Tail Modeling in Reliability-Based Design Optimization for Highly Safe Structural Systems

Nam H. Kim* and Palaniappan Ramu†
University of Florida, Gainesville, Florida, 32611

and

Nestor V. Queipo‡
University of Zulia, Venezuela

This paper presents an approach for the reliability–based design optimization of highly safe structural systems where a tail–model is used for computing the reliability constraint during design optimization. It is generally accepted that using central models (e.g., moment–based method or stochastic response surfaces) for estimating large percentiles such as those required in reliability constraint calculations can lead to significant inaccuracies in the result. The tail–model is an adaptation of a powerful result from extreme value theory in statistics related to the distribution of exceedances. The conditional excess distribution above a certain threshold is approximated using the generalized Pareto distribution (GPD). The shape and scale parameters in the GPD are estimated using the least–square method. The tail–modeling technique is utilized to approximate the performance measure in inverse reliability analysis. The accuracy and convergence properties are studied using an analytical function. The effectiveness and efficiency of the proposed approach are demonstrated using benchmark problems in structural design under uncertainty.

I. Introduction

WHEN a system contains uncertainty in input parameters, the performance function of the system also shows a probabilistic characteristic. In reliability analysis of structural systems, the cumulative distribution of the performance function is one of the most important criteria in determining the safety level of the system. In evaluating the reliability of the system, engineers are often interested in the probability of failure of the performance function. Many techniques have been to proposed to model the probability of failure, such as moment matching method (Parkinson *et al*., 1993), first–order reliability method (Enevoldsen and Sorensen, 1994), Monte Carlo simulation (Qu *et al*., 2003), stochastic response surface (Kim *et al*., 2004), and worst–case analysis (Sundaresan *et al*., 1993). All methods have their own advantages and disadvantages in terms of accuracy, computational cost, and robustness.

Reliability–based design optimization (RBDO) involving a computationally demanding model has been limited by the relatively high number of simulations required for evaluating the reliability constraints, in particular, for highly safe structural systems (e.g., three–sigma and six–sigma designs). Traditional approaches based on Monte Carlo methods for these tasks often fail to meet constraints (computational resources, cost, time, etc.) typically present in industrial environments. To overcome this issue, several approaches have been proposed, including moment–based methods, response surface methods, and stochastic response surface methods. The moment–based methods (FORM, SORM) approximate the uncertainty propagation to be a linear or quadratic relation. The construction of stochastic response surfaces (e.g., polynomial chaos expansion) coupled with Monte Carlo methods has been proposed; see, for example, Kim *et al*. (2004).

Even though the stochastic response surface method provides an efficient approach for uncertainty quantification, it has the drawback that it represents a central model and not those required (namely, tail–models) for evaluating reliability constraints where the interest lies in the occurrence of rather exceptional events. It is generally accepted

* Assistant Professor, Mechanical and Aerospace Engineering, PO Box 116250, AIAA Member.
† Graduate Student, Mechanical and Aerospace Engineering, PO Box 116250, Student Member.
‡ Professor, Applied Computing Institute, Faculty of Engineering, University of Zulia, Venezuela.

that using central models (e.g., stochastic response surfaces) for estimating large percentiles such as those required in reliability constraint calculations can lead to significant inaccuracies in the RBDO results (e.g., Maes and Huyse, 1995).

This paper presents an RBDO approach for highly safe structural systems where the reliability constraint is computed from rather general tail–models available from extreme value theory in statistics (Castillo, 1988). The conditional excess distribution above a certain threshold is approximated using the generalized Pareto distribution (GPD). The parameters in GPD are calculated using either the maximum likelihood or the least square method. By incorporating the tail–modeling technique with the probability of failure, the reliability analysis and optimization of a structure can be solved with highly safe reliability constraints. The proposed method does not approximate the functional expression of the model output; rather approximates the tail of the cumulative distribution. Thus, it has an advantage of the system reliability analysis and design in which no single form of functional expression is available.

The paper is structured as follows. In Section 2, the tail of the cumulative distribution function is modeled using the generalized Pareto distribution. The application of the tail–model to the reliability analysis and inverse reliability analysis is presented in Section 3. The RBDO framework using the tail–modeling technique is presented in Section 4. Two numerical examples are presented in Section 5, followed by conclusions in Section 6.

II. Tail Modeling and Generalized Pareto Distribution

The cumulative probability distribution of a random variable associated with reliability constraints in RBDO can be viewed as consisting of three parts: a lower tail, a central part, and an upper tail. Identifying a probabilistic model for large (extreme) values of the random variables is then a key for a more accurate evaluation of the reliability constraints. At this point, the extreme value theory in statistics can prove very helpful as it provides a powerful result related to the distribution of exceedances called generalized Pareto distribution (Pickands, 1975) that can be adapted for solving the problem of interest.

The fundamental idea of the tail–modeling technique stems from the property of tail equivalence. Two distribution functions $F(x)$ and $G(x)$ are called *tail equivalent* (Maes and Breitung, 1993) if

$$\lim_{x\to\infty}\frac{1-F(x)}{1-G(x)}=1\,. \tag{1}$$

As far as the extreme behaviors of the two distributions are equivalent, the tail–model of $F(x)$ can be used to approximate the upper (or lower) tail of $G(x)$. This approach does not take into account the central behavior of the distribution. Rather, it focuses on the upper or lower tail behavior, which fits for the purpose of structural reliability analysis.

Let $\mathbf{x}$ be the vector of input random variables. Due to the uncertainty propagation, the performance function, $y(\mathbf{x})$, also shows random distribution. Let the performance function be a random variable and g be a large threshold of y (see Figure 1). For the region that y is greater than g, the GPD represents a rather general approximation of the conditional excess distribution $F_g(z)$ where $z=y-g$; that is, the distribution of values of random variable y above the threshold g. Specifically, a theorem from extreme value theory establishes that for large values of g, the conditional distribution $F_g(z)$ can be well approximated by:

$$F_g(z)\approx\hat{F}_{\xi,\sigma}(z)\,, \tag{2}$$

where

$$\hat{F}_{\xi,\sigma}(z)=\begin{cases}1-\left\langle 1+\dfrac{\xi}{\sigma}z\right\rangle_+^{-\frac{1}{\xi}} & \text{if}\quad \xi\neq 0\\ 1-\exp\left(-\dfrac{z}{\sigma}\right) & \text{if}\quad \xi=0\end{cases}\,. \tag{3}$$

In Eq. (3), $\langle A \rangle_+ = \max(0, A)$ and $z \geq 0$. $\hat{F}_{\xi,\sigma}(z)$ in Eq. (3) is called the generalized Pareto distribution (GPD), and ξ and σ are the shape and scale parameters, respectively, which need to be determined. Note that the conditional excess distribution $F_g(z)$ is related to the cumulative distribution of interest $F(y)$ through the following expression:

$$F_g(z) = \frac{F(y) - F(g)}{1 - F(g)}. \tag{4}$$

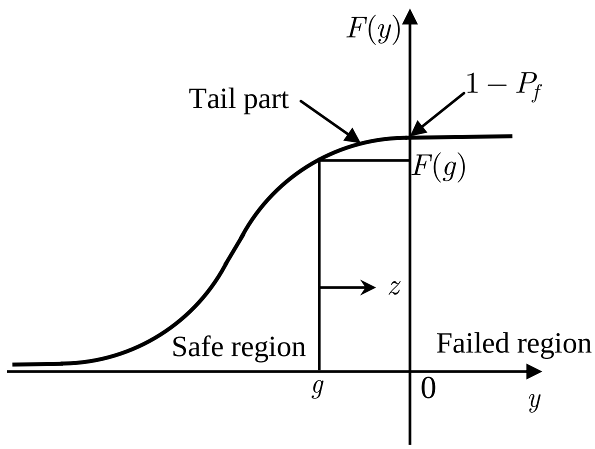

Figure 1: Tail–modeling of $F(y)$ using the threshold value of g.

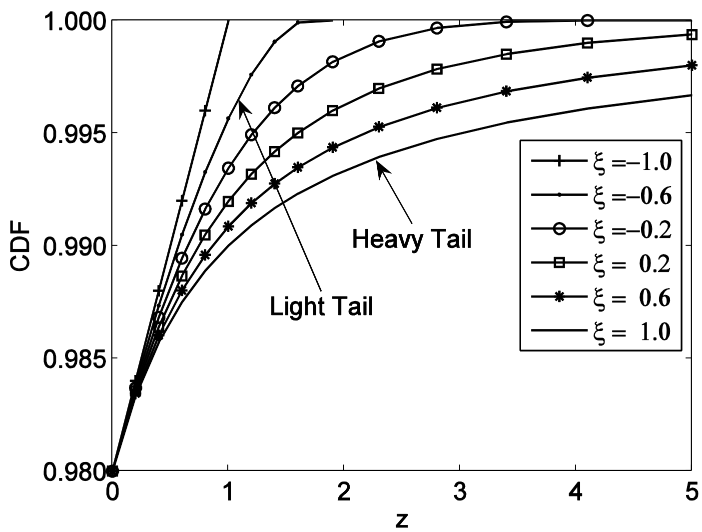

Figure 2: Generalized Pareto distributions for different shape parameters.

The flexibility of the GPD in Eq. (3) can be examined by changing its parameters and plotting the distribution above the threshold. Figure 2 shows the different cumulative distributions that are generated from the GPD when the scale parameter, σ, is fixed to one, and the threshold, g, is selected such that $F(g) = 0.98$. When the shape parameter $\xi > 0$, it represents the heavy tail behavior, such as Pareto distribution. On the other hand, when $\xi < 0$, it represents the light tail behavior, such as the beta distribution. Note that the uniform distribution can also be modeled using $\xi = -1$.

Tail–modeling can be performed in three stages. First, a set of samples of the random performance function is generated. A Monte Carlo simulation or a Latin Hypercube Sampling is often used for this purpose. In structural problems, the samples of the input random variables are first generated from the given distribution types. The samples of the random performance function are then calculated through the structural analysis. However, tail–modeling is not limited for the case when the distribution types of input random variables are known. As far as the samples of the random performance function can be obtained, either from analysis or experiment, this method can be applied.

Second, a threshold value, g , is selected from the distribution of the performance function y . The appropriate value g , that is, the specification of the beginning of the upper tail, has been the subject of extensive research, and empirical values for it have been proposed (e.g., Boos, 1984; Hasofer, 1996; Caers and Maes; 1998). In Hasofer study, for example, the use of $N_g \approx 1.5\sqrt{N}$ is suggested where N_g is the number of data that belong to the tail part and N is the total number of data. In this paper, the CDF $F(g)$ of the threshold is prescribed, from which g and N_g can be obtained. This method works better because interest region is given in terms of the probability of failure.

Last, the shape and scale parameters in the GPD are estimated by fitting the tail–model with the empirical CDF. Prescott and Walden (1980) and Hosking (1985) used the maximum likelihood method to estimate the parameters. In this paper, the least square method is employed to estimate the two parameters, in which the error between the tail–model and the empirical CDF is minimized by

$$\text{minimize} \quad \sum_{i=N-N_g}^{N} \left[p_i - F(y_i | \sigma, \xi) \right]^2 , \tag{5}$$

where the empirical CDF is given by

$$p_i = \frac{i-0.5}{N}, \quad i = 1, \ldots, N . \tag{6}$$

Note that only the tail part of the data is used in estimating the parameters. A new method will be introduced to estimate the tail parameters using the inverse measure in the following section.

In general, two sources of errors are involved in tail–modeling: (a) lack of modeling capability, and (b) errors in random sampling and in the empirical CDF. The former is related to the flexibility of the tail–model in representing various tail behaviors, and the latter is related to the number of samples and to the appropriate selection the threshold. The effects of these two sources of errors will be discussed in the numerical examples.

III. Reliability Analysis Using Tail–Modeling

Reliability analysis in structural problems often means the evaluation of the probability of failure. In this section, the tail–model will be used to calculate the probability of failure analytically. In addition, the inverse reliability analysis can easily be performed because the analytical expression of the reliability is available. The accuracy and convergence of the tail–model will be discussed using various distribution types.

A. Probability of Failure

In structural reliability analysis, the probability of failure, P_f , is often used as a constraint, so that it should be less than the prescribed target probability of failure, $P_{f,\text{target}}$. An analytical expression for the constraint value is now developed in three steps based on the GPD approximation and the available data. First, an explicit expression for $F(y)$ is obtained from Eq. (4), as

$$F(y) = [1 - F(g)] F_g(z) + F(g) . \tag{7}$$

Second, in the above expression $F_g(z)$ is substituted by the corresponding GPD in Eq. (3), and for the prescribed $F(g)$, the threshold is interpolated using

$$g = y_{j-1} + \left(y_j - y_{j-1} \right) \frac{F(g) - p_{j-1}}{p_j - p_{j-1}} , \tag{8}$$

where p_j is the empirical CDF from Eq. (6). After the substitutions, $F(y)$ can be written as:

$$F(y) = 1 - \left(1 - F(g)\right)\left\langle 1 + \frac{\xi}{\sigma}(y - g)\right\rangle_{+}^{-\frac{1}{\xi}}. \tag{9}$$

When the performance function, y, is defined such that the structural system is failed when $y > 0$ and safe when $y \le 0$, the probability of failure can be written as

$$P_f := 1 - F(y = 0) = \left(1 - F(g)\right)\left\langle 1 - \frac{\xi}{\sigma} g\right\rangle_{+}^{-\frac{1}{\xi}}. \tag{10}$$

Equation (10) provides an analytical expression of the probability of failure, which can be directly used in evaluating the constraints in RBDO.

The estimation of the probability of failure in Eq. (10) is only valid when the threshold $g < 0$, which means that $P_f < 1 - F(g)$. Equivalently, the limit state $(y = 0)$ must belong to the tail part. When the safety margin of the structural system is small, the probability of failure does not belong to the tail part, and the above formula cannot be used for estimating the probability of failure. The requirement of the structural safety is usually given in the range of small probability of failure so that the above requirement is satisfied. During the process of design optimization, however, it may be possible that a design may produce a relatively unsafe configuration. In such a case, a special treatment is required to estimate the probability of failure below the threshold. However, the estimation does not have to be accurate because it is not the final design.

B. Reliability Index and Inverse Reliability Analysis

In reliability–based design optimization, two methods are often referred: the reliability index approach and the performance measure approach, or often called the inverse measure approach. An inverse measure is the value of the performance function that corresponds to the given value of the probability, while a reliability index is the index of the standard normal distribution, corresponding to the specific value of the performance function. These two approaches work well with the first–order reliability method (FORM), where the performance function is assumed to be normally distributed after linearization.

For the estimated probability of failure in Eq. (10), the reliability index, β, can be calculated using

$$\beta = -\Phi^{-1}(P_f), \tag{11}$$

where $\Phi(\bullet)$ is the CDF of the standard normal random variable. The reliability constraint is then imposed using the reliability index, as

$$\beta \ge \beta_{\text{target}} := -\Phi^{-1}(P_{f,\text{target}}), \tag{12}$$

where β_{target} is the target reliability index that corresponds to the target probability of failure, $P_{f,\text{target}}$.

On the other hand, the inverse measure approach calculates the value of the performance function, y^*, corresponding to the target probability of failure. Using tail–modeling in Eq. (10) with $P_f = P_{f,\text{target}}$, the inverse measure can be obtained by

$$y^*(\sigma, \xi) = g + \frac{\sigma}{\xi}\left[\left(\frac{P_{f,\text{target}}}{1 - F(g)}\right)^{-\xi} - 1\right]. \tag{13}$$

The reliability constraint is then imposed using the performance function as

$$y^* \le 0. \tag{14}$$

When the tail is heavy, i.e., $\xi > 0$, the above formula can be used to find the value of the performance function $y^* (\ge g)$ that has probability of failure P_f. On the other hand, when the tail is light, i.e., $\xi < 0$, the value of the performance function can be found up to $y^* = g - \sigma / \xi$, at which $P_f = 0$.

In the view of the inverse measure approach, it is possible to formulate the least square method in Eq. (5) in terms of the performance function, as

$$\text{minimize} \quad \sum_{i=N-N_g}^{N} (1-p_i)\left[y_i - y_i^*(\sigma,\xi) \right]^2 , \tag{15}$$

where $(1-p_i)$ is used for the weight.

In the literature (Lee *et al*., 2002; Youn *et al*., 2003), it has been presented that the inverse measure approach is more stable than the reliability index approach. When the probability of failure is zero, the latter shows a singularity. The difficulty in the reliability index approach is related to the transformation in Eq. (11). The reliability index approaches to the value of infinity as the probability of failure is reduced. Thus, it is difficult to calculate the reliability index when the target reliability is far from the failure surface. On the other hand, the inverse reliability analysis always yields a finite value of performance function that satisfies the target reliability. Ramu *et al*. (2004) presented an inverse measure, called probabilistic sufficiency factor (PSF), when sampling–based methods are used.

C. Accuracy and Convergence Study

In order to see the capability of tail–modeling, a simple function, $y = x$, is considered with x being a random variable with various distribution types. The error related to random sampling is removed by using the Latin Hypercube sampling with equal space on the probability scale: $p_i = (i-0.5)/N$. The sampling points can be found using the inverse CDF. First, $N = 500$ samples are generated using the Latin Hypercube sampling method, and then sorted in the ascending order. The threshold is selected at $F(g) = 0.95$, and corresponding threshold value is found through the interpolation in Eq. (8). Using the data above g, two parameters are estimated using the least square method, as in Eq. (5). Using the estimated parameters, the inverse measure (performance function) are calculated for a given target probability of failure using Eq. (13).

The accuracy of the inverse measures from tail–modeling is compared with that from the exact CDF in Table 1. Based on the results at $P_{f,\text{target}} = 10^{-4}$, the tail model is accurate for light tails, such as beta distribution, and relatively inaccurate for the heavy tails, such as lognormal distribution. It fact the error in the gamma distribution increases as the first parameter is increased, in which the distribution approaches to the normal distribution. The results in Table 1 are repeatable because the equal space on the probability scale is used in the Latin Hypercube sampling (lhsdesign function in MATLAB). In such a case, the empirical CDF is given by $p_i = (i-0.5)/N$. It is observed that the percent error in Table 1 remains constant when the standard deviation of the distribution is changed.

Table 1: Tail–modeling accuracy of the inverse measure for various distributions (N = 500, $P_{f,\text{target}} = 10^{-4}$)

Distribution	y^*_{exact}	y^*_{tail}	Error in y (%)	P_f	Error in P_f (%)
Normal (0,1)	3.7190	3.5578	4.335E+0	3.878E−5	6.122E+1
Lognormal (0,1)	41.2238	44.8401	–8.772E+0	1.336E–4	–3.357E+1
Exponential (1)	9.2103	9.2184	–8.700E–2	1.008E–4	–8.020E–1
Uniform (0,1)	0.9999	0.9999	–5.272E–11	1.000E–4	–5.271E–7
Gamma (1,1)	9.2103	9.2184	–8.700E–2	1.008E–4	–8.020E–1
Weibull (2,1)	18.4207	18.4367	−8.700E−2	1.008E−4	−8.020E−1
Beta (1,2)	0.9900	0.9900	–4.948E–3	1.010E–4	–9.806E–1

The errors in Table 1 are contributed from the lack of modeling capability of the tail–model. Even if the number of sampling is dramatically increased, the errors will not change significantly. For example, when 10^6 sampling points are used, the tail–model for the normal distribution still has 4.412% error. Figure 3 further illustrates the modeling errors of the probability of failure compared with the exact values. Except for normal and lognormal distributions, all other distributions show good agreements. It turns out that the normal distribution overestimates the probability, while the lognormal distribution underestimates it.

The tail-model in Table 1 is based on the threshold that corresponds to the CDF of $F(g) = 0.95$. It is possible that the selection of the threshold may affect the accuracy of the tail–model. In order to see the effect of the threshold, the value of $F(g)$ is selected between 0.90 and 0.98 for the normally distributed one. Table 2 presents the percent error of the inverse measure at different target probability of failure with different threshold values. It is noted that the accuracy is improved as the threshold is close to one. However, it is necessary to ensure that enough number of points should be included above the threshold, so that the lease square optimization in Eq. (5) is stable and robust.

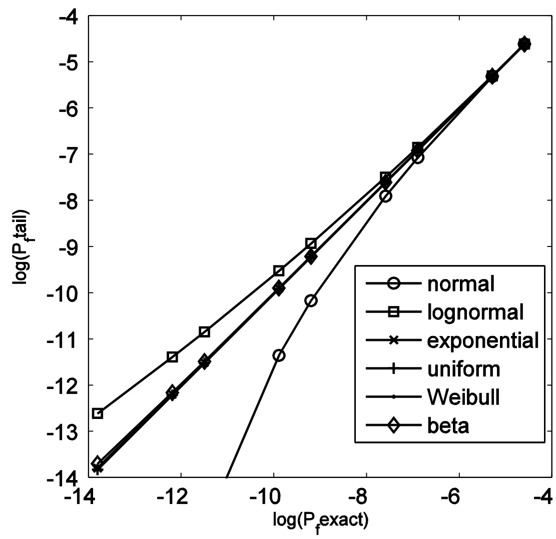

Figure 3. Comparison of probability of failure for various distribution types

Table 2: Effect of threshold in tail–modeling (normal distribution, percent error of the inverse measure)

$P_{f,\text{target}}$	10^{-2} (y = 2.326)	10^{-3} (y = 3.090)	10^{-4} (y = 3.719)	10^{-5} (y = 4.265)	10^{-6} (y = 4.753)
$F(g)$ = 0.90	1.86E−1	3.33E+0	8.24E+0	1.36E+1	1.88E+1
$F(g)$ = 0.91	1.04E−1	2.90E+0	7.49E+0	1.26E+1	1.77E+1
$F(g)$ = 0.92	3.73E−2	2.48E+0	6.73E+0	1.16E+1	1.65E+1
$F(g)$ = 0.93	−1.43E−2	2.06E+0	5.95E+0	1.06E+1	1.53E+1
$F(g)$ = 0.94	−4.90E−2	1.65E+0	5.16E+0	9.45E+0	1.39E+1
$F(g)$ = 0.95	−6.51E−2	1.26E+0	4.34E+0	8.27E+0	1.25E+1
$F(g)$ = 0.96	−6.06E−2	8.77E−1	3.48E+0	7.00E+0	1.09E+1
$F(g)$ = 0.97	−3.49E−2	5.20E−1	2.58E+0	5.59E+0	9.06E+0
$F(g)$ = 0.98	4.17E−3	1.98E−1	1.59E+0	3.94E+0	6.82E+0

Next, the error related to random sampling and empirical CDF is tested using the same function. In order to see the uncertainty related to the sample size, the previous study is repeated for different number of samples. Instead of using the equally spaced CDF in the Latin Hypercube sampling, a random sampling method in the uniform interval is used. Accordingly, the sampling method will produce uncertainty. Table 3 summarizes the convergence study results according to the number of samples. Each data is obtained from 500 repetitions. The mean values of the errors are not reduced as the number of samples is increased, but the standard deviations are. In order to obtain the same level of standard deviation at N = 500 and $P_{f,\text{target}} = 10^{-4}$, more than 7×10^{6} sampling points will be required in the Monte Carlo simulation. Figure 4 shows the empirical CDF and the tail–model of the normally distributed function when N = 500.

Table 3: Convergence study of tail–modeling errors for normally distributed function (percent error of the inverse measure)

$P_{f,\text{target}}$	10^{-2} (y = 2.326)		10^{-3} (y = 3.090)		10^{-4} (y = 3.719)	
	Mean	Stdv	Mean	Stdv	Mean	Stdv
N = 100	−6.7984E−2	5.4947E−2	7.4895E−1	5.6264E−1	3.2011E+0	1.2126E+0
N = 200	−6.0369E−2	2.0830E−2	1.1909E+0	1.6473E−1	4.1786E+0	3.3356E−1
N = 500	−6.2226E−2	8.9495E−3	1.2759E+0	5.4919E−2	4.3709E+0	1.0366E−1
N = 1000	−6.2146E−2	3.6247E−3	1.2903E+0	2.2619E−2	4.4024E+0	4.2405E−2
N = 5000	−6.2235E−2	3.2867E−4	1.2942E+0	2.0410E−3	4.4113E+0	3.8114E−3

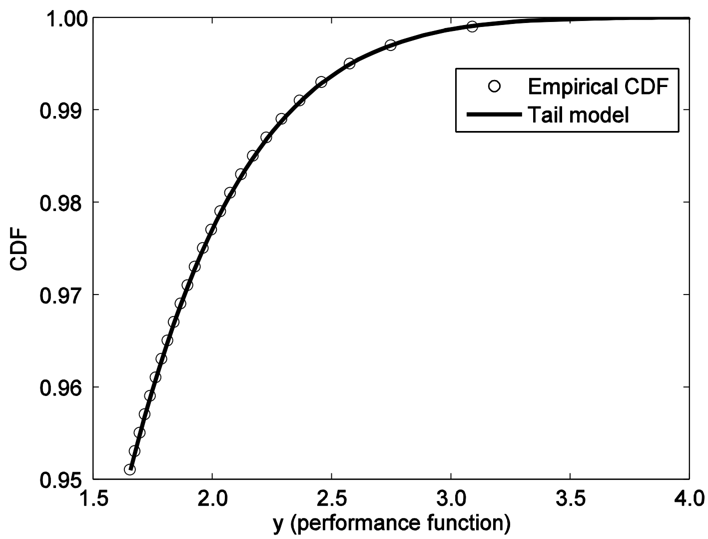

Figure 4. Cumulative distribution of normally distributed function from tail–modeling

IV. Reliability–base Design Optimization – Problem Formulation

In order to illustrate the use of tail–modeling in the RBDO framework, a simple optimization formulation (Enevoldsen and Sorensen, 1994) is discussed in this section. The cost function is assumed to be easily evaluated using the mean values of random variables and the constraints are defined using probabilistic distributions of the performance functions. Specifically, consider the following form of the RBDO problem:

$$\begin{array}{ll} \text{minimize} & c(\mathbf{x},\mathbf{d}) \\ \text{subject to} & P(y(\mathbf{x},\mathbf{d})>0) \le P_{f,\text{target}} , \\ & \mathbf{d}_L \le \mathbf{d} \le \mathbf{d}_U \end{array} \tag{16}$$

where $\mathbf{x}$ denotes the vector of random variables, $\mathbf{d}$ represents the design variables, and $c(\mathbf{x},\mathbf{d})$ identifies the cost function evaluated using the mean values. The system performance criterion is described by the performance functions $y(\mathbf{x},\mathbf{d})$ such that the system fails if $y(\mathbf{x},\mathbf{d}) > \mathbf{0}$. Using the tail–model in Eq. (10), the probability constraint can be calculated by

$$P\left[y(\mathbf{x},\mathbf{d})>0\right] = \left[1-F(g)\right]\left\langle 1-\frac{\xi}{\sigma}g\right\rangle_+^{-\frac{1}{\xi}} \le P_{f,\text{target}} . \tag{17}$$

The prescribed failure probability limit $P_{f,\text{target}}$ is often represented by the reliability target index as $\beta_{\text{target}} = -\Phi^{-1}(P_{f,\text{target}})$, where Φ is the cumulative distribution function of standard normal random variable. In the reliability index approach, the reliability constraint in Eq. (17) is imposed using Eq. (12). In the performance measure approach, on the other hand, it is imposed using Eq. (14).

In many engineering applications, the target reliability index is usually greater than 3.0, which corresponds to 0.13% of the distribution. Thus, tail–modeling is important to accurately estimate the probability in this region.

V. Numerical Example

Consider the cantilevered beam design problem, shown in Figure 5 (Wu *et al*., 2001). The objective is to minimize the weight or equivalently the cross sectional area, $A = w \cdot t$, subject to two reliability constraints, which require the reliability indices for strength and deflection constraints to be larger than three. The expressions of two performance functions are given as

Strength:
$$y_s = S - R = \left(\frac{600}{w^2 t} F_X + \frac{600}{wt^2} F_Y \right) - R \tag{18}$$

Tip Displacement:
$$y_d = D - D_O = \frac{4L^3}{Ewt} \sqrt{\left(\frac{F_Y}{t^2} \right)^2 + \left(\frac{F_X}{w^2} \right)^2} - D_O \tag{19}$$

where R is the yield strength, F_X and F_Y are the horizontal and vertical loads and w and t are the design parameters. L is the length and E is the elastic modulus. R, F_X, F_Y, and E are random in nature and are defined in Table 4.

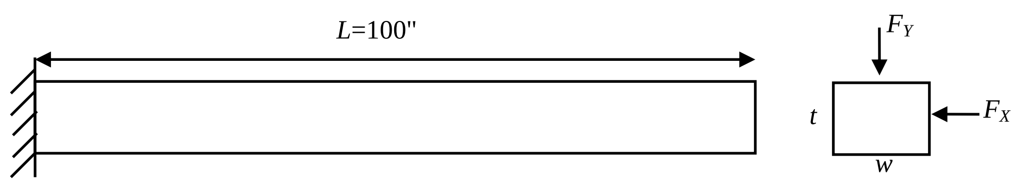

Figure 5: Cantilever beam subjected to horizontal and vertical random loads

Table 4: Random variables for the cantilevered beam problem

Random Variable	F_X	F_Y	R	E
Distribution	Normal (500,100)lb	Normal (1000,100)lb	Normal (40000,2000) psi	Normal (29E6,1.45E6) psi

In order to model the tail part of the cumulative distribution of the stress function, 1,000 samples are used with the upper part of tail is modeled using $F(g) = 0.95$. Optimization sub-problem is solved to find the shape and scale parameters that minimize the error between the empirical CDF and the CDF from the proposed tail–modeling. After that, the probability of failure is calculated from Eq. (10).

Design optimization problem is formulated and solved to minimize the weight of the cantilevered beam subject to the probability of failure of stress being less than 0.00135 $(\beta = 3.0)$. Figure 6 shows the history of objective function (cross-sectional area) during design optimization. Since the initial design (w = 1 and t = 2) violates the constraint significantly, the objective function increases at first four iteration. Table 5 shows the optimization results. The results are also compared with other methods using FORM and MCS. Figure 7 shows the tail–model at the optimum design.

Table 5: Optimization results of the beam problem

	FORM (Ramu, 2004)	MCS (Qu, 2003)	Tail model
ξ	–	–	–0.171
σ	–	–	1,671
Width (w)	2.446	2.453	2.455
Height (t)	3.892	3.884	3.843
Obj. fn (wt)	9.520	9.527	9.435
Cons. fn (y^*)	–	–	–6.68E-6
β	3.00	3.016	3.000
P_f	0.00135	0.0013	0.00135

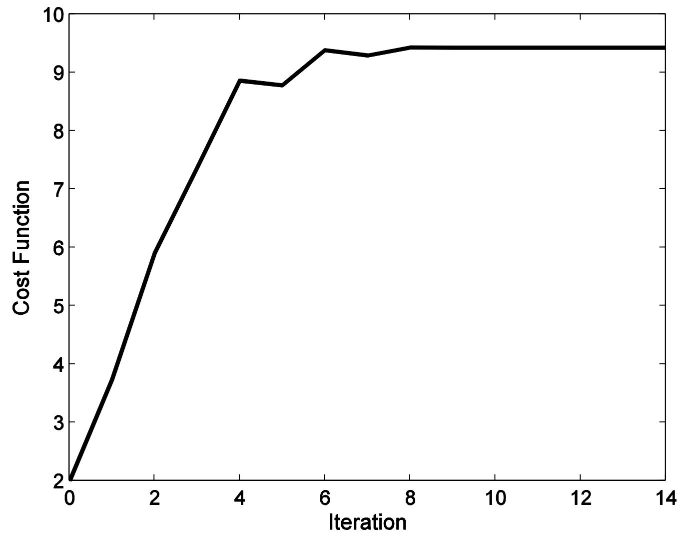

Figure 6: History of objective function during optimization.

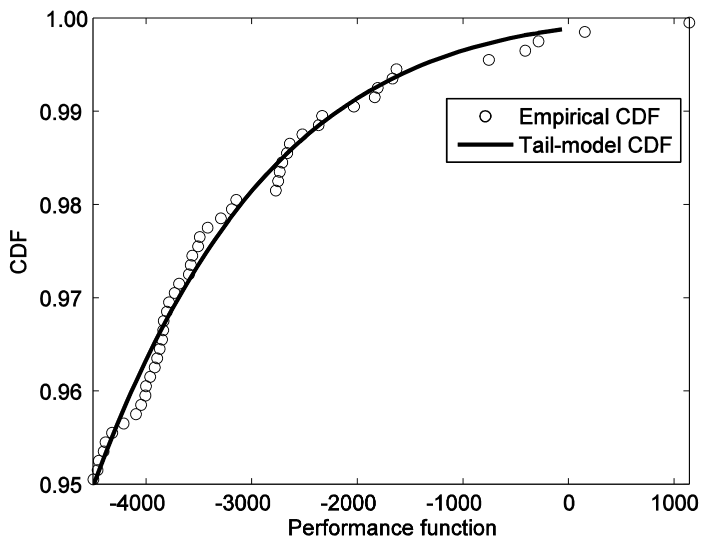

Figure 7: Tail–modeling result at the optimum design.

VI. Conclusions

A tail–modeling technique is utilized to estimate the high reliability of structural systems. The tail–modeling allows to focusing on the behavior of the tail with equivalent tail behavior. The generalized Pareto distribution provides a convenient tool for estimating high probability data with much less number of samplings than the conventional Monte Carlo simulation. Difficulty in convergence for the gradient-based optimization algorithm (all sampling-based methods) requires further research in this field.

References

[1]Kim, N. H., Wang, H., and Queipo, N. V., Efficient shape optimization technique using stochastic response surfaces and local sensitivities, ASCE Joint Specialty Conference on Probabilistic Mechanics and Structural Reliability, July 26 - 28, 2004, Albuquerque, New Mexico.

[2]Enevoldsen, I. and Sorensen, J. D. (1994), Reliability-based optimization in structural engineering, Structural Safety, 15 169-196.

[3]Kim, N. H., Choi, K. K., and Botkin, M. E. (2003), Numerical method for shape optimization using meshfree method, Struct. Multidisc. Optim. 24 418-429.

[4]Maes, M., and Huyse, L. (1995), Tail effects of uncertainty models in QRA, Proceedings of the Third International Symposium on Uncertainty Modeling and Analysis, IEEE Computer Society Press, 133-138.

[5]Castillo, E. (1988), Extreme value theory in engineering, Academic Press, San Diego, California, USA.

[6]Pickands, J. III (1975), Statistical inference using extreme order statistics, Annals of Statistics, 3 119-131.

[7]Boos, D. (1984), Using extreme value theory to estimate large percentiles, Technometrics, 26 (1) 33-39.

[8]Hasofer, A. (1996), Non-parametric estimation of failure probabilities, Mathematical Models for Structural Reliability, Eds. F. Casciati, and B. Roberts, CRC Press, Boca Raton, Florida, USA, 195-226.

[9]Caers, J., and Maes, M. (1998), Identifying tails, bounds, and end-points of random variables, Structural Safety, 20 1-23.

[10]Prescott, P., and Walden, A. (1980), Maximum likelihood estimation of the parameters of the generalized extreme-value distribution, Biometrika, 67 (3) 723-724.

[11]Hosking, J. (1985), Algorithm AS 215: Maximum-likelihood estimation of the parameters of the generalized extreme-value distribution, Applied Statistics, 34 (3) 301-310.

[12]Maes, M.A. and Breitung, K. (1993), Reliability-Based Tail Estimation, Proceedings, IUTAM Symposium on Probabilistic Structural Mechanics (Advances in Structural Reliability Methods), San Antonio, Texas, June, 335-346

[13]Lee, J. O., Yang, Y. S., Ruy, W. S., A Comparative Study on Reliability-index and Target-performance-based Probabilistic Structural Design Optimization, Computers and Structures, Vol. 80, No. 3-4, 2002, pp. 257-269.

[14]Youn, B. D., Choi, K. K., and Park, Y. H., Hybrid Analysis Method for Reliability-based Design Optimization, Journal of Mechanical Design, ASME, Vol. 125, No. 2, 2003, pp. 221-232.

[15]Ramu, P., Qu, X., Youn, B.D., Haftka, R.T., Choi, K.K., Safety Factor and Inverse Reliability Measures, 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Palm Springs, CA, April 2004 - paper AIAA-2004-1670

[16]Parkinson, A., Sorensen, C., and Pourhassan, N., 1993, A General Approach for Robust Optimal Design, ASME J. Mech. Des., 115, pp. 74–80.

[17]Sundaresan, S., Ishii, K., and Houser, D. R., 1993, A Robust Optimization Procedure with Variations on Design Variables and Constraints, Advances in Design Automation, ASME DE-Vol. 69-1, pp. 379–386.

[18]Qu, X., Haftka, R.T., Venkataraman, S., and Johnson, T.F., "Deterministic and Reliability-Based Optimization of Composite Laminates for Propellant Tanks," AIAA Journal, 41(10), pp. 2029-2036, 2003.

[19]Wu, Y T., Shin Y., Sues, R., and Cesare, M., "Safety Factor Based Approach for Probability–based Design Optimization," Proceedings of 42nd AIAA/ ASME/ ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Seattle, WA, 2001, AIAA Paper 2001-1522.