Uncertainty Management by Safety Measures in Probabilistic Product Design

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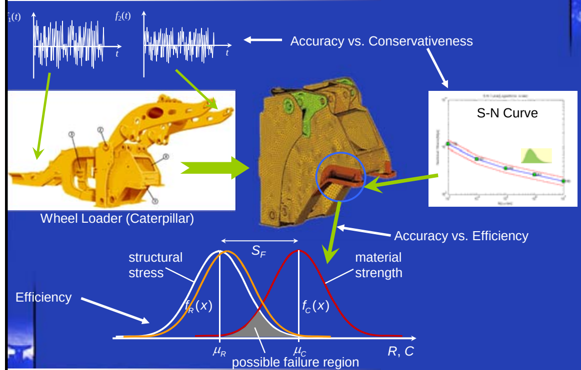
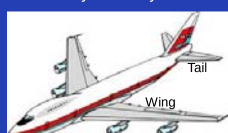
Motivation



Real advantages of probabilistic design

- Risk allocation for multiple failure modes
- Tradeoff for cheapest safety measures to satisfy reliability constraint
- Wing example

$$P_F = 0.5^{100\frac{\Delta W}{W_0}} \cdot P_{F0}$$

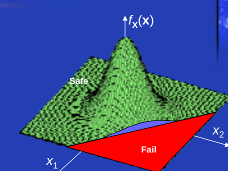


	Wing		Tail		System	
	Weight	P_F	Weight	P_F	Weight	P_F
Initial	10,000lb	1%	1,000lb	1%	11,000lb	2%
Det. opt	10,200lb	0.25%	1,020lb	0.25%	11,220lb	0.5%
RBDO	10,120lb	0.435%	1,080lb	0.004%	11,200lb	0.439%

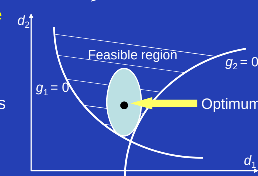
Well, RBDO seems good, but …

- Conventional RBDO formulation

Minimize $C(X,d)$
Subject to $P[g(X,d) \le 0] \le P_{target}$



- can only incorporate **uncertainties that are given in the design stage**
- Too large uncertainty without considering safety measures
- Industry developed safety measures by trial-and-error



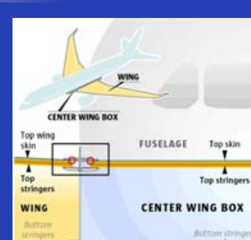
Types of uncertainty

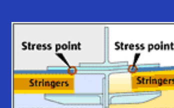
- **Aleatory uncertainty**
  - Inherent variation of the physical system or the environment under consideration
  - Variability, type A, irreducible, or stochastic uncertainty
- **Epistemic uncertainty**
  - Potential inaccuracy in any phase or activity of the modeling process due to a lack of knowledge
  - Subjective, type B, reducible, or cognitive uncertainty

Type of uncertainty	Definition	Causes	Reduction measures
Error	Departure of average from model	Simulation errors, construction errors	Testing and model refinement
Variability	Departure of individual sample from average	Variability in material properties, construction tolerances	Tighter tolerances, quality control

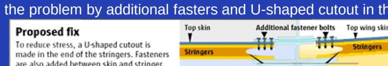
Boeing 787 failed certification test

- Boeing expect I-stringers evenly distribute its load to the joint of the center wing box
- Excessive load is transferred through the upper part of stringers and wing surface and the joint is overloaded
- **Error in internal load calculation!**





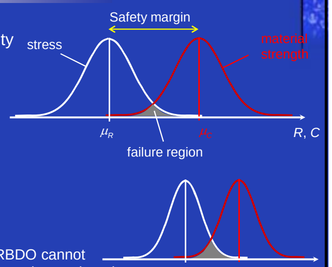
- Fixed the problem by additional fasters and U-shaped cutout in the stringers



Design under Uncertainty

- How to design under uncertainty

1. Live with it
   - Conservative design with a large safety margin
   - For aleatory uncertainty
2. Improve knowledge
   - More accurate models
   - Tests
   - For epistemic uncertainty
   - Without shaping uncertainty, RBDO cannot satisfy reliability constraint in complex engineering systems

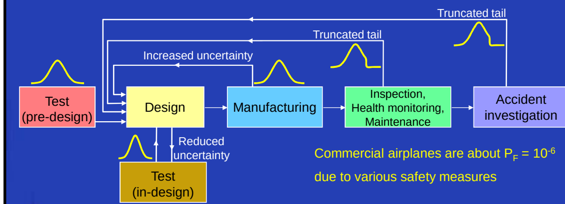


- Designer needs to consider both uncertainties

Need a tradeoff between conservative design and more tests

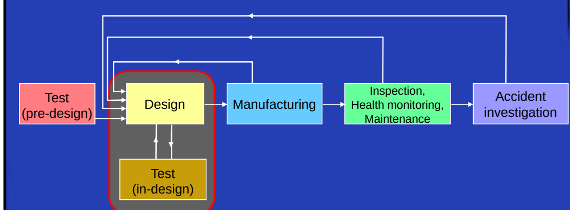
Safety Measures

- How can the safety of a complex system be maintained?
- Activities of reducing uncertainty to make the system safer



Commercial airplanes are about $P_F = 10^{-6}$ due to various safety measures

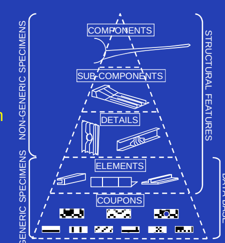
- How to allocate resources to provide public safety most efficiently?
  - Need to quantify the contribution of each safety measure to the system safety



Uncertainty Quantification in the Building-Block Process

Complex Engineering System Development (Building-Block Test Process)

- Building-block test process: Systematic design and test evaluation at different stages
- Impossible to satisfy system reliability without shaping uncertainty at multi-level process
- Accumulation of uncertainties in the building-block process
- Calibration and redesign at each level based on calculation and test evaluation
- Want to find/correct design errors at a low-level (need to consider test variability)

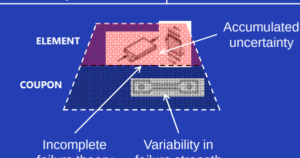


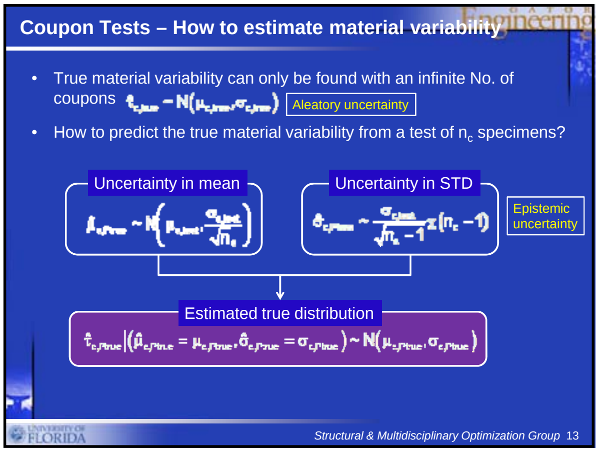
Difficulties in the Current Building-Block Process

- Developed based on trial-and-error and adding safety margins
- Reducing epistemic uncertainty (error) at different levels through tests
- Current technology cannot quantify how much errors at each level are accumulated and contributed to the system
- Unable to identify how much tests at each stages can effectively reduce uncertainties in the system level
- Need a probability-based calculation methodology
- Possible to make the most efficient test evaluation plan within budget
- Possible to estimate the level of confidence at certificate

Basic Purpose of Coupon and Element Tests

Test stage	Objectives	Uncertainty sources
Coupon test	Estimate nominal value and variability of material strength	Variability in material strength and sampling error due to a finite number of coupons
Element design	Estimate multi-axial strength based on a failure theory	Incomplete knowledge of failure mechanism: error in failure theory
Element test	Reduce uncertainty in the multi-axial strength	Sampling error due to a finite number of elements




Coupon Tests – How to estimate material variability
True material variability can only be found with an infinite No. of coupons
Aleatory uncertainty
How to predict the true material variability from a test of n_c specimens?
Uncertainty in mean
Uncertainty in STD
Epistemic uncertainty
Estimated true distribution
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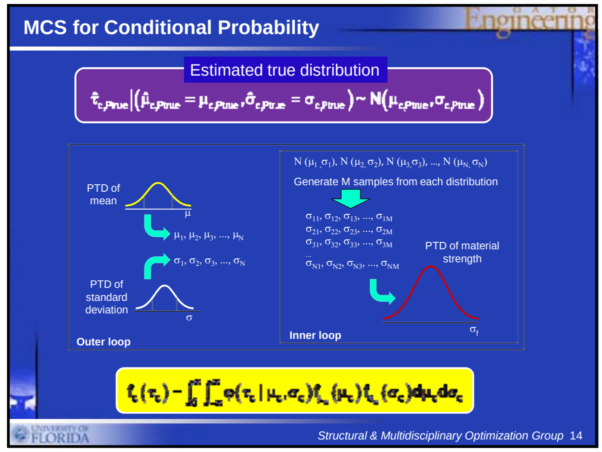
MCS for Conditional Probability
Estimated true distribution
Generate M samples from each distribution
PTD of mean
PTD of standard deviation
PTD of material strength
Outer loop
Inner loop
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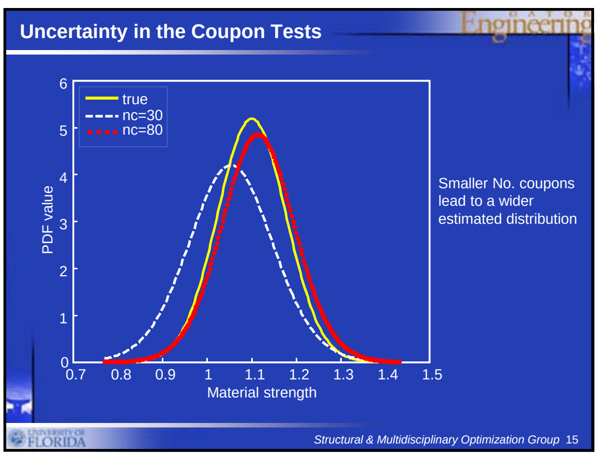
Uncertainty in the Coupon Tests
true
nc=30
nc=80
PDF value
Material strength
Smaller No. coupons lead to a wider estimated distribution
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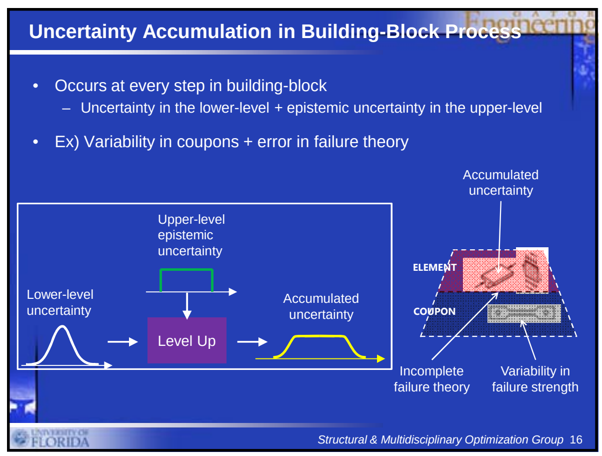
Uncertainty Accumulation in Building-Block Process
Occurs at every step in building-block
Uncertainty in the lower-level + epistemic uncertainty in the upper-level
Ex) Variability in coupons + error in failure theory
Upper-level epistemic uncertainty
Lower-level uncertainty
Level Up
Accumulated uncertainty
ELEMENT
COUPON
Incomplete failure theory
Variability in failure strength
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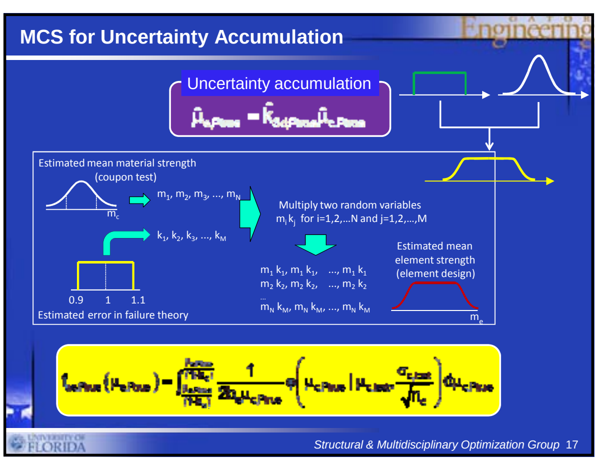
MCS for Uncertainty Accumulation
Uncertainty accumulation
Estimated mean material strength (coupon test)
Multiply two random variables
Estimated mean element strength (element design)
Estimated error in failure theory
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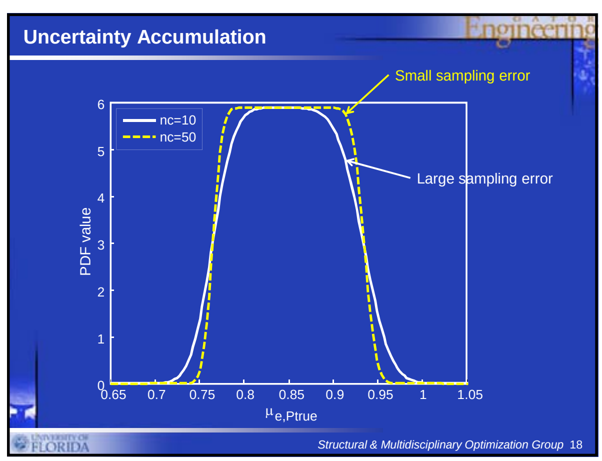
Uncertainty Accumulation
Small sampling error
Large sampling error
nc=10
nc=50
PDF value
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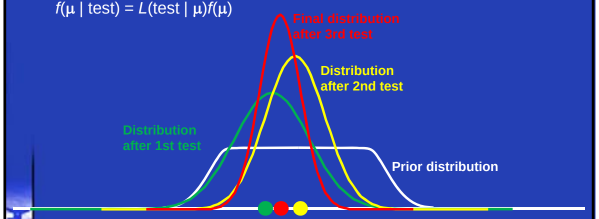
Uncertainty Reduction by Tests

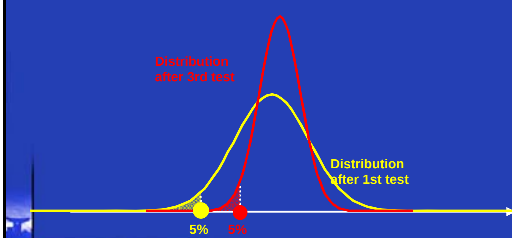
- Accumulated uncertainty between levels as a prior
- Likelihood based on experimental error
- Posterior distribution based on Bayesian inference after tests

$f(\mu \mid \text{test}) = L(\text{test} \mid \mu) f(\mu)$



Effect of the Number of Tests

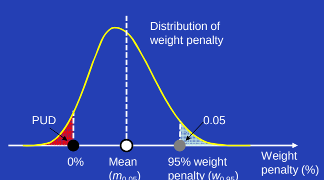
- Distribution depends on the number of tests
- Possible to estimate the effect of system weight based on 5% conservative allowables



Weight Penalty

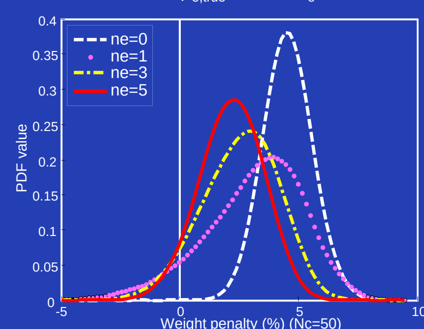
- A way of accessing the merits of the number of coupon and element tests
- Weight penalty is calculated by conservative 5th percentile against the true distribution

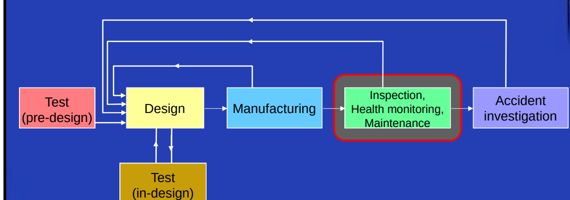
$$w_1 = \left(\frac{A'_{0.05}}{A_e} - 1\right) \times 100 = \left(\frac{F/\sigma'_{0.05}}{F/\mu_{e,true}} - 1\right) \times 100 = \left(\frac{\mu_{e,true}}{\sigma'_{0.05}} - 1\right) \times 100\ (\%)$$



Ex) Unconservative Failure Theory

nc = 50, $\mu_{e,true}$ = 0.95, b_e = 10%

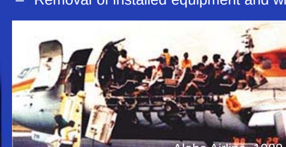




Prognostics and Health Management

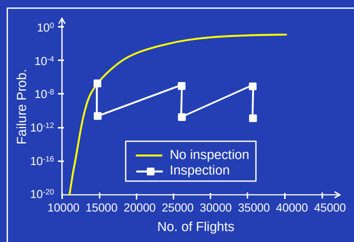
Damage Tolerance Design and Preventive Maintenance

- Damage-tolerance design
  - Flaws can exist and propagate with usage as long as they can be detected and repaired through preventive maintenance
- Preventive maintenance (PM)
  - Inspection (NDI) for all panels & stiffeners
  - Repair/replace for detected cracks
  - Type C : Cost ~ $2M
  - Downtime loss
  - Removal of installed equipment and wires



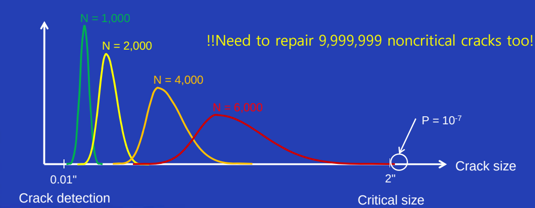
What's the Effect of Inspection on Safety?

- PM removes dangerous tail part and recovers reliability (safety)
- Without inspection, the current panel thickness must be increased by 30%



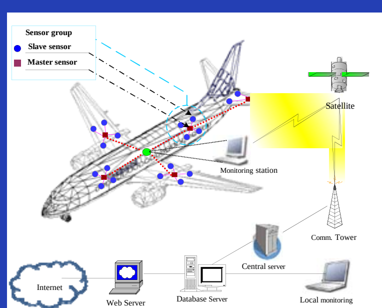
Preventive Maintenance and Uncertainty

- Periodic NDI -> repair & replace
  - FAA regulation: every 6,000 flights, repair cracks larger than 0.01"
  - Not causing fracture before next inspection
- Uncertainty in crack growth
  - Uncertainty in material properties, applied loadings, etc
  - Probability to grow to the critical size before next inspection: 10^{-7}



Structural Health Monitoring (SHM) System

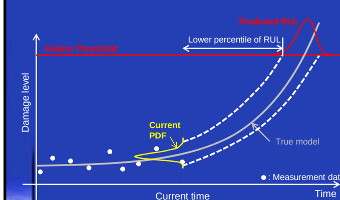
- Damage detection using sensors installed on panels

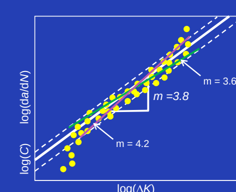


Online SHM

What Is Prognostics?

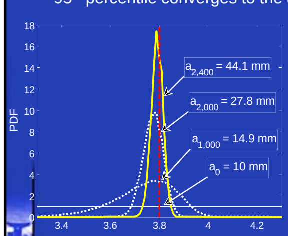
- To predict the behavior of damage (crack size, performance degradation) by combining measurement data with physical model
- To Identify model parameters using noisy data: $\frac{da}{dN} = C(\Delta K)^m$
- To predict RUL (remaining useful life, remaining time before maintenance)

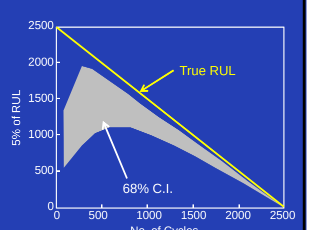




Parameter Identification (with noise and bias)

- Initial distribution: m ~ U(3.3, 4.3)
- Updated m converges to m_{true} = 3.8
- Fast convergence as crack grows fast
- RUL: remaining cycles until critical size a_C
- 95th percentile converges to the accurate RUL from a conservative side



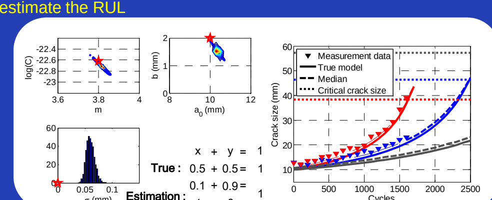


Multiple Parameters – Challenge in Correlation

- In general multiple parameters need to be identified
- Some of them are correlated
  - Strong correlation between m and C
  - Strong correlation between a_0 and bias

D. An, et al, Identification of correlated damage parameters under noise and bias using Bayesian inference, Structural Health Monitoring 11(3), 292-302, 2012

- Difficult to identify exact values of parameters, but good enough to estimate the RUL



Summary

- Conventional RBDO needs to include safety measures, which shape uncertainty before/after design
- Bayesian inference is used to quantify the effect of uncertainty reduction by tests during building-block process
- Bayesian inference is also used to reduce uncertainty in Inspection & maintenance process to accurately predict the remaining useful life

Contributors and Supporters

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Thank you for your attention