SHAPE DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION OF CONTACT PROBLEM USING MESHFREE METHOD

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K. K. Choi, Nam H. Kim, and J. S. Chen

The Center for Computer-Aided Design
Department of Mechanical Engineering
College of Engineering
The University of Iowa
Iowa City, IA52242

INTRODUCTION

⟹ **Frictional Contact Problem**

- Continuum-Based Contact Formulation
- Penalty Regularization
- Regularized Coulomb Friction Model

⟹ **Structural Problem**

- Finite Deformation Elastoplasticity Using Multiplicative Decomposition of Deformation Gradient
- Mooney-Rivlin Type Hyperelasticity with Nearly Incompressible Constraint

⟹ **Meshfree Discretization**

- Reproducing Kernel Particle Method (RKPM)
- Direct Transformation Method for Essential B. C.

⟹ **Design Sensitivity Analysis (DSA)**

- Material Derivative Approach is Used for Shape DSA
- Shape Function of RKPM Depends on Shape Design
- Material Derivative is Taken to the Regularized Contact Variational Equation

REPRODUCING KERNEL PARTICLE METHOD (RKPM)

Reproduced Displacement Function

$$z^R(x) = \int_\Omega C(x; y-x)\phi_a(y-x)z(y)\,dy$$

$z^R(x) \to z(x)$ as $a \to 0$ Dirac Delta Measure

Correction Function

$$C(x; y-x) = \mathbf{q}(x)^T \mathbf{H}(y-x)$$

$$\mathbf{H}(y-x)^T = [1, (y-x), (y-x)^2, \cdots, (y-x)^n]$$

$$\mathbf{q}(x)^T = [q_0(x), q_1(x), \cdots, q_n(x)]$$

N-th Order Completeness Requirement (Reproducing Condition)

$$z^R(x) = \int_\Omega C(x; y-x)\phi_a(y-x)z(y)\,dy$$

$$= \overline{m}_0(x)z(x) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\overline{m}_n(x)\frac{d^n z(x)}{dx^n}$$

$\overline{m}_0(x) = 1$ $\overline{m}_k(x) = 0$ $k = 1, \ldots, n$

RKPM (cont.)

Reproducing Condition

$$\mathbf{M}(x)\mathbf{q}(x) = \mathbf{H}(0) \qquad \mathbf{H}(0)^T = [1,0,\ldots,0]$$

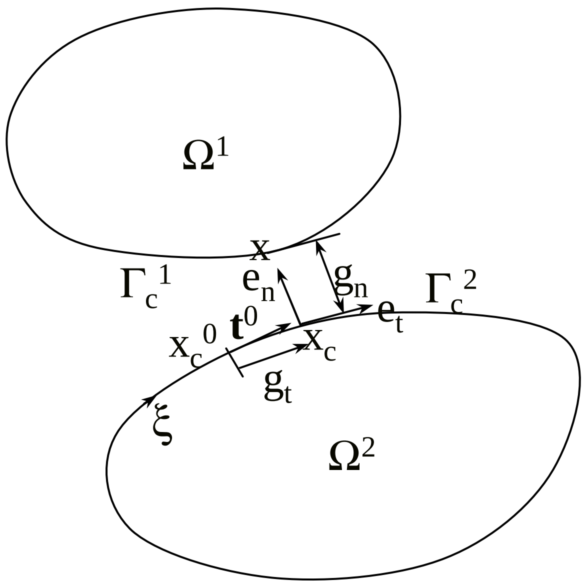
$$\mathbf{M}(x) = \begin{bmatrix} m_0(x) & m_1(x) & \ldots & m_n(x) \\ m_1(x) & m_2(x) & \ldots & m_{n+1}(x) \\ . & . & \ldots & . \\ m_n(x) & m_{n+1}(x) & \ldots & m_{2n}(x) \end{bmatrix}$$

$$C(x; y-x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \mathbf{H}(y-x)$$

$$z^R(x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \int_\Omega \mathbf{H}(y-x)\phi_a(y-x)z(y)\,dy$$

$$z^R(x) = \sum_{I=1}^{NP} C(x; x_I - x)\phi_a(x_I - x)z_I\Delta x_I = \sum_{I=1}^{NP} \Phi_I(x)d_I$$

PENALTY METHOD FOR FRICTIONAL CONTACT



Impenetration Condition

$$g_n \equiv (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_n(\xi_c) \geq 0, \quad \mathbf{x} \in \Gamma_c^1, \mathbf{x}_c \in \Gamma_c^2$$

Tangential Slip Function

$$g_t \equiv \left\| \mathbf{t}^0 \right\| (\xi_c - \xi_c^0)$$

Contact Consistency Condition

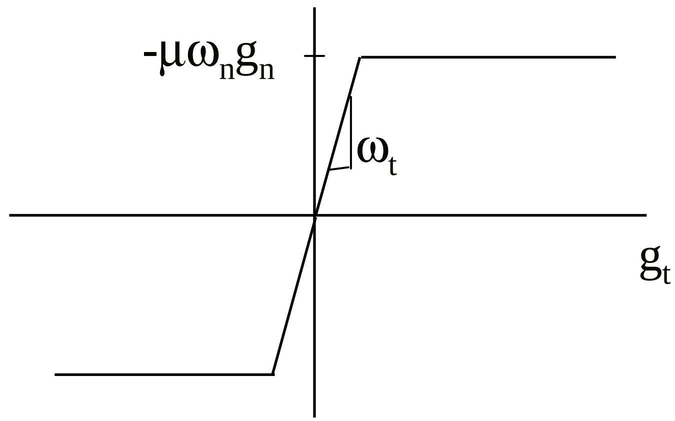
$$\varphi(\xi_c) = (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_t(\xi_c) = 0$$

Contact Penalty Function

$$P = \frac{1}{2}\omega_n \int_{\Gamma_C} g_n^{\,2} d\Gamma + \frac{1}{2}\omega_t \int_{\Gamma_C} g_t^{\,2} d\Gamma$$

where integration is performed only on the region where $g_n < 0$ on Γ_C

PENALTY METHOD (cont.)



- Stick Condition: $|\omega_t g_t| \le |\mu\omega_n g_n|$

$$b_\Gamma(\mathbf{z},\overline{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n (\overline{\mathbf{z}} - \overline{\mathbf{z}}_c)^T \mathbf{e}_n \, d\Gamma + \omega_t \int_{\Gamma_C} g_t \left[\nu(\overline{\mathbf{z}} - \overline{\mathbf{z}}_c)^T \mathbf{e}_t + (g_n \|\mathbf{t}^0\| / c) \overline{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \right] d\Gamma$$

- Slip Condition : $|\omega_t g_t| > |\mu\omega_n g_n|$

$$b_\Gamma(\mathbf{z},\overline{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n (\overline{\mathbf{z}} - \overline{\mathbf{z}}_c)^T \mathbf{e}_n \, d\Gamma - \mu\omega_n \, \mathrm{sgn}(g_t) \int_{\Gamma_C} g_n \left[\nu(\overline{\mathbf{z}} - \overline{\mathbf{z}}_c)^T \mathbf{e}_t + (g_n \|\mathbf{t}^0\| / c) \overline{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \right] d\Gamma$$

VARIATIONAL FORMULATION

Governing Variational Equation

$$a_{\Omega}({}^{n+1}\mathbf{z},\bar{\mathbf{z}}) + b_{\Gamma}({}^{n+1}\mathbf{z},\bar{\mathbf{z}}) = \ell_{\Omega}(\bar{\mathbf{z}}), \qquad \forall \bar{\mathbf{z}} \in Z$$

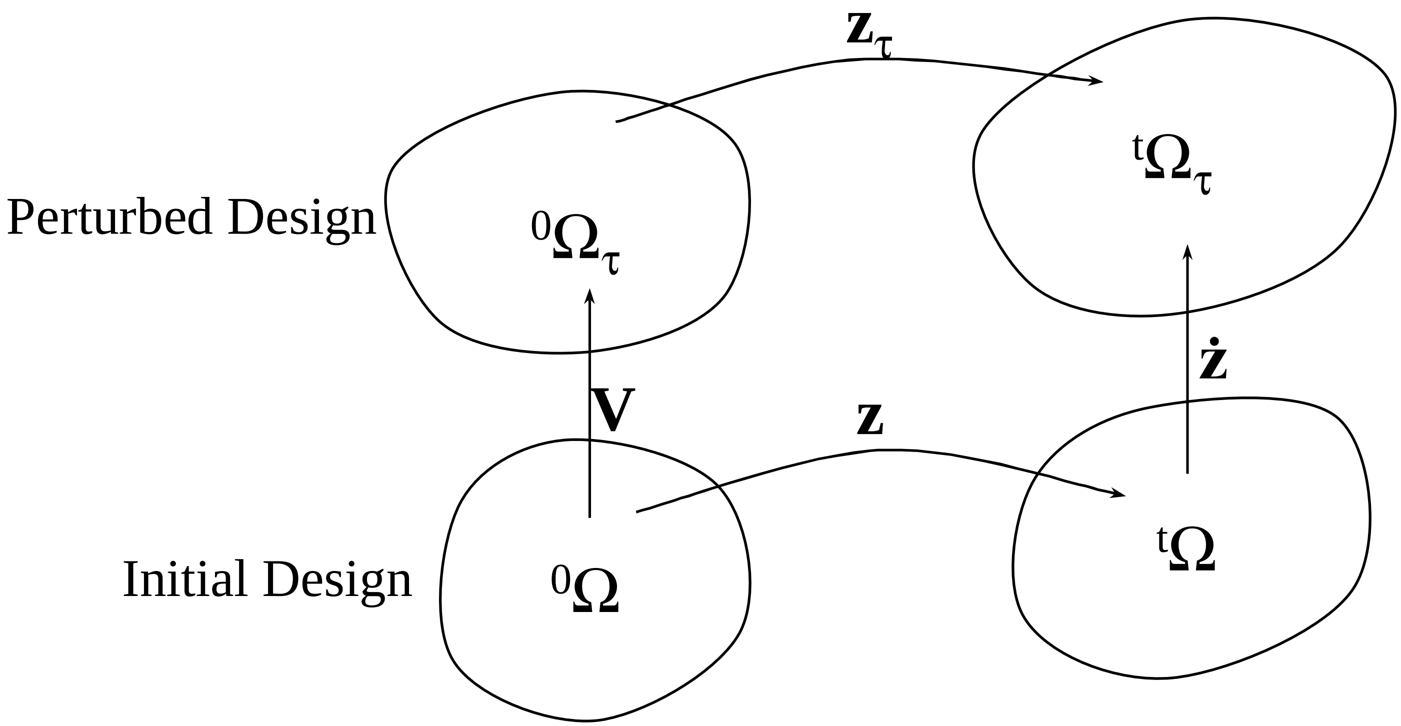
$$a_{\Omega}(\mathbf{z},\bar{\mathbf{z}}) \begin{cases} = \displaystyle\int_{\Omega} \boldsymbol{\tau} : \bar{\boldsymbol{\varepsilon}}\, d\Omega & \text{Elastoplasticity, Updated Lagrangian} \\ = \displaystyle\int_{\Omega} (\mathbf{S} : \bar{\mathbf{E}} + p\bar{H})\, d\Omega & \text{Hyperelasticity, Total Lagrangian} \end{cases}$$

$$\ell(\bar{\mathbf{z}}) = \int_{\Omega} \bar{\mathbf{z}}^{T}\mathbf{f}^{B}\, d\Omega + \int_{\Gamma_T} \bar{\mathbf{z}}^{T}\mathbf{f}^{S}\, d\Gamma$$

Linearization

$$\begin{aligned} &a^{*}_{\Omega}({}^{n+1}\mathbf{z}^{k}; \Delta\mathbf{z}^{k+1}, \bar{\mathbf{z}}) + b^{*}_{\Gamma}({}^{n+1}\mathbf{z}^{k}; \Delta\mathbf{z}^{k+1}, \bar{\mathbf{z}}) \\ &\quad = \ell_{\Omega}(\bar{\mathbf{z}}) - a_{\Omega}({}^{n+1}\mathbf{z}^{k}, \bar{\mathbf{z}}) - b_{\Gamma}({}^{n+1}\mathbf{z}^{k}, \bar{\mathbf{z}}) \qquad \forall \bar{\mathbf{z}} \in Z \end{aligned}$$

DESIGN SENSITIVITY ANALYSIS



V : Design velocity vector, direction of design perturbation

τ : Design perturbation parameter

z : Displacement vector

$\dot{\mathbf{z}}$: Material Derivative of displacement

DESIGN SENSITIVITY ANALYSIS (cont.)

Material Derivative of Variational Equation

$$\tfrac{d}{d\tau}\left[a_{\Omega}({}^{n+1}\mathbf{z},\bar{\mathbf{z}})\right]+\tfrac{d}{d\tau}\left[b_{\Gamma}({}^{n+1}\mathbf{z},\bar{\mathbf{z}})\right]=\tfrac{d}{d\tau}\left[\ell_{\Omega}(\bar{\mathbf{z}})\right], \qquad \forall\bar{\mathbf{z}}\in Z$$

DSA Equation

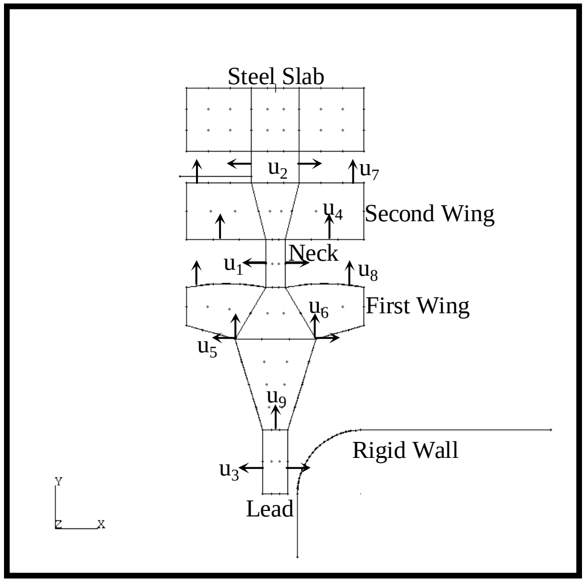
$$a_{\Omega}^{*}(\mathbf{z};\dot{\mathbf{z}},\bar{\mathbf{z}})+b_{\Gamma}^{*}(\mathbf{z};\dot{\mathbf{z}},\bar{\mathbf{z}})=\ell'_{V}(\bar{\mathbf{z}})-a'_{V}(\mathbf{z},\bar{\mathbf{z}})-b'_{V}(\mathbf{z},\bar{\mathbf{z}}), \qquad \forall\bar{\mathbf{z}}\in Z$$

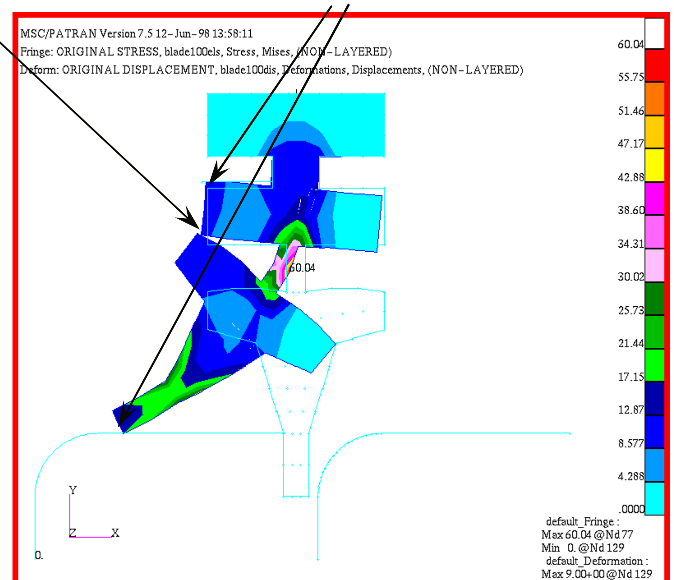
Remarks

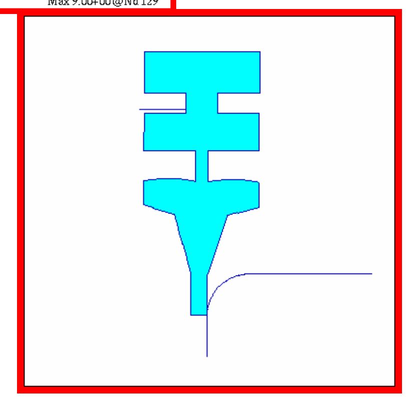
- Total form of sensitivity equation
- No iteration is required
- DSA needs to be carried out at each converged load step
  - Direct Differentiation Method is used
- Update path-dependent variables (intermediate configuration, plastic internal variables, frictional effect)

WINDSHIELD WIPER MODEL

Multi-body Contact Flexible-rigid Body Contact







Material Constant D_{10} = 80 kPa, Bulk Modulus K = 80 MPa

D_{01} = 20 kPa

Frictional Coefficient $\mu = 0.15$

Windshield Blade = 128 Particle Points

Rigid Wall = 26 Piecewise Linear Master Segments

100 Load Steps for Analysis with Displacement Driven Procedure

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ACCURACY OF SHAPE DSA RESULTS

Analysis: 633 Sec, Sensitivity: 133 Sec for 6 DV = 22.2 Sec

Performance	$\Delta\Psi$	Ψ'	$(\Delta\Psi/\Psi')\times 100$
u_1			
area	.28406E-05	.28406E-05	100.00
vm_{53}	.19984E-03	.19984E-03	100.00
vm_{54}	.28588E-03	.28588E-03	100.00
F_{cx}	-.83099E-06	-.83098E-06	100.00
F_{cy}	.55399E-05	.55399E-05	100.00
u_2			
area	.20000E-05	.20000E-05	100.00
vm_{53}	.32324E-05	.32316E-05	100.03
vm_{54}	.50379E-05	.50380E-05	100.00
F_{cx}	-.80829E-07	-.80826E-07	100.00
F_{cy}	.53886E-06	.53884E-06	100.00
u_3			
area	.68663E-05	.68663E-05	100.00
vm_{53}	.19410E-03	.19410E-03	100.00
vm_{54}	.68832E-04	.68832E-04	100.00
F_{cx}	-.65963E-05	-.65963E-05	100.00
F_{cy}	.43976E-04	.43976E-04	100.00
u_4			
area	-.50000E-05	-.50000E-05	100.00
vm_{53}	.28830E-04	.28829E-04	100.00
vm_{54}	-.60316E-05	-.60305E-05	100.02
F_{cx}	.33493E-05	.33493E-05	100.00
F_{cy}	-.22328E-04	-.22329E-04	100.00

Design Optimization
Problem Definition

Min Area(39)

s.t. $\sigma_{53}(75) \le 55$

$\sigma_{54}(45) \le 55$

$\sigma_{76}(32) \le 55$

$\sigma_{84}(34) \le 55$

$F_{y128}(5) \ge 5.5$

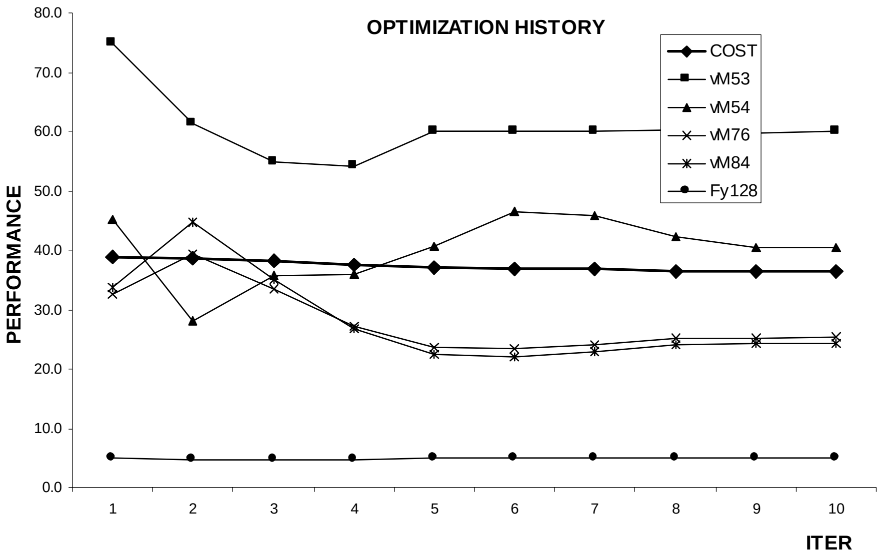
$-0.2 \le u_i \le 0.2 \quad i = 1,3,7,8$

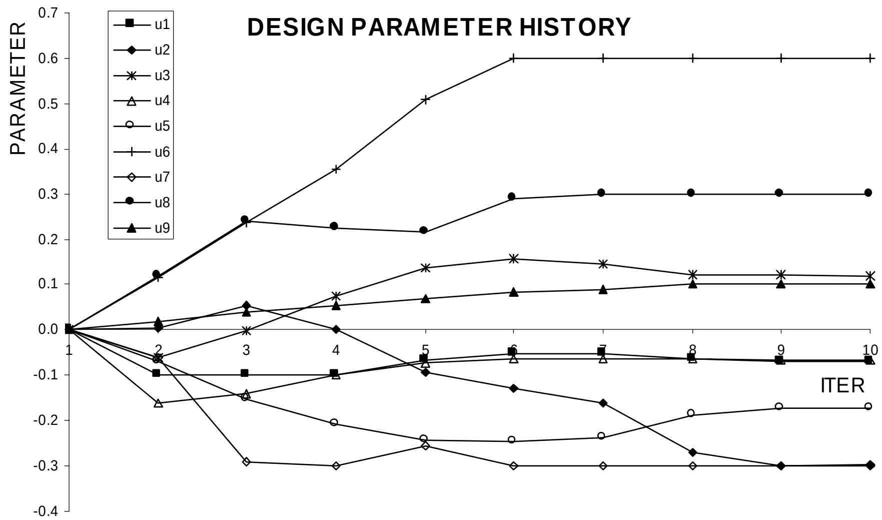
$-0.3 \le u_i \le 0.3 \quad i = 2,4$

$-0.6 \le u_i \le 0.6 \quad i = 5,6$

$-0.1 \le u_i \le 0.1 \quad i = 9$

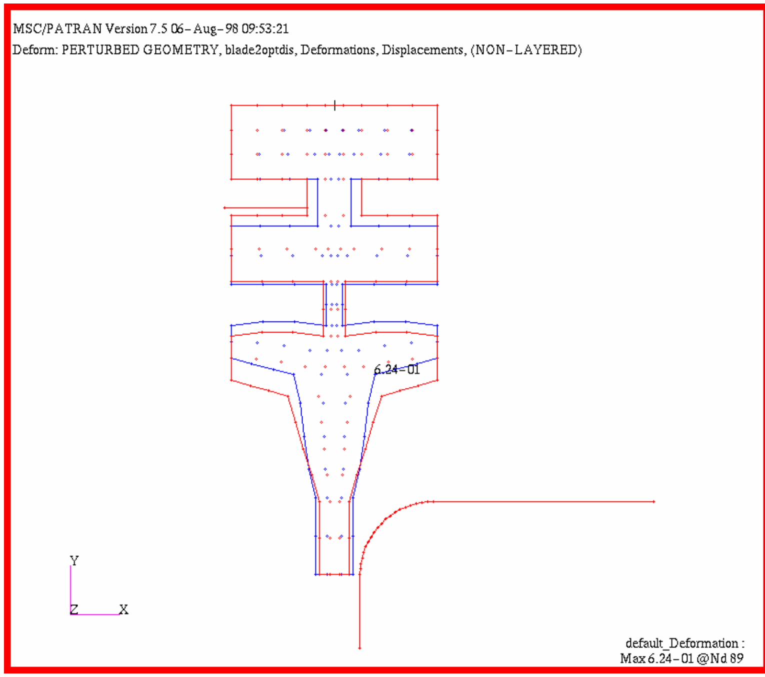
OPTIMIZATION HISTORY



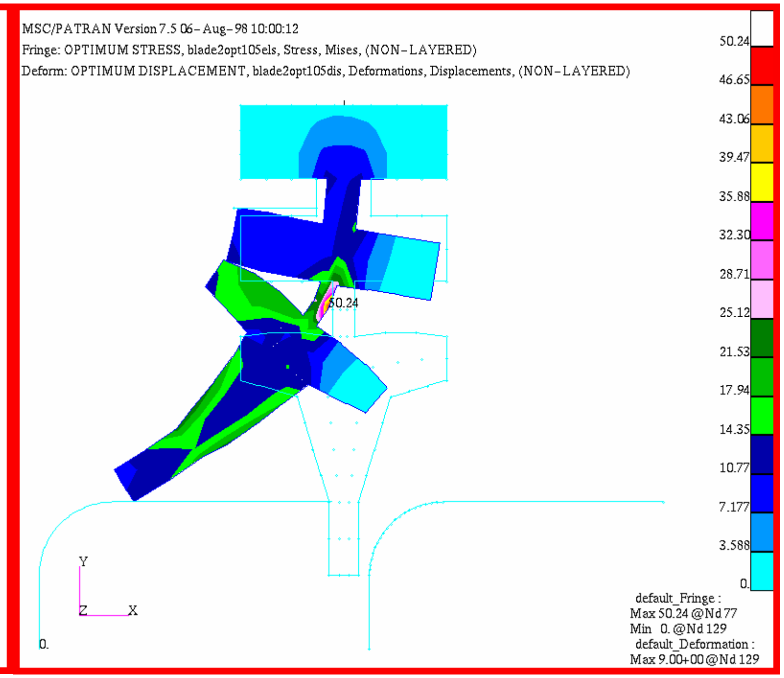


OPTIMIZED WINDSHIELD BLADE

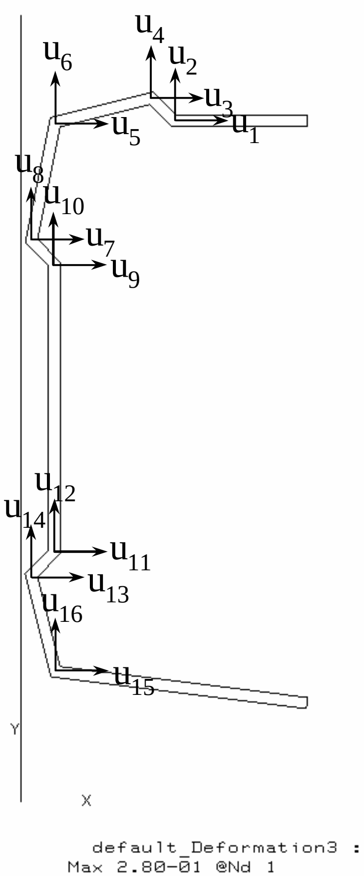
Optimum Shape

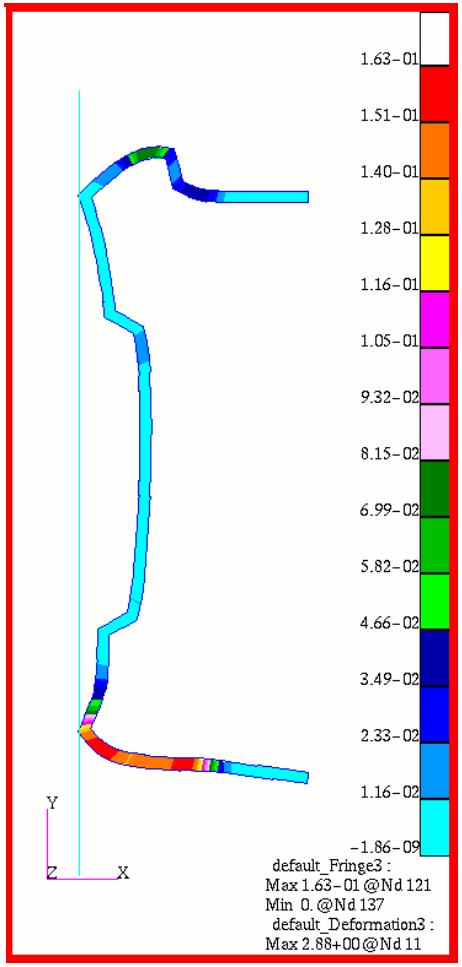


Analysis of Optimum Design

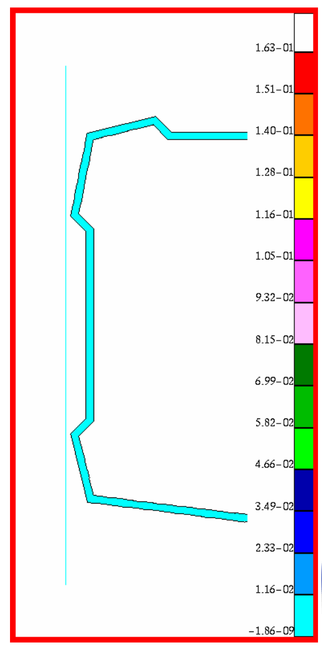


BUMPER IMPACT PROBLEM





Mounting Displ.	d = 2.8 cm
Thickness	h = 0.5 cm
Contact Penalty No.	w_n = 1,000
Friction Coeff.	μ_f = 0.4
Lame's Constants	λ = 110.8 GPa
	μ = 80.2 GPa
Plastic Hardening	H = 1.1 GPa
Initial Yield Stress	σ_Y = 500 MPa



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SENSITIVITY ANALYSIS RESULTS

Response Analysis : 275 sec
Sensitivity Analysis : 116 / 16 sec

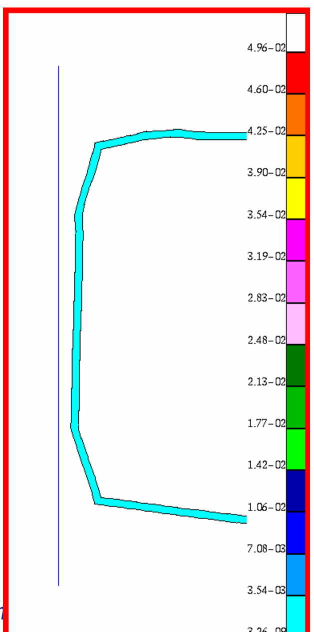
Performance (Ψ)		ΔΨ	Ψ′	(ΔΨ / Ψ′)×100%
u_2				
$\hat{e}^p_{15}$	.680005E-01	-.179756E-07	-.179757E-07	100.00
$\hat{e}^p_{65}$	.164338E+00	.311392E-08	.311393E-08	100.00
$\hat{e}^p_{29}$	.126643E-01	-.901637E-10	-.901545E-10	100.01
Z_{x39}	.429139E+00	.120943E-06	.120940E-06	100.00
F_{Cx100}	.379375E+01	.473864E-07	.473865E-07	100.00
u_4				
$\hat{e}^p_{15}$	.680005E-01	.246181E-07	.246181E-07	100.00
$\hat{e}^p_{65}$	.164338E+00	.105172E-08	.105173E-08	100.00
$\hat{e}^p_{29}$	.126643E-01	.589794E-09	.589795E-09	100.00
Z_{x39}	.429139E+00	-.295825E-06	-.295824E-06	100.00
F_{Cx100}	.379375E+01	.335517E-09	.335511E-09	100.00
u_6				
$\hat{e}^p_{15}$	.680005E-01	-.170857E-07	-.170857E-07	100.00
$\hat{e}^p_{65}$	.164338E+00	-.237257E-08	-.237256E-08	100.00
$\hat{e}^p_{29}$	.126643E-01	-.720239E-10	-.720198E-10	100.01
Z_{x39}	.429139E+00	.167699E-06	.167698E-06	100.00
F_{Cx100}	.379375E+01	-.176290E-07	-.176292E-07	100.00
u_8				
$\hat{e}^p_{15}$	.680005E-01	.581799E-09	.581877E-09	99.99
$\hat{e}^p_{65}$	.164338E+00	-.635253E-09	-.635254E-09	100.00
$\hat{e}^p_{29}$	.126643E-01	-.185890E-08	-.185890E-08	100.00
Z_{x39}	.429139E+00	-.397143E-07	-.397141E-07	100.00
F_{Cx100}	.379375E+01	.250196E-07	.250194E-07	100.00

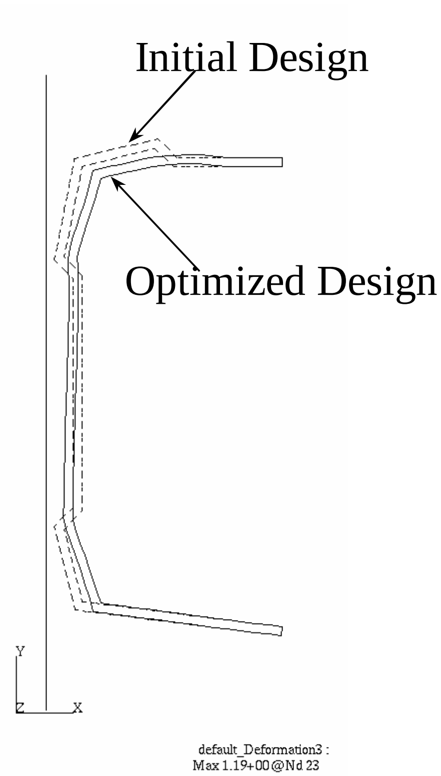
Design Optimization Problem Definition

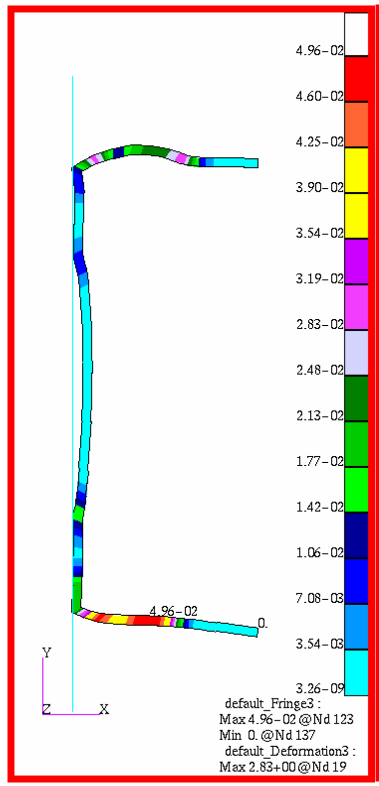
MIN Area

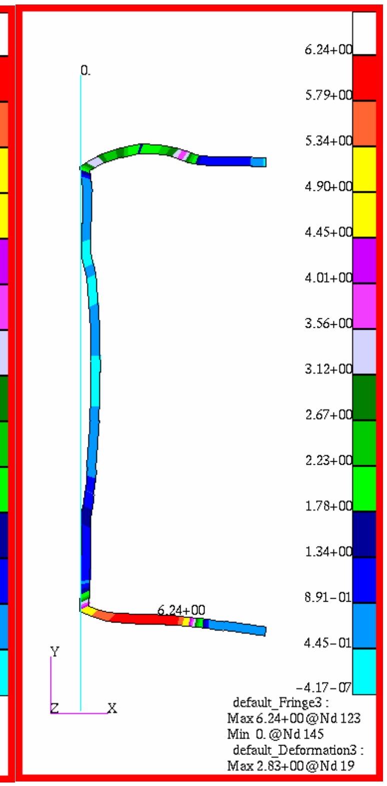
S.T.
$\hat{e}^p_{16}(0.10) \le 0.05$
$\hat{e}^p_{62}(0.15) \le 0.05$
$\hat{e}^p_{65}(0.16) \le 0.05$
$\hat{e}^p_{66}(0.16) \le 0.05$
$\hat{e}^p_{67}(0.15) \le 0.05$
$\hat{e}^p_{60}(0.15) \le 0.05$
$\hat{e}^p_{61}(0.14) \le 0.05$
$F_{Cx}(4.55) \ge 4.55$
$-1.0 \le u_i \le 1.0 \qquad i = 1,16$

DESIGN OPTIMIZATION



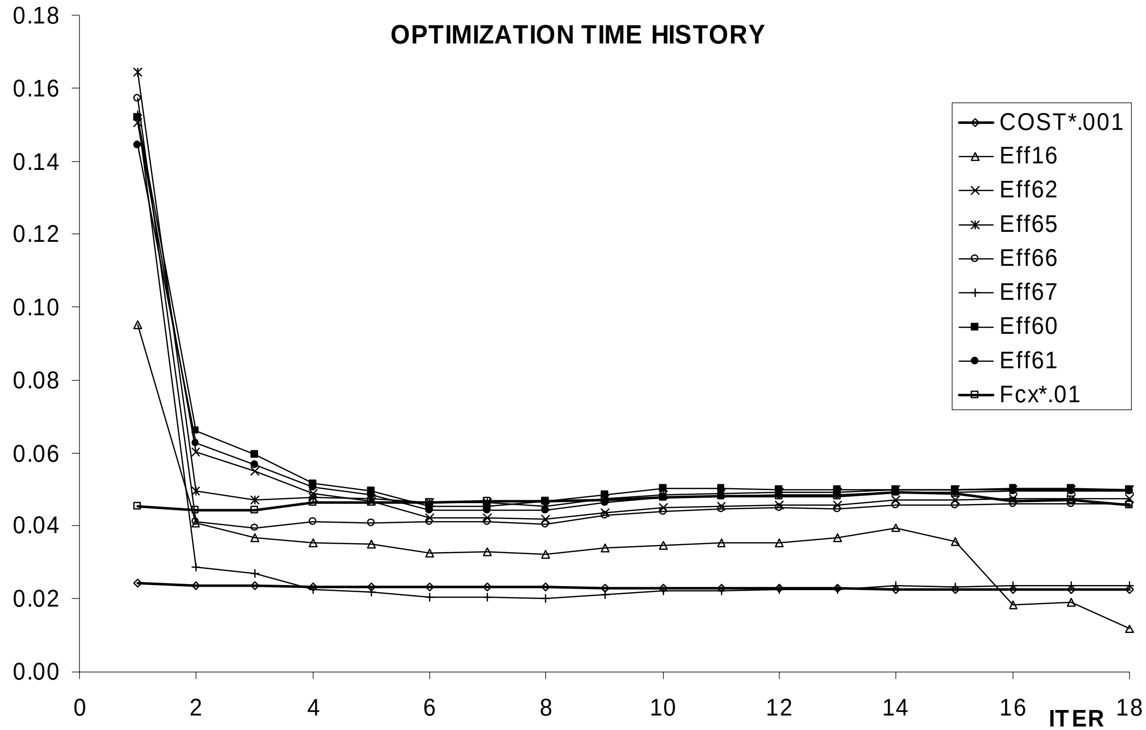






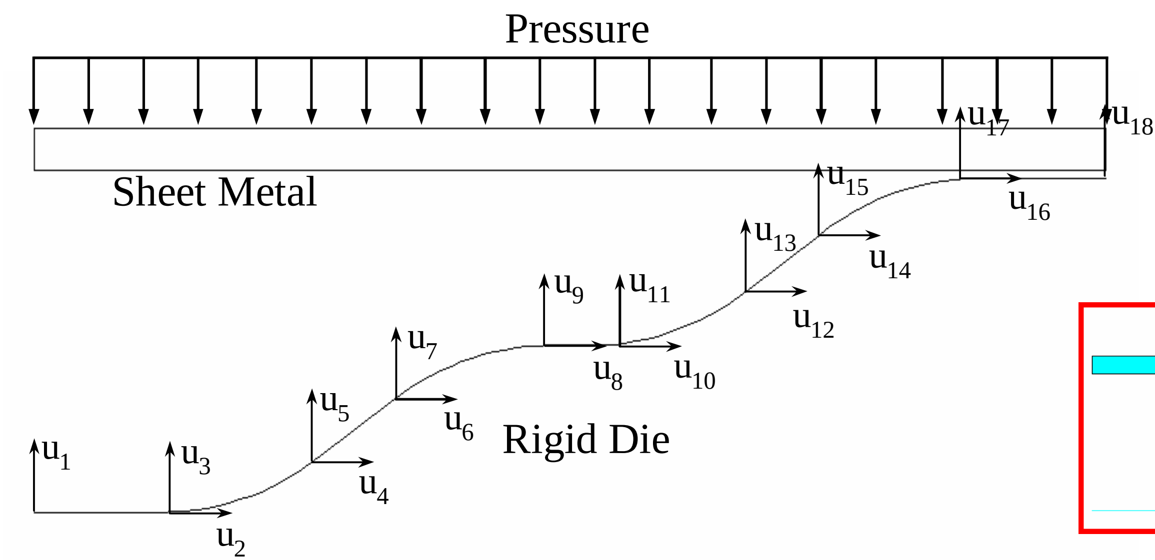
Effective Plastic Strain   Von Mises Stress

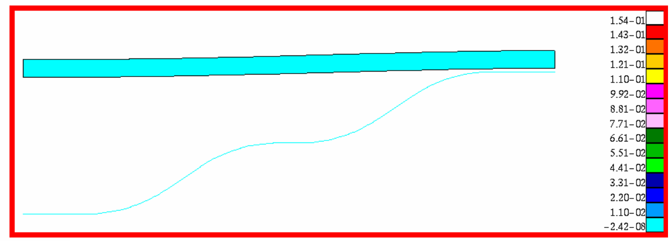
DESIGN OPTIMIZATION HISTORY

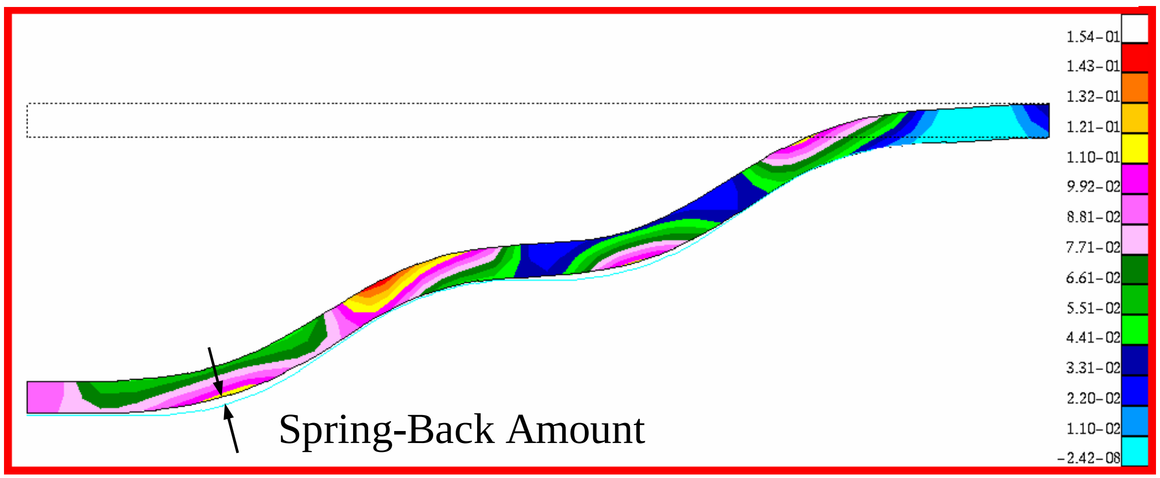


Response Analysis : 37
Sensitivity Analysis : 18

SHEET METAL STAMPING PROBLEM







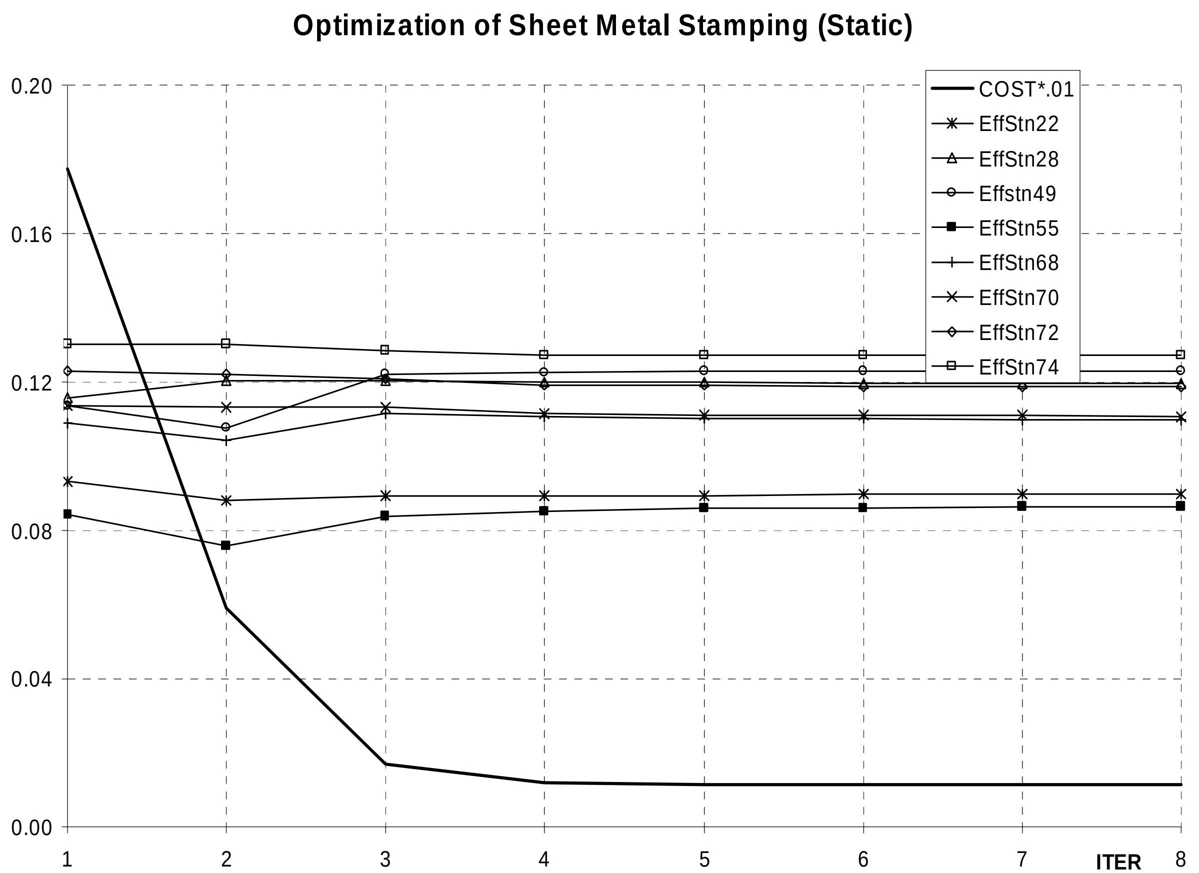
DESIGN OPTIMIZATION

$$\text{MIN} \quad G = \int_{\Gamma} \left\| \pi(\mathbf{x}) - \mathbf{x} \right\|^2 d\Gamma$$

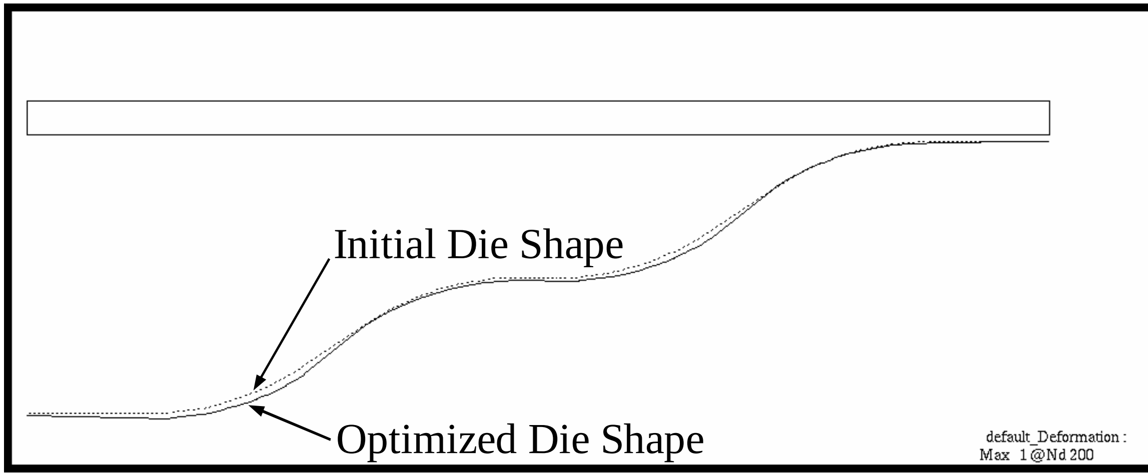
$$\text{S.T.} \quad \hat{e}^p{}_i \le 0.13$$

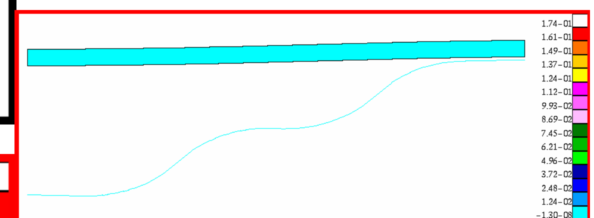
$$-2.0 \le u_i \le 2.0$$

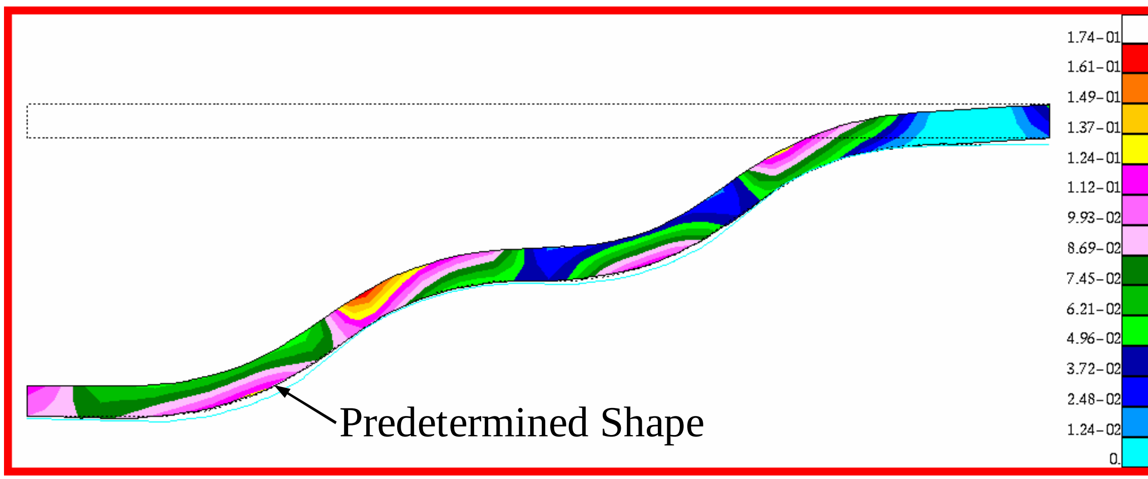
Response Analysis : 24
Sensitivity Analysis : 8



OPTIMUM DIE SHAPE







CONCLUSIONS

⇒ An efficient and accurate nonlinear DSA is proposed for path-dependent structural problems.

⇒ Sensitivity equation always uses the same tangent stiffness matrix as the response analysis at each converged configuration.

⇒ Path-dependency of DSA is from the intermediate configuration and internal plastic variables as well as frictional contact effect.