and rigorous axioms of the expected utility theory were established by von Neumann and Morgenstern in the 20<sup>th</sup> [15]. Expected utility theory assumes that a rational person wants to maximize utility rather than profits, and utility functions express their preferences and risk attitudes. The theory has been broadly used in economics and recently is applied to engineering applications [16-20].

In expected utility theory, a person is risk-averse when he prefers a certain prospect to any uncertain prospect with the same expected prospect. For example, suppose there is a bet with 50% chance to win \$10 and 50% chance for nothing. The other choice is \$5 for certain. The expected gain for both cases is \$5, but a risk-averse person is willing to choose the latter one, i.e., \$5 for certain. With a von Neumann-Morgenstern utility function, risk-averse attitude is modeled by a concave utility function. Figure 3 illustrates the framework of the decision making of this example. The utility of the certain outcome (\$5) is denoted as  $U_2 = U(5) = E[U]_2$ , and the expected utility of betting is obtained as  $E[U]_1 = 0.5U(10) + 0.5U(0)$ . Because of the concavity of the utility function,  $E[U]_2 > E[U]_1$ . Thus, the utility maximizer will choose the risk-free \$5.

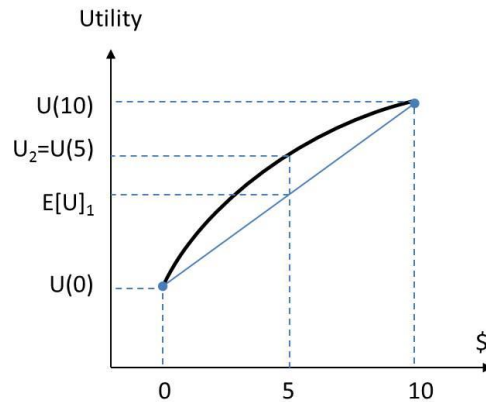


Figure 3. Risk-averse attitude by concave utility function

Arrow-Pratt coefficients are commonly used to measure the degree of risk aversion. Absolute risk aversion (ARA) represents the curvature of the curve and is defined as

$$R_A(w) = -\frac{U''(w)}{U'(w)} \quad (1)$$

where  $U(\cdot)$  and  $w$  represent a utility function and the level of an attribute, such as wealth, income, system performance, respectively. A higher value of ARA represents a more risk-averse person. ARA can also be

interrupted as the reciprocal of amount of wealth that a person accepts to expose to risk. If ARA decreases with increasing  $w$ , the amount of wealth that a person willing to expose to risk increases with wealth. In other words, betting over a certain amount becomes more attractive as person's wealth increases. On the other hand, the preference of a person with a constant ARA will not change with wealth. Exponential utility, e.g.,  $U(w) = 1 - \exp(-\rho w)$ , is the unique form of constant absolute risk aversion (CARA) and  $R_A(w) = \rho$ . Relative risk aversion (RRA) is associated with the percentage of wealth a person is willing to expose to risk and is calculated by Eq. (2).

$$R_R(w) = -w \frac{U''(w)}{U'(w)} = wR_A(w) \quad (2)$$

Hyperbolic absolute risk aversion (HARA) captures a general risk attitude that a person becomes less reluctant to risk when he becomes wealthier. Power utility function, e.g.,  $U(w) = w^{1-r}/(1-r)$ , is a form of HARA, and  $R_A(w) = r/w$  and  $R_R(w) = r$ .

While expected utility theory has been attractive because its tractability and well-modeled risk attitude, some limitations of the theory have been discussed. Kahneman and Tversky [21] argued that expected utility does not reflect people's tendency to overvalue a certain outcome than a merely probable outcome, so-called the certainty effect. This leads to inconsistent preferences even when the forms of expected utilities are essentially the same. Rabin and Thaler [22] pointed out that diminishing marginal utility is not necessarily a plausible explanation of risk aversion. In the context of engineering design, Thurston [20] summarized limitations and benefits of utility analysis, focusing on the issues on multi-attribute utility.

#### IV. Design Tradeoff of Thermal Protection System

##### (1) Design problem

Thermal protection systems protect a space vehicle from extreme temperatures during atmospheric reentry. Integrated thermal protection system (ITPS) is a concept for reusable vehicles, which can provide structural load bearing function and insulation function simultaneously, and is intended to save the mass of the system [5, 23] (Fig 4). In this paper, only one failure, material deterioration of the bottom face sheet due to high temperature starting losing the load bearing function, is taken into account.

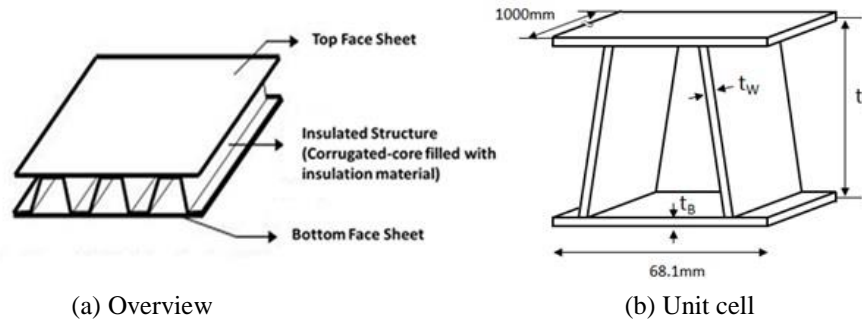


Figure 4. Integrated Thermal Protection System (ITPS)

For design optimization, we minimize the mass of unit cell of ITPS (Fig. 4.(b)) under the constraint of probability of failure of the material deterioration of the bottom face sheet. The thicknesses of web ( $t_w$ ), bottom face sheet ( $t_B$ ), and foam ( $t_F$ ) are selected as design variables  $\mathbf{d}$ . Thus, the design optimization is formulated as

$$\begin{aligned} \min_{\mathbf{d}} m(\mathbf{d}) \\ \text{s.t. } P(\mathbf{d}) \leq P_{target} \end{aligned} \quad (3)$$

where  $P(\mathbf{d})$  represents the estimated probability of failure and  $P_{target}$  is a target probability of failure here set at  $1 \times 10^{-4}$ . For the estimated probability of failure, one can select the percentile value of the distribution depending on the desired level of conservativeness.

Once an optimal design is obtained from Eq. (3), its risk of redesign after a certification test is evaluated. Taking an advantage of the capability of probabilistic design, our earlier work proposed a method to simulate multiple possible future outcomes out of post-design processes, including test observation, error calibration, redesign decision, and redesign [5, 24]. The method deploys uncertainty propagation throughout the future processes and allows us to calculate the probability of redesign and the expected mass after redesign if needed for any given design candidate. In this simulation of multiple possible futures, redesign decisions and redesign are implemented by using the updated probability of failure estimate by the test observation,  $P^{up}$ , by Eq. (4). For more details, the reader refers to Refs. [5, 24].

Redesign is needed, if  $P^{up} > P_{target}$

Then

$$\begin{aligned} \min_{\mathbf{d}} m(\mathbf{d}) \\ \text{s.t. } P^{up}(\mathbf{d}) < P_{target} \end{aligned} \quad (4)$$

For the present study, we first optimize the ITPS with various percentiles of the probability of failure estimate from 50<sup>th</sup> percentile to 100 percentile, as well as the mean. Then, we calculate the probability of redesign and the prospect masses of the optimal designs by using the method of multiple future simulations. For uncertainty, we model errors in temperature prediction of the bottom face sheet ( $\pm 10\%$  uniform distribution) and test observation ( $\pm 5K$  uniform distribution) as epistemic uncertainty, and variability in geometry, flight condition, and material properties as aleatory uncertainty [5].

Figure 5 shows the tradeoff between the probability of redesign and the mass of ITPS associated with the percentile value of probability of failure estimate. As expected, as the percentile value increases, the probability of redesign decreases accompanied by mass increase. Even with the most conservative design using 100<sup>th</sup> percentile, there is a chance of redesign (7.7%) because of possible error in test observation. The optimal solution for the mean design, the design using the mean value of the distribution of probability of failure, lies between the 60<sup>th</sup> and 70<sup>th</sup> percentile designs. The percentile value corresponding to the mean for the mean design is 65.7.

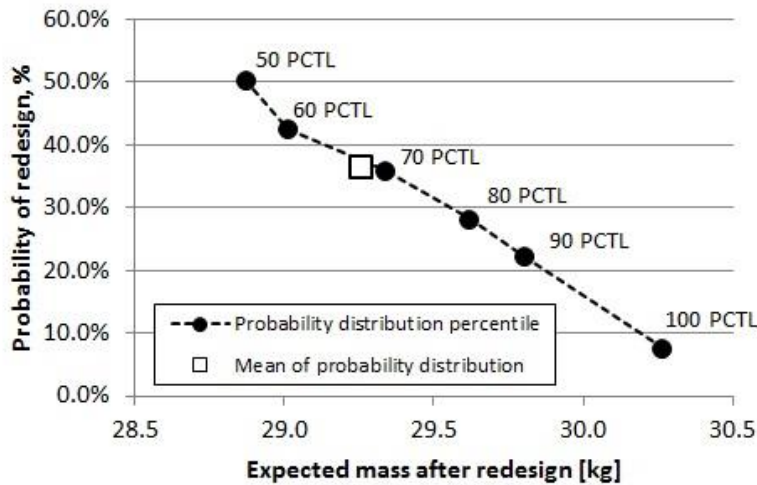


Figure 5. Effect of conservativeness in probability of failure estimate on expected mass and risk of requiring redesign

## (2) Cost Model

For investigating the tradeoff between the mass reduction and redesign risk, we assume that both attributes can be measured by monetary value. Mass reduction is assumed to yield profit since a smaller mass reduces manufacturing cost and material cost. Redesign requires additional engineering cost; therefore it is modeled as a monetary loss. These assumptions also allow for a single utility function with respect to the monetary value.

The profit due to mass reduction ( $W_m$ ) is assumed to be proportional to the mass reduction ( $\Delta m$ ) as shown in Fig. 6. The mass reduction is measured from a reference point (30.5kg), the maximum prospect mass in the future simulation of the most conservative design (100<sup>th</sup> percentile design). Note that the mass used in this cost model is the mass after redesign if needed, which is optimized by Eq. (4). If redesign is judged by the test to be unnecessary, the initial mass obtained from Eq. (3) remains.

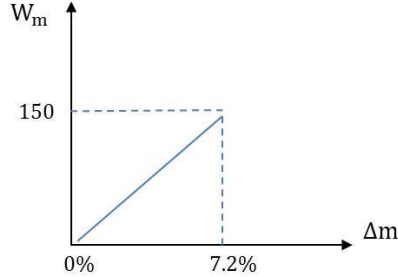


Figure 6. Monetary value of mass reduction

Cost impact of redesign  $W_{RE}$  is varied from -700 to 0 in order to examine the effect of the severity of redesign. For the highest cost ( $W_{RE} = -700$ ), the cost impact of redesign is far more significant than the potential profit from mass reduction (maximum = 150). Finally, the total wealth due to the two possible outcomes is obtained with the initial budget ( $W_0$ ) as in Eqs. (5) and (6).

$$\text{When no redesign needed: } W = W_0 + W_m \quad (5)$$

$$\text{When redesign needed: } W = W_0 + W_m + W_{RE} \quad (6)$$

Since multiple futures are simulated for a given optimal design, profit based decision maximizes the expected wealth by calculating the average wealth of the possible futures as in Eq. (7).

$$E[W] = \frac{1}{N} \sum_{i=1}^N W_i \quad (7)$$

where  $N$  is the number of simulated futures and  $N=1000$  is used for this study. In the same manner, expected utility theory maximizes the expected utility of the outcomes as in Eq. (8).

$$E[U] = \frac{1}{N} \sum_{i=1}^N U(W_i) \quad (8)$$

We assume that the decision maker follows the constant absolute risk aversion (CARA), and the utility function is modeled as an exponential utility,  $U(W) = 1 - \exp(-\rho W)$ . Then, the initial budget is set as  $W_0 = 700$  so that the total wealth is always remains positive. Since we use a utility function with a constant absolute risk coefficient (ARA), the selection of initial budget does not affect the decision-making (preference of choice) as mentioned earlier.

## V. Selection of Design Conservativeness

We first evaluate the design conservativeness selection based on the expected wealth. Figure 7 shows the expected wealth as a function of the percentile value of the probability of failure estimate and redesign cost. If the redesign cost is high ( $W_{RE} = -500$  and  $-700$ ), the most conservative 100<sup>th</sup> percentile design is preferred. For the case of  $W_{RE} = -300$ , all the percentile selections are comparable. As the redesign cost becomes less significant

( $W_{RE} = -100$  and  $0$ ), the 50<sup>th</sup> percentile design will be preferable. Thus, the severity of redesign penalty is the main driver of choice.

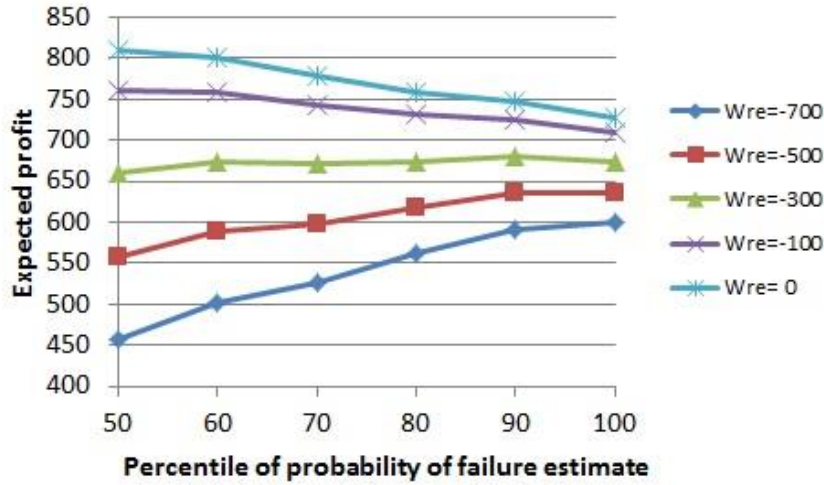


Figure 7. Expected profit versus the selection of percentile value of probability of failure for different values of redesign cost.

Next, we examine how the conservativeness of probability of failure estimate is interpreted in the context of risk aversion of expected utility theory. For doing this, we identify the level of risk aversion for a given decision. That is, if a person prefers a more conservative design using a higher percentile value for the probability of failure estimate to a less conservative design using a lower percentile value, what is the level of risk aversion of this person? We elicit a coefficient of risk aversion for such decision-making. The elicitation of the coefficient of risk aversion  $R_A = \rho$  can be implemented by solving the following inequality of the expected utilities for  $\rho$ .

$$E[U(\rho)]_{high} > E[U(\rho)]_{low} \quad (9)$$

where  $E[U(\rho)]_{high}$  and  $E[U(\rho)]_{low}$  are the expected utilities of a high percentile design and a low percentile design, respectively.

We first compare the two extreme designs, the 100<sup>th</sup> percentile design (conservative design) and the 50<sup>th</sup> percentile design (less conservative design). Figure 8 shows the difference in their expected utilities with respect to  $\rho$ . In Fig. 8, a positive value indicates that the 100<sup>th</sup> percentile design is preferable to the 50<sup>th</sup> percentile design. When the redesign cost is less severe ( $W_{RE} = -100$  and  $0$ ), the 100<sup>th</sup> percentile design will not be chosen by any risk-averse person or comparable to the 50<sup>th</sup> percentile design for a highly risk-conscious person ( $\rho$  is higher).

To examine this trend further, Figure 9 illustrates the decision framework for the case using  $W_{RE} = -100$ , which is abstracted as a binomial outcome problem, redesign or no-redesign. For converting the multiple-future problem ( $N=1000$ ) into a binomial problem, the expected wealth for each of the redesign and no-redesign cases is separately calculated by Eq. (8). In this abstraction, the levels of wealth of the 50<sup>th</sup> percentile design ( $W=682$  and  $840$ ) are both higher than corresponding wealth of the 100<sup>th</sup> percentile design ( $W=617$  and  $715$ ). Even though the 50<sup>th</sup> percentile design is exposed to the higher chance of redesign (50%), its expected wealth ( $E[W]_{50}=762$ ) is higher than that of the 100<sup>th</sup> percentile design ( $E[W]_{100}=707$ ) because of the small redesign cost. Moreover,  $E[W]_{50}$  is even higher than the profit of the 100<sup>th</sup> percentile design when no redesign is required ( $W=715$ ). As a result, for all  $\rho$ , the 50<sup>th</sup> percentile design is superior to the 100<sup>th</sup> percentile design, which is consistent with the decision based on the expected wealth.

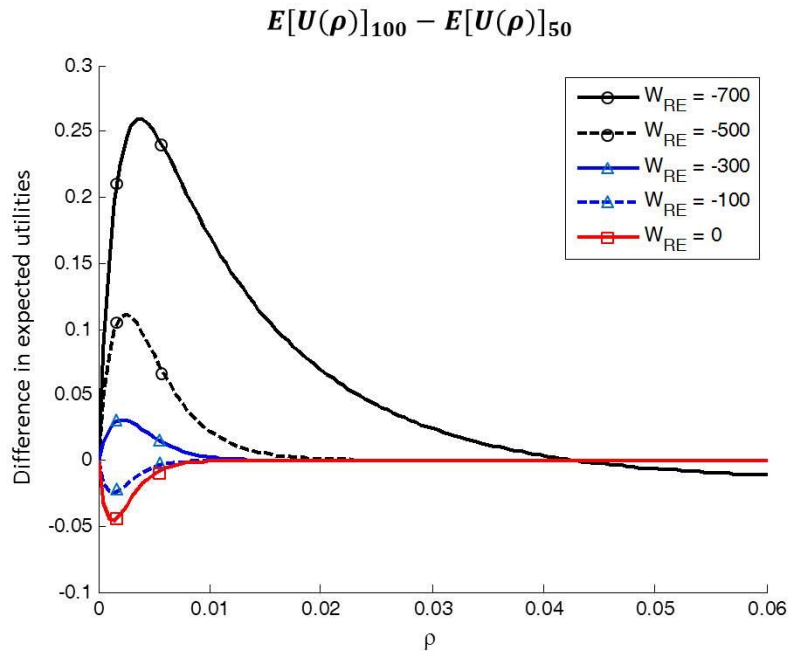


Figure 8. Difference in the expected utilities: 100<sup>th</sup> percentile design vs. 50<sup>th</sup> percentile design.

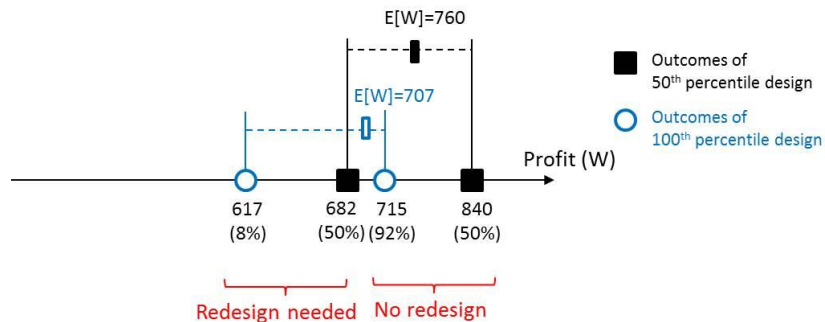


Figure 9. Abstracted binomial decision framework when  $W_{RE} = -100$ .  
(The numbers on this figure do not exactly match the numbers on Fig. 7 because of the simplification.)

On the contrary, if the redesign cost is significant ( $W_{RE} \leq -300$ ), the 100<sup>th</sup> percentile design is preferred, which is also consistent with the decision based on the expected wealth in Fig. 7. However, for a highly risk-averse person ( $\rho > 0.0425$ ), the less conservative design (50<sup>th</sup> percentile design) is more attractive, and this counterintuitive behavior is more clearly seen for the case of  $W_{RE} = -700$ .

To analyze further this behavior, Figs. 10 and 11 show the abstracted binary outcomes and the breakdown of the expected utility corresponding to each of the outcomes, respectively for the case of  $W_{RE} = -700$ . As shown in Fig. 10, the higher expected wealth of the 100<sup>th</sup> percentile design is a main driver of the preference when  $\rho$  is small. However, as shown in Fig. 11, as  $\rho$  increases, the slowest pace of increase in the expected utility corresponding to the smallest wealth of the 100<sup>th</sup> percentile design ( $W=17$  and the dashed blue line at the bottom in Fig. 11) allows the expected utility of the 50<sup>th</sup> percentile design surpass. In other words, an extremely risk-averse person tends to avoid a choice with the worst-case scenario even though its likelihood is small.

In spite of the mathematical-based explanation, this behavior of expected utility hating the worst case scenario creates an inconsistency with the preference for the low stake. That is, for example, when  $\rho = 0.06$ , Fig. 8 tells that the person who is indifferent when the redesign cost is zero turns out to prefer the less conservative design that is exposed to a high probability of redesign when the redesign cost is severe. In addition, according to the fair bet



problem in Table 1,  $\rho = 0.06$  under a high stake is deemed as extremely risk-averse demanding 100% chance to win. From both considerations, the trend of utility theory observed for a high risk-aversion may be considered impractical.

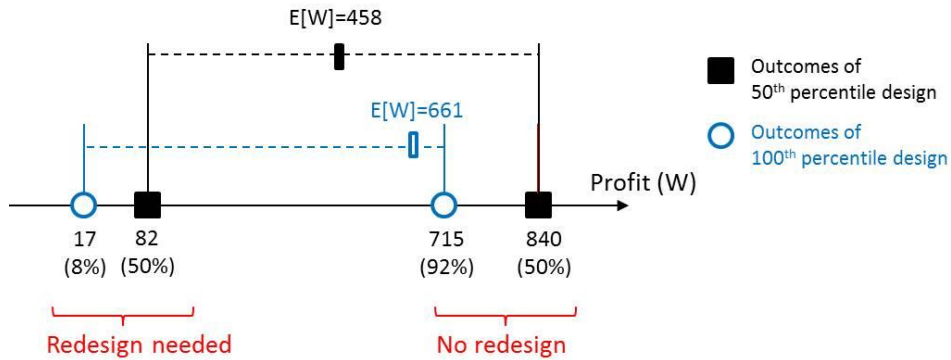


Figure 10. Abstracted binomial decision framework when  $W_{RE} = -700$ .  
 (The numbers on this figure do not exactly match the numbers on Fig. 7 because of the simplification.)

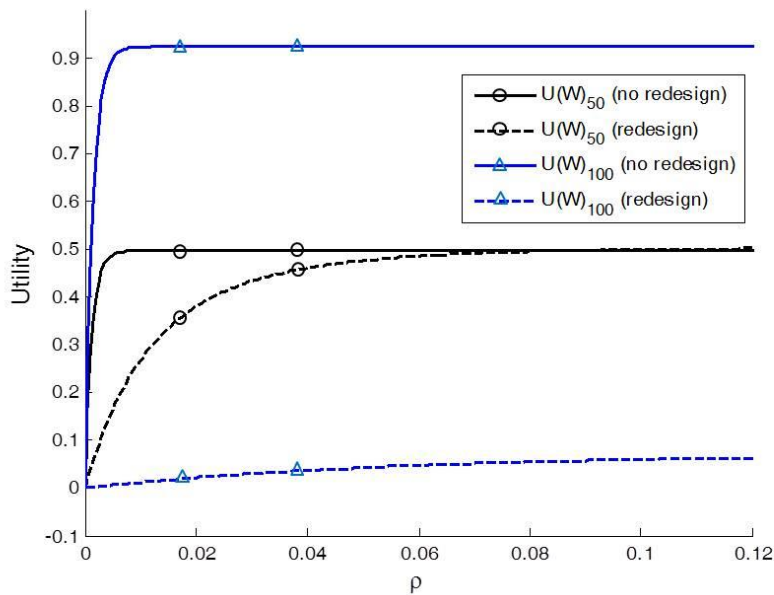


Figure 11. Convergence of utilities corresponding to the binary outcomes when  $W_{RE} = -700$

To understand further the effects of level of risk-aversion, Fig. 12 depicts the utility function with various values of  $\rho$ . As seen in the figure, when the coefficient of risk aversion is small (i.e.,  $\rho = 0.0001$  or  $0.001$ ), the utility functions are almost linear over the range of wealth; therefore the design conservativeness selection coincides with the decision based on the expected wealth. As  $\rho$  is increases, while the value (utility) of wealth at lower wealth becomes more distinctive (steeper slope), the value of wealth at higher wealth does not much. Similar trend is also observed for another type of utility function with hyperbolic absolute risk aversion (HARA). In the context the design conservativeness selection, the problem boils down to a binomial outcome problem, i.e., two outcomes apart (redesign or no-redesign). Therefore, decision for a risk-averse person tends to be dominated by the comparison of outcomes with smaller wealth, i.e., the wealth of redesign cases. In Appendix, additional study on the level risk aversion is conducted.

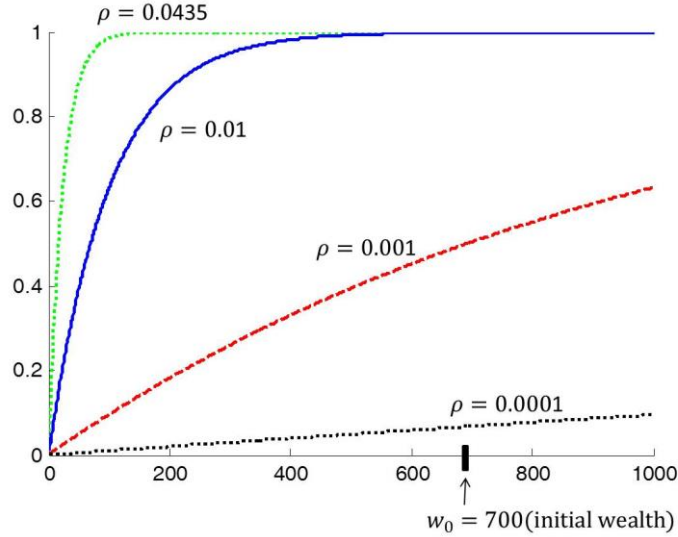


Figure 12. Utility functions with various levels of risk-aversion

We carried out the same study for all the combinations of the percentile of the probability of failure estimate, and the similar trend is observed for all cases. In summary, first, the study shows a danger of expected utility theory facilitating an impractical decision-making. An extremely risk-averse person might make a riskier choice when the redesign cost is high. Second, a moderately risk-averse person concurs with the preferences based on expected value theory unless he is highly risk-averse. This indicates that the main driver of the selection of conservativeness is not the level of risk aversion, but the severity of redesign.

## VI. Concluding Remarks

This paper investigates the tradeoff between redesign risk after a certification test and performance (mass reduction) associated with the conservativeness of probability of failure estimate. Expected utility theory is used to shed light on key features of the tradeoff. The study reveals that the key factor to the conservativeness selection is not the level of risk aversion of the decision maker, but the cost impact of redesign. In addition, it is found that for extremely high levels of risk aversion, expected utility theory might promote an impractical decision which paradoxically leads to a riskier situation. Therefore, it is important when using utility theory for making such decision to select reasonable values of the risk aversion coefficient.

## VII. Appendix

To intuitively understand the level of risk aversion, we exercise a fair bet (lose  $w$ / win  $w$  under the initial wealth of  $w_0$ ). We are interested in for what probability of winning ( $p$ ), the decision maker is willing to bet. This can be evaluated by solving the following equation, where the expected utility of betting is the same as the utility of not betting.

$$pU(w_0 + w) + (1 - p)U(w_0 - w) = U(w_0) \quad (A1)$$

To solve this equation, we employ the Taylor-expansion at  $w_0$ , then

$$p \left[ U(w_0) + wU'(w_0) + \frac{1}{2}w^2U''(w_0) \right] + (1 - p) \left[ U(w_0) - wU'(w_0) + \frac{1}{2}w^2U''(w_0) \right] = U(w_0) \quad (A2)$$

By neglecting the higher order terms, we obtain

$$(2p - 1)wU'(w_o) + \frac{1}{2}w^2U''(w_o) \cong 0 \quad (A3)$$

If the utility function is linear,  $U'' = 0$ , and then  $p = 1/2$ . If the utility function is not linear, by using the coefficient of risk aversion defined by Eq.(1)

$$p \cong \frac{1}{2} - \frac{1}{4}w \frac{U''(w_o)}{U'(w_o)} = \frac{1}{2} + \frac{1}{4}wR_A(w) \quad (A4)$$

Table A1 shows the probability to win for which a risk-averse person is willing to bet. For example, when  $w=50$ , which corresponds to the stake of the ITPS problem when  $W_{RE} = -100$ , a person with coefficient of risk aversion  $R_A=0.035$  will not bet unless the probability of winning is higher than 0.938. This person can be deemed as a highly risk-averse. For a larger stake ( $w=350$ ) corresponding to  $W_{RE} = -700$ , the reluctance of betting is intensified, and even  $R_A=0.01$  is considered as extremely risk-averse, requiring 100% chance of winning.

Table A1. Probability of winning for which a risk-averse person is willing to bet a fair bet (lose  $w$ / win  $w$ ).

Risk aversion $R_A$	Probability of winning		
	$w=50$	$w=100$	$w=350$
0.001	>0.13	>0.525	>0.5875
0.005	>0.563	>0.625	>0.9375
0.010	>0.615	>0.750	1
0.015	>0.688	>0.875	1
0.020	>0.750	1	1
0.035	>0.938	1	1
0.040	1	1	1

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