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CONCEPTUAL DESIGN OPTIMIZATION WITH ECONOMIC UNCERTAINTY: AN APPLICATION TO INTERACTIONS BETWEEN DESIGNERS AND AIRLINES

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ABSTRACT

Many engineering problems involve interactions between multiple decisions makers, or stakeholders, each with their own objectives and uncertainties. Considering these interactions during design optimization allows us to account for new sources of uncertainty, which we refer to as economic uncertainty. In this paper, we consider an application of optimization considering interactions between aircraft designers and airlines based on the design of a commercial transport aircraft wing. We consider that the aircraft designer makes their design decisions first, and therefore must predict the reaction of the airline. We focus on the effect of two economic uncertainties: uncertainty that would normally only affect the airline and uncertainty due to asymmetric information, or errors in the designers' understanding of the airlines' preferences. We find that these uncertainties play a significant role in the optimal decisions by both airlines and designers. We also show that asymmetric information may actually be beneficial for both stakeholders in certain cases, where both players benefit from the aircraft designer underestimating the operating costs of the airline.

INTRODUCTION

Design optimization frequently deals with uncertainty due to variations in material properties, operating conditions, and design specifications. One often overlooked source of uncertainty is in the way designers determine tradeoffs between multiple objectives. These tradeoffs affect the value of the design to customers, regulators, and other interested stakeholders in the design, which ultimately determines the profitability of the design. Each of these stakeholders are dynamic decision makers performing their own optimization to maximize their profits. However, traditional multi-objective design optimization rarely considers these dynamic interactions, and when it does it models the preferences of other stakeholders using heuristic methods.

Aircraft design is often viewed as a characteristic multi-objective or multi-disciplinary problem. Aircraft design is also subject to complex relationships between stakeholders; these stakeholders include the airlines who buy the aircraft and the passengers who buy tickets. Additionally, because of the large time gap between design and entire service life, changing market conditions can play a major role in the success or failure of a design; for instance changes in fuel prices or public demand for air travel.

Designers set their objective function based on data regarding the preferences of airlines and the public, either learned from past experience or provided directly by these stakeholders. The interaction between stakeholders in the communication of these preferences is subject to uncertainties because stakeholders do not have perfect knowledge of their own interests now or in the future, but also because it may be advantageous for them to provide misleading information, leading to what is known as information asymmetry. We propose to utilize game theory to model how each of these stakeholders will interact with one another as they make strategic decisions to maximize their own welfare. The framework for reformulating a multidisciplinary design problem considering multiple stakeholder interactions using game theory was described in Waycaster et al [1]. In this work, we will focus on quantifying the importance of considering these economic uncertainties relative to other sources of uncertainty in a characteristic problem.

NOMENCLATURE

C_{fix}	Airline fixed operating cost
C_{fuel}	Jet fuel cost
C_{ini}	Initial design fixed cost

C_{pen}	Cost of failing design certification
FC	Aircraft lifetime fuel consumption
K_{min}	Minimum certification knockdown factor
K_{pen}	Certification knockdown factor
K_{rate}	Effect of number of tests on knockdown factor
N_{air}	Number of aircraft purchased
N_{flight}	Aircraft service life in flights
N_{pax}	Number of passengers per flight
N_{test}	Number of design certification tests
N_{tix}	Number of tickets sold per year
P_{air}	Aircraft purchase price
P_{max}	Maximum airline ticket price
P_{pen}	Probability of certification penalty
P_s	Ticket demand slope
P_{tix}	Ticket price
PF	Probability of failure
SF	Design safety factor
VSL	Value of statistical life
Π_a	Airline profit function
Π_d	Aircraft designer profit function

DESIGN PROBLEM DESCRIPTION

The goal of our simple example problem is to represent a characteristic multidisciplinary design optimization problem using simple analytical formulas. The problem we look at is the design of a commercial transport aircraft wing, which provides a mixture of structural and aerodynamic performance goals. The aircraft designer specifies two configuration design variables: the wing aspect ratio and the design safety factor beyond what is required by regulations.

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The aircraft designer makes a third decision on the number of structural tests to perform which will affect the minimum acceptable knockdown factor for certification of the aircraft, similar to the A-basis criteria specified by the FAA [2]. Based on the probability of not meeting this certification criteria, the designer will be assessed some monetary penalty. Additionally, each test performed will have some fixed cost. This means the designer may choose to have higher design cost (more tests) in order to improve performance or reduce certification cost by allowing for a less conservative certification criteria.

The wing is idealized using a trapezoidal shape, where sweep, planform area, and taper ratio are based on the dimensions of a Boeing 737-700 [3] and are constant across all designs, meaning that changing the aspect ratio will scale the span and chord proportionally. The wing box is constrained by the wing cross-section, meaning increasing the aspect ratio will decrease the maximum possible design safety factor due to longer, more slender wings.

Once these design variables are determined, a structural designer optimizes the wing box for minimum weight subject to constraints on stress and deflection. Details on the models used to estimate aerodynamic and structural characteristics of the wing are provided in Annex A.

This simple design problem introduces some basic tradeoffs similar to those seen in a true multidisciplinary design problem. By increasing the aspect ratio, the aircraft designer can reduce the aircraft fuel consumption, but this change cause penalties structural weight and probability of failure. The aircraft designer can reduce the probability of this design penalty either by increasing safety factor or increasing the number of tests performed. These trade-offs are summarized in Fig. 1 Design problem trade-off chartFig. 1.

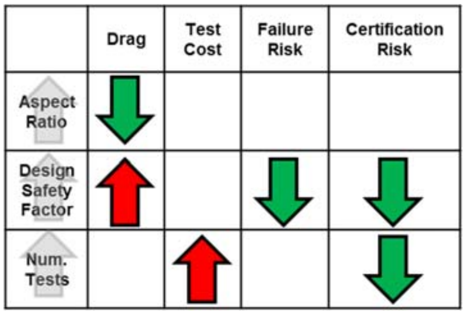

Fig. 1 Design problem trade-off chart

REFORMULATING OPTIMIZATION

To be able to consider interactions in optimization, we must describe how decisions are made among stakeholders and how information is shared between them. In this case, we will consider that the designer is interacting with an airline, who determines the number of aircraft to purchase based on the number of tickets the airline is able to sell. To deal with these interactions, we use common terminology and techniques utilized in game theory [4].

One key concept is the idea of a *best reply function*, which defines a player's optimal strategy given the strategies of all other players. We can calculate a best reply function by taking the partial derivative of a player's objective function with respect to each of their decision variables, setting the result to zero and solving for the optimal value of that decision variable, as shown in Equations 1 and 2.

$$\Pi_2(X_1, X_2) \tag{1}$$

$$\frac{\partial \Pi_2}{\partial X_2} = 0 \Longrightarrow X_2^* = f(X_1) \tag{2}$$

Second order conditions are guaranteed by the fact that any meaningful profit function for a player should be concave in each player's own decision variables. The resulting expression will provide the optimal value of that decision as a function of the actions of all other players.

For our example problem, we will consider that the stakeholders play a sequential game, where the aircraft designer will act first to determine the nature of the aircraft available. After learning what the aircraft designer does, the airline will determine how many aircraft to purchase. In order for the aircraft designer to act first, they must estimate the best reply function of the airline which can then be inserted into the aircraft designer's own profit function. In doing so, the aircraft designer will incorporate the uncertainties faced by the airline directly into their own optimization problem.

The estimation of this best reply function may itself be subject to some error or uncertainty. We consider that we may have a case of asymmetric information, meaning one player has more available information than another. For instance in our example problem, the airline may know their own profit function exactly, while the aircraft designer may estimate some elements of the airline profit function with some error.

It can then be determined whether or not the airline has an incentive to signal, or to communicate information to the aircraft designer, that would either increase or decrease the error of this estimation by the aircraft designer. Similarly, we can see if the designer has an incentive to screen, or try to gather more information from the airline about their preferences. In some cases, errors in the aircraft designer's estimation of the airline's preferences might be good for both players, bad for both, or might increase one player's profit at the expense of the other. Understanding the situations that give rise to these cases is an important factor in understanding airline and aircraft designer relations.

ECONOMIC INTERACTION MODEL

In order to model the interactions between aircraft designers and airlines, we must first develop reasonable ways to express the profits of each group. We attempt to specify objective functions that capture some important trade-offs and interactions for both stakeholders without excessive complexity. The interactions of stakeholders with the design problem and their exchange of information are summarized in Fig. 2.

For the aircraft designer, revenues are based on the number of aircraft sold to airlines and the price the aircraft designer decides to charge. The aircraft designer's costs are based on a fixed initial cost of a new project, the number of tests they perform which each have a fixed cost, and the probability of a certification penalty which we assume will also have a fixed cost associated with making the design safety compliant. The aircraft designer profit function is then given as

$$\Pi_d = N_{air}P_{air} - N_{test}C_{test} + P_{pen}C_{pen} - C_{ini} \qquad (3)$$

The airline's revenue is based on the price and quantity of tickets sold. We assume that each aircraft has a useful life of 60,000 flights with an average passenger load of 100 passengers per flight. Additionally, we consider that the number of aircraft is significant enough as to not face scheduling and route constraints. The demand for air travel is defined using a simple linear demand function, such that price is determined for a given number of flights, such that

$$P_{tix} = P_{max} - P_s N_{tix} \qquad (4)$$

Airlines have four different sources of costs; the first is based on the fuel consumption over the life of the aircraft. The second cost is based on the acquisition price of the aircraft they choose to purchase. The third cost component is a fixed cost for each aircraft, based on the labor, taxes, fees, and passenger services required for each aircraft. The final cost component is based on the level of safety of the aircraft being utilized, where the cost is equal to the product of the probability of failure, the number of passengers, and a penalty per life at risk; for this penalty we use the value of statistical life specified by the Department of Transportation. This cost term is intended to reflect the increased safety and maintenance costs related to flying less safe aircraft. Combining these components, the airline's profit function is

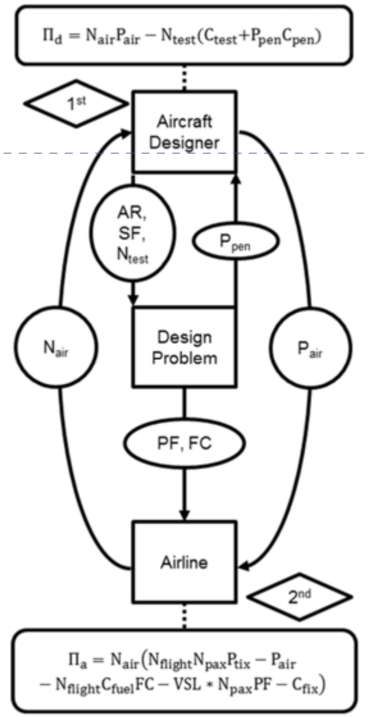

Fig. 2 Complete interactions between stakeholders and design

$$\Pi_a = N_{air}\big(N_{flight}N_{pax}P_{tix} - P_{air} - N_{flight}C_{fuel}FC \\ - VSL * N_{pax}PF - C_{fix}\big) \qquad (5)$$

We relate the number of tickets sold and the number of aircraft purchased by assuming each aircraft is capable of 5 flights per day.

$$N_{air} = \frac{N_{tix}}{5 * 365N_{pax}} \qquad (6)$$

Since we have specified a sequential game, the aircraft designer will need to estimate the actions of the airline using a best reply function. We can calculate the airline's best reply function by taking the

Commented [HT1]: We are dealing with a special case that only the designer needs a best reply function, while the airline does not. So the main asymmetry is there. Notice that even if the airline and the builder have the same information, the same issue exists, because the designer does not need information on the airline cost now, but on the cost when the airplane will fly. So it is not clear that asymmetry is the main culprit. So yo may want to tone down the asymmetry as the single cause and have it as one of the two possible causes.

Commented [GW2R1]: While it is true that only the designer is using a best reply function, this comes about due to the sequential nature of our problem. The term asymmetric information deals specifically with a case where one player has better information about a certain parameter than another. I have tried to distinguish these two issues by discussing the sequential game as well as the asymmetric information.

Tab. 1: Fixed coefficient values

Coefficient	Value
Half-span	17.16 m
Taper ratio	0.159
Required lift	80,000 kg
Zero lift drag coefficient	0.01
Planform area	68 m^2
Maximum ticket price, P_{max}	\$400 – \$600
Ticket demand slope, P_s	0.2 – 0.3 \$ per million tickets
Passengers per flight, N_{pax}	100
Test cost, C_{test}	\$1M
Initial design cost, C_{ini}	\$30B
Penalty cost, C_{re}	\$500M – \$5B
Fuel cost, C_{fuel}	\$0.53 – \$1.33 per liter
Lifetime fixed operating cost, C_{fix}	\$780M – \$900M
Value of statistical life, VSL	\$9.1M
Yield stress	450 MPa – 550 MPa
Elastic Modulus	65 GPa – 80 GPa
Thrust specific fuel consumption	0.06 $^{kg}/_{N \cdot hr}$
Nominal wing weight	5,000 kg
Nominal wing volume	2.0 m^3
Test limit load	3 g
Critical deflection/span	0.25
Designer error in C_{fix}	-10% – 10%

first derivative of their profit with respect to the number of tickets sold. Combining Equations (4 – 6), we can rewrite the profit function as

$$\Pi_a = \frac{N_{tix}}{5 * 365 * N_{pax}} \big(N_{flight} N_{pax} (P_{max} - P_s N_{tix}) - P_{air} - N_{flight} C_{fuel} FC - VSL * N_{pax} PF - C_{fix}\big) \qquad (7)$$

Taking the derivative with respect to N_{tix}, setting the result equal to zero, and solving for the optimal value of N_{tix} yields

$$N_{tix}^{*} = \frac{1}{2 N_{flight} N_{pax} P_s} (N_{flight} N_{pax} P_{max} - C_{fix} - P_{air} - C_{fuel} N_{flight} FC - VSL * N_{pax} PF) \qquad (8)$$

We may now use the best reply function to allow the aircraft designer to anticipate N_{air} by using the relation in equation (4). In doing so, we have now directly incorporated information that previously only affected the airline, such as fuel cost and demand for air travel, directly into the optimization formulation for the other stakeholders.

PROBLEM ANALYSIS

Now that we have defined the formulation of the design problem and the profit functions for each of our stakeholders, we can calculate the profit maximizing solution for the aircraft designer who is anticipating the reaction of the airline. To do this, we must first find reasonable estimates for some of the coefficients present in the design and profit functions. Table 1 provides a summary of these coefficients, their assumed values or a range of values; these values and ranges are estimated based on various sources [5 - 21].

Note that for eight of these variables, we have assumed a range of values. This is either due to uncertainty in the true values (e.g. fixed operating cost), or actual randomness in the true values (e.g. yield strength.) To understand the effect of these variations, we perform a case study in which we take each of these uncertainties as an interval variable. We propagate this uncertainty in order to understand the effect of changes in these values on the optimal decisions made by the designer and airline. Even though the change in profits may be significant, if the optimal decisions are relatively constant with respect to variation in one of these coefficients, it will be reasonable to neglect it. We perform an optimization using the interval values as decision variables in order to minimize and maximize the optimal values of each of the four decisions variables between both stakeholders (Table 2)

Tab. 2: Range of optimal decisions

Decision Variable	Range of Optimal Values
Aspect Ratio	7.5 – 14.0
Safety Factor	1.17 – 2
Number of Tests	9 – 11
Number of Tickets	0 – 3.2B

We see that there is a large variation in the optimal values of each of the decisions, except for number of tests. We also find that for some combinations of variables, the airline will elect not to fly at all meaning that for some combinations of parameters it is impossible for the airline to be profitable. The primary change for the designer comes from the aspect ratio, the optimal value of which varies completely between the upper and lower bounds specified in the optimization problem. In order to understand how much of this variation in optimal decisions is due to economic uncertainty, we can compare to the case where each economic variability is fixed and only variability in material properties remains. The results of this analysis can be seen in Table 3. It can clearly be seen that the addition of economic uncertainties has a significant effect on the decisions.

Tab. 3: Range of optimal decisions without economic uncertainty

Decision Variable	Range of Optimal Values
Aspect Ratio	11.5 – 14.0
Safety Factor	1.17 – 1.30
Number of Tests	10
Number of Tickets	765M – 772M

To consider asymmetric information, we look at the relationship between error in the designer's estimate of airline fixed costs and profits for both stakeholders across all cases of our other uncertainties. We find an interesting result; both stakeholders benefit from the designer underestimating the airline's fixed costs. When the designer believes the airline fixed costs are low, they will build a more efficient and more expensive aircraft, as they believe airlines will have higher profit margins for the same number of tickets and may be willing to pay more for aircraft that can reduce fuel consumption. The net effect of this change will be a reduction in in the cost per flight due to increased fuel efficiency, leading the airline to sell more tickets at lower prices (and in turn buying more aircraft).

The reason that this can happen is that the designer's decision based on the best reply function is not guaranteed to be Pareto optimal for either player. Were the designer allowed to change the design after learning the true number of aircraft purchased by the airline, they would choose a different design which would provide even higher profits for the designer, but lower profits for the airline. This is a phenomenon known as double marginalization, where two firms each add some profit margin to the price of a good, in this case air travel. The net effect of this double marginalization is actually a reduction in profits for both firms. When the designer has error in assessing airline fixed prices, they essentially reduce their own profit margin, and the benefits of this action are passed to the airline. In this case, the designer benefits as well due to the increase in aircraft sales.

Finally, we consider an example that demonstrates some unexpected results of this study. It is commonly known among aircraft designers that fuel prices are an important consideration, and that aspect ratio can provide a trade-off between fuel consumption and increased weight (and therefore increasing aircraft purchase price). Another important consideration that emerges from this work is the effect of consumer demand for air travel.

We consider that the aircraft designer has already designed the optimal aircraft at the current level of demand for air travel where the demand curve intercept is $380; this design is shown in Table 4, Case 1. After some time, the demand for air travel drops such that the new intercept is $250. The aircraft designer can now choose to keep the same aircraft and update the price, or design a brand new aircraft; these are shown in case 2A and 2B respectively.

We observe a significant change in the optimal aspect ratio and safety factor for the redesign case, which also provides more than 60% greater aircraft design profits as compared to using the same design. The airline also sees a 40% increase in profits, with more than 50 million more tickets sold each year. This occurs because the lower demand causes the airline to be more sensitive to aircraft prices and to buy fewer aircraft. Taking this into account, the designer uses a lower aspect ratio and safety factor, sacrificing fuel efficiency and reliability for reduced cost. Without incorporating the effects of changes in demand into the optimization framework, an aircraft designer would not be aware of this potential change and might lose profits as a result.

Tab. 4: Demand shift case study

Parameter	Case 1[1]	Case 2A[2]	Case 2B[3]
Demand Curve Intercept	$375	$250	$250
Aircraft Designer Profits	$8.08B per year	$0.8B per year	$1.36B per year
Airline Profits	$28.4B per year	$1.7B per year	$4.60B per year
Aspect Ratio	12.28	12.28	11.68
Design Safety Factor	1.38	1.38	1.27
Number of Tests	11	11	10
Aircraft Price	$71.6M	$49.3M	$41.7M
Number of Tickets Sold	840M per year	291M per year	339M per year

[1] Initial optimal design with initial demand level
[2] Initial design at new demand with updated price
[3] New optimal design at new demand level

CONCLUSIONS AND FUTURE WORK

An example problem has been put forward that uses a basic multidisciplinary design problem and a simple model of economic interaction between stakeholders to investigate the relative importance of engineering and economic uncertainty on design decisions and outcomes. We have used a basic model of a wing structure and aerodynamics and simple expressions to describe profit functions for aircraft designers and airlines. Interactions between these stakeholders are modeled using game theory, where we have a sequential game with the aircraft designer moving first and asymmetric information regarding the airline's profit function.

We conduct a case study in which we vary the values of eight important model inputs that are likely to be subject to variability or uncertainty. The range of optimal decision sets across all of these cases is computed and indicates that changes in market conditions can have a large impact in these decision values. We find that variabilities related to market conditions and stakeholder profit functions have a much greater impact on design decisions and outcomes than traditional design variabilities such as material properties. This finding indicates that understanding customer preferences and market variability is as much if not more important than understanding uncertainty in design parameters and operating conditions. Additionally, we show that designers acting with errors or limited information may actually produce a more profitable design for both the airline and the designer.

Future work on this topic will include extending our simulation to consider inputs as uncertain at the time of design, rather than deterministic but varying across different cases. This will pose the problem as a robust optimization where each stakeholder seeks to maximize their expected profit. We will also consider the addition of competition among multiple aircraft designers and multiple airlines with possibilities for product differentiation, as well as the potential for different kinds of interaction models than the one proposed in this paper.

REFERENCES

[1] Waycaster, G., et al., 2014, “The effect of stakeholder interactions on design decisions,” Conference on Engineering and Applied Sciences Optimization (OPT-i), Kos Island, Greece.

[2] Federal Aviation Administration. 2003. “Material Strength Properties and Material Design Values.” (Advisory Circular 25.613-1)

[3] The Boeing 737 Technical Site, “Boeing 737 Detailed Technical Data,” URL: http://www.b737.org.uk/techspecsdetailed.htm [cited 7 April 2014].

[4] D. Fudenberg, J. Tirole, *Game Theory*, Cambridge, MA, MIT Press, 1991.

[5] Bureau of Transportation Statistics, “TranStats Database: Air Carrier Summary: T2: U.S. Air Carrier TRAFFIC And Capacity Statistics by Aircraft Type,” URL: http://www.transtats.bts.gov/Fields.asp?Table_ID=254 [cited 7 April 2014].

[6] Airlines for America, “Average round-trip fares and fees,” URL: http://www.airlines.org/Pages/Annual-Round-TripFares-and-Fees-Domestic.aspx [cited 7 April 2014].

[7] van Es, G. W. H., July 2002, “Rapid Estimation of the Zero-Lift Drag Coefficient of Transport Aircraft,” Journal of Aircraft, **39**, pp. 597-599.

[8] Gates, D., “Boeing celebrates 787 delivery as development costs top $32B,” Seattle Times, September 24, 2011, URL: http://seattletimes.com/html/businesstechnology/2016310102_boeing25.html [cited 7 April 2014].

[9] Airlines for America, “Annual Crude Oil and Jet Fuel Prices,” URL: http://www.airlines.org/Pages/Annual-Crude-Oil-and-Jet-Fuel-Prices.aspx [cited 7 April 2014].

[10] Jiang, H., “Key Findings on Airline Economic Life,” Boeing Technical Report, March, 2013.

[11] McCartney, S., “How Airlines Spend Your Airfare,” *The Wall Street Journal*, June 6, 2012, URL: http://online.wsj.com/news/articles/SB10001424052702303296604577450581396602106 [cited 7 April 2014].

[12] Trottenberg, P., Rivkin, R. S., February 28, 2013, Memo to secretarial officers, modal administrators, “Guidance on Treatment of the Economic Value of a Statistical Life in U.S. Department of Transportation Analyses,” Washington, DC.

[13] Snyder, B., “Bend Not Break: Boeing 787 Passes Wing Load Test,” CBS News, March 31, 2010, URL: http://www.cbsnews.com/news/bend-not-break-boeing-787-passes-wing-load-test/ [cited 7 April 2014].

[14] Stanford University, “Data on Large Turbofan Engines,” URL: http://adg.stanford.edu/aa241/propulsion/largefan.html [cited 7 April 2014].

[15] Ardema, M.D., et al., May 1996, “Analytical Fuselage and Wing Weight Estimation of Transport Aircraft,” NASA Technical Memorandum 110392.

[16] Rustenburg, J., Skinn, D., Tipps, D. O., August 1998, “Statistical Loads Data for Boeing 737-400 Aircraft in Commercial Operations,” Technical Report DOT/FAA/AR-92/28.

[17] Boeing Archived Quarterly Earnings Releases. URL: http://www.boeing.com/boeing/companyoffices/financial/quarterly.page [cited 7 April 2014].

[18] EADS Archived Financial Results. URL: http://www.eads.com/eads/int/en/investor-relations/key-financial-information/Financial-Statements-and-Presentations/archived-results.html [cited 2 June 2013].

[19] Bureau of Transportation Statistics. “TranStats Database: Air Carrier Summary: Schedule P-1.1.” URL: http://www.transtats.bts.gov/Fields.asp?Table_ID=290 [cited 7 April 2014].

[20] Bureau of Transportation Statistics. “TranStats Database: Air Carrier Summary: Schedule P-1.2.” URL: http://www.transtats.bts.gov/Fields.asp?Table_ID=295 [cited 7 April 2014].

[21] Boeing, “Boeing 737-700 Technical Characteristics,” URL: http://www.boeing.com/boeing/commercial/737family/pf/pf_700tech.page [cited 7 April 2014].

[22] Raymer, D. P., 2006, “Aircraft design: a conceptual approach,” American Institute of Aeronautics and Astronautics, Reston.

[23] Airfoil Tools, “NACA 22112”, URL: http://airfoiltools.com/airfoil/details?airfoil=naca22112-jf [cited 10 April 2014].

ANNEX A

AIRCRAFT MODEL DESCRIPTION

The total induced drag and lift distribution are approximated using lifting line theory [22], and can be calculated as

$$L(y) = \rho V_\infty \Gamma(y) \tag{A1}$$

$$D = \rho V_\infty \int_{-s}^{s} \Gamma \sin\left(\sum_n \frac{nA_n \sin(n * s * \cos y)}{\sin(s \cos y)}\right) dy \tag{A2}$$

Where ρ is the air density, V_∞ is the free stream velocity, s is the wing half-span, and Γ is the circulation given as

$$\Gamma = 4sV_\infty \sum_n A_n \sin(n * s * \cos y) \tag{A3}$$

And the terms A_n can be determined by taking some finite n and solving the system of equations given by

$$\sum_n A_n \sin(n * s \cos y)\left(\sin(s \cos y) + \frac{nC_{l\alpha}c}{8s}\right) = \frac{C_{l\alpha}c}{8s} \sin(s \cos y)(\alpha - \alpha_0) \tag{A4}$$

Where c is the local chord length, $\boldsymbol{\alpha}$ is the angle of attack, α_0 is the zero-lift angle of attack, and $\boldsymbol{C_{l\alpha}}$ is the slope of the lift coefficient, approximated from thin airfoil theory as $2\pi/rad$ [22].

Based on the drag determined above, we calculate the fuel burn for the aircraft using the thrust specific fuel consumption for the engines. Fuel burn serves as our primary aerodynamic discipline performance measure.

We then consider the structure of the wing as a tapered box beam subjected to the distributed load described by the lift distribution in equation (A1). Since the wing structure must fit inside the wing, we constrain the outer dimensions of our box beam based the dimensions of a NACA 22112 airfoil, similar to those used on commercial transport aircraft. Based on this airfoil, we restrict the width and height of the box beam to 60% and 10% of the chord length, respectively, as shown in figure A1.

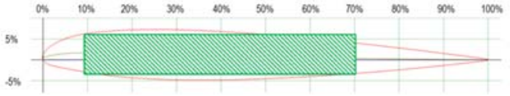

Fig. A1: NACA 22112 airfoil and approximate box beam dimensions [23]

The box beam has two design variables: the horizontal member thicknesses at the wing root and wing tip. Because the wing considered in our example is only subjected to pure bending, the dimensions of the vertical members are not significant. Figure A2 shows the dimensions of the box beam.

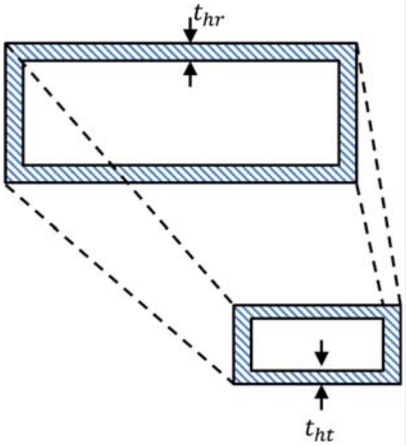

Fig. A2: Box beam dimension definitions

We optimize this structure in order to minimize weight (volume) subject to maximum stress and deflection constraints at some limit load, with the aircraft designer defined safety factor, SF. The weight of the wing structure is then calculated based on the volume of the design using the ratio of the nominal volume of a Boeing 737-700 wing structure and wing weight. The angle of attack is then updated based on equation (A1) such that the total weight of the aircraft with the new structure is equal to the total lift at cruise.

The design is then subjected to a certification test subject to material property uncertainty, where the design must meet a specified knockdown factor against constraint violation determined as a function of the number of tests performed; the probability of not meeting this criteria will cause the aircraft designer to face a certification penalty. This penalty knockdown factor is given as

$$K_{pen} = K_{min} + \frac{\mathrm{K_{rate}}}{N_{test}} \tag{A5}$$

This is intended to represent design requirements such as A-basis and B-basis used by the FAA, where increased number of tests would reduce the 95% confidence bounds, thereby reducing the required knockdown factor.

To calculate the probability of failure and probability of certification penalty, we calculate the design stress and design critical elastic modulus where the maximum deflection/span is achieved and compare to a prescribed variation in yield stress and elastic modulus based on a 5% COV in both properties. Assuming that both yield stress and elastic modulus follow a normal distribution, we calculate the probability of failure and certification penalty directly from these properties' cumulative distribution functions.