STRUCTURAL SHAPE DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION USING MESHFREE METHOD

Workshop on Meshfree Methods

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INTRODUCTION

Meshfree Discretization

- Reproducing Kernel Particle Method Is Used
- Stable Solution for Large Shape Changing Problem
- Accurate Result for Finite Deformation Problem
- Direct Transformation/Mixed Transformation/Boundary Singular Kernel Methods for Essential B. C.

Frictional Contact Analysis

- Continuum-Based Contact Formulation
- Penalty Regularization of Variational Inequality
- Regularized Coulomb Friction Model

DSA of Frictional Contact Problem

- DSA Variational Inequality Is Approximated Using the Same Penalty Method
- Die Shape Change Is Considered by Perturbing Rigid Surface
- Path Dependent Sensitivity Results for Frictional Problem

INTRODUCTION cont.

Structural Analysis of Elastoplasticity

- Finite Deformation Elastoplasticity Using Multiplicative Decomposition of Deformation Gradient
- Return Mapping Algorithm in Principal Stress Space
- Stress Is Computed Using Hyper-Elasticity w.r.t. Stress-Free Intermediate Configuration
- Exact Linearization Is Required for Quadratic Convergence of Analysis and Accuracy of DSA

Structural Design Sensitivity Analysis (DSA)

- Material Derivative Approach Is Used for Shape DSA
- Updated Lagrangian Formulation Is Used for Elastoplasticity
- Shape Function of RKPM Depends on Shape Design
- Direct Differentiation Method Is Used to Solve Displacement Sensitivity
- DSA Equation Is Solved at Each Converged Configuration without Iteration
- Material Derivative of Intermediate Configuration Is Updated at Each Load Step Instead of Stress in Conventional Method

REPRODUCING KERNEL PARTICLE METHOD

Reproduced Displacement Function

$$z^R(x) = \int_\Omega C(x; y-x)\,\phi_a(y-x)\,z(y)\,dy$$

$$\begin{cases} \phi_a(y-x) > 0 & \text{if } |y-x| < a \\ \phi_a(y-x) = 0 & \text{otherwise} \end{cases}$$

$z^R(x) \to z(x)$ as $a \to 0$ Dirac Delta Measure

Correction Function

$$C(x; y-x) = \mathbf{q}(x)^T \mathbf{H}(y-x)$$

$$\mathbf{H}(y-x)^T = [1, (y-x), (y-x)^2, \cdots, (y-x)^n]$$

$$\mathbf{q}(x)^T = [q_0(x), q_1(x), \cdots, q_n(x)]$$

n-th Order Completeness Requirement (Reproducing Condition)

$$z^R(x) = \int_\Omega C(x; y-x)\,\phi_a(y-x)\,z(y)\,dy$$

$$= \overline{m}_0(x) z(x) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \overline{m}_n(x) \frac{d^n z(x)}{dx^n}$$

$\overline{m}_0(x) = 1 \qquad \overline{m}_k(x) = 0 \qquad k = 1, \ldots, n$

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RKPM cont.

Reproducing Condition

$$\mathbf{M}(x)\,\mathbf{q}(x) = \mathbf{H}(0) \qquad \mathbf{H}(0)^T = [1, 0, \ldots, 0]$$

$$\mathbf{M}(x) = \begin{bmatrix} m_0(x) & m_1(x) & \ldots & m_n(x) \\ m_1(x) & m_2(x) & \ldots & m_{n+1}(x) \\ . & . & \ldots & . \\ m_n(x) & m_{n+1}(x) & \ldots & m_{2n}(x) \end{bmatrix}$$

$$C(x; y-x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \mathbf{H}(y-x)$$

$$z^R(x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \int_\Omega \mathbf{H}(y-x)\phi_a(y-x)z(y)\,dy$$

$$z^R(x) = \sum_{I=1}^{NP} C(x; x_I - x)\phi_a(x_I - x)z_I \Delta x_I = \sum_{I=1}^{NP} \Phi_I(x) d_I$$

RKPM cont.

- Shape Function $\Phi_I(x_I)$ Depends on Current Coordinate Whereas FEA Shape Functions Depend on Coordinate of the Reference Geometry
- Does Not Satisfy Kronecker Delta Property: $\Phi_I(x_J) \neq \delta_{IJ}$
- Lagrange Multiplier Method for Essential B.C.

$$\Pi = U - \int_{\Gamma_D} \lambda^T (z - \zeta)\, d\Gamma$$

  – First-order variation is

$$\bar{\Pi} = \bar{U} - \int_{\Gamma_D} \lambda^T \bar{z}\, d\Gamma - \int_{\Gamma_D} \bar{\lambda}^T (z - \zeta)\, d\Gamma$$

- Direct Transformation Method, Mixed Transformation Method, and Singular Kernel Methods Are Available.

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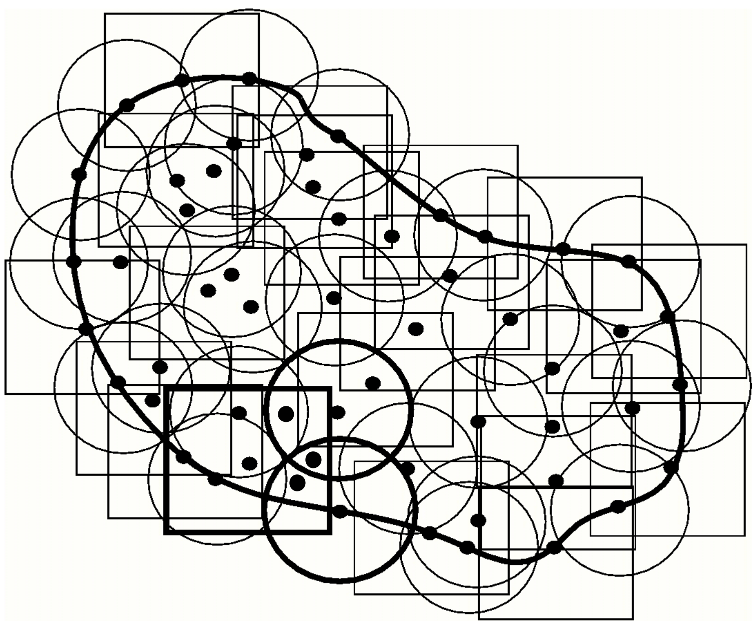
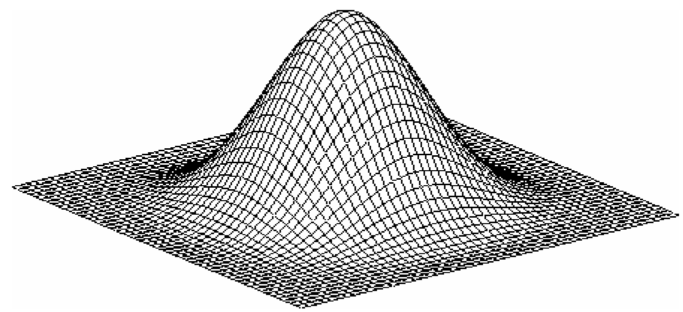
DOMAIN DISCRETIZATION

Meshfree Discretization





Meshfree Shape Function

MESHFREE METHOD

Advantages

Construction of Shape/Interpolation Function in Global Level

Mesh Independent Solution Accuracy Control

Versatile hp-Adaptivity

A Remedy to Mesh Distortion in Shape Optimization

Accurate Solution to Large Deformation Problem

Disadvantages

Difficulties in Imposing Essential Boundary Conditions

Expensive Computational Cost

Larger Bandwidth of Stiffness Matrix Than FEM

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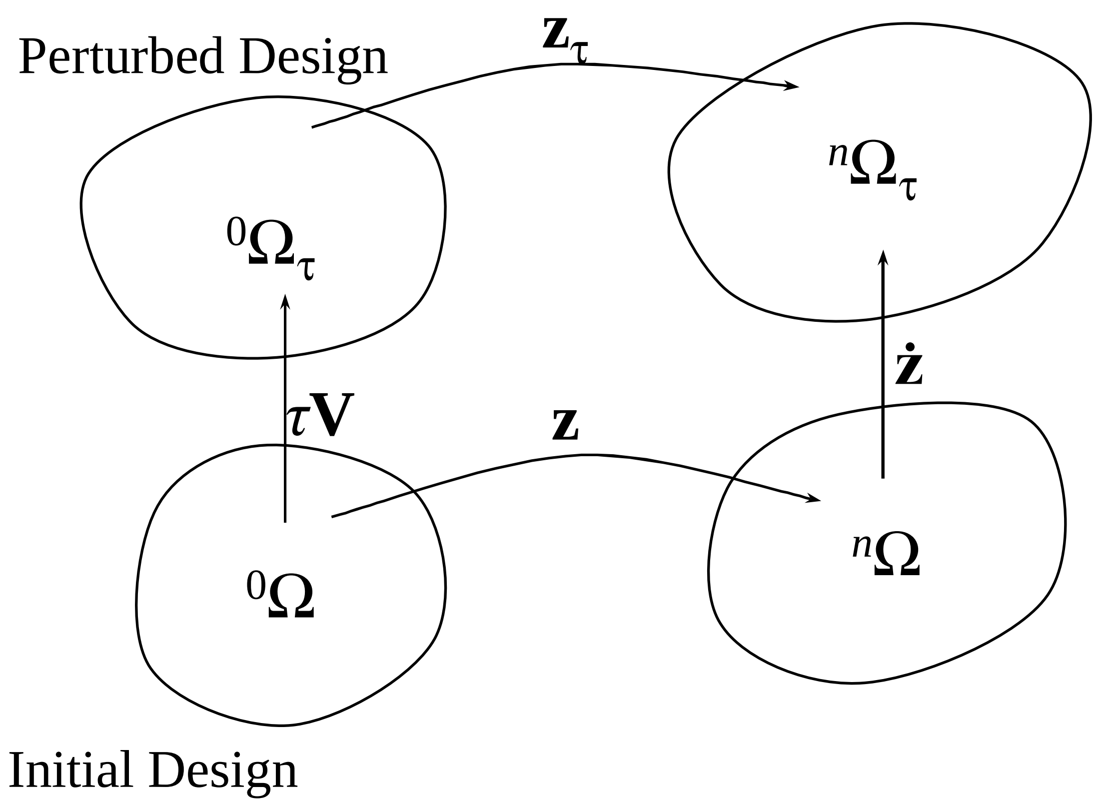
SHAPE DESIGN SENSITIVITY ANALYSIS



$$\textit{mapping } \mathbf{T} : \mathbf{x} \rightarrow \mathbf{x}_\tau(\mathbf{x}), \mathbf{x} \in \Omega$$

$$\mathbf{T}(\mathbf{x}, \tau) = \mathbf{x} + \tau\mathbf{V}(\mathbf{x}) + H.O.T.$$

$$\dot{\mathbf{z}} = \frac{d}{d\tau}\mathbf{z}_\tau(\mathbf{x} + \tau\mathbf{V})$$

$$= \lim_{\tau \to 0}\left[\frac{\mathbf{z}_\tau(\mathbf{x} + \tau\mathbf{V}) - \mathbf{z}(\mathbf{x})}{\tau}\right]$$

$\mathbf{V}$: Design velocity vector, direction of design perturbation

τ : Design perturbation parameter

$\mathbf{z}$: Displacement vector

$\dot{\mathbf{z}}$: Material Derivative of displacement

NONLINEAR STRUCTURAL ANALYSIS

- Nonlinear Variational Equation
  (Updated Lagrangian Formulation)

$$a_\Omega({}^n\mathbf{z},\bar{\mathbf{z}}) + b_\Gamma({}^n\mathbf{z},\bar{\mathbf{z}}) = \ell_\Omega(\bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z$$

$$a_\Omega(\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega \boldsymbol{\tau} : \bar{\boldsymbol{\varepsilon}}\, d\Omega$$ Structural Energy Form

$$b_\Gamma(\mathbf{z},\bar{\mathbf{z}})$$ Contact Variational Form

$$\ell_\Omega(\bar{\mathbf{z}}) = \int_\Omega \bar{\mathbf{z}}^T \mathbf{f}^B\, d\Omega + \int_{\Gamma^S} \bar{\mathbf{z}}^T \mathbf{f}^S\, d\Gamma$$ Load Linear Form

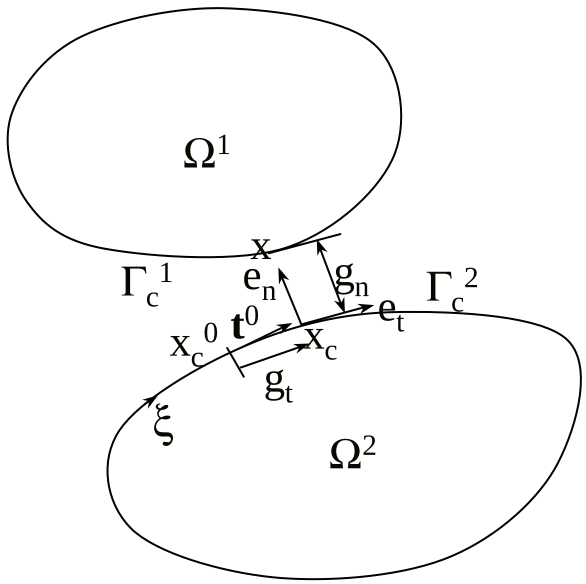
- Linearization

$$a^*_\Omega({}^n\mathbf{z}^k;\Delta\mathbf{z}^{k+1},\bar{\mathbf{z}}) + b^*_\Gamma({}^n\mathbf{z}^k;\Delta\mathbf{z}^{k+1},\bar{\mathbf{z}})$$
$$= \ell_\Omega(\bar{\mathbf{z}}) - a_\Omega({}^n\mathbf{z}^k,\bar{\mathbf{z}}) - b_\Gamma({}^n\mathbf{z}^k,\bar{\mathbf{z}}), \qquad \forall \bar{\mathbf{z}} \in Z$$

$$a^*_\Omega(\mathbf{z};\Delta\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega [\bar{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}(\Delta\mathbf{z}) + \boldsymbol{\tau} : \boldsymbol{\eta}(\Delta\mathbf{z},\bar{\mathbf{z}})]\, d\Omega$$

$$\mathbf{c} = \frac{\partial\boldsymbol{\tau}}{\partial\boldsymbol{\varepsilon}} = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^{\mathrm{alg}} \mathbf{m}^i \otimes \mathbf{m}^j + 2\sum_{i=1}^{3} \tau_i^p \hat{\mathbf{c}}^i$$

FRICTIONAL CONTACT PROBLEM



Impenetration Condition

$$g_n \equiv (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_n(\xi_c) \geq 0, \;\; \mathbf{x} \in \Gamma_c^1, \mathbf{x}_c \in \Gamma_c^2$$

Tangential Slip Function

$$g_t \equiv \left\| \mathbf{t}^0 \right\| (\xi_c - \xi_c^0)$$

Contact Consistency Condition

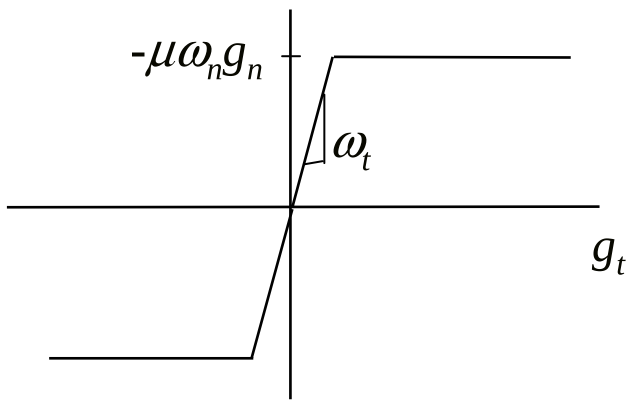
$$\varphi(\xi_c) = (\mathbf{x} - \mathbf{x}_c(\xi_c))^T \mathbf{e}_t(\xi_c) = 0$$

Contact Penalty Function

$$P = \frac{1}{2}\omega_n \int_{\Gamma_C} g_n{}^2 \, d\Gamma + \frac{1}{2}\omega_t \int_{\Gamma_C} g_t{}^2 \, d\Gamma$$

Contact Variational Form

$$b_\Gamma(\mathbf{z}, \overline{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n \overline{g}_n \, d\Gamma + \omega_t \int_{\Gamma_C} g_t \overline{g}_t \, d\Gamma S$$



Modified Coulomb Friction Model

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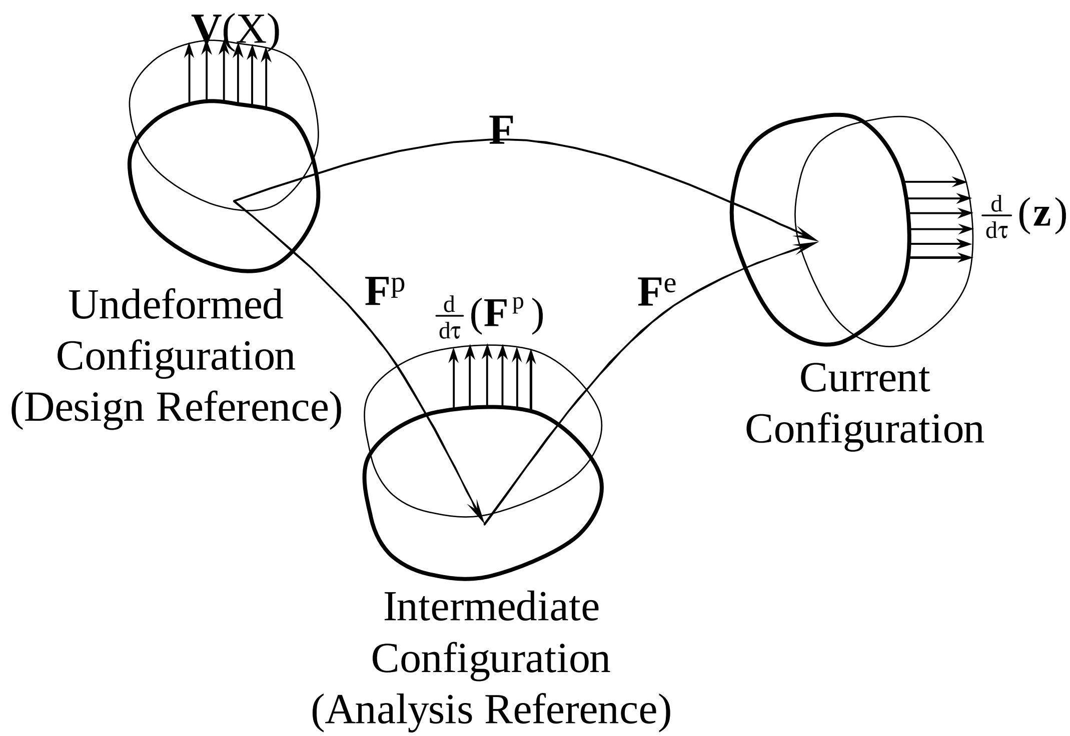
DESIGN SENSITIVITY ANALYSIS



- Updated Lagrangian Formulation
- Finite Deformation Elastoplasticity
- No Need to Update Velocity Fields
- Updating Sensitivity Information of Intermediate Configuration and Plastic Variables

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FINITE DEFORMATION DSA

Material Derivative of Variational Equation

$$\frac{d}{d\tau}\left[a_{\Omega_\tau}({}^{n}\mathbf{z}_\tau, \bar{\mathbf{z}}_\tau)\right]_{\tau=0} + \frac{d}{d\tau}\left[b_{\Gamma_\tau}({}^{n}\mathbf{z}_\tau, \bar{\mathbf{z}}_\tau)\right]_{\tau=0} = \frac{d}{d\tau}\left[\ell_{\Omega_\tau}(\bar{\mathbf{z}}_\tau)\right]_{\tau=0}, \quad \forall \bar{\mathbf{z}}_\tau \in Z_\tau$$

Design Sensitivity Equation

$$a^{*}_{\Omega}({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}}) + b^{*}_{\Gamma}({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}, \bar{\mathbf{z}}) = \ell'_{V}(\bar{\mathbf{z}}) - a'_{V}({}^{n}\mathbf{z}, \bar{\mathbf{z}}) - b'_{V}({}^{n}\mathbf{z}, \bar{\mathbf{z}})$$

Remarks:

- The same tangent operator is used as analysis -- Need accurate computation of tangent operator
- Direct Differentiation -- DSA needs to be carried out at each converged load step
- Update sensitivity information: intermediate configuration (analysis reference), plastic internal variables, and frictional effect
- Total form of sensitivity equation
- No iteration is required to solve the sensitivity equation

FINITE DEFORMATION DSA cont.

Fictitious load

$$a'_V(\mathbf{z},\overline{\mathbf{z}}) = \int_\Omega \left(\overline{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}_V(\mathbf{z}) + \overline{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}_P(\mathbf{z}) + \boldsymbol{\tau}^{fic} : \overline{\boldsymbol{\varepsilon}}\right) d\Omega$$
$$+ \int_\Omega \left(\boldsymbol{\tau} : \boldsymbol{\eta}_V(\mathbf{z},\overline{\mathbf{z}}) + \boldsymbol{\tau} : \boldsymbol{\eta}_P(\mathbf{z},\overline{\mathbf{z}}) + \boldsymbol{\tau} : \overline{\boldsymbol{\varepsilon}} div\mathbf{V}\right) d\Omega$$

$$\boldsymbol{\varepsilon}_V(\mathbf{z}) = -sym(\nabla_0 \mathbf{z} \nabla_n \mathbf{V})$$
$$\boldsymbol{\eta}_V(\mathbf{z},\overline{\mathbf{z}}) = -sym\left(\nabla_n \overline{\mathbf{z}}^T \nabla_0 \mathbf{z} \nabla_n \mathbf{V}\right) - sym(\nabla_0 \overline{\mathbf{z}} \nabla_n \mathbf{V})$$
$$\boldsymbol{\varepsilon}_P(\mathbf{z}) = -sym(\mathbf{G})$$
$$\boldsymbol{\eta}_P(\mathbf{z},\overline{\mathbf{z}}) = -sym(\nabla_n \overline{\mathbf{z}}^T \mathbf{G})$$

$$\boldsymbol{\tau}^{fic} = \sum_{i=1}^{3} \left[\frac{\partial \tau_i^p}{\partial \boldsymbol{\alpha}} \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \frac{\partial \tau_i^p}{\partial \hat{e}^p} \tfrac{d}{d\tau}(e_n^p) \right] \mathbf{m}^i$$

$$\mathbf{G} = \mathbf{F}^e \tfrac{d}{d\tau}(\mathbf{F}^p) \mathbf{F}^{-1}$$

Updating path-dependent terms

$$\tfrac{d}{d\tau}(\boldsymbol{\alpha}_{n+1}) = \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \left(H_\alpha + \sqrt{\tfrac{2}{3}} H'_\alpha \gamma\right) \tfrac{d}{d\tau}(\gamma) \mathbf{N} + H_\alpha \gamma \tfrac{d}{d\tau}(\mathbf{N})$$
$$\tfrac{d}{d\tau}(e_{n+1}^p) = \tfrac{d}{d\tau}(e_n^p) + \sqrt{\tfrac{2}{3}} \tfrac{d}{d\tau}(\gamma)$$
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^p) = \tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\ -1}) \mathbf{F}_{n+1} + \mathbf{F}_{n+1}^{e\ -1} \tfrac{d}{d\tau}(\mathbf{F}_{n+1})$$
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^e) = \tfrac{d}{d\tau}(\mathbf{f}^p) \mathbf{F}_{n+1}^{e\ tr} + \mathbf{f}^p \tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\ tr})$$
$$\mathbf{f}^p = \sum_{j=1}^{3} \exp(-\gamma N_j) \mathbf{m}^j$$

Incremental Plastic Deformation Gradient

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CONTACT DSA

- **Material Derivative of Contact Variational Form**

$$\frac{d}{d\tau}\Big[b_{\Gamma_\tau}(\mathbf{z}_\tau,\bar{\mathbf{z}}_\tau)\Big] = b_\Gamma^*(\mathbf{z};\dot{\mathbf{z}},\bar{\mathbf{z}}) + b'_V(\mathbf{z},\bar{\mathbf{z}})$$

$$b'_V(\mathbf{z},\bar{\mathbf{z}}) = b'_N(\mathbf{z},\bar{\mathbf{z}}) + b'_T(\mathbf{z},\bar{\mathbf{z}})$$

- **Normal Contact Fictitious Load Form for DSA**

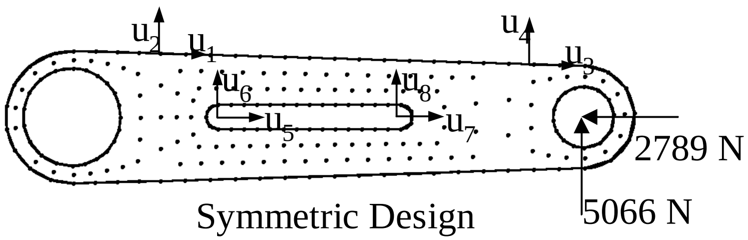
$$\begin{aligned} b'_N(\mathbf{z},\bar{\mathbf{z}}) = {} & \omega_n \int_{\Gamma_C} (\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T \mathbf{e}_n \mathbf{e}_n{}^T (\mathbf{V}-\mathbf{V}_c)\, d\Gamma - \omega_n \int_{\Gamma_C} \big(\alpha g_n / c\big)(\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T \mathbf{e}_t \mathbf{e}_t{}^T (\mathbf{V}-\mathbf{V}_c)\, d\Gamma \\ & -\omega_n \int_{\Gamma_C} \big(g_n \|\mathbf{t}\| / c\big)(\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T \mathbf{e}_t \mathbf{e}_n{}^T \mathbf{V}_{c,\xi}\, d\Gamma - \omega_n \int_{\Gamma_C} \big(g_n \|\mathbf{t}\| / c\big) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \mathbf{e}_t{}^T (\mathbf{V}-\mathbf{V}_c)\, d\Gamma \\ & -\omega_n \int_{\Gamma_C} \big(g_n{}^2 / c\big) \bar{\mathbf{z}}_{c,\xi}{}^T \mathbf{e}_n \mathbf{e}_n{}^T \mathbf{V}_{c,\xi}\, d\Gamma + \omega_n \int_{\Gamma_C} \kappa g_n (\bar{\mathbf{z}}-\bar{\mathbf{z}}_c)^T \mathbf{e}_n (\mathbf{V}^T \mathbf{n})\, d\Gamma \end{aligned}$$

CONTACT DSA cont.

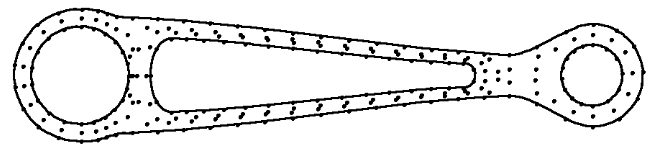
- **Tangential Stick Fictitious Load Form for DSA**

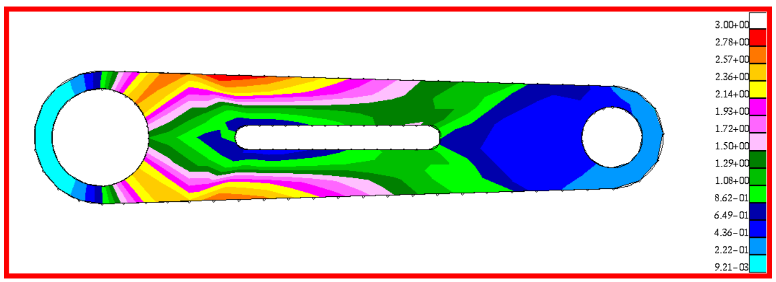
$$
\begin{aligned}
& b_T'({}^n\mathbf{z},\overline{\mathbf{z}}) = b_T^*({}^n\mathbf{z};\mathbf{V},\overline{\mathbf{z}}) \\
& +\omega_t \int_{\Gamma_C} {}^n\left(2g_t\left\|\mathbf{t}\right\|/c\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left({}^n\nu(2\,{}^{n-1}\beta\,{}^n g_t-\left\|{}^{n-1}\mathbf{t}\right\|^2\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V}+{}^{n-1}\dot{\mathbf{z}}-\mathbf{V}_c-{}^{n-1}\dot{\mathbf{z}}_c)\,d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta\,{}^n g_n\,{}^n g_t\left(\left\|{}^{n-1}\mathbf{t}\right\|+\left\|{}^n\mathbf{t}\right\|\right)\Big/{}^n c\,{}^{n-1}c\right](\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
& -\omega_t \int_{\Gamma_C} \left[{}^n g_n\left\|{}^n\mathbf{t}\right\|\left\|{}^{n-1}\mathbf{t}\right\|^2\Big/{}^n c\,{}^{n-1}c\right](\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{t-\Delta t}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\quad d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^n g_n\left\|{}^{n-1}\mathbf{t}\right\|\left(2\,{}^{n-1}\beta\,{}^n g_t-\left\|{}^{n-1}\mathbf{t}\right\|^2\right)\Big/{}^n c\,{}^{n-1}c\right]\overline{\mathbf{z}}_{c,\xi}^{\,T}\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V}+{}^{n-1}\dot{\mathbf{z}}-\mathbf{V}_c-{}^{n-1}\dot{\mathbf{z}}_c)\,d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^n g_n\,{}^{n-1}g_n\left(2\,{}^{n-1}\beta\,{}^n g_t-\left\|{}^{n-1}\mathbf{t}\right\|^2\right)\Big/{}^n c\,{}^{n-1}c\right]\overline{\mathbf{z}}_{c,\xi}^{\,T}\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi}+{}^{n-1}\dot{\mathbf{z}}_{c,\xi})\,d\Gamma \\
& +\omega_t \int_{\Gamma_C} \kappa\left\{{}^n\nu\,{}^n g_t(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t+\left({}^n g_n\,{}^n g_t\left\|{}^{n-1}\mathbf{t}\right\|/c\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\right\}(\mathbf{V}^T\mathbf{n})\,d\Gamma
\end{aligned}
$$

MESHFREE METHOD WITH LARGE SHAPE CHANGING PROBLEM

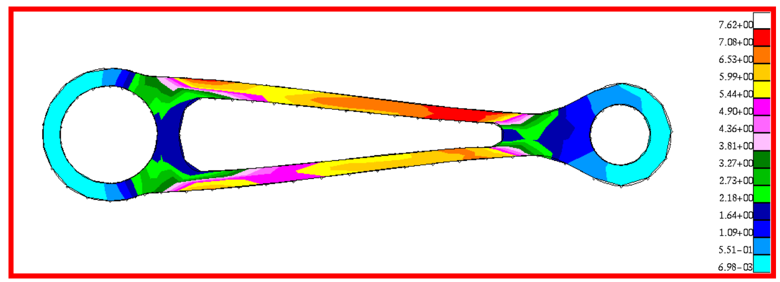


Symmetric Design





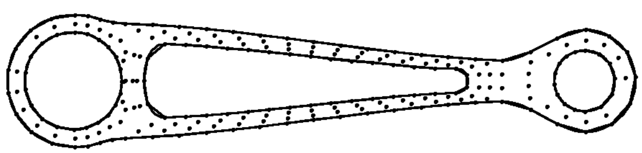
Initial Design

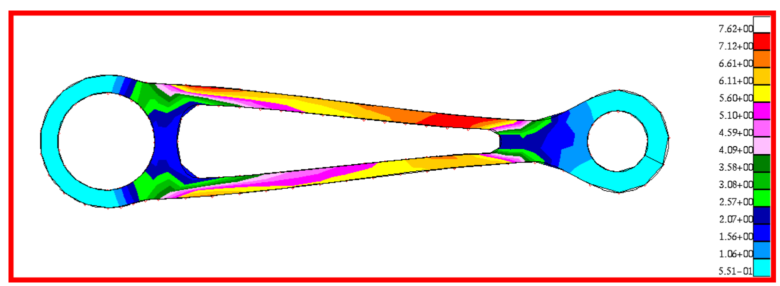


Optimum Design

FEM with remodeling : 45 iterations

Meshfree w/o remodeling : 20 iterations





Analysis Result after Remeshing at Optimum Design

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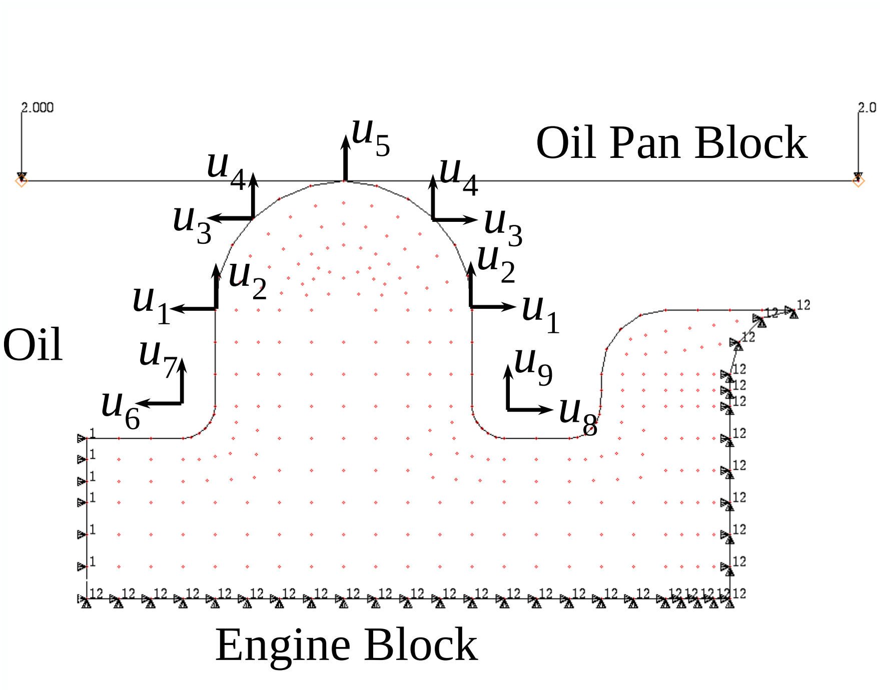
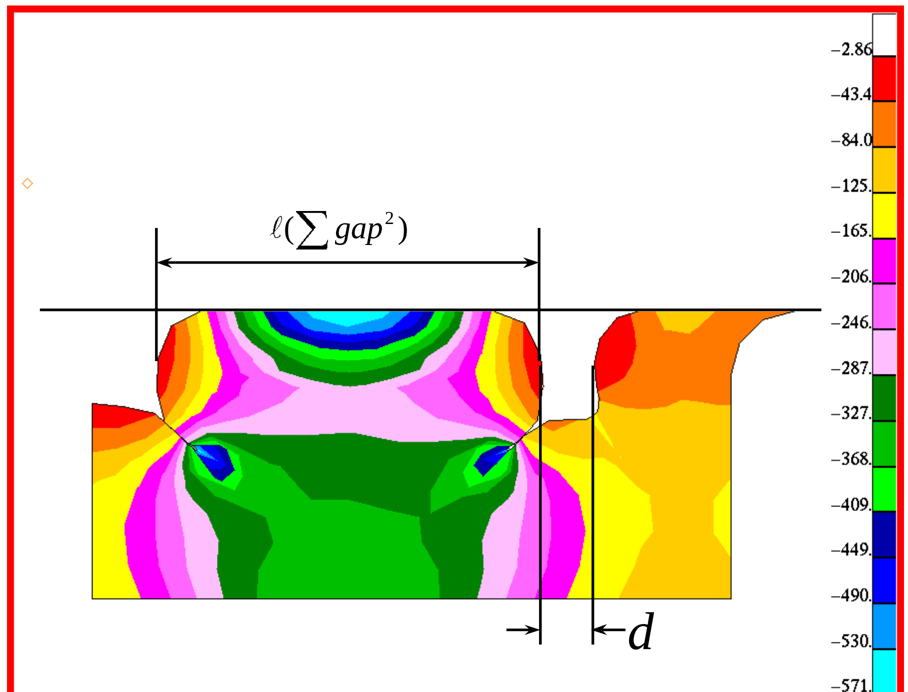
GASKET SHAPE DESIGN

- Oil Pan Gasket to Reduce Leakage
- Mooney-Rivlin Rubber Material
- Flexible-Rigid Body Contact and Self-Contact Conditions
- Significant Distortion in Self-Contact Regions





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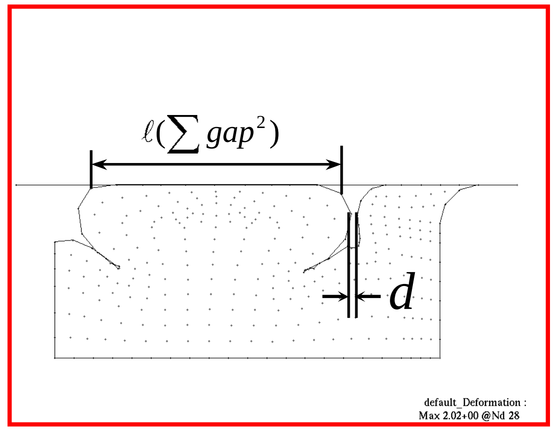
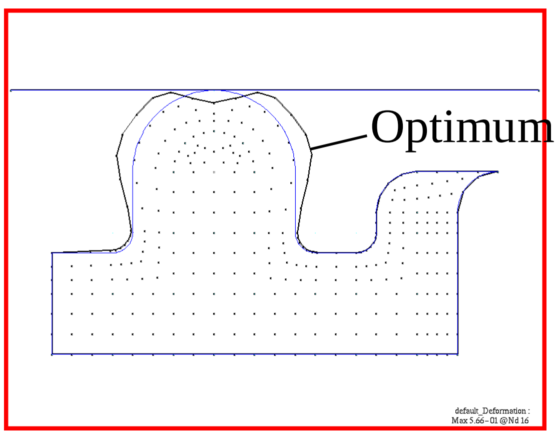
DESIGN OPTIMIZATION

Optimization Problem 1

$$
\begin{aligned}
\min \quad & d \\
s.t. \quad & |F_C| \geq 300\ kN \\
& \sigma \leq 1700\ kPa \\
& \sum gap^2 \leq 1.0\ mm \\
& -0.5 \leq u_i \leq 0.5
\end{aligned}
$$

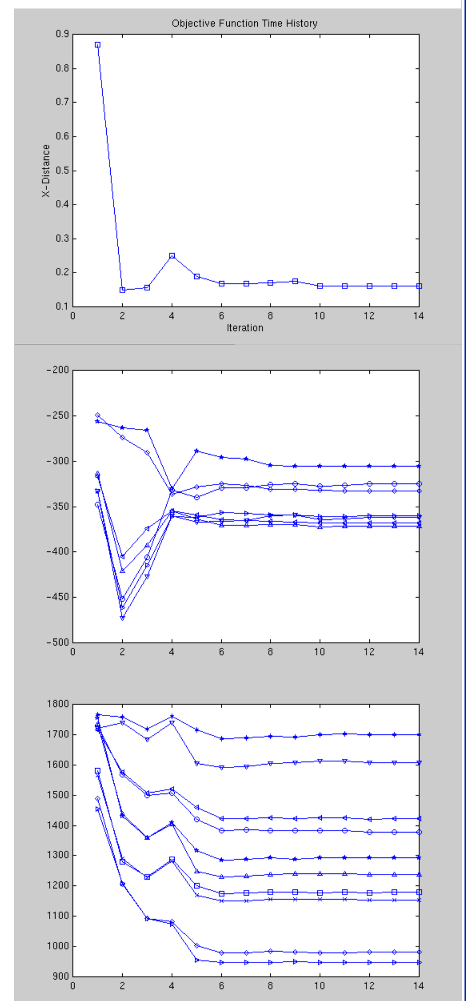




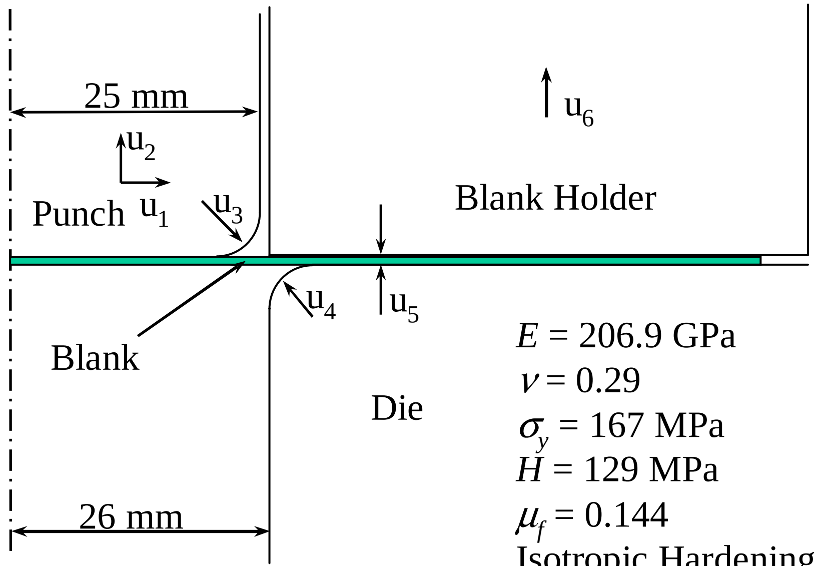
Optimum Design

Pressure Distribution

Optimization History

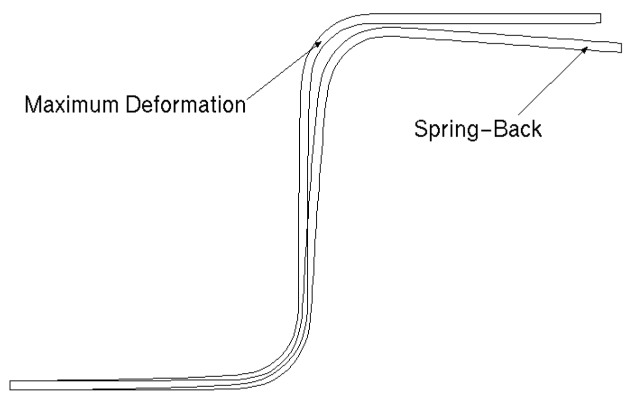


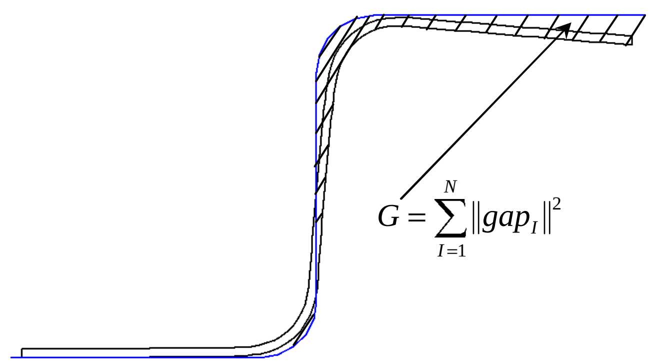
DESIGN OPTIMIZATION OF DEEPDRAWING



E = 206.9 GPa
ν = 0.29
σ_y = 167 MPa
H = 129 MPa
μ_f = 0.144
Isotropic Hardening

Performance(Ψ)		ΔΨ	Ψ'Δτ	RATIO(%)
u_1				
e^p_{31}	.189554+1	-.147351-6	-.147312-6	100.03
e^p_{32}	.160840+1	-.458731-6	-.458733-6	100.00
e^p_{33}	.113010+1	-.268919-6	-.268949-6	99.99
e^p_{34}	.812794+0	-.217743-6	-.217764-6	99.99
e^p_{35}	.568991+0	-.144977-6	-.144996-6	99.99
e^p_{36}	.383581+0	-.802032-7	-.802123-7	99.99
G	.510092+2	-.159478-4	-.159503-4	99.98
u_2				
e^p_{31}	.189554+1	-.727109-7	-.726800-7	100.04
e^p_{32}	.160840+1	.764579-7	.764732-7	99.98
e^p_{33}	.113010+1	.118855-6	.118849-6	100.01
e^p_{34}	.812794+0	.829857-7	.829822-7	100.00
e^p_{35}	.568991+0	.700201-7	.700157-7	100.01
e^p_{36}	.383581+0	.345176-7	.345177-7	100.00
G	.510092+2	.494067-4	.494016-4	100.01





$$G = \sum_{I=1}^{N} \|gap_I\|^2$$

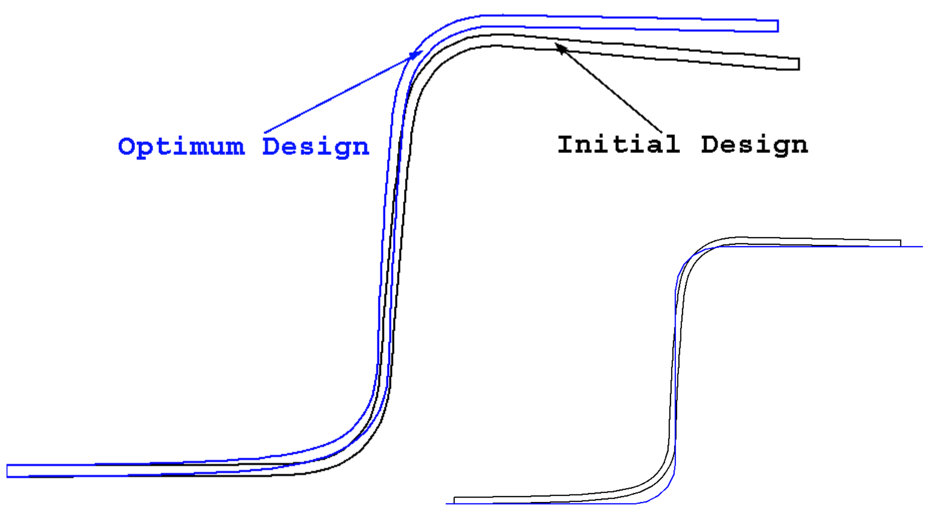
DESIGN OPTIMIZATION OF DEEPDRAWING (CONT.)

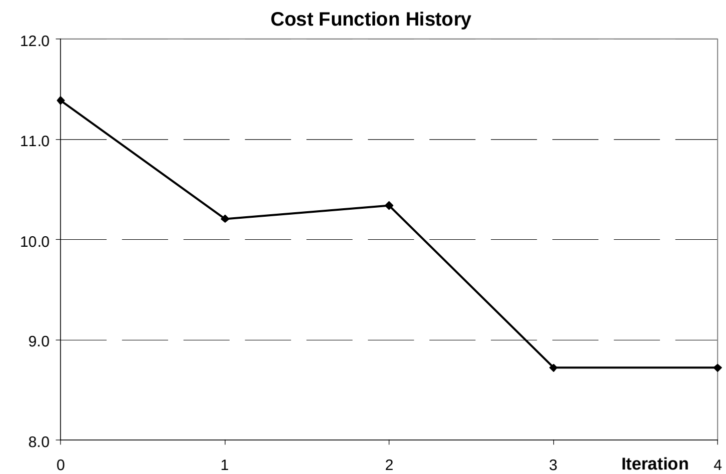
Optimization Problem

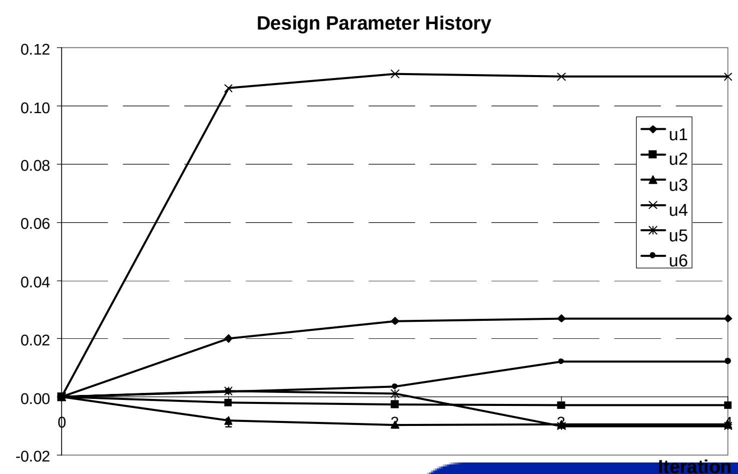
$$\min \quad \sum gap^2$$

$$\text{s.t.} \quad \bar{e}_p \leq 0.2$$

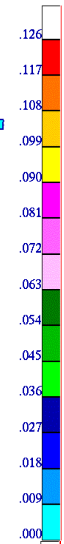
DOT : Sequential Quadratic Programming



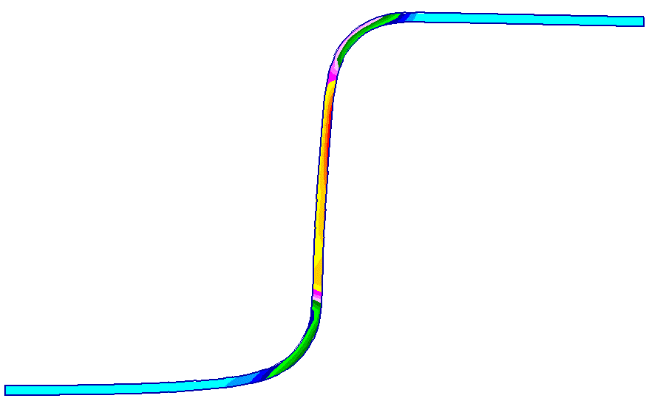


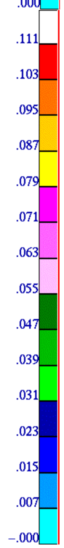


DESIGN OPTIMIZATION OF DEEPDRAWING (CONT.)

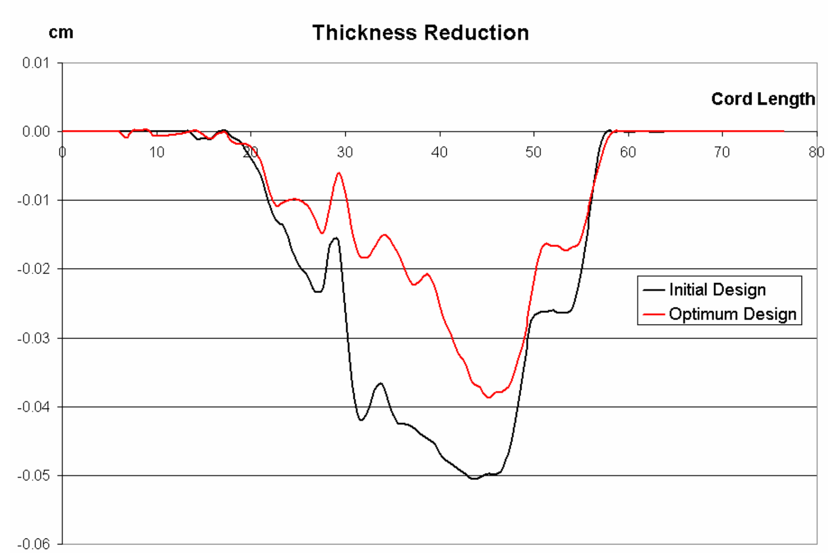


Plastic Strain (Initial Design)





Plastic Strain (Optimum Design)



SUMMARY

- Meshfree Method Is Effective in Shape Optimization
- Developed Accurate and Efficient Shape DSA Method for Finite Deformation Elastoplasticity Using Multiplicative Decomposition of Deformation Gradient
- Die Shape Design Sensitivity Formulation Is Developed for the Frictional Contact Problem
- Deepdrawing Optimization Is Successfully Carried out to Reduce the Springback Amount
- Very Accurate DSA Results Made Optimization Problem Converged in a Small Number of Iterations