Statistical Characterization of Damage Propagation Properties in Structural Health Monitoring

Alexandra Coppe[1], Raphael T. Haftka[2] and Nam-Ho Kim[3]
University of Florida, Gainesville, FL, 32611

and

F. G. Yuan[4]
North Caroline State University, Raleigh, NC, 27695

Structural health monitoring may provide readings that follow fatigue induced damage growth in service. This information may in turn be used to improve the characterization of the material properties that govern damage propagation for the monitored structure. These properties are often widely distributed between nominally identical structures because of differences in manufacturing and aging. The improved accuracy in damage growth characteristics allows more accurate prediction of the remaining useful life (RUL) of the structural component. In this paper, a probabilistic approach using Bayesian statistics is employed to progressively narrow the uncertainty in damage growth parameters in spite of variability and error in sensor measurements. Starting from an initial, wide distribution of damage parameters that are obtained from coupon tests, the distribution is progressively narrowed using the damage growth between consecutive measurements. Detailed discussions on how to construct the likelihood function under given variability of sensor data and how to update the distribution are presented. The approach is applied to crack growth in fuselage panels due to cycles of pressurization and depressurization. It is shown that the proposed method rapidly converges to the accurate damage parameters when the initial damage size is 20mm and the variability in sensor data is 1mm. It is observed that the distribution narrows down rapidly when the damage grows fast, while slows down when the damage grows slowly. This property works in favor because more accurate information will be obtained when the damage is dangerous. Using the identified damage parameters, the RUF is predicted with 95% confidence in order to obtain conservative prediction. The proposed approach may have the potential of turning aircraft into flying fatigue laboratories.

Nomenclature

ΔK	=	Range of stress intensity factor
σ	=	Applied stress
a	=	Crack size
a'	=	Measured crack size
a_0	=	Initial crack size
a_C	=	Critical crack size
a_N	=	Crack size at Nth inspection

---

[1] Graduate research assistant, Mechanical and Aerospace Engineering department, University of Florida, Gainesville, FL, 32611, AIAA Student Member.
[2] Distinguished Professor, Mechanical and Aerospace Engineering department, University of Florida, Gainesville, FL, 32611, AIAA Fellow.
[3] Associate Professor, Mechanical and Aerospace Engineering department, University of Florida, Gainesville, FL, 32611, AIAA Member.
[4] Professor, Mechanical and Aerospace Engineering department, North Carolina State University, Raleigh, NC, 27695, AIAA Member.

a_{true}	=	True crack size
C	=	Paris law parameter
e	=	Error in damage size measurement
f_{ini}	=	Initial (or prior) probability density function (PDF)
$f_{i,test}$	=	Likelihood function
f_{updt}	=	Updated (or posterior) PDF
g	=	Crack growth
g'	=	Measured crack growth
K_{IC}	=	Fracture toughness
m	=	Paris law exponent
M	=	Monte-Carlo simulation sample size
N	=	Step between inspections
p	=	pressure
r	=	Fuselage radius
t	=	Panel thickness
v	=	Variability in damage size measurement

I. Introduction

STRUCTURAL health monitoring (SHM) will have a significant impact on increasing safety as well as reducing the operating and maintenance costs of structures by providing an accurate quantification of degradation and damage at an early stage to reduce or eliminate malfunctions. Furthermore, SHM will allow predictions of the structure's health status and remaining useful life (RUL) without intrusive and time consuming inspections. Continual on-line SHM is based on dynamic processes through the diagnosis of early damage detection, then prognosis of health status and remaining life.

Prognosis techniques can be categorized based on the usage of information: (1) physics-based, (2) data-driven, and (3) hybrid methods. The physics-based method, or model-based method[1], assumes that the system behavior is known. Dynamic stochastic equation, lumped-parameter model[2], and functional models[3] correspond to this category. In the case of SHM, crack growth model[2,4,5] or spall growth models are often used for micro-levels, and first principle models[6] are used for macro-levels. The data-driven method[7] uses information from collected data to predict future status of the system, and includes least-square regression[8,9], Gaussian process regression[10,11], neural network[6,10,11], and relevance vector machine[10,12]. This method has advantages when the system is so complex that no simple physical model is available. The hybrid method[13] uses the advantages from both methods, and includes particle filters[14] and Bayesian techniques[15,16]. It is generally accepted that uncertainty is the most challenging part for prognosis[16,17]. Sources of uncertainty are initial state estimation, current state estimation, failure threshold, measurement, future load, future environment, and models. In order to address the uncertainty, various methods have been proposed, such as confidence intervals[18], relevance vector machine[10], Gaussian process regression[10,11], and particle filers[14,19].

The current technology of sensor-based SHM anticipates difficulties associated with uncertainties in variability of sensor data, errors in damage growth models, and material and geometric properties. Compared to manual inspections, the accuracy of SHM is still poor. Thus, the major challenge is how to accurately predict the damage growth when the measured data include variability and errors. However, unlike manual inspection, SHM may provide frequent measurement of damage, allowing us to follow damage growth. This in turn, should allow us to narrow the uncertainty in the material properties that govern damage growth. The uncertainty in these properties is normally large because of variability in manufacturing and ageing of the monitored structured. The main objective of this paper is to demonstrate the reduction in uncertainty that may be achieved. In this paper, a probabilistic approach using Bayesian statistics is employed to progressively improve the accuracy of predicting damage parameters under variability and error of sensor measurements.

The approach is demonstrated for a through-the thickness crack in an aircraft fuselage panel which grows through cycles of pressurization and de-pressurization. A simple damage growth model, Paris model, with two damage parameters is utilized. However, more advanced damage growth models can also be used, which usually comes with more parameters. Using this simple model we aim to demonstrate that SHM can be used to identify the damage parameters of each particular panel. This process can be viewed as turning every aircraft into a flying fatigue laboratory. The narrowing of uncertainty in damage growth parameters can narrow in turn the uncertainty in predicting remaining useful life (RUL), i.e. in prognosis.

The paper is organized as follows. In Section 2, a simple damage growth model based on Paris model is presented. The current paper is based on data obtained from a previous work from Kale *et al*.[20,21,22], which describes the fatigue crack growth in a fuselage panel of 7075-T651 aluminum alloy. In Section 3, the probability distribution of damage parameters are progressively narrowed using the Bayesian technique. Detailed discussions on how to construct the likelihood function under given variability of sensor data and how to update the distribution are presented. In Section 4, the RUL of the panel is predicted using the identified damage parameters. Since the damage parameters are randomly distributed, the RUL is calculated using a certain level of confidence. Conclusions are presented in Section 5 along with future plans.

II. Damage growth model

Although damages in a structure start from the micro-structure level, such as dislocations, they are generally accumulated to the level of detectable macro-cracks through nucleation and growth. Damages in the micro-structure level grow slowly, are not dangerous in the viewpoint of structural safety, and are difficult to detect. Thus, SHM often focuses on macro-cracks, which grow relatively quickly in repeated loading cycles.

Once the damage is in the detectable size, various SHM techniques can be used to evaluate the current state of the damage[23]. In the physics-based prognosis techniques, it is necessary to incorporate the measured data into a damage growth model to predict the future behavior of the damage. Since the damage growth model is not the main focus of the paper, the simplest model by Paris[24] is used in this paper. However, more advanced models can also be applied using the same concept.

We consider a fatigue crack growth in a fuselage panel with an initial crack size a_i subjected to load cycles with constant amplitude. We assume that the main fatigue loading is due to pressurization, with stress varying between a maximum value of σ to a minimum value of zero in one flight. One cycle of fatigue loading consists of one flight. Like many other researchers (e.g., Harkness *et al*.[25] and Lin *et al*.[26]), we use the damage growth model by Paris[24] as

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where a is the crack size in *meters*, N the number of cycles (flights), da/dN the crack growth rate in *meters/cycle*, and ΔK the range of stress intensity factor in $MPa\sqrt{meters}$. The above model has two damage parameters, C and m. Accurate prediction of these parameters is important in predicting the remaining useful life of a particular panel.

The range ΔK of stress intensity factor for a center-cracked panel is calculated as a function of the stress σ and the crack length a in Eq. (2), and the hoop stress due to the pressure differential p is given by Eq. (3)

$$\Delta K = \sigma\sqrt{\pi a} \tag{2}$$

$$\sigma = \frac{pr}{t} \tag{3}$$

where r is the fuselage radius and t is the panel thickness.

The number of cycles N of fatigue loading that makes a crack to grow from the initial crack size a_i to the final crack a_N can be obtained by integrating Eq. (1) between the initial crack a_i and the final crack a_N.

$$N = \int_{a_0}^{a_N} \frac{da}{C\left(\sigma\sqrt{\pi a}\right)^m} = \frac{a_c^{1-\frac{m}{2}} - a_0^{1-\frac{m}{2}}}{C\left(1-\frac{m}{2}\right)\left(\sigma\sqrt{\pi}\right)} \tag{4}$$

Alternatively, the crack size a_N after N cycles of fatigue loading can be obtained by solving Eq. (4) for a_N.

$$a_N = \left(NC\left(1-\frac{m}{2}\right)\left(\sigma\sqrt{\pi}\right)^m + a_0^{1-\frac{m}{2}}\right)^{\frac{2}{2-m}} \tag{5}$$

In SHM, the inspection is performed at a specific interval of flights; i.e., with a fixed N. If the damage size a_N obtained from sensors does not include errors or variability, then the curve fitting technique can be used to determine the damage parameters of the monitored panel with at least two measured data. Due to errors and variability in the measurements, however, it is unclear how to estimate the damage parameters. Especially, this is more significant for SHM because the errors and variability in SHM is much larger than that of manual inspection.

III. Statistical Characterization of Damage Properties using Bayesian Updating

As mentioned in the previous section, the damage parameters, *C* and *m*, are critical factors to determine the growth of damage. These parameters are normally measured using fatigue tests. However, uncertainty in these parameters is normally large because of variability in manufacturing and ageing of the specific panel. As can be found in Fig. 1, the parameter *C* corresponds to y-intercept of the fatigue curve, while the exponent *m* is the slope of the curve in the log-log scale. Kale et al.[20,21,22] showed that the effect of exponent *m* is more significant than that of *C*. To simplify the idea of the paper, we assumed that the accurate value of *C* is known, while that of *m* is uncertain. Since the range of the exponent *m* is generally known, it is possible to initially assume that the exponent is uniformly distributed between the lower- and upper-bounds. Then, the goal is to narrow the distribution of the exponent using the Bayesian statistics with measured damage sizes.

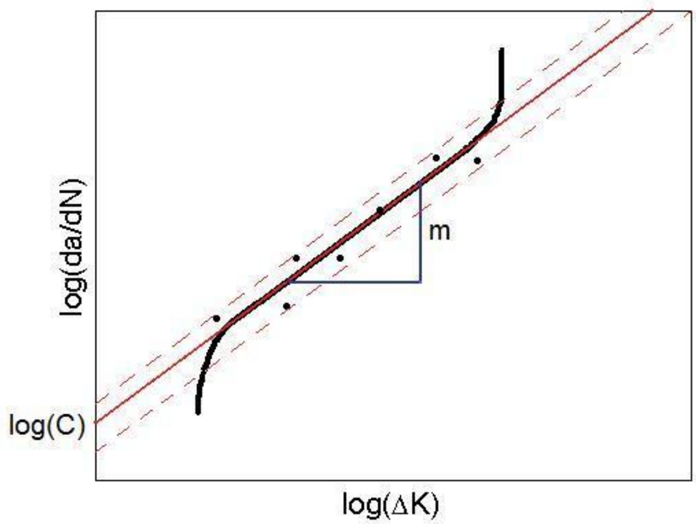

Figure 1. Illustration of Paris law parameter in a log-log plot of crack growth rate.

Bayesian updating[27] is a technique commonly used to obtain updated (also called posterior) distribution of a random variable by using new information obtained about the variable. In this paper, we used the following form of Bayes' theorem that can fit for our purpose:

$$f_{updt}(\theta) = \frac{f_{i,test}(\theta) f_{ini}(\theta)}{\int_{-\infty}^{+\infty} f_{i,test}(\theta) f_{ini}(\theta) d\theta} \qquad (6)$$

where θ is the variable (i.e., the damage exponent *m* here), f_{ini} the assumed (or prior) probability density function (PDF) of θ, f_{updt} the updated (or posterior) PDF of θ and $f_{i,test}$ is called the likelihood function, which is the probability of obtaining the measured damage size, given the errors and variability of sensor measurement.

The likelihood function is designed to integrate the information obtained from inspection to the knowledge we have about the distribution of *m*. In this paper, we choose the damage growth between two consecutive inspections as a likelihood function. Thus, it is the probability to have the measured damage growth for a given *m*. In general, the measured damage growth includes the effect of errors and variability of the sensor measurement. With a large number of measurements, however, it is possible to narrow down the distribution of *m* from its initial wide distribution.

We denote the damage growth between two consecutive measurements as g and the likelihood function as$P(g|m)$. Then, the Bayesian updating formula for the distribution of damage exponent *m* can then be written as

$$f_{updt}(m) = \frac{P(g|m) f_{ini}(m)}{\int_{-\infty}^{+\infty} P(g|m) f_{ini}(m) dm} \tag{7}$$

The likelihood function is constructed based on the fact that there is variability and error in the measured crack size. Let a_N be the measured crack size, *e* the error, and *v* the variability. The true crack size, $a_{N,}$ is then defined as

$$a_{true} = a_N + e + v \tag{8}$$

The measurement error *e* reflects a deterministic bias, such as calibration error, while the variability *v* reflects random noise. For different measurements, the error *e* remains constant, while the variability *v* will vary.

At a given inspection, assuming that the previous and the current crack sizes are uniformly distributed and the crack growth is triangularly distributed, the respective distributions can be found below:

$$a'_i \sim U(a_N + e - v \,;\, a_N + e + v) \tag{9}$$

$$g'_i \sim T(g - 2v \,;\, g \,;\, g + 2v) \tag{10}$$

The likelihood function is then defined as the probability for a given *m* that the growth belong to that distribution. Since the measured data (crack growth) is different from the updated distribution (Paris exponent m), Monte Carlo simulation (MCS) is used to calculate the likelihood function. Let a_i and a_{i+1} be the true crack sizes at (i)-th and (i+1)-th inspections, respectively. In addition, let *N* be the interval of these two inspections. Then, the true crack growth during this interval is given as $g = a_{i+1} - a_i$. Due to sensor error and measurement variability, the measured crack sizes are different from the true ones. The measured crack sizes, $a_i^{'}$ and $a_{i+1}^{'}$, are determined from Eq. (9) and a given value of *m*. Then, the measured crack growth is $g^{'} = a_{i+1}^{'} - a_i^{'}$. The likelihood value at the particular *m* is the value of PDF in Eq. (10) at g'. Since this value will change due to error and variability, this process is repeated *M* times and the averaged value is used for the likelihood value at a given m. The numerical experiments showed that *M* = 1,000 is enough to obtain a smooth distribution of the likelihood function.

One of the major advantages of SHM is that measurements can be performed frequently. Thus, the update in Eq. (7) can be performed as frequently as possible. However, since the damage grows slowly and the errors and variability of measurements are in general large, too frequent measurements may not help to narrow down the distribution of damage parameters. We will investigate the effect of measurement intervals on the rate of narrowing the distribution of damage parameters.

IV. Numerical application

In this section, we present how the distribution of the damage parameter for a fuselage panel can be narrowed using SHM and Bayesian update. Typical material properties for 7075-T651 aluminum alloy are presented in Table 1. The applied fuselage pressure differential is 0.06 *MPa*, obtained from Niu[28] and the stress is given by Eq. (3). Paris law parameters *m* and *C* are obtained using a crack growth rate plot published by Paris *et al.*[29]. Note that due to scatteredness of the crack growth rate, the exponent m is assumed to be uniformly distributed between the lower- and upper-bounds.

Table 1. Fatigue properties of 7075-T651 Aluminum alloy

Property	Pressure, *p* (*MPa*)	Fracture toughness, K_{IC} ($MPa\sqrt{meters}$)	Fuselage radius, *r* (*meters*)	Paris law exponent, *m*	Material parameter, *C*
Distribution type (mean, standard deviation)	Lognormal (0.06, 0.003)	Deterministic 36.58	Deterministic 3.25	Uniform (3.2, 4.6)	Deterministic 3.8×10^{-11}

The damage parameters and their distributions in Table 1 are for generic 7075-T651 aluminum alloy. Depending on manufacturing and assembly processes, the actual damage parameters for individual aircraft can be different. For a specific panel, we assume that there exists a true value of deterministic damage parameters. In the following numerical simulation, the true damage will grow according to the true value of damage parameters. On the other hand, the measured damage size will have errors and variability of the measurements. As mentioned before, it is assumed that the y-intercept C is already known and the distribution of exponent m will be considered in the section.

From the preliminary damage growth analysis, it is found that the effect of variability in pressure p has negligible effect of damage growth because the effect of randomness is averaged out. Thus, in the following analysis, the applied pressure is assumed to be deterministic. **(If this is the case, I suggest to delete the pressure column in Table 1)**

In general, the minimum size of detectable damage using SHM is much larger than that of the manual inspection. Although different SHM techniques may have different minimum detectable size, we choose an initial crack size $a_0 = 20\,mm$, which is large enough to be detected by most SHM methods. In addition, this size of damage will provide significant crack growth data between the two consecutive inspections.

V. Updating of material property *m*

In order to test the updating procedure, we assume that the actual value m of a particular panel is deterministic, m_{true}, which governs the crack growth. However, the available information is the initial distribution of m is assumed to be uniform for simplicity. The likelihood function is based on the crack growth so in order to have information we need for the crack to be large enough to be detectable because no detection means no crack size, so no information and we don't use the absence of data as an information. This is why we choose the initial crack size to be 20 mm.

In the example, we assume that the inspection has been performed at every 200 cycles (i.e., N = 200). For the results presented here inspections have been performed on a single panel that contains a crack size that is initially 20 mm until failure of the given panel. The variability in crack detection is assumed to be of 1 mm and the error to be zero. The inspection are conducted until 50,000 cycles or until the crack reaches its critical size, a_C.

$$a_C = \left(\frac{K_{IC}}{\sigma\sqrt{\pi}}\right)^2 \qquad (11)$$

Figure 2 shows the updated distributions of m for different deterministic values of m_{true}. The initial distribution of m is in red, the dotted blue curves are updated distribution (every 2,000 cycles) and the solid blue curve is the final updated distribution.

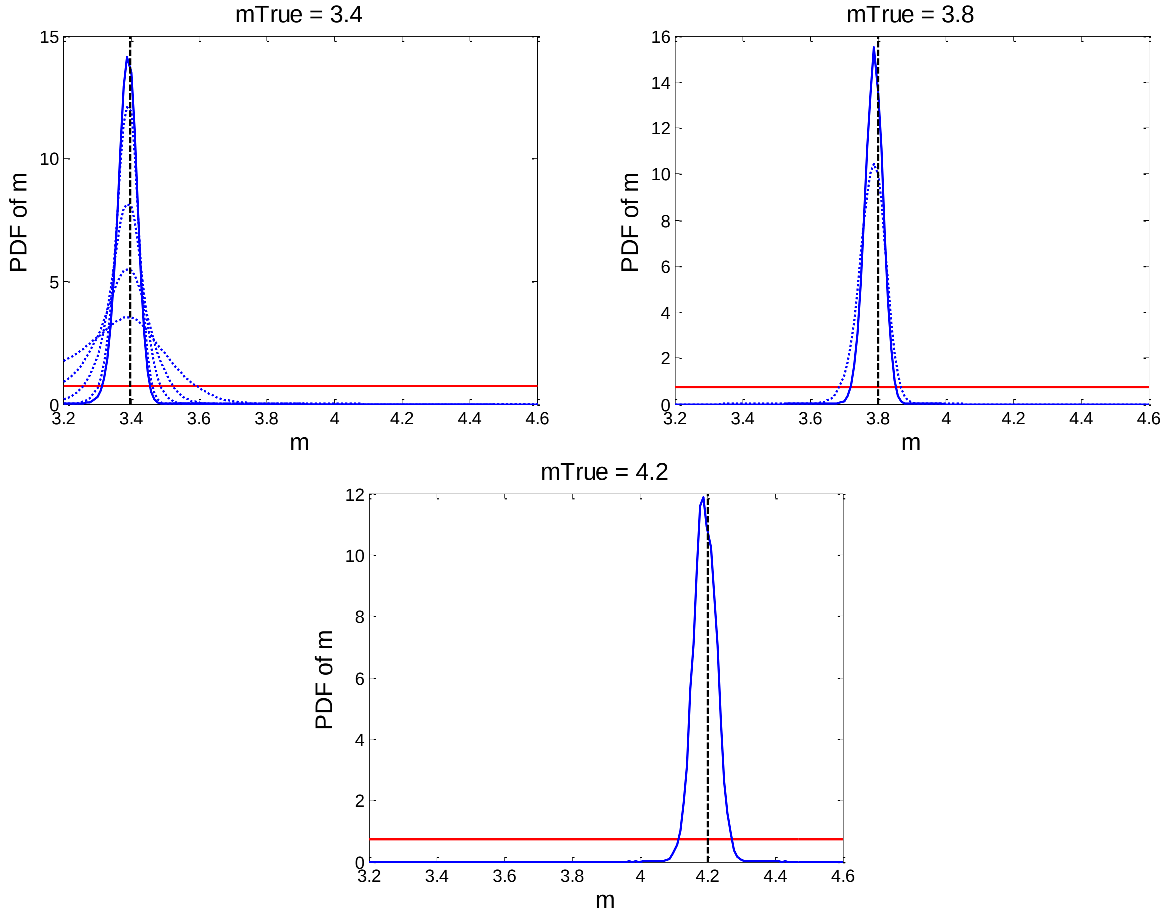

Figure 2. Updated distribution of material parameter *m* for different actual values m_{true}

It can be observed that no matter the value of m_{True} the updating is very accurate, the larger m is, the faster the crack growth, but is m is smaller the reduced crack growth rate is compensated by the fact that we have more inspection results.

VI. Remaining useful life using updated distribution of *m*

An interest of updating the distribution of m is to improve the accuracy of prognosis for a structure. We will here discuss the application to the calculation of the remaining useful life (RUL) at every inspection. In order to show the interest of our method we compare RUL calculated using the actual value of m, m_{true}, the and the critical value for both the initial and the updated distributions. We define the critical value of m, m_C, such that $P(m \leq m_C) = 0.95$.

In order to simplify the model we use a deterministic value for the crack size, we assume the worst possible crack which means that we use $a = a_N + v + e$. Figure 3 shows the comparison of the remaining useful life for the previous estimates of m.

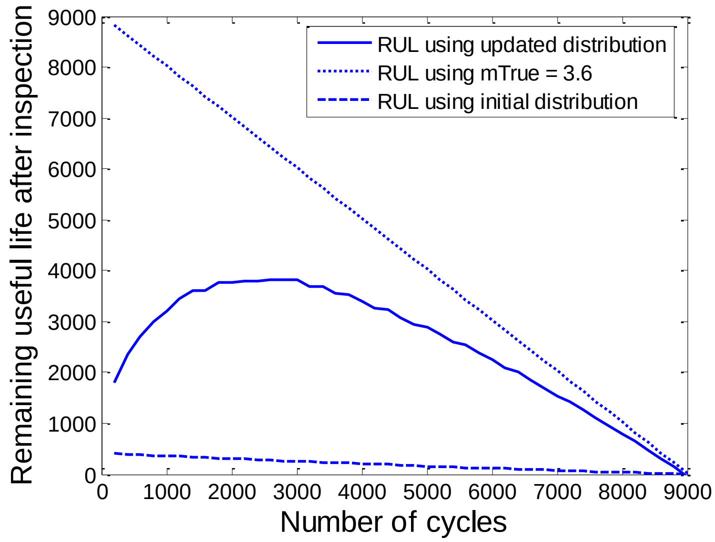

Figure 3 – Remaining useful life after inspection

It can be observed that the updated distribution of m allows us to have conservative estimate of the RUL that converges to the actual value, whereas the initial distribution gives an estimate that is way too conservative and obviously not reasonable since after the first inspection we have a RUL of approximately 500 cycles but after 500 cycles we can observe that the RUL has not change much.

VII. Conclusion

This paper shows that the amount of data obtained from SHM compensates for the uncertainties in measurements involved in that kind of inspection. Another observation resulting from this work is that the method presented here is actually insensitive to the amplitude of variability and error in damage size measurement.

That method allows to narrow down the distribution of material parameters, in this case Paris law exponent, no matter what the value of the parameter is, it is here tested only for a deterministic value but the goal is to extend the work to be able to narrow down on a distribution for an entire aircraft, not only a single panel.

It can be observed that the improvement in statistical knowledge of material properties improves significantly the estimation of remaining useful life not only it converges to the right value and is much better than the estimation obtained from the initial distribution, it also stays conservative even close from failure. The model presented here is very simplified, the actual result is a distribution instead of a deterministic value but that simplified model gives already a good estimation of the expected behavior and it is conservative since it is the worst possible case.

Acknowledgments

This work was supported by the U.S. Air Force under Grant FA9550-07-1- and by the NASA under Grant NNX08AC334.

References

[1] J. Luo, M Namburu, K. Pattipati, L. Qiao, M. Kawamoto and S. Chigusa, "Model-based prognostic techniques," in *Proceedings of the IEEE Systems Readiness Technology Conference*, Sep. 2003, pp.330–340.

[2] C. Li and H. Lee "Gear fatigue crack prognosis using embedded model gear dynamic model and fracture mechanics," *Mech. Syst. Signal Process.*, vol. 20, pp. 836-846, 2005.

[3] Berruet, P. and Toguyeni, AKA and Craye, E.," Structural and functional approach for dependability in FMS", *IEEE International Conference on Systems, Man, and Cybernetics*, 1999. IEEE SMC'99 Conference Proceedings, Vol. 4, 1999.

[4] A. Ray and S. Tangirala "Stochastic modeling of fatigue crack dynamics for online failure prognostics," *IEEE Trans. Control Syst. Technol.*, vol. 4, pp. 443, Jul. 1996.

[5] A. Ray, and R. Patankar, 1999, "Stochastic Modeling Fatigue Crack Propagation under Variable Amplitude Loading," *Engineering Fracture Mechanics*, Vol. 62, pp. 477-493.

[6] Jaw L. C., "Neural network for model-based prognostics", *Proceedings of the IEEE Aerospace Conference*, Vol. 3, 1999, pp.21-28.

[7] Schwabacher, M., “A Survey of Data-Driven Prognostics”, *Proceedings of the AIAA Infotech@Aerospace Conference*. Reston, VA: American Institute for Aeronautics and Astronautics, Inc., 2005.

[8] G. C. Montanari, "Aging and Life Models for Insulation Systems Based on PD Detection", IEEE Trans. on Diel. and El. Ins., Vol. 2, pp. 667-675, August 1995.

[9] Y. Xue, D.L. McDowell and M.F. Horstemeyer, Microstructure-based multistage fatigue modeling of aluminum alloy 7075-T651, *Eng Fract Mech* **74** (2007), pp. 2810–2823.

[10] Goebel, K. and Saha, B. and Saxena, A. and MCT, N. and RIACS, N., “A Comparison of Three Data-Driven Techniques for Prognostics”.

[11] Srivastava, A.N. and Das, S.,” Detection and Prognostics on Low Dimensional Systems”, IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, Vol. 39, No. 1, pp 44-54,2009.

[12] M. E. Tipping, “The Relevance Vector Machine, in Advances in Neural Information Processing Systems”, Morgan Kaufmann, San Mateo, CA, 2000.

[13] J. Yan, J. Lee, A hybrid method for on-line performance assessment and life prediction in drilling operations, in: Proceedings of the IEEE International Conference on Automation and Logistics, Jinan, 2007, pp. 2500–2505.

[14] Orchard, M. and Kacprzynski, G. and Goebel, K. and Saha, B. and Vachtsevanos, G., “Advances in Uncertainty Representation and Management for Particle Filtering Applied to Prognostics”, *International Conference on Prognostics and Health Management*, Denver, CO, 2008, pp. 1-6.

[15] Sheppard, John W and Mark Kaufman, “Bayesian Diagnosis and Prognosis Using Instrument Uncertainity,”, *AUTOTESTCON 2005 Conference Record*, IEEE Press, February 2005.

[16] Saha, B. and Goebel, K., “Uncertainty Management for Diagnostics and Prognostics of Batteries using Bayesian Techniques”, *2008 IEEE Aerospace Conference*, 2008.

[17] S.J. Engel, B.J. Gilmartin, K. Bongort, and A. Hess, “Prognostics, the real issues involved with predicting life remaining,” *Proceedings of the 2000 IEEE Aerospace Conference*, Big Sky, Montana, March 18-25, 2000.

[18] Gu, J. and Barker, D. and Pecht, M., “Uncertainty Assessment of Prognostics of Electronics Subject to Random Vibration”.

[19] M. Orchard, B. Wu, and G. Vachtsevanos, “A Particle Filter Framework for Failure Prognosis,” *Proceedings of WTC2005, World Tribology Congress III*, Washington D.C., USA, 2005.

[20] Kale, A. A, Haftka, R. T., Papila, M., Sankar, B. V., “Tradeoff for Weight and Inspection Cost for Fail-Safe Design,” *44th AIAA/ASME/SDM conference*, AIAA-2003-1501, Norfolk, 2003.

[21] Kale, A. A., Haftka, R. T., Sankar, B. V., “Tradeoff of Structural Weight and Inspection Cost in Reliability Based Optimum Design Using Multiple Inspection Types,” *10th AIAA/ ISSMO Multidisciplinary analysis and optimization conference*, AIAA-2004-4404, Albany, 2004.

[22] Kale, A. A., Haftka R. T., Sankar B. V., “Reliability Based Design and Inspections of Stiffened Panels,” *46th AIAA/ASME/SDM Conference*, AIAA-2005-2145, Austin, 2005.

[23] Wang, L. and Yuan, F. G., “Damage identification in a composite plate using prestack reverse-time migration technique”, Struct. Health Monit., Vol. 4, pp. 195–211, 2005.

[24] P.C. Paris, H. Tada and J.K. Donald, “Service load fatigue damage - a historical perspective.”, International Journal of Fatigue **21**, pp. S35–S46, 1999.

[25] Harkness, H. H, “Computational Methods for Fracture Mechanics and Probabilistic Fatigue,” Ph.D. Dissertation, Dept. of Mechanical Engineering, Northwestern Univ., Evanston, IL, 1994.

[26] Lin, K. Y., Rusk, D. T., Du J., “An Equivalent Level of Safety Approach to Damage Tolerant Aircraft Structural Design,” *41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit*, AIAA-00-1371, Atlanta, 2000.

[27] J. An, E. Acar, R. T. Haftka, N. H. Kim, P. G. Ifju, and T. F. Johnson, Being Conservative with a Limited Number of Test Results, Journal of Aircraft Structure, Vol. 45, No. 6, pp. 1969-1975, 2008

[28] Niu, M., C., “Airframe Structural Design, In: Fatigue, Damage Tolerance and Fail-Safe Design,” Conmilit Press Ltd., Hong Kong, pp. 538-570, 1990.

[29] Paris P. C., Lados D., Tada H., “Reflections on identifying the real ΔKeffective in the threshold region and beyond*”,* Engineering Fracture Mechanics, Elsevier, 2008.