Safety Envelope for Load Tolerance and Its Application to Fatigue Reliability Design

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A concept of safety envelope for load tolerance is introduced, which shows a capacity of the current design, a future reference for design upgrade, maintenance and control. The safety envelope is applied to estimate load tolerance of a structural part with respect to the reliability of fatigue life. First, the dynamic load is decomposed into the average value and amplitude, which are modeled as random variables. Through the fatigue analysis and uncertainty propagation, the reliability of fatigue life is calculated for a given distribution of random variables. The effect of different distribution types of random variables is investigated. In order to improve finding the boundary of the envelope, sensitivity information is utilized. When the relationship between the safety of system and applied loads is linear or mildly nonlinear, linear estimation of the safety envelope turns out to be efficient. During the application of the algorithm, a stochastic response surface of fatigue life with respect to load capacity coefficient is constructed, and Mote Carlo Simulation is used to calculate the reliability and sensitivities.

Nomenclature

f = load history
G = system response
α, γ = load capacity coefficients
β = reliability index
P_f = probability of failure
μ = mean
σ = standard deviation
T = transformation from any random space to standard normal space
u = standard normal random variable
L = logarithmic fatigue life
Γ_p = multidimensional Hermite polynomials of degree p
a = coefficient of polynomial
Ω_u = standard normal space
Ω_x = random space

I. Introduction

TRADITIONALLY structural design under uncertainty includes structural dimension, shape, and material properties as uncertainty parameters. These parameters are relatively well controlled so that the variability is usually small. However, the uncertainty in load or force is much larger than that of others. The variability of the load is often ignored in the design stage and is very difficulty to quantify it. Without knowing the accurate uncertainty characteristics of input, it is hard to rely on the reliability of the output. In this paper, a different approach is taken by asking how much load a system can support. The amount of load, which a structure system can support, becomes

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important information for evaluating a design. Traditional design concerns load capacity by introducing safety factor, which suppose to give a safety margin for the uncertain load conditions. Kwak and Kim[2] proposed a concept of allowable load set, where deterministic loads are used without considering uncertainties involved in it. In linear systems, the allowable load set becomes piecewise linear and convex.

In this paper, the idea of allowable load set is extended to the fatigue life estimation under uncertainty in the applied dynamic loads through the stochastic response surface technique and sensitivity information. The dynamic load is parameterized such that the uncertainties in the parameters are considered. Since the problem at hand includes fatigue life of dynamic system, it is computationally intensive, without mentioning the probability of failure. Thus, it is important to calculate the uncertainty propagation efficiently. Instead of searching the load tolerance directly, an estimation method using the data at the current load and its sensitivity is proposed. This idea can be further extended to the multi-dimensional case, in which the load tolerance becomes a safety envelope.

With reference to Figure 1, the analysis procedure can be decomposed into three different levels: (1) Calculate the fatigue life of the system when a dynamic load history is provided. In this particular application, a commercial program, FE-Safe, is used to calculate the fatigue life. (2) Construct the stochastic response surface to calculate the reliability of the system's fatigue life due to the uncertainty in load parameters. (3) Predict the load tolerance and construct the safety envelope using the path following continuation algorithm.

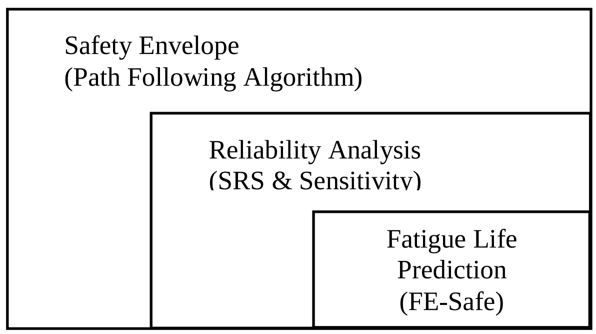

Figure 1. Analysis procedure of constructing safety envelope

II. Parameterization of dynamic loads and concept of safety envelope

Safety of the system strongly depends on the assumptions given in input conditions. Among them, the assumption in the applied load may be the most important factor. Thus, it would not make any sense to analyze and design a structure without considering the variability of the load. The same design can be safe or failed based on input loads. However, input loads are often unknown, especially for dynamic systems. In addition, it is subjective. The load characteristic of one operator may completely different from that of the other operator. In order to perform reliability analysis, it is necessary to know uncertainty characteristics of inputs. However, distribution type and parameters of loads are often unknown. As a partial remedy for this difficulty, it is assumed that the representative dynamic load history $f_0(t)$ is available either from experiment or from computer simulation. This dynamic load can be decomposed into the average value and amplitude. The parameterization of the dynamic load can then be introduced by changing the average value and amplitude as

$$f(t) = \alpha f_{\mathrm{ave}} + \gamma \left(f_0(t) - f_{\mathrm{ave}} \right) \tag{1}$$

where f_{ave} is the average value of $f_0(t)$, and α and γ are load capacity coefficients (LCC) for the average value and amplitude, respectively. When $\alpha = \gamma = 1$, the applied load is identical to the initial load history. In Eq. (1), γ can not be negative.

In the reliability analysis, α and γ are considered as random variables that can represent the statistical behavior of the applied dynamic load. In traditional reliability-based design, variability in parameters is usually modeled by assuming specific type of random distribution. In this paper, the effect of different distribution types on the system response is investigated by introducing the concept of conservative distribution type, which provides a safer way to model uncertainties.

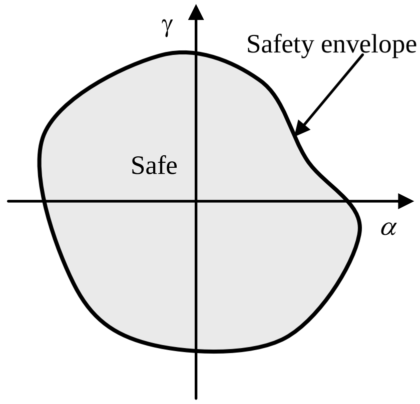

Figure 2. Safety envelope for two variables

When the two LCCs are gradually increased from zero, the initially safe system becomes unsafe at certain values of LCCs. If all combinations of LCCs that make the system unsafe are collected and connected, a closed envelope can be constructed. Figure 2 shows a schematic illustration of the safety envelope when two variables are involved. However, search for all possible LCCs are time consuming and, in many applications, impractical. In this paper, a systematic way of searching the boundary of the safety envelope is proposed using a Euler-Newton continuation method[2], an effective path following algorithm.

When the relationship between the safety of the system and the applied loads is linear or mildly nonlinear, this approach can produce an effective way of estimating the safety envelope once sensitivity information is provided. In context of reliability based safety measure, the target of safety envelope is that failure probability cannot reach over the prescribed value. Thus, a reliability based safety envelope has been introduced.

III. Fatigue life prediction

The computational model is the front loader frame of civil construction equipment. The model consists of 172,000 finite elements. Dynamic loads are measured in 26 different channels; i.e., 26 DOFs. More than 9,000 peak-and-valleys of dynamic loads are sampled during 46 min. In fatigue analysis, first a unit static load is applied per each channel or load degree-of-freedom to calculate the stress influence coefficient. The stress influence coefficients are multiplied by dynamic load history to calculate the dynamic stress.

Based on different stress amplitude at different time, the fatigue damage is linearly accumulated, which is proposed by Miner. The stress-life method is used to determine the fatigue life because the primary concern is not the base material, but the fabricated joints; that is, weld joints. Since the stress state is not uni-axial, critical plane algorithm is used to convert it to uni-axial fatigue data. In addition, the Goodman model is used to compensate non-zero mean stress. The design goal is to maintain the operation for 60,000 hrs. Since load data are measured for 40 min., this corresponds to about 78,000 cycles. The target probability of failure is 0.1.

IV. Reliability Analysis and Probability Sensitivity using SRS

Stochastic response surface method[1] is used to predict the relationship of fatigue life and load capacity in standard Gaussian space. The uncertainty propagation is based on constructing a particular family of stochastic response surfaces known as polynomial chaos expansion. This kind of SRS[5] can be view as an extension of classical deterministic response surfaces for model outputs constructed using uncertain inputs and performance data collected at heuristically selected collocation points. Let n be the number of random variables and p the order of polynomial. The model output can then be expressed in terms of standard random variables $\{u_i\}$ as:

$$G^p = a_0^p + \sum_{i=1}^{n} a_i^p \Gamma_1(u_i) + \sum_{i=1}^{n}\sum_{j=1}^{i} a_{ij}^p \Gamma_2(u_i, u_j) + \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} a_{ijk}^p \Gamma_3(u_i, u_j, u_k) + \cdots \tag{2}$$

where G^p is the model output, the $a_i^p, a_{ij}^p, \ldots$ are deterministic coefficients to be estimated, and the $\Gamma_p(u_1,\ldots,u_p)$ are multidimensional Hermite polynomials of degree p. In general, the approximation accuracy increases with the order of the polynomial, which should be selected reflecting accuracy needs and computational constraints. The accuracy and convergence of SRS can be found in Kim et al[5].

In this paper, the fatigue life of a structural part is considered as a model output. However, the range of the model output changes over several order of magnitude. Accordingly, Logarithmic fatigue life is approximated using the SRS in Eq. (2).

Reliability analysis is then carried out by the Monte Carlo simulation operated on the SRS. In MCS, probability of failure is calculated by

$$P_f = \int_{\Omega_x} I(G(\mathbf{x}) \le 0)\, f(\mathbf{x})\, d\mathbf{x} \tag{3}$$

where $G(\mathbf{x}) \le 0$ is the failure region, $f(\bullet)$ the joint probability density function, and $I(G(\mathbf{X}) \le 0)$ the indication function such that I = 1 if $G(\mathbf{X}) \le 0$ and I = 0 otherwise. In Eq. (3), Ω_x denotes the entire random design space. In the SRS, however, all input random variables are transformed into the standard random variable space $\Omega_{\mathbf{u}}$. Since the explicit expression of the model output is given in terms of Hermite polynomial as in Eq. (2), the MCS is not expensive even with 10^5 samples.

In estimating the safety envelope, the sensitivity information is very important. When moment-based methods are used, the sensitivity of the reliability index can be calculated without requiring additional computation. However, sensitivity calculation in sampling-based methods, such as in Eq. (3), is not trivial due to the uncertainty involved in the Monte Carlo integral. Let θ be a statistical parameter. Then, the sensitivity of failure probability can be obtained by following a similar Monte Carlo integral as

$$\frac{\partial P_f}{\partial \theta} = \int_{\Omega_{\mathbf{u}}} \left\{ I(G(\mathbf{x}) \le 0) \left[\frac{\partial f(\mathbf{x})}{f(\mathbf{x}) \partial \theta} \right]_{\mathbf{x}=\mathbf{T}^{-1}(\mathbf{u})} \varphi(\mathbf{u}) \right\} d\mathbf{u} \tag{4}$$

where $\varphi(\mathbf{u})$ is the joint PDF of standard random variables.

As an illustration of the accuracy of sampling based probability sensitivity analysis, consider a simple linear analytical function $G(x) = 1.6 - 3x$, with x being a random variable that is normally distributed according to $N(0,0.4^2)$. When the input variable is normally distributed, sensitivity with respect to random parameters in Eq. (4) can be obtained by

$$\frac{\partial P_f}{\partial \mu} = \frac{1}{N} \sum_{j=1}^{N} I_j \frac{1}{\sigma_i} u_i^j \tag{5}$$

$$\frac{\partial P_f}{\partial \sigma} = \frac{1}{N} \sum_{j=1}^{N} I_j \frac{1}{\sigma_i} (u_i^j u_i^j - 1) \tag{6}$$

The accuracy of the sampling-based sensitivity calculation in the above equations can be evaluated by comparing with the sensitivity from FORM. Since the function is linear and the input is normally distributed, the reliability and its sensitivity from FORM will yield the exact values. Table 1 compares the probability of failure and its sensitivity with respect to random parameters. The proposed sensitivity calculation results agree with that from FORM.

Table 1. Accuracy of sampling based sensitivity analysis

	FORM	MCS on SRS (10^5 samples)	Ratio (%)
P_f	0.0915	0.0914	100.11
$dP_f/d\mu$	0.4100	0.4109	99.78
$dP_f/d\sigma$	0.5469	0.5484	99.73

V. Estimation of safety envelope using sensitivity

First, a single parameter is selected to estimate the safety envelope of the structure using sensitivity information. Suppose the average value of the dynamic load maintains constant, while the amplitude is changed randomly. From Eq. (1), the uncertainty of the amplitude can be represented using the following decomposition of the dynamic load:

$$f(t) = f_{\text{ave}} + \gamma\left(f_0(t) - f_{\text{ave}} \right) \tag{7}$$

When $\gamma = 1$, we can recover the original load history. When $\gamma = 0$, the dynamic load becomes a static load with the average value. In this definition, γ cannot take a negative value.

Since no accurate information is available for the dynamic load, we assume that γ is a random variable. In order to simplify the problem, we further assume that the parameter γ shows a normal distribution. Since $\gamma = 1$ represents the original dynamic load, we assume that γ is normally distributed with the mean of one and the standard deviation of 0.25 (COV=0.25). The random variable γ can be converted into the standard random variable u by

$$\begin{aligned} \gamma &= \mu_\gamma + \sigma_\gamma u \\ &= 1 + 0.25u \end{aligned} \tag{8}$$

where $u \sim N(0,1^2)$, $\gamma \sim N(1,0.25^2)$, μ_γ = mean, σ_γ = standard deviation. In order to see the effect of mean change, we fix the standard deviation. Thus, the only variable is the mean value of random variable γ. The goal is to find the value of μ_γ that the system fails.

For any given sample point u corresponding γ can be obtained from Eq. (8), and using γ a new dynamic load history can be obtained from Eq. (7). By applying this dynamic load history, the fatigue life of the system can be obtained.

Because the fatigue life changes in several orders of magnitudes, it is better to construct the response surface for the logarithmic fatigue life. A cubic stochastic response surface is constructed as a polynomial chaos expansion for the logarithmic fatigue life as

$$L(\gamma(u)) = \log_{10}(\text{Life}) = 5.7075 - 0.7223u - 0.0581(u^2 - 1) + 0.0756(u^3 - 3u)\,. \tag{9}$$

Since the required life by the working component is 60,000 hours and each cycle corresponding to 46 minutes, the target of the fatigue life can be written in logarithmic scale by

$$\begin{aligned} L_{\text{target}} &= \log_{10}(60{,}000 \text{ hours}) \\ &= \log_{10}(78{,}261 \text{ cycles}) \\ &\approx 4.9 \end{aligned} \tag{10}$$

The system is considered to be failed when the predicted logarithmic life in Eq. (9) is less than the target logarithmic life in Eq. (10). Accordingly, we can define the probability of failure as

$$P_f\left[L(\gamma) - L_{\text{target}} \le 0 \right] \le P_{\text{target}}\,, \tag{11}$$

where P_{target} is the target probability of failure. For example, when $P_{\text{target}} = 0.1$, the probability of failure should be less than 10%. Even though the interpretation of Eq. (11) is clear, it is often inconvenient because the probability changes in several orders of magnitudes. In reliability analysis, it is more common to use the reliability index, which uses the notion of the standard random variable. Equation (11) can be rewritten in terms of the reliability index as

$$P_f \equiv \Phi(-\beta) \le P_{\text{target}} \equiv \Phi(-\beta_{\text{target}})\,, \tag{12}$$

where β is called the reliability index and Φ is the cumulative distribution function of the standard random variable. When $P_{\text{target}} = 0.1$, $\beta_{\text{target}} \approx 1.3$. The advantage of using the reliability index will be clear in the following numerical results.

Using the response surface in Eq. (9), reliability analysis is carried out using the SRS at μ_γ = 1. The results of reliability analysis are as follows:

$$\begin{aligned} P_f &= 17.81\% \\ \beta &= 0.922456 \\ \frac{\partial \beta}{\partial \mu_\gamma} &= -3.972 \end{aligned} \tag{13}$$

where $\partial\beta/\partial\mu_\alpha$ is the sensitivity of the reliability index with respect to μ_γ. Since P_{target} = 0.1 and β_{target} = 1.3, the current system does not satisfy the reliability requirement.

It is obvious that for a deterministic, linear system, the system response is linear to the applied load. Thus, estimating the safety envelope is trivial. However, the fatigue reliability of a system is not linear with respect to the applied load history. When the fatigue reliability is mild nonlinearity, it is still possible to estimate the safety envelope using sensitivity information. Based on the result from Eq. (13), the value of μ_γ that satisfies the required reliability can be estimated using a linear approximation. The linear approximation of μ_γ can be obtained by

$$\mu_\gamma^{\text{estmated}} = 1 - \frac{(\beta_{\mu_\gamma=1} - \beta_{\text{target}})}{\dfrac{\partial \beta_{\mu_\gamma=1}}{\partial \mu_\gamma}} = 0.9049\,, \tag{14}$$

which means that μ_γ needs to be decreased by 10% from the original load amplitude in order to satisfy the required reliability.

In order to verify the accuracy of the estimated result, several sampling points are taken and reliability analyses are performed. Figure 3 shows the reliability index with respect to μ_γ, while Figure 4 shows the probability of failure P_f with respect to μ_γ. The solid line is linearly approximated reliability using sensitivity information. When γ is normally distributed, the reliability index is almost linear and the estimation using sensitivity is close to the actual reliability index. When the target probability of failure is 0.1 and γ has the distribution of N(μ_γ, 0.25^2), the safety envelope can be defined as

$$0 \le \mu_\gamma \le 0.9049 \tag{15}$$

Thus, the current design, considering 25% standard deviation in the load amplitude, is not enough to achieve 90% reliability. The structure should be operated under milder working conditions, which means either lower the mean of the load amplitude by about 10%.

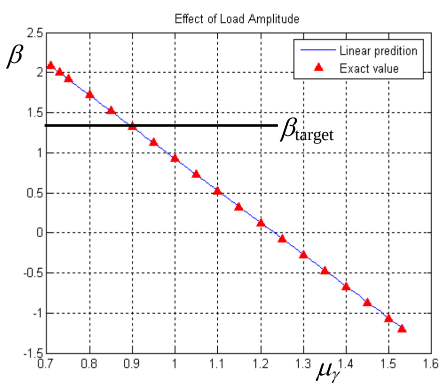

Figure 3. Reliability index β with respect to μ_γ

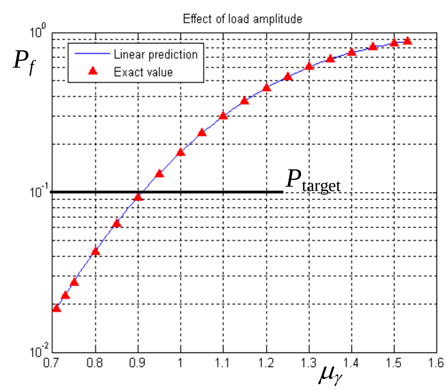

Figure 4. Probability of failure P_f with respect to μ_γ

VI. Influence of different distribution type on load tolerance and conservative distribution type

In the previous section, LCC is assumed to be normally distributed. However, in many cases, the distribution is unknown and it is difficult to identify it accurately. In addition, different distribution types may yield completely different results in load tolerance estimation. Figure 5 and 6 shows the difference between normal and lognormal distributions. Note that lognormal distribution shows higher nonlinearity in the relation of reliability indices and the mean of LCC. The linear prediction of load tolerance for lognormal LCC cannot be accurate enough, but it is still possible to apply piecewise linear prediction to load tolerance design by restrict step size to acceptable range.

Identifying the load distribution is one of the most difficult tasks in the uncertainty analysis because different operating conditions will yield completely different distribution types. Thus, design engineers often look for a conservative distribution type. For example, in Figures 5 and 6, lognormal distribution is more important when μ_α is large, whereas normal distribution is important when μ_α is small. Using sensitivity and linear approximation, it would be possible to predict which distribution type has a significant effect on the load tolerance. Once dominant distribution type is selected, the detailed load tolerance can be constructed.

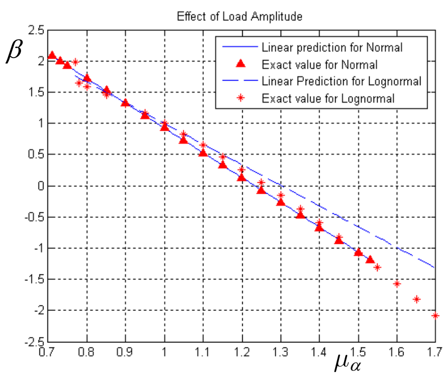

Figure 5. Reliability index β with respect to μ_α

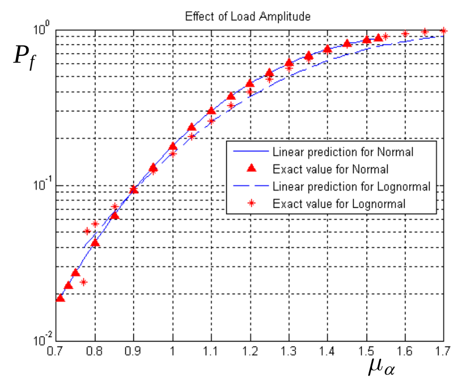

Figure 6. Probability of failure P_f with respect to μ_α

VII. Multi-dimensional safety envelope

When more than one parameter is involved in load tolerance estimation, the safe region of the parameters is called the safety envelope. The technical challenge is how to find the boundary of the envelope without trial-and-error approach. In this paper, an efficient search algorithm is proposed based on Euler-Newton continuation method[4]. For the illustration purpose, consider two parameters, α and γ, as random variables. Furthermore, it is assumed that both parameters show normal distribution. It is clear that the two parameters must have non-negative values. The capacity of the system with respect to the mean values of α and γ is interested. If the required probability of failure is 10% ($\beta_{\text{target}} = 1.3$), following steps can been taken to construct the safety envelope:

Step 1: By fixing $\mu_\gamma = 0$ and increasing μ_α from zero, find the initial boundary point ($\mu_\alpha \geq 0$) on the envelope ($\beta(\mu_\alpha, \mu_\gamma) = 1.3$);

Step 2: Using sensitivity information, find the next solution on safety envelope using Euler-Newton continuation to meet the constraint $\beta = 1.3$;

Step 3: Since only $\mu_\alpha, \mu_\gamma \geq 0$ is meaningful, continue Step 2 until the curve end in this region;

Figure 7 shows the two-dimensional safety envelope for the loader frame while LCCs are both normally distributed. It is clear for the figure that the system has much more safety margin in the average value than that of the amplitude.

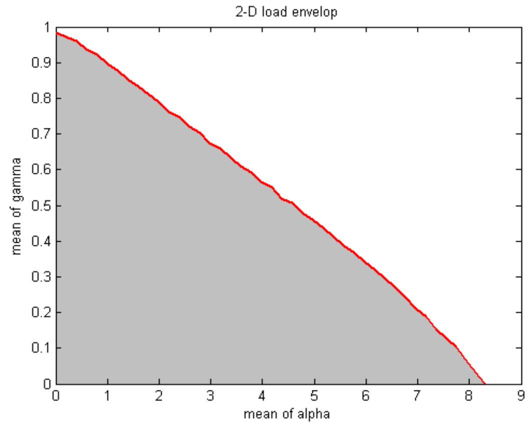

Figure 7. 2-D Safety envelop for fatigue reliability

VIII. Conclusion

In this paper, a systematic road map of safety envelope has been presented. FE-based fatigue evaluation, SRS-based reliability and sensitivity analysis, path following algorithm are integrated to construct a design reference for a structure. Conservative distribution type will be considered to give safer design of load without complete knowledge of uncertainty properties. Complete work will be done by constructing a multi-dimensional safety envelope for load tolerance by considering conservative distribution type.

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