APPLICATION OF TAIL MODEL IN HIGH SAFETY STRUCTURE SYSTEM DESIGN

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The reliability analysis and design in structural systems is often focused on estimating the behavior of the system at very low probability of failure. For high safety structure (e.g., three-sigma and six-sigma designs), the main concern in using statistical data to estimate the reliability of a system is the accuracy at the tail. Traditional reliability-based design optimization (RBDO) methods using first- and second-order reliability method (FORM/SORM) or central models (e.g., stochastic response surface) can lead to significant inaccuracies in RBDO results when they are used to estimate large percentiles required in high safety structure design. In this paper, an RBDO method which uses the extreme value theory to evaluate the uncertainties is presented.

In statistics of extremes, it is a common practice to approximate the distribution of absolute exceedances of a random variable x above a high-enough threshold g using a Generalized Pareto Distribution (GPD):

$$\widehat{F}(y) = \begin{cases} 1-\left(1+\dfrac{\gamma y}{\sigma}\right)^{-\frac{1}{\gamma}} & y>0;\gamma>0 \\ 1-e^{-\frac{y}{\sigma}} & y>0;\gamma=0 \end{cases} \tag{1}$$

where $y = x - g$, γ and σ are shape and scale parameters, respectively. Given a value of threshold g and the number of data N_g from the original sample $x_1, \ldots, x_N$ exceeding g, the estimation of the parameters γ and σ can be performed in a variety of ways, such as regression method, maximum likelihood method, probability-weighted moment method, and elemental percentile method.

The probability constraint in RBDO can be evaluated based on the GPD approximation and the available data. Note that the conditional excess distribution $F(y)$ is related to the cumulative distribution of interest $F(x)$ through the following expression:

$$F(y) = \frac{F(y+g)-F(g)}{1-F(g)} = \frac{F(x)-F(g)}{1-F(g)} \tag{2}$$

where $F(g)$ can be approximated by $F(g) \approx (N - N_g) / N$. From Equations (2) and (3), expression for probability of failure P_f can be obtained as

$$P_f = P(x<0) = F(0) = 1-\frac{N_g}{N}\left(1-\frac{\gamma}{\sigma}g\right)^{-\frac{1}{\gamma}} \tag{3}$$

An RBDO problem is formulated and solved using the probability of failure in Eq. (4) for high safety structural designs. The performance of the method is evaluated in terms of the stability, accuracy, and efficiency.

References

[1] J.Beirlant, et al, “Statistics of Extremes: Theory and Applications”, Wiley Series in Probability and Statistics, 2004, *John Wiley & Sons, Ltd.*