Bayesian Approach for Structural Health Monitoring –Application to Migration Technique

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ABSTRACT

There have been many attempts to predict remaining-useful-life of aircraft structures. Many non-destructive damage evaluation and testing techniques have been developed for this purpose in structural health monitoring. Among these, a migration technique based on ultrasonic wave propagation and reflection in structural components holds promise not only for locating the damage but also for quantitatively imaging it. Recently, an f-k migration technique showed more reliable results in estimating the location and size of the damage than the reverse-time migration technique. However, the damage diagnosis is often hampered by poor resolution due to a limited number of embedded sensors and ambient noise. In order to achieve higher resolution of damage profile, in this paper a Bayesian approach is proposed, which can enhance the damage profile obtained from the migration technique. Moreover, it allows more accurate estimation of the location and size of cracks, which is crucial for more accurate prediction of structural life from the enhanced image. The proposed Bayesian approach is applicable to any damage imaging method. The location and size of damage are progressively enhanced using likelihood estimators that are constructed based on the image. Two ways of estimating and constructing likelihood functions are discussed in detail.
Keywords: Structural health monitoring, Non-destructive evaluation, Migration technique, Bayesian approach, Likelihood function

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INTRODUCTION

Structural Health Monitoring (SHM) can be potentially designed to convert maintenance of structural components to an automated procedure. The procedure of SHM begins with damage diagnosis followed by prognosis. Diagnosis uses damage identification techniques to assess the current health status of a structure, and then prognosis uses the diagnosed damage and damage progression theory to estimate the remaining useful life (RUL) of the structure and indicate the need for maintenance and repair. Diagnosis is composed of two parts, obtaining sensor information and interpreting the results to identify the damage. Therefore, two main issues of damage diagnosis are sensor development and techniques that transform the raw data into estimates of damage profile. We focus on developing techniques to collect the sensor information and increase the accuracy of our inspection method.

Among many non-destructive evaluation (NDE) techniques, ultrasonic inspection is well established in the engineering community for several decades (Giurgiutiu and Cuc, 2005). Ultrasonic wave based approaches employ piezo sensors and actuators for inspection. Furthermore, because of reasons including setup convenience, quantifiability, and capacity for real time monitoring, we focus on improving damage profile obtained by the migration technique suggested by Yuan and his colleagues (Lin et al, Wang et al, 2005).

One advantage of using migration technique is that the damage image can be readily obtained using the sensor to collect reflected waves from damages. With the image, the size and shape of the damage as well as its location can be quantified. That is, more accurate and quantifiable damage profile using multiple actuator-sensor pairs can be obtained. However, the back propagation part of migration technique is done by finite difference scheme, and numerical or experimental noise often obscures the damage imaging results.

Because of the uncertainty associated with the sensor data, Bayesian techniques are appealing for improving the resolution of the damage detection by migration. Bayes rule was developed in 1763, but most advances in Bayesian techniques have occurred in the twentieth century due to many practical problems (Gelman et al., 2004). In Bayes' rule, two key factors are prior distribution and likelihood function. Compared to conventional statistical approaches, Bayesian approaches enjoy better performance by incorporating better prior information and by a successive updating procedure. The computational cost for Bayesian approach is rather high, but due to recent advances in computing, Bayesian technique is widely developed to handle many difficult engineering applications (An et al., Gogu et al., 2008).

The main objective of this paper is to improve the accuracy of detecting crack location by using Bayesian update, but we will also discuss how to obtain an estimate of the crack size from the updated damage profile. The paper is organized as follows. In Section 2, we briefly introduce the migration technique, and Section 3 describes the Bayesian approach and two ways to approximate the likelihood estimator. In Section 4, we provide illustrative numerical results, and Section 5 will give concluding remarks.

SIMULATION SETUP FOR MIGRATION TECHNIQUE

The migration technique uses elastic wave propagation and reflection for imaging damages in the structure. In this study, a horizontal crack centered at (-9, -11) cm with length of

4 cm is considered. A three-peaked toneburst narrowband excitation with center frequency 150 kHz (duration of 20 μs) is emitted from an actuator, after the flexural waves propagate in the plate, the reflected signals from the damage are detected by the sensors. Seven actuators are excited sequentially and 200 sensors located along the *x*-axis collect reflected waves (see Figure 1). Under this excitation frequency, only the first fundamental flexural mode propagates in the plate. In an isotropic aluminum material, the group velocity for the center frequency is 2437.7 m/s. The phase velocity is 1413.4 m/s with wavelength 9.4 mm. We use finite difference method based on Mindlin plate theory for modeling the wave propagation. An Al-6061 aluminum square plate with dimension $50cm \times 50cm \times 0.32cm$ is discretized by 200 × 200 plate elements. Simulated sensor data is collected from 2500 time steps (total time span: 250 μs). Using the simulated sensor data, the migration technique is then used for imaging the crack.

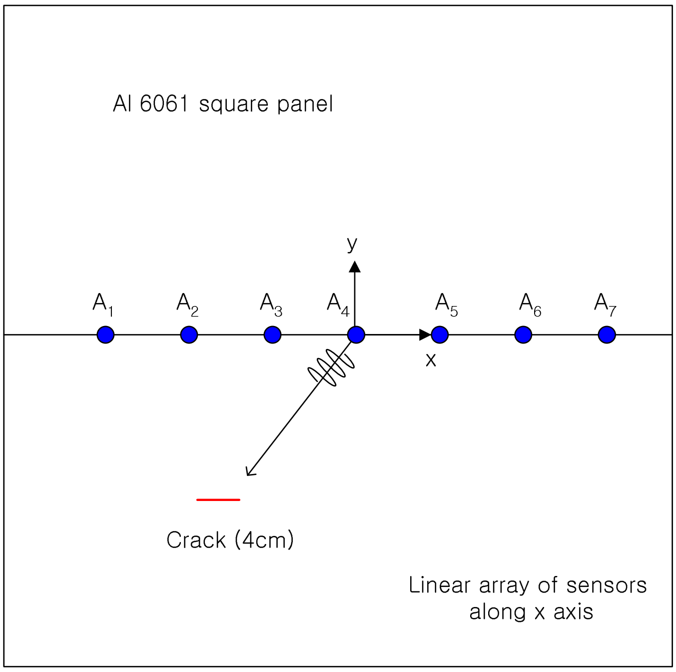


Figure1. Linear array of actuators/sensors on an aluminum plate

<u>Time domain migration</u>

A previous approach in migration technique is called reverse-time migration in time domain (Lin et al, Wang et al, 2005). This technique time-correlates the back propagated sensor data and the forward propagating waves from the actuator. Figure 2 shows the image intensity obtained from excitation from actuator A_1. The image intensity is calculated by the strength of cross-correlation at nodal location. The term “image intensity” refers to the pixel values of resultant image. It will be discussed later in this section.

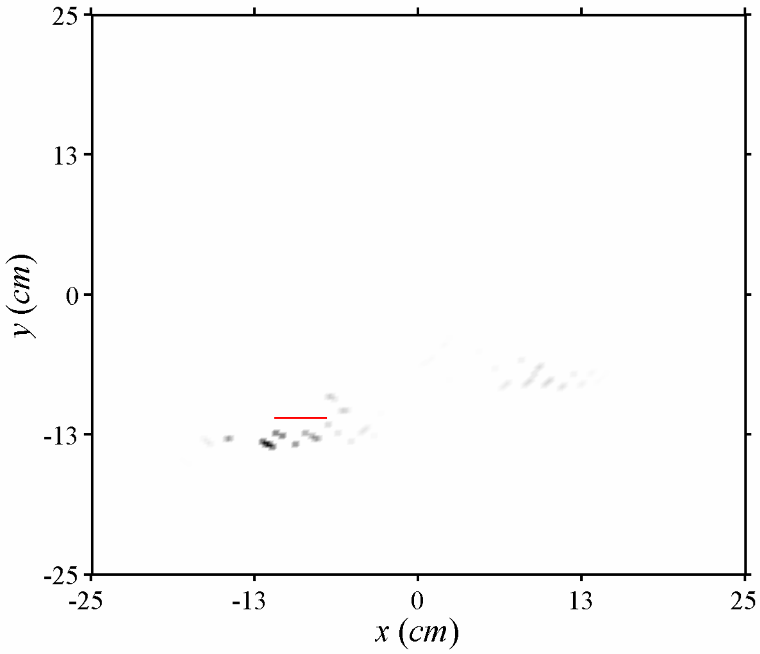


Figure 2. Damage imaging from the first actuator by time domain based migration

Frequency domain migration (F-k migration)

Frequency domain migration technique was first developed in geophysics (Stolt, Gadjag, 1978) and extended for SHM (Yuan, 2008). This technique is computationally more efficient than the reverse-time migration.

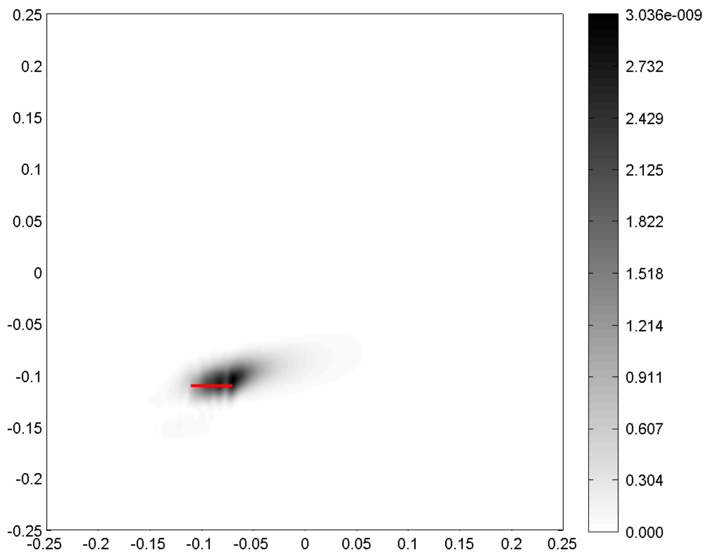


Figure 3. Damage imaging from the first actuator by f-k migration

Figure 3 shows the image intensity excited from the same actuator A_1. These two variants of migration technique basically use the same sensor information, but in different domains. However, since an f-k migration considers the entire excitation frequency band, while Time domain migration uses only the center frequency information. Clearly the f-k migration shows much higher damage fidelity. The remainder of the paper will apply the Bayesian approach with f-k migration images.

From Figures 2 and 3, the image intensity is defined by the strength of cross correlation between the actuator signal and migration. It is calculated at each node in finite difference grid.

$$I^i(x,y) = Image\ intensity\ obtained\ at\ x,y\ from\ i^{th}\ actuator\ result \tag{1}$$

The previous approach of combining the images is to superimpose the image intensities from all the actuators to get a new image of the damage (Lin et al, Wang et al, 2005). This is called prestack migration and can be represented by Eq (2). The stacked image result of all seven actuators is shown in Figure 4.

$$f(x,y) = \sum_{i=1}^{7} I^i(x,y) \tag{2}$$

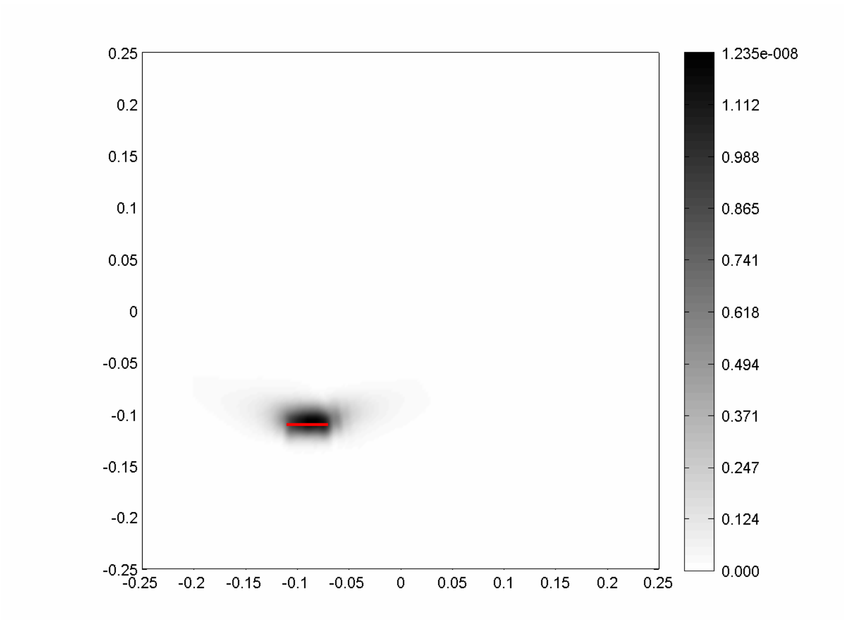


Figure 4. Stacked image results of all seven images from actuators 1~7

BAYESIAN APPROACH

The Bayesian framework provides an approach for incorporating the image intensity data with previous information. Here we limit ourselves to identifying the location of the of the crack. The general form of the Bayes' rule for finding a location of a point in the given straight crack with given image intensity is shown in Eq (3).

$$f^{upd}(crack\,location \mid I(x,y)) = \frac{f^{init}(crack\,location)\,l(I(x,y) \mid crack\,location)}{\int f^{init}(crack\,location)\,l(I(x,y) \mid crack\,location)\,d\Omega} \tag{3}$$

where $I(x,y)$ is the image intensity obtained from a test, $f^{init}(crack\,location)$ is the prior distribution of the location derived from previous knowledge, and l is the likelihood of $crack\,location$ given by current test data. In Bayesian approach, the likelihood function is defined by the relation between actual crack location and what we obtained. For this particular example, if the center of crack is *x*, *y* then the probability density that we can obtain the image intensity from the migration simulation is defined as the likelihood.

To begin with, the likelihood function or likelihood estimator, l need to be calculated. If we have knowledge about the relationship between the image intensity and the crack location, then we can accurately estimate $l(I(x,y) \mid crack\,location)$. After that, by multiplying the prior distribution and likelihood function, we calculate the probability distribution of $crack\,location$ given the test data, which is $f^{upd}(crack\,location \mid I(x,y))$, and this can be used as the prior distribution for the next step. The image intensity $I(x,y)$ is obtained by analyzing each migration results as an image.

To clarify this two dimensional problem further, we assume that the image intensity and the *x* and *y* coordinates of obtained images are independent, or the randomness of the two are not correlated. We want to find the crack location *x, y* by using given image intensity information as in Eq (3). In Eq (4), we rewrote Eq (3) using the coordinate information and image intensity from the i^{th} test.

$$f^{upd}(x,y \mid I^i) = \frac{f^{init}(x,y)\,l(I^i \mid x,y)}{\int f^{init}(x,y)\,l(I^i \mid x,y)\,dxdy} \tag{4}$$

Where *x, y* are the coordinates of a possible location of the center of the crack. That is, the left hand side of Eq (4) is the probability density that the center of the crack is at *x, y*, given the image we obtained and our prior distribution.

We start the process to determine the center of the crack location by assuming a non-informative prior or initial distribution of *x* and *y* on the plate. Here we choose a uniform distribution function as our prior distribution for plates that have never been inspected before. Assuming that the size of the square panel is $X \times Y$ and the origin is at the center of the plate, we can construct a prior distribution as Eq (5), which implies that we have no information for the plate before inspection.

$$f^{init}(x,y) = Uniform(x;-X/2,X/2) \times Uniform(y;-Y/2,Y/2) \tag{5}$$

With the given prior distribution, the next task is the selection of appropriate likelihood estimators for our next iteration. The latter part of the numerator of Eq (4), called likelihood estimator, is defined by the relation between the real center of crack location and the image. This is expressed by the probability that we have the test result when the true value lies at *x, y*,. In this case, the test result I^i indicated by the image intensity is a deterministic and known value obtained in the simulation. If the center of crack is *x*, *y* then the probability density that we can obtain the image intensity from the migration simulation is defined as the likelihood. Since we use simulation, the likelihood function depends on the relation between the simulation and the real life, but it usually requires a many simulations to obtain an explicit expression for the discrepancy between detected location and actual location. Generally, when the relation is known as some type of distribution, we can say that the likelihood is the probability density of the distribution when we have the expected value at *x, y*.

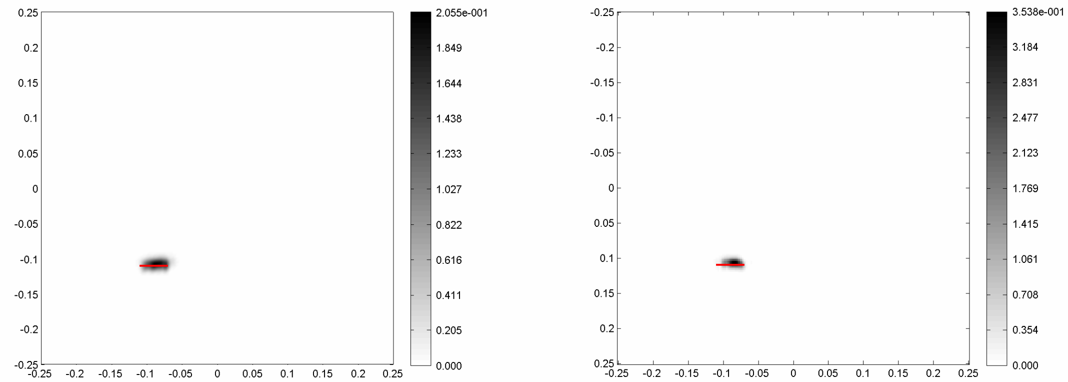
However, due to computational cost of the migration techniques, accurate definition of likelihood estimator is almost impossible. Instead, we suggest several ways to assume and estimate l through error analysis. In the following we suggest two basic ways to construct the likelihood estimators, direct image intensity and multivariate normal distribution.

<u>Direct image intensity</u>

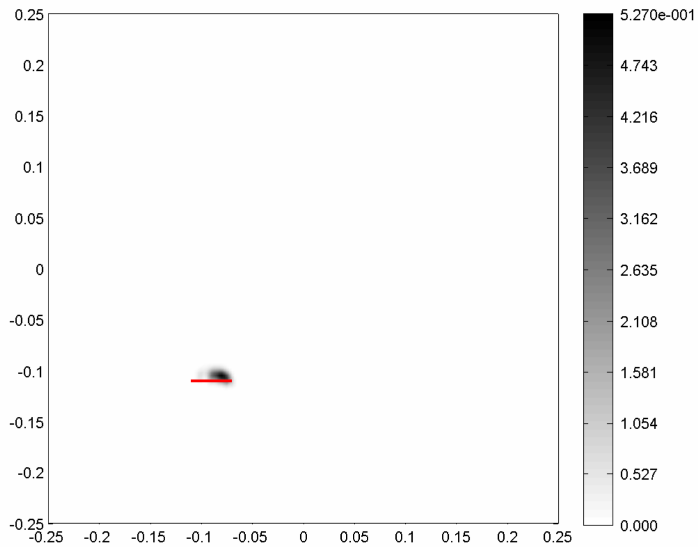
The simplest way to construct a likelihood function out of given image is directly converting the given image intensity as our likelihood function. In other words, the probability that we will get the image when the center of the crack is at point (x,y) is proportional to the image intensity at (x,y). In this case the likelihood estimator is defined by Eq (6).

$$l(I^i \mid x, y) = I^i(x, y) \tag{6}$$

Using this likelihood function with Bayesian updating with all seven images from the seven actuators, we obtained as as the final distribution shown as an image in Figure 5.



(a) (b)



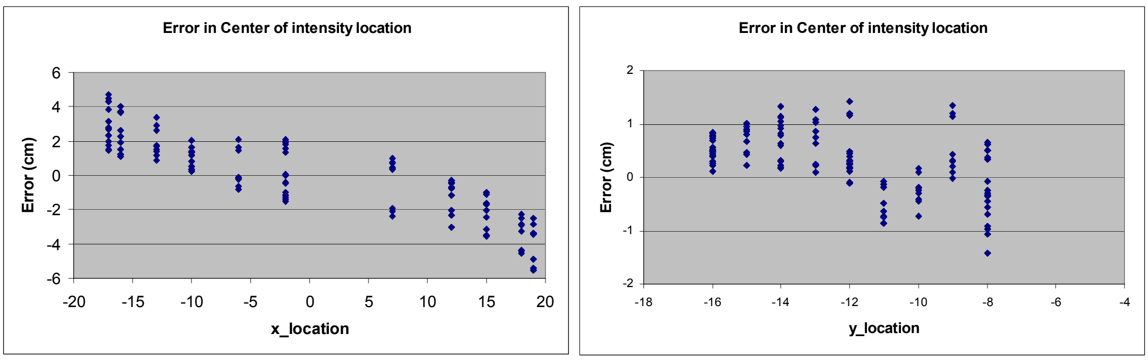
(c)

Figure 5. Combining image using direct image implementation (a) Up to 3rd actuator (b) Up to 5th actuator (c) Up to 7th actuator (final)

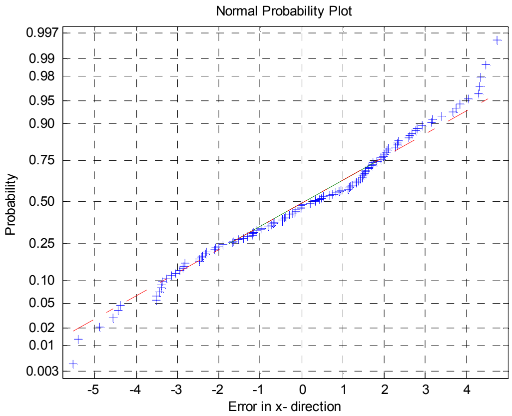
Multivariate normal model about center of intensity

The center of intensity of an image obtained from the i-th actuator is given in Eq (7).

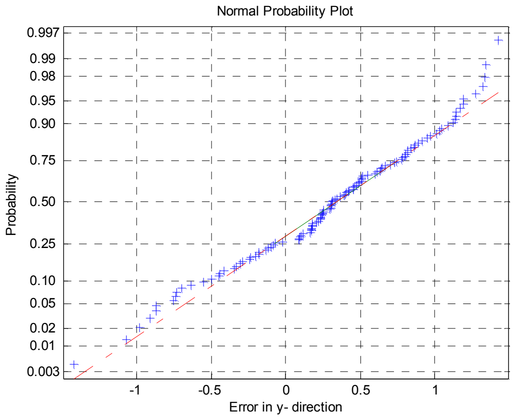
$$x_{c,i} = \frac{\iint x I^i(x,y)dxdy}{\iint I^i(x,y)dxdy}, \; y_{c,i} = \frac{\iint y I^i(x,y)dxdy}{\iint I^i(x,y)dxdy} \qquad (7)$$



(a) (b)



(c)



(d)

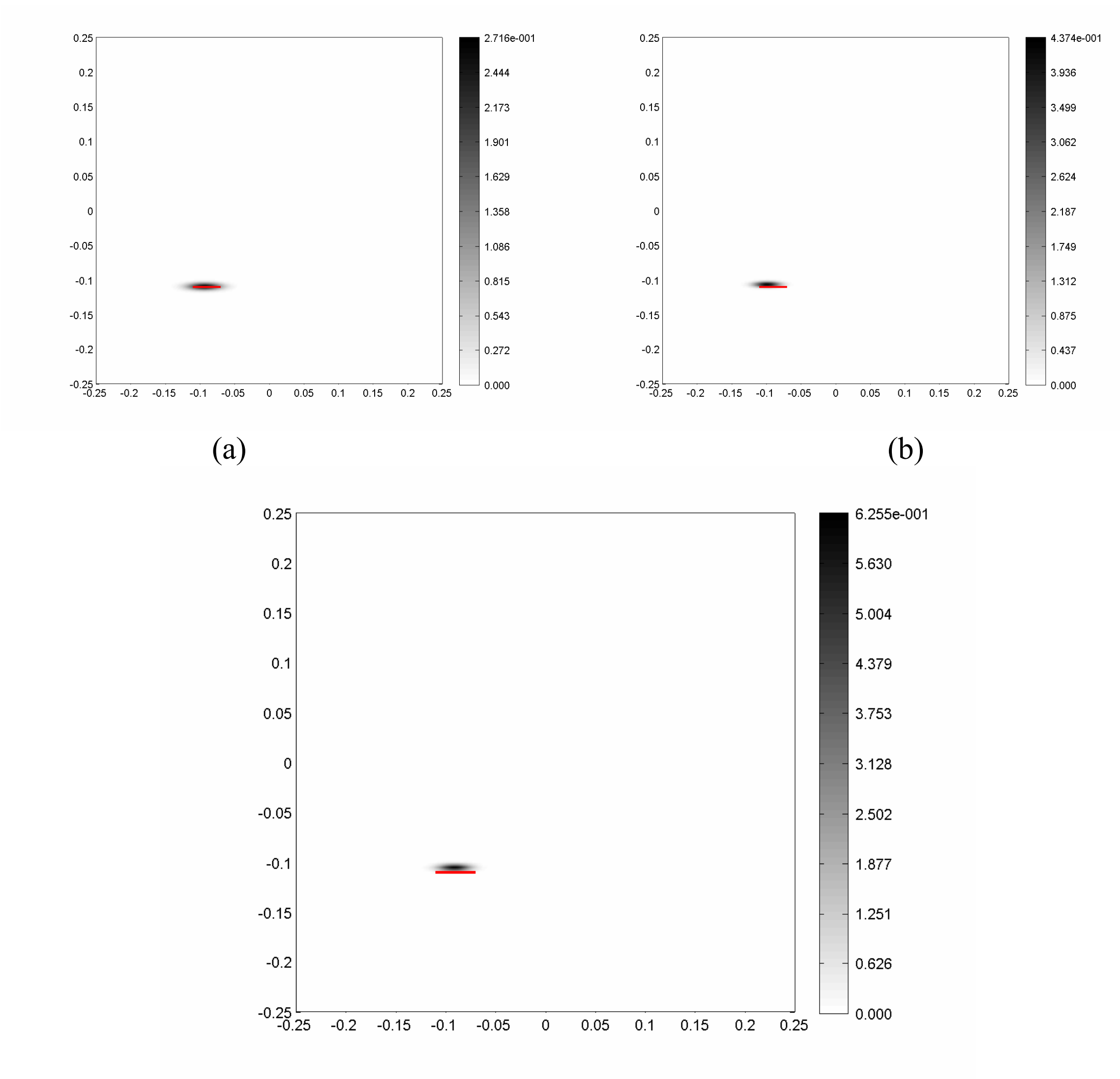
Figure 6. Estimation error in x and y directions. We tested 117 different crack configurations with different actuators and calculated the difference between actual locations and the estimated locations. (a) and (b) shows estimation error associated with location of crack center, and (c) and (d) shows normal fit of the error distribution.

Instead of using the entire image data, we elect to use only the center of intensity an estimate where the center of the crack is. This estimation has errors associated with the migration technique. By simulating a large number of cracks, we found that the error in estimated location from each actuator follows a simple trend defined by a bivariate normal distribution, and the variation due to location of crack center is observed (Figure 6). So, we defined a multivariate normal model for the likelihood and define the parameters as $\sigma_x = 2.42$, $\sigma_y = 0.60$, and linear coefficient in x direction as $c_1 = -0.162$ from Figure 6(a) and the bias in y direction as $\hat{y} = 0.307$ cm from Figure 6(d).

The bivariate distribution gives the probability of the center of intensity being at a point, given that the center of the crack is at (x,y). So the likelihood function is given by Eq (8).

$$\begin{aligned} l^i(I^i \mid x, y) &= l(x_{c,i} \; from \, I^i \mid x) \times l(y_{c,i} \; from \, I^i \mid y) \\ &= Normal(x_{c,i}; x - c_1 x, {\sigma_x}^2) \times Normal(y_{c,i}; y + \hat{y}, {\sigma_y}^2) \end{aligned} \tag{8}$$

Using Bayesan updating with this likelihood function with all seven images from the seven actuators, we obtained as the final distribution shown as an image in Figure 7.



(a) (b)

(c)

Figure 7. Combined image using multivariate normal model. (a) Up to 3rd actuator (b) Up to 5th actuator (c) Up to 7th actuator (final)

CRACK LENGTH

Most cracks are straight under simple loading conditions, and the ultrasonic wave can be reflected from any point inside the crack length with equal probability. This suggests that the crack length may be estimated from the standard deviation of the distribution of the center of the crack. If we assume that the distribution is approximately uniform, we get Eq. 9

$$estimated\ crack\ length(a) = \frac{2\sigma}{1/\sqrt{3}}(cm) \tag{9}$$

Table 1 shows estimated crack location and size.

Table 1. Estimation error due to several different techniques

Likelihood estimators	Estimated location (cm)	Error (cm)	Estimated crack length (cm)
Actual	-9.00, -11.00	-	4
Prestack migration	-8.11, -10.40	1.073	Not available
Direct image	-8.27, -10.57	0.847	3.08
Multivariate normal	-9.14, -10.55	0.471	3.86

CONCLUDING REMARKS

We have discussed how to improve the estimation of damage profile, specifically the center location and size of crack damage, by combining Bayesian approach with the migration technique. We have examined two likelihood estimators for the Bayesian updating based on image results. In applying Bayesian framework, two key procedures are: constructing a proper prior distribution and applying an appropriate likelihood function. The prior distribution before results from the first actuator is non-informative. In typical SHM situation, prior distribution based on previous inspections may further improve the resolution of the technique.

The choice of likelihood function is the main focus of the paper. As seen in the results, setting an appropriate likelihood function is important in that it forms the basis for the updating Bayesian procedure. However, practical situation is more complicated and identifying all sources of uncertainties explicitly is much more difficult. We compared the efficiency of identifying one crack by center location in the plate through two approaches for constructing likelihood estimators.

First, we applied Bayesian approach with direct implementation. Here we simplified the types of uncertainty involved in the inspection procedure as the uncertainty already included in the migration images. This procedure is very efficient in terms of enhancing the image results by suppressing the noise effect, and the resulting image is much clearer than the image produced by stacking procedure.

The second approach involves multivariate normal distribution over the region. Usually, a normal distribution is a convenient parameterization of the error. By error analysis based on many image results, we concluded that a multivariate normal distribution is suitable for this case, and by simulating multiple cracks and correlating to images we were able to refine a likelihood function to account for image bias. As our likelihood function improves, the accuracy of detection by Bayesian approach will also improve with it.

Next, we suggested a size estimation scheme based on the width of posterior distribution based on the fact that the detected location has a variation due to finite crack size within crack length. By comparing the standard deviation of a uniform distribution and the standard deviation of the posterior distribution, we were able to estimate the crack size more accurately.

The accuracy of inspection is very crucial for the accuracy of our prediction and the error or uncertainty involved in diagnosis greatly amplifies when applied to prognosis. So, the accuracy of inspection is important, and Bayesian approach ensures a higher accuracy when we have a better interpretation of relation between real life and our experiment.

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