SHAPE DESIGN SENSITIVITY FORMULATION FOR FINITE DEFORMATION ELASTO-PLASTICITY WITH CONTACT

3rd World Congress of
Structural and Multidisciplinary Optimization

K. K. Choi, Nam H. Kim, and J. S. Chen

The Center for Computer-Aided Design
Department of Mechanical Engineering
College of Engineering
The University of Iowa
Iowa City, IA52242

INTRODUCTION

⟹ Finite Deformation Elastoplasticity

- Multiplicative Decomposition of **F**
- Stress-Free Intermediate Configuration
- Return Mapping in Principal Stress Space
- Exact Tangent Operator

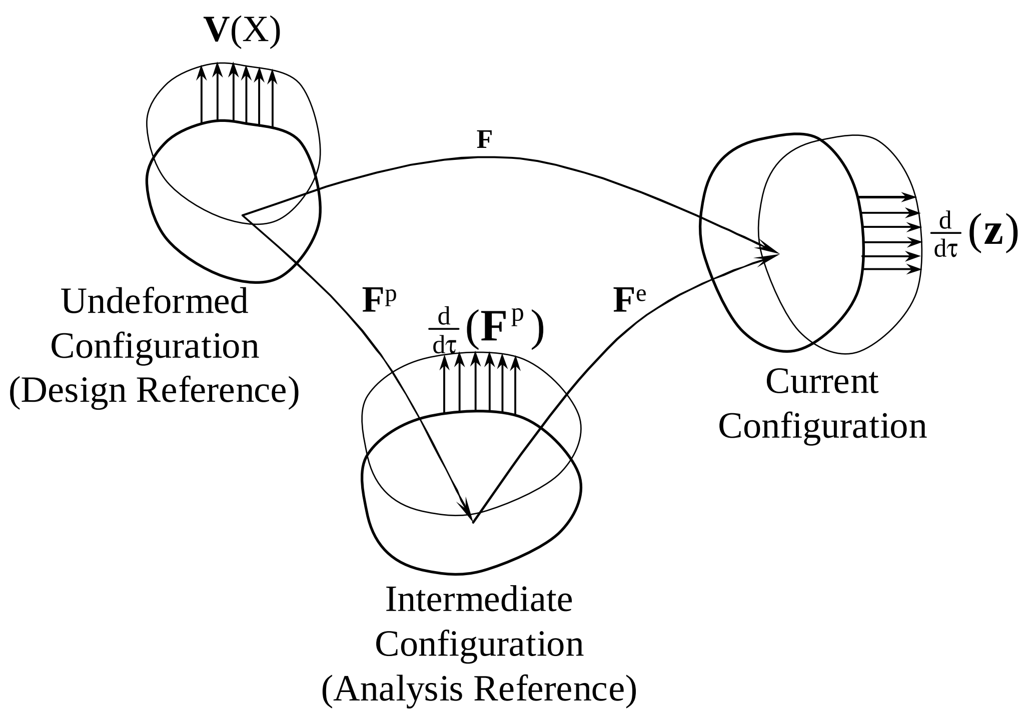
⟹ Frictional Contact Problem

- Continuum-Based Contact Formulation
- Penalty Regularization
- Regularized Coulomb Friction Model

⟹ Meshfree Discretization

- Reproducing Kernel Particle Method (RKPM)
- Direct Transformation Method for Essential B. C.

ANALYSIS AND DESIGN PROCEDURE



VARIATIONAL FORMULATION

Governing Equation & Linearization

$$a_\Omega({}^{n+1}\mathbf{z},\bar{\mathbf{z}}) = \ell_\Omega(\bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z \qquad a_\Omega(\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega \boldsymbol{\tau} : \bar{\boldsymbol{\varepsilon}}\, d\Omega$$

$$a^*_\Omega({}^{n+1}\mathbf{z}^k; \Delta\mathbf{z}^{k+1}, \bar{\mathbf{z}}) = \ell_\Omega(\bar{\mathbf{z}}) - a_\Omega({}^{n+1}\mathbf{z}^k, \bar{\mathbf{z}}) \qquad \forall \bar{\mathbf{z}} \in Z$$

$$a^*_\Omega({}^{n+1}\mathbf{z}^k; \Delta\mathbf{z}^{k+1}, \bar{\mathbf{z}}) = \int_\Omega \left[\bar{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}(\Delta\mathbf{z}^{k+1}) + {}^{n+1}\boldsymbol{\tau}^k : \boldsymbol{\eta}(\Delta\mathbf{z}^{k+1}, \bar{\mathbf{z}})\right] d\Omega$$

Consistent Tangent Operator

$$\mathbf{c} = \frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\varepsilon}} = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^{alg}\, \mathbf{m}^i \otimes \mathbf{m}^j + 2\sum_{i=1}^{3} \tau_i^p \hat{\mathbf{c}}^i$$

$$\mathbf{c}^{alg} \equiv \frac{\partial \boldsymbol{\tau}^p}{\partial \mathbf{e}^{tr}} = \mathbf{c}^e - 4\mu^2 A \mathbf{N} \otimes \mathbf{N} - \frac{4\mu^2 \hat{\gamma}}{\left\|\boldsymbol{\eta}^{tr}\right\|}\left[\mathbf{I}_{dev} - \mathbf{N} \otimes \mathbf{N}\right]$$

DESIGN SENSITIVITY ANALYSIS

DSA Equation

$$\tfrac{d}{d\tau}\left[a_{\Omega}({}^{n+1}\mathbf{z},\bar{\mathbf{z}})\right]=\tfrac{d}{d\tau}\left[\ell_{\Omega}(\bar{\mathbf{z}})\right], \qquad \forall\bar{\mathbf{z}}\in Z$$

$$a^{*}_{\Omega}(\mathbf{z};\dot{\mathbf{z}},\bar{\mathbf{z}})=\ell'_{V}(\bar{\mathbf{z}})-a'_{V}(\mathbf{z},\bar{\mathbf{z}}), \qquad \forall\bar{\mathbf{z}}\in Z$$

Fictitious Load

$$a'_{V}(\mathbf{z},\bar{\mathbf{z}})=\int_{\Omega}\left(\bar{\boldsymbol{\varepsilon}}:\mathbf{c}:\boldsymbol{\varepsilon}_{V}(\mathbf{z})+\bar{\boldsymbol{\varepsilon}}:\mathbf{c}:\boldsymbol{\varepsilon}_{P}(\mathbf{z})+\boldsymbol{\tau}^{fic}:\bar{\boldsymbol{\varepsilon}}\right)d\Omega$$
$$+\int_{\Omega}\left(\boldsymbol{\tau}:\boldsymbol{\eta}_{V}(\mathbf{z},\bar{\mathbf{z}})+\boldsymbol{\tau}:\boldsymbol{\eta}_{P}(\mathbf{z},\bar{\mathbf{z}})+\boldsymbol{\tau}:\bar{\boldsymbol{\varepsilon}}\,\mathrm{div}\mathbf{V}\right)d\Omega$$

Design velocity dependent terms

$$\boldsymbol{\varepsilon}_{V}(\mathbf{z})=-sym(\nabla_{0}\mathbf{z}\nabla_{n}\mathbf{V})$$
$$\boldsymbol{\eta}_{V}(\mathbf{z},\bar{\mathbf{z}})=-sym\left(\nabla_{n}\bar{\mathbf{z}}^{T}\nabla_{0}\mathbf{z}\nabla_{n}\mathbf{V}\right)-sym(\nabla_{0}\bar{\mathbf{z}}\nabla_{n}\mathbf{V})$$

DESIGN SENSITIVITY ANALYSIS (cont.)

Path-dependent terms

$$\boldsymbol{\varepsilon}_P(\mathbf{z}) = -sym(\mathbf{G}) \qquad \mathbf{G} = \mathbf{F}^e \tfrac{d}{d\tau}(\mathbf{F}^p)\mathbf{F}^{-1}$$

$$\boldsymbol{\eta}_P(\mathbf{z},\bar{\mathbf{z}}) = -sym(\nabla_n \bar{\mathbf{z}}^T \mathbf{G})$$

$$\boldsymbol{\tau}^{fic} = \sum_{i=1}^{3}\left[\frac{\partial \tau_i^p}{\partial \boldsymbol{\alpha}}\tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \frac{\partial \tau_i^p}{\partial \hat{e}^p}\tfrac{d}{d\tau}(\hat{e}_n^p)\right]\mathbf{m}^i$$

Updating DSA Variables

$$\tfrac{d}{d\tau}(\boldsymbol{\alpha}_{n+1}) = \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \left(H_\alpha + \sqrt{\tfrac{2}{3}}H'_\alpha\hat{\gamma}\right)\tfrac{d}{d\tau}(\hat{\gamma})\mathbf{N} + H_\alpha\hat{\gamma}\tfrac{d}{d\tau}(\mathbf{N})$$

$$\tfrac{d}{d\tau}(\hat{e}_{n+1}^p) = \tfrac{d}{d\tau}(\hat{e}_n^p) + \sqrt{\tfrac{2}{3}}\tfrac{d}{d\tau}(\hat{\gamma})$$

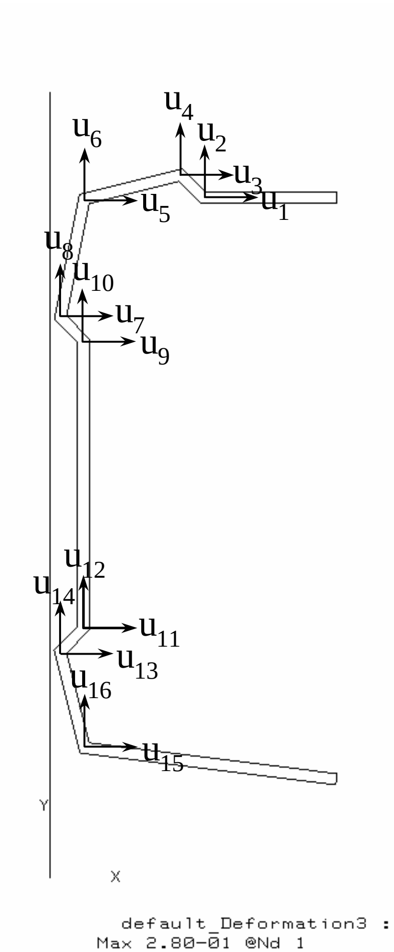
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^p) = \tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\,-1})\mathbf{F}_{n+1} + \mathbf{F}_{n+1}^{e\,-1}\tfrac{d}{d\tau}(\mathbf{F}_{n+1})$$

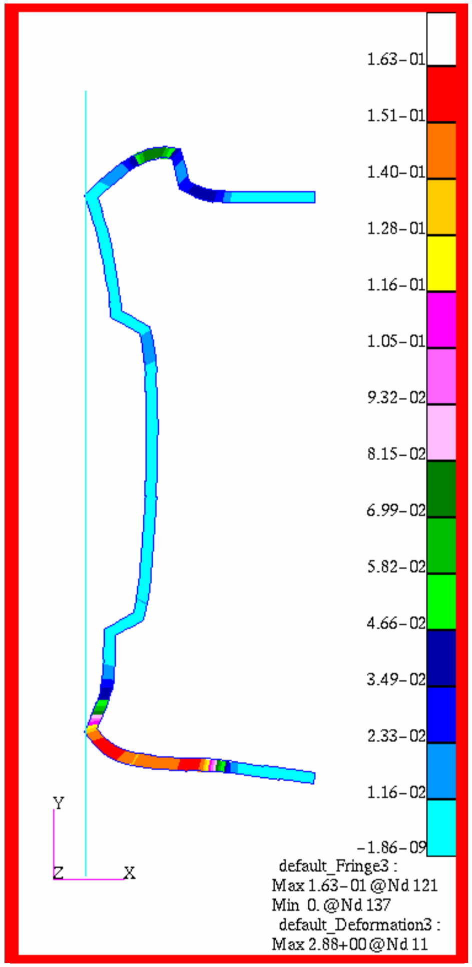
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^e) = \tfrac{d}{d\tau}(\mathbf{f}^p)\mathbf{F}_{n+1}^{e\,tr} + \mathbf{f}^p\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\,tr})$$

$$\mathbf{f}^p = \sum_{j=1}^{3}\exp(-\hat{\gamma}N_j)\,\mathbf{m}^j$$

Incremental Plastic Deformation Gradient

BUMPER IMPACT PROBLEM





Density	ρ = 7,800 kg/m^3
Initial Velocity	v0 = 10 km/hr
Analysis Time	t = 0 ~ 10 msec
Time Increment	Δt = 0.1 msec
Mounting Displ.	d = 2.8 cm
Thickness	h = 0.5 cm
Contact Penalty No.	w_n = 1,000
Friction Coeff.	μ_f = 0.4
Lame's Constants	λ = 110.8 GPa
	μ = 80.2 GPa
Plastic Hardening	H = 1.1 GPa
	Isotropic Hardening
Initial Yield Stress	σ_Y = 500 MPa
Newmark Parameters	γ = 0.26
	β = 0.5

SENSITIVITY ANALYSIS RESULTS

Response Analysis
275 sec

Sensitivity Analysis
116 / 16 sec

Performance (Ψ)		$\Delta\Psi$	Ψ'	$(\Delta\Psi/\Psi')\times 100\%$
u_2				
$\hat{e}^P_{15}$	.680005E-01	-.179756E-07	-.179757E-07	100.00
$\hat{e}^P_{65}$	.164338E+00	.311392E-08	.311393E-08	100.00
$\hat{e}^P_{29}$	.126643E-01	-.901637E-10	-.901545E-10	
Z_{x39}	.429139E+00	.120943E-06	.120940E-06	100.00
F_{Cx100}	.379375E+01	.473864E-07	.473865E-07	100.00
u_4				
$\hat{e}^P_{15}$	.680005E-01	.246181E-07	.246181E-07	100.00
$\hat{e}^P_{65}$	.164338E+00	.105172E-08	.105173E-08	100.00
$\hat{e}^P_{29}$	.126643E-01	.589794E-09	.589795E-09	100.00
Z_{x39}	.429139E+00	-.295825E-06	-.295824E-06	100.00
F_{Cx100}	.379375E+01	.335517E-09	.335511E-09	100.00
u_6				
$\hat{e}^P_{15}$	.680005E-01	-.170857E-07	-.170857E-07	100.00
$\hat{e}^P_{65}$	.164338E+00	-.237257E-08	-.237256E-08	100.00
$\hat{e}^P_{29}$	.126643E-01	-.720239E-10	-.720198E-10	
Z_{x39}	.429139E+00	.167699E-06	.167698E-06	100.00
F_{Cx100}	.379375E+01	-.176290E-07	-.176292E-07	100.00
u_8				
$\hat{e}^P_{15}$	.680005E-01	.581799E-09	.581877E-09	99.99
$\hat{e}^P_{65}$	.164338E+00	-.635253E-09	-.635254E-09	100.00
$\hat{e}^P_{29}$	.126643E-01	-.185890E-08	-.185890E-08	100.00
Z_{x39}	.429139E+00	-.397143E-07	-.397141E-07	100.00
F_{Cx100}	.379375E+01	.250196E-07	.250194E-07	100.00
u_{10}				
$\hat{e}^P_{15}$	.680005E-01	-.262956E-09	-.262932E-09	100.01
$\hat{e}^P_{65}$	.164338E+00	.136684E-09	.136687E-09	100.00
$\hat{e}^P_{29}$	.126643E-01	.873228E-09	.873231E-09	100.00
Z_{x39}	.429139E+00	-.168128E-06	-.168129E-06	100.00
F_{Cx100}	.379375E+01	-.431408E-07	-.431402E-07	100.00
u_{12}				
$\hat{e}^P_{15}$	.680005E-01	-.186975E-09	-.186998E-09	99.99
$\hat{e}^P_{65}$	.164338E+00	-.129114E-08	-.129115E-08	100.00
$\hat{e}^P_{29}$	.126643E-01	-.311886E-09	-.311878E-09	100.00
Z_{x39}	.429139E+00	-.304683E-07	-.304689E-07	100.00
F_{Cx100}	.379375E+01	.490867E-07	.490870E-07	100.00

Design Optimization Problem Definition

MIN Area

S.T. $\hat{e}^P{}_{16}(0.10) \le 0.05$

$\hat{e}^P{}_{62}(0.15) \le 0.05$

$\hat{e}^P{}_{65}(0.16) \le 0.05$

$\hat{e}^P{}_{66}(0.16) \le 0.05$

$\hat{e}^P{}_{67}(0.15) \le 0.05$

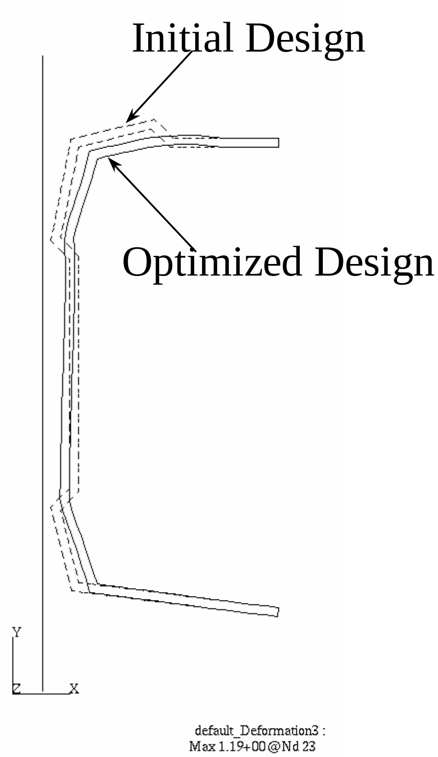
$\hat{e}^P{}_{60}(0.15) \le 0.05$

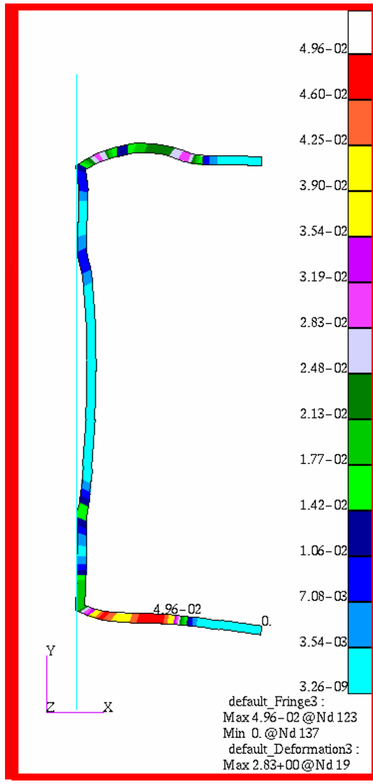
$\hat{e}^P{}_{61}(0.14) \le 0.05$

$F_{Cx}(4.55) \ge 4.55$

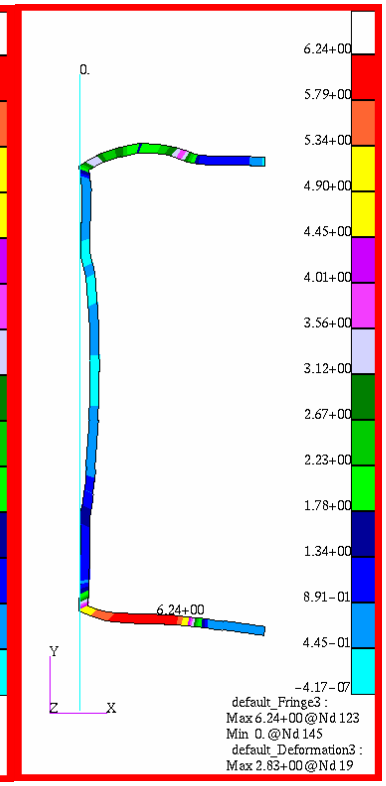
$-1.0 \le u_i \le 1.0 \qquad i = 1,16$

DESIGN OPTIMIZATION



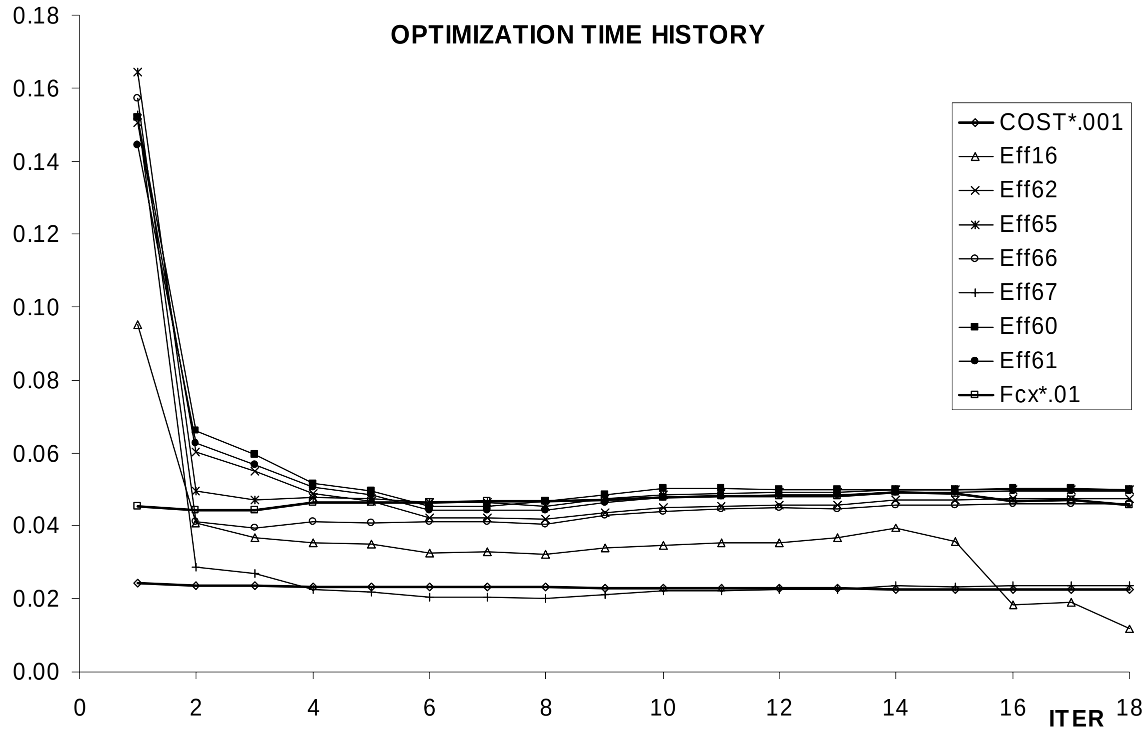


Effective Plastic Strain



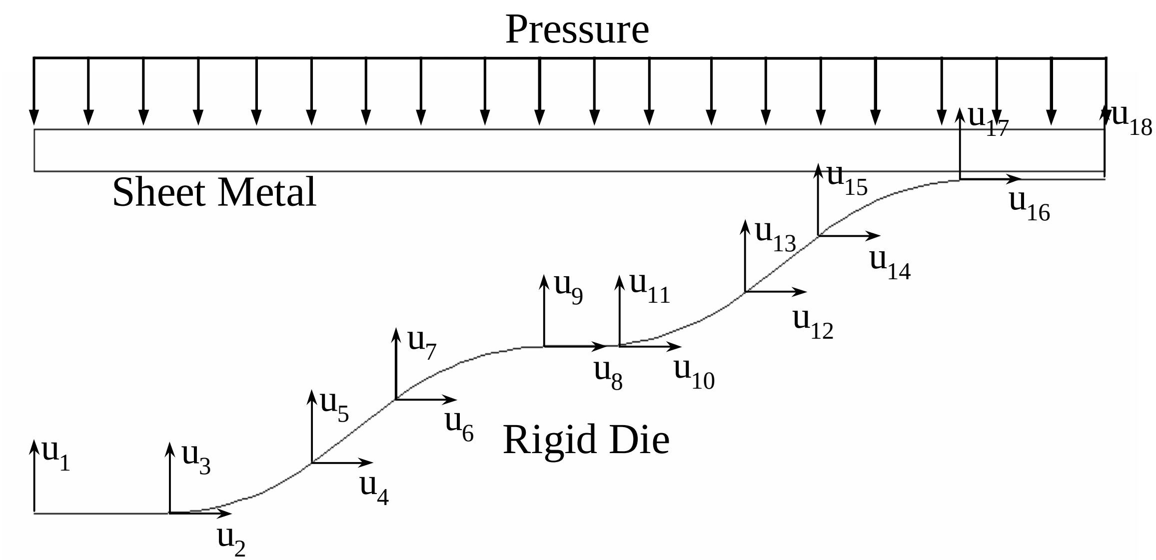
Von Mises Stress

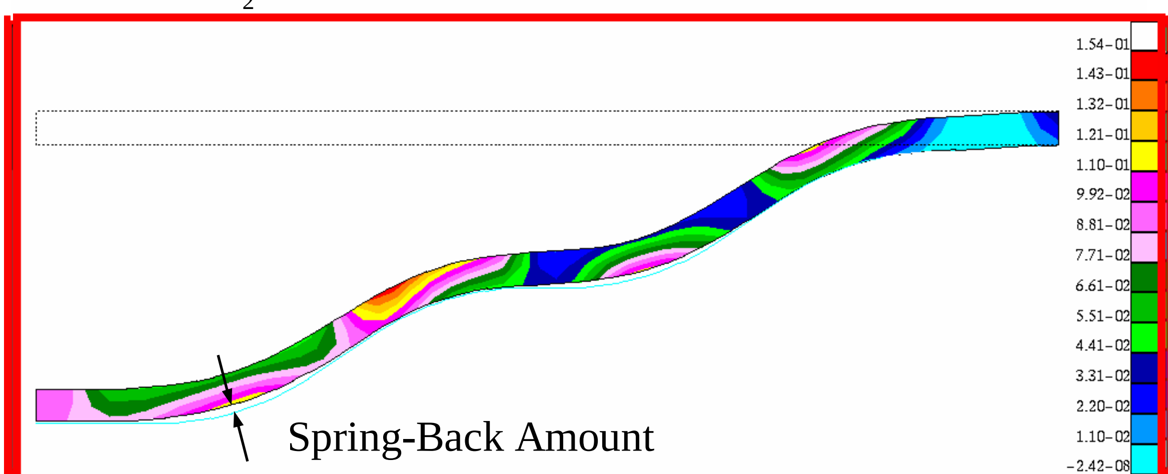
DESIGN OPTIMIZATION HISTORY



Response Analysis : 37
Sensitivity Analysis : 18

SHEET METAL STAMPING PROBLEM





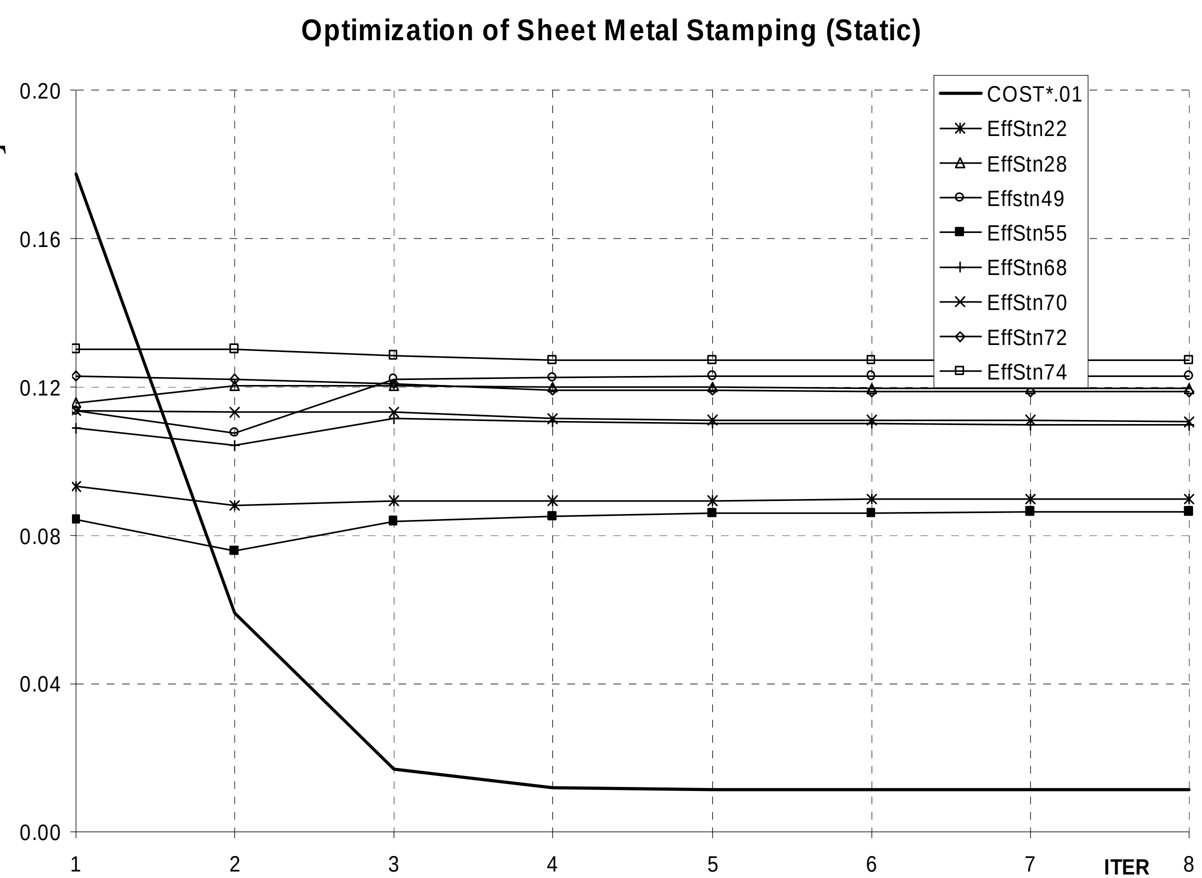
DESIGN OPTIMIZATION

$$\text{MIN} \quad G = \int_{\Gamma} \|\pi(\mathbf{x}) - \mathbf{x}\|^2 \, d\Gamma$$

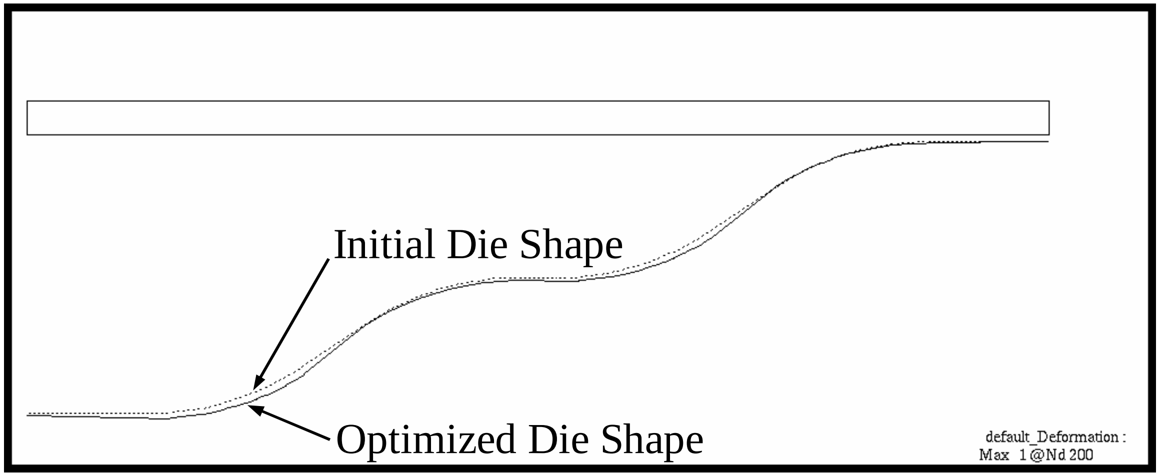
$$\text{S.T.} \quad \hat{e}^{p}{}_{i} \leq 0.13$$

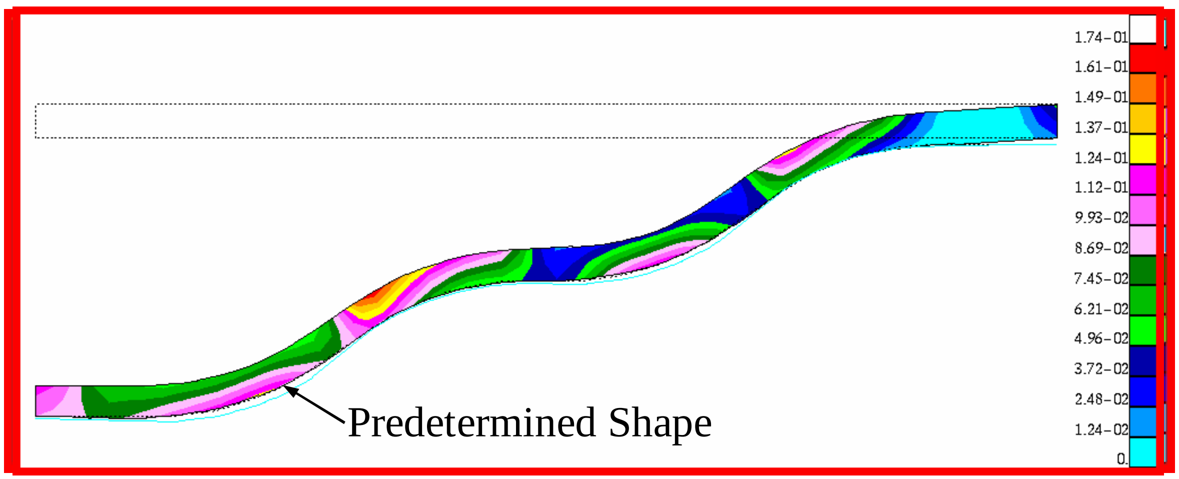
$$-2.0 \leq u_i \leq 2.0$$

Response Analysis : 24
Sensitivity Analysis : 8



OPTIMUM DIE SHAPE





CONCLUSIONS

⇒ An efficient and accurate nonlinear DSA is proposed for path-dependent structural problems.

⇒ Sensitivity equation always uses the same tangent stiffness matrix as the response analysis at each converged configuration.

⇒ Path-dependency of DSA is from the intermediate configuration and internal plastic variables as well as frictional contact effect.

⇒ Intermediate configuration is required to be specified for DSA purpose such that the incremental plastic rotation is ignored.