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A Comparison of Spur Gear Response under Non-Ideal Loading Conditions

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ABSTRACT

The current practice of gear design is based on the Lewis bending and Hertzian contact models. The former provides the maximum stress on the gear base, while the latter calculates pressure at the contact point between gear and pinion. Both calculations are obtained at the reference configuration and ideal condition; i.e., zero tolerances. The first purpose of this paper is to compare these two analytical models with the numerical results, in particular, using finite element analysis. It turns out that the estimations from the two analytical equations are closely matched with that from the numerical analysis. The numerical analysis also estimates the variation of contact pressure and bending stress according to the change in the relative position between gear and pinion. It has been shown that both the maximum bending stress and contact pressure occur at non-reference configuration, which should be considered in the calculation of safety factor. In reality, the pinion-gear assembly is under the tolerance of each part and clearance in assembly. The second purpose of this paper is to estimate the effect of these uncertain parameters on the maximum bending stress and contact pressure. For the case of the selected gear-pinion assembly, it turns out that due to a 0.57% increase of clearance, the maximum bending stress is increased by 4.4%. Due to a 0.57% increase of clearance, the maximum contact pressure is increased by 17.9%.

INTRODUCTION

A gear is one of the most common mechanisms that transfer power from one machine to the other. In the design viewpoint, the fatigue strength and wear are the most important criteria because each gear tooth may experience billions of load cycles. Thus, the gear design tends to incorporate a large safety margin and is usually over conservative. However, for space applications, the weight of the system is an important constraint, and accordingly many researches have been performed to reduce the system weight. For example, a deployable space structure has more than a hundred gear-pinion pairs. In such a case, a small weight reduction in each gear can reduce a significant amount of the total system weight. On the other hand, a small reduction in gear stress can cause significant change in expected fatigue life. For the case of mild steel, 10% reduction in stress range can cause about a 50% change in fatigue life [1]. Thus, accurate calculation of stress in gears is crucially important. The objective of the paper is to evaluate the accuracy of the traditional, code-based gear design using computer-aided engineering tools and then evaluate the margin of safety under uncertainties that can happen during manufacturing and assembly. By providing for and designing gear trains that account for these uncertainties, a more accurate and predictable product can be produced.

The design of gear strength is based on two models: Lewis bending stress and Hertzian contact stress models. The former is related to the stress at the gear base, while the latter is related to the wear at the contact surface. The Lewis bending equation was introduced by Wilfred Lewis in 1892. Since then, this equation remains the standard for gear design to this date [1, 4]. Since Lewis introduced his original equation many modifications have been made [9]. The current practice involves the use of the American Gear Manufacturers Association (AGMA) to calculate the bending stress. This calculation assumes that the load is applied at the location of the pitch radius (i.e., reference configuration) [9]. In practice, however, the contact between gear and pinion occurs at various locations during the rotation. Depending on the relative location between gear and pinion, the magnitude and angle of the applied load may vary, affecting the bending stress in the base. This topic is extensively studied in this paper to illustrate the correlation that a numerical analysis can provide.

In addition to the Lewis bending equation, Heinrich Hertz was working on the spur gear problem around the year 1895. He developed an expression for the surface contact stress based on the maximum contact pressure between two cylinders [2]. He realized that the tooth profiles of the gear and pinion were similar to the convex shape of two cylinders in contact. By applying the necessary geometrical conditions of a spur gear to his model based on two cylinders in contact, a method for calculating the maximum contact stress of a spur gear was formulated [1]. In practice, however, the tooth profile is not a perfect cylinder, and the deflection of the tooth makes the contact condition non-Hertzian. A numerical analysis will be introduced for the comparison of the Hertzian contact stress to validate the results.

The current trend of gear design has begun to focus on innovative methods to design gears which are capable of handling higher loads [10]. Although the involute gear discussed in this paper is one of the most widely used gears in the industry, a number of other gear types exist which have recently come into use. A cycloidal gear profile offers a distinct advantage as far as efficiency is concerned, but lacks the necessary load carrying capabilities [10]. Another design implements a gear profile based on a circular arc. These gears are capable of transmitting higher loads between gears but are very sensitive to manufacturing errors [10]. The deviation function has been introduced into the gear design industry to alleviate both the contact pressure and bending stresses in the gear teeth by modifying the surface profile. This method analytically solves for the amount of sliding between contact points due to tooth profile modification. The amount of deviation from the pure rolling design can be included into the function to design a new profile which reduces the amount of sliding.

In addition to the deviation function method for gear design, the optimization of existing designs has also been studied. One method uses the “multi-variable” approach to determine the best gear design possible [11]. Several design variables which can be optimized to reduce both the contact pressure and bending stresses include the center distance, face width and pressure angle [11, 12]. By minimizing the amount of noise produced by gears in mesh, the authors were able to consider millions of designs which could potentially lead to a quieter, more efficient gear set [11]. Another optimization approach attempts to reduce the bending stress at the base of the gear tooth by optimizing the fillet radius in conjunction with finite element analysis [12]. By extending the limitations of the Lewis bending equation with the addition of FEA, along with fillet optimization, a reduction of bending stress on the order of 10-30% was realized [12].

The AGMA bending and Hertzian contact stress models have typically been utilized at the reference configuration and ideal conditions of the spur gear model [5]. The equations assumed that all tolerances in assembly and manufacture were zero. In the application of these gears to real world circumstances this assumption cannot be valid. The common thread which unites all of these developing designs is the attempt to reduce the contact pressure and bending stress while maintaining a high level of efficiency. Thus, the pressure falls upon the manufacturer to produce gears which maintain a high level of accuracy in order to maintain the geometric constraints imposed by the designer. This paper studies the detrimental effects that manufacturing errors can have on both the contact pressure and bending stress. Certain manufacturing and assembly errors will always exist which will unavoidably lead to errors in static and dynamic behavior [3]. In this paper the contribution of assembly errors to static behavior of the gear model will be discussed. Specifically, the influence of the relative position between the gear and pinion and resulting effect on maximum bending stress and contact pressure will be analyzed.

ANALYTICAL METHODS OF GEAR DESIGN

BENDING STRESS AT THE BASE

Lewis considered the tooth as a cantilevered beam and calculated the maximum stress at the root [1]. Although the ratio between the length and thickness is small, this approximation proves a reasonable estimate of the maximum stress, along with stress concentration factor. Consider a cantilevered beam with length L, thickness t, and width s. When a contact force w is applied at the tip, the maximum bending stress of the beam can be found as

$$\sigma_{\text{bending}} = \frac{6wL}{st^2} \tag{1}$$

The above formula is for a beam with a rectangular cross-section under an applied tip load. Although this serves as a good basis for the gear design problem, both the geometry of the gear and appropriate loading must be introduced. The American Gear Manufacturers Association (AGMA) approached the problem by realizing that the load “w” would have to be applied at a point other than at the tip. In addition, the geometric parameters at the contact point must be taken into account in order to properly calculate the bending stress. In Figure 1 the point A_w is where the contact force is applied. The point where the normal to the profile A_w cuts the tooth center line is denoted as “D”. It is at this point that the force is considered to act. Due to this loading there will be two resultant stresses, one acting perpendicular to the center line, the other acting along it. These stresses are known as the bending and radial stresses respectively and when combined, lead to a tensile and compressive stress at points A and A′.

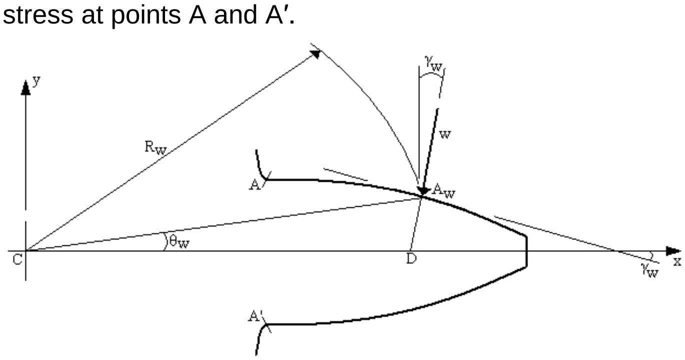

Figure 1: Loading configuration at the tooth of spur gear

In more recent years the AGMA has developed formulas for the bending stress which better predict the effects that stress concentrations such as fillet radius add to the analysis. The tensile stress which is calculated at A will be inaccurate due to the tooth shape which is dissimilar from a beam. Dolan and Broghamer [14] proposed a set of concentration factors that more accurately predict the bending stress based on photoelastic experiments. Because gear application is applied to designs which experience millions of cycles these stress concentrations will lead to a higher overall stress value, causing the fatigue life of the gear to decrease. The AGMA has provided the following equations to incorporate the stress concentration into the gear bending stress. First, the radius of curvature at the root of the tooth must be considered.

$$r_f = r_{rT} + \frac{(a_r - e - r_{rT})^2}{R_{sg} + (a_r - e - r_{rT})} \tag{2}$$

where r_{rT} is the radius of curvature of the circular tip of the rack cutter, a_r is the addendum radius, e is the profile shift and R_{sg} is the radius of the pitch center of the gear. Then, the stress concentration factor can be given as

$$K_f = k_1 + \left(\frac{2y}{r_f}\right)^{k_2}\left(\frac{2y}{x_D - x}\right)^{k_3} \tag{3}$$

where x and y are coordinates on the fillet radius and x_D is the x-coordinate of D, the point where the normal intersects the tooth center-line. Equation (3) utilizes three constants k_1, k_2 and k_3. These constants were developed by Dolan and Broghamer [14]. The determination of x and y are an iterative process along with the gear bending equation to maximize the bending stress within the allowable range. The allowable range is defined as all feasible points along the fillet radius profile.

Once the stress concentration factor is known the maximum bending stress is given as

$$\sigma_t = \frac{w}{m}\cos\gamma_w \left[K_f\left(\frac{1.5m(x_D - x)}{y^2} - \frac{0.5m\tan\gamma_w}{y}\right)\right]_{\max} \tag{4}$$

where m is the gear module, a constant, w is the load intensity, and γ_w is the angle between involute tangent and the tooth center-line at the load point. Equation (4) is evaluated along the entire length of the fillet radius and the maximum value is obtained. This approach is considered to be the most accurate because it accounts for the higher stresses which will invariable result from the change in radius of the involute curve to the base of the gear [14]. This maximum value will be compared with the finite element analysis to validate the results.

HERTZIAN CONTACT STRESSES

In addition to the bending stress at the base, the contact stress is an important design criterion. Hertz observed that two curved surfaces in contact could be modeled by two cylinders that are pressed together, creating a contact pressure [2]. The Hertzian contact stress was developed by utilizing the maximum pressure on the surface of two cylinders which are in contact. By a similar derivation as in the Lewis bending equation the maximum contact pressure of two cylinders can be modified to introduce the geometry of the gear and pinion teeth in contact. When this geometry is introduced the Hertzian contact stress is calculated as:

$$\sigma_C^2 = \frac{w}{\pi \cdot s \cdot \cos\phi} \frac{(1/r_1) + (1/r_2)}{\left[(1-\nu_1^2)/E_1\right] + \left[(1-\nu_2^2)/E_2\right]} \tag{5}$$

where $\phi = \cos^{-1}(r_b / r)$ is the pressure angle, r_b is the radius of base circle, r is the pitch radius, $r_1 = \frac{1}{2}d_p \sin\phi$, d_p is the pinion diameter, $r_2 = \frac{1}{2}d_g \sin\phi$, d_g is the gear diameter, ν_1 and ν_2 are Poisson's ratio of pinion and gear, and E_1 and E_2 are the elastic modules of pinion and gear. By utilizing this formula the compressive stress on the surface of the gear tooth can be solved for.

Although the Hertzian contact stress in Eq. (5) provides a reasonable estimate of contact stress on the gear, it is based on two assumptions. In practice, the involute curve of the gear is not an exact cylinder and the effect of neighboring non-cylindrical regions deviate the results from Hertzian contact conditions. In addition, when applied to a spur gear the Hertzian contact stress assumes that the pitch radius point of the spur gear is in contact with that of the pinion at the nominal clearance value. In application, tolerances in manufacturing and clearance in assembly affects the contact locations. In this paper, we call this a non-ideal condition. Because of these assumptions, errors in the computation of safety factors can easily manifest themselves in assembly error [6]. When the point of contact is altered in the spur gear mating it will be possible for several disadvantageous conditions to occur.

The most important of these come into consideration when multiple teeth are in mesh. At certain points during the rotation of the gear, load sharing will occur between consecutive gear teeth. This effect is desirable as it reduces the maximum bending stresses. The gear designer counts on this phenomenon when designing a gear pair. This paper will show that due to non-ideal loading conditions the number of load sharing teeth may reduce from three to two and cause an increase in bending stress.

NUMERICAL ANALYSIS USING NONLINEAR FINITE ELEMENT METHOD

The Lewis bending equation was developed more than 100 years ago and remains the basis for all gear design to this day [1]. The Lewis bending equation provides a reasonable estimate of the bending stress at the root of the gear tooth for many different gear dimensions and designs. Because of its simplicity and accuracy the methods developed remain popular to this day. However, with the advent of modern numerical analysis tools the design and evaluation of engineering concepts and devices can be explored more readily. In this paper, we use a finite element analysis program, ANSYS, to evaluate the accuracy of the two analytical models for gear design. The advantage of using finite element analysis software is that it can accurately consider the effect of detailed geometry, as well as complicated loading conditions. Especially, it allows us to calculate the bending stress and contact pressure during the rotation of gears. In addition, it also allows us to calculate these two criteria under non-ideal conditions.

ANSYS Parametric Design Language (APDL) is used to create the gear and pinion model, to apply contact and boundary conditions, to control nonlinear solution sequence, and interpret analysis results. The code that was developed for this problem is separated into four different files, each of which serves its own purpose in creating the model, performing analysis, and interpreting the results.

The numerical analysis begins by defining the geometrical characteristics of the gear. These characteristics define the entire geometry of the gear and pinion model in two dimensions. Table 1 shows the required parameters and their values used in the numerical study. These parameters are stored in a separate data file so that the modeling program can refer to it while creating finite element models.

Table 1: Input parameters for gear FEA analysis

Pinion (unit: mm)		Gear (unit: mm)	
No. of teeth	25	No. of teeth	31
Pitch diameter	79.38	Pitch diameter	98.43
Involute diameter	74.59	Involute diameter	92.49
Addendum	4.43	Addendum	4.18
Dedendum	5.05	Dedendum	5.30
Tooth thickness	4.90	Tooth thickness	4.90
Face width	31.75	Face width	31.75
Root fillet radius	1.04	Root fillet radius	0.99
Tooth fillet radius	0.78	Tooth fillet radius	0.99
Elastic modulus		206.80 GPa	
Poisson's ratio		0.33	
Distance b/w axes		88.9	
Element size for teeth		0.1	
Element size for body		1.0	
Friction coefficient		0.0	
Applied torque		800 N-m	

Once the most basic parameters have been defined the next task is to generate the numerical model of spur gears. First, additional parameters that are necessary to create the spur gear will be implemented. Once these calculations have been performed the program first models the pinion. By starting with the involute curve and finishing with the deletion of the gear tooth slots the pinion can be generated. To further simplify the analysis only a portion of the pinion needs to be created. Because of the symmetry of the pinion only three teeth are modeled. This will significantly reduce the computational effort required to perform the finite element analysis on the model.

The next step is to generate the gear geometry that will mesh with the pinion. By using the same type of method the gear geometry is created with the same symmetry characteristics being utilized. The gear has only three teeth modeled to allow for faster computation of the problem.

The next step is to generate finite elements for the spur gear. Plane182 (2D solid) elements were used in ANSYS. This element is a 4-node plane stress element. After the appropriate thickness of the spur gear is read into the file, along with the material constants, the size of the elements can be chosen. Depending on the type of analysis to be performed the element size can be input into the parameter file. For more accurate and also computationally intensive calculations the density of the mesh can be increased. For quick simulations the modification of two parameters can significantly reduce the amount of time needed to run the simulation. Figure 2 shows finite element for gear and pinion. A small mesh size is used for contact surface for accurate calculation.

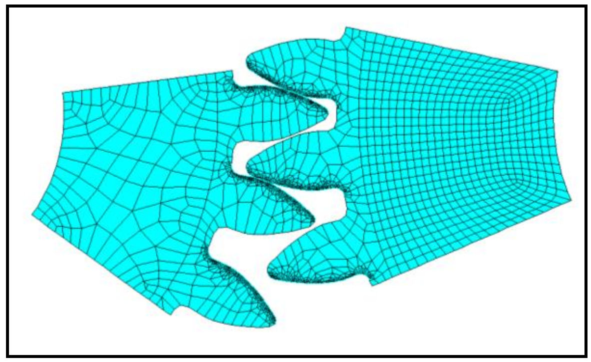

Figure 2: Finite element models for gear and pinion

After the gear and pinion has been meshed the contact elements are generated. In ANSYS the method of determining contact is performed in a two step process. First, a "target" element needs to be specified. For this program the type of target element is a two-node element. Next, the contact element is specified in much the same way. ANSYS will only detect contact when a contact element penetrates a target element by a small amount.

Finally, rigid elements are generated on the rest of the available surface area which is not critical to the analysis. Now, the pinion is rotated to make initial contact with the gear. The rigid elements are unselected along with their accompanying nodes. The pinion is rotated in the pre-processor and the amount of contact generation is recorded. If the contact penetration is greater than zero the program ends the pinion rotation and records the pinion rotation angle. This value is stored and all data is written to a database file which stores all relevant data necessary for the next step.

The next step in the process is the simulation file. The purpose of this file is to impose the necessary constraints on the pinion and gear and to rotate the pinion with a torque applied. First, the gear is constrained so that no rotation is possible and the pinion has a torque applied. The torque is applied as a ramp load until the maximum value is reached. Next, the torque is held constant while the gear is allowed to rotate. The gear is rotated over a prescribed angle and the reactions are recorded for a specified number of load steps. Once the load steps are completed the LSSOLVE command is invoked and the simulation mode is over. The next step is to post-process the results.

The post-processing module is responsible for obtaining the desired results. For this paper these results are defined to be the maximum bending stress and the maximum contact pressure. These values are obtained in the post-processor file and recorded. With these results the numerical analysis is now complete.

COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS

Now that both the analytical and numerical methods have been thoroughly described the results will be contrasted. First, the necessary parameters of the AGMA bending equation and the Hertzian stress equation are presented in Table 2. These parameters were used for both the analytical solutions as well as the numerical solutions. Following the presentation of the parameters the results obtained will be shown.

Table 2: Bending stress analysis parameters

Torque	800.00	N-m
k_1	0.18	
k_2	0.15	
k_3	0.45	
W	827.81	N/mm
M	2.59	mm
γ_W	45.51	degrees
K_f	1.18	
x_D	43.44	mm
x	34.45	mm
y	3.71	mm

Table 3: Bending stress comparison

AGMA	FEA	Difference (%)
578.7 MPa	581.4 MPa	0.46

From Table 3 it is apparent that the values for the AGMA bending stress and the numerical analysis closely coincide. It is important to note that the AGMA bending stress assumes that the maximum bending stress occurs at the location given above for *x* and *y*. This is the practical value which maximizes the bending stress equation. A practical value is one whose location lies on the fillet radius in the appropriate section of the pinion. Because most failures occur through tension in the gear and not compression [14] the tension side of the pinion was chosen as the sampling point.

Figure 3 was generated by recording the bending stress at two nodes located at the base of the pinion as a function of rotation angle. The plot which is shown in red corresponds to the node which is in tension and the plot in blue is the node which experiences compression. Besides the correlation between the AGMA stress and the numerical values there is another region of interest in Figure 3. At around eight degrees of rotation the bending stress in both tension and compression drops significantly due to multiple pinion teeth being in contact during the rotation of the gear set. Up until the point where the bending stress decreases there are only two teeth in contact. For a few degrees of rotation there will be three teeth in contact which reduces the bending stress by as much as 13.9%. Once the gear further rotates and two teeth are in contact the bending stress increases further. This will be an important phenomenon when the non-ideal loading conditions are introduced.

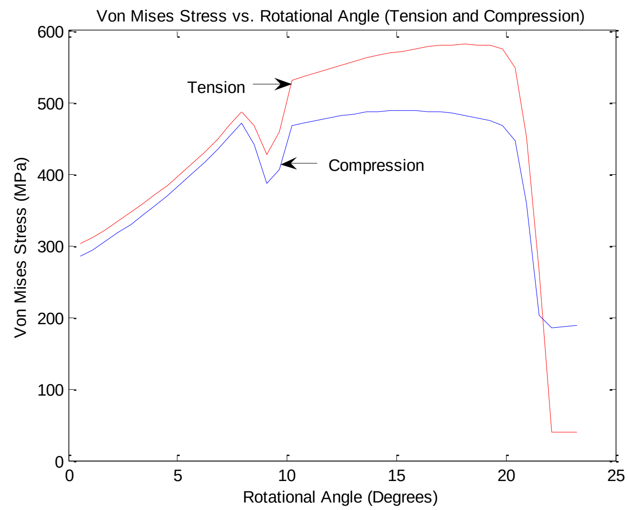

Figure 3: Variation of bending stress as a function of rotational angle

Next, the contact pressure between the gear and the pinion was analyzed. Table 4 shows the parameters that were utilized.

Table 4: Hertzian contact stress analysis parameters

w	26,283	N
s	31.75	mm
r_b	30.44	mm
r	79.38	mm
Φ	20.00	deg
d_p	88.24	mm
d_g	106.78	mm
R_1	24.29	mm
R_2	32.76	mm

The analysis of the contact between the gear and pinion is more complicated than the case of the bending stress. As the contact evolves between the spur gear and pinion a few characteristics of gears must first be explained. The first consideration is that the finite element analysis generates nodes along the contact profile for the gear and pinion. Because of this the values for the contact stress can only be obtained at these points. Correspondingly the true values and the accuracy of the analysis depend solely on the validity of the mesh that is used.

For the first part of the analysis the maximum contact stress at any one point in the entire model was obtained.

Table 5: Contact stress comparison

Hertz	FEA	Difference (%)
1,511 MPa	1,574 MPa	4%

Table 5 shows that the Hertzian contact pressure equation calculates the maximum value to within 4%. Figure 4 is a plot of the contact stress versus the rotational angle. Because the finite element model is discrete there are only contact stress values for a few rotational angles. In addition, the contact pressure function in ANSYS has only two values for the entire contact situation. This disparity of data is not enough to accurately compare with the Hertzian contact value so the von Mises stress was used instead. At the true point of contact this value should correspond with the Hertzian contact pressure.

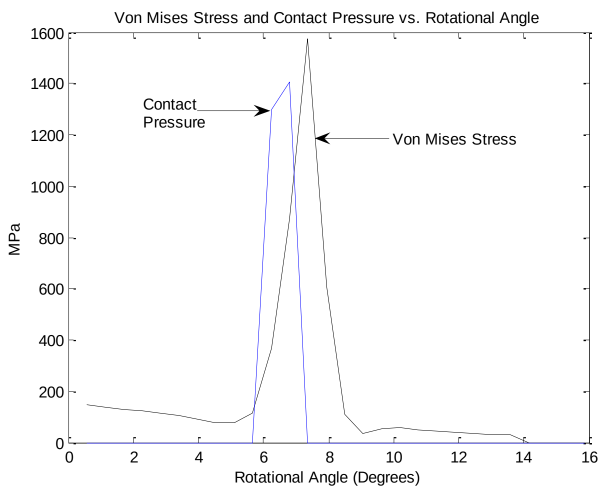

Figure 4: Variation of contact stress as a function of rotational angle

The most significant aspect of this plot is the high magnitudes of stress that can develop in the spur gear. These high levels of stress are only present for a small range of rotational angles. Over time the oscillatory effect of these values can cause pitting and erosion due to fatigue stress [1]. Further, if these values are increased over any portion of the gear or pinion the amount of wear will increase. This increase in contact stress will be further investigated in the following section.

As a design engineer a tool that is capable of accurately and quickly evaluating these spikes in contact pressure must be available. The method presented in this paper allows for a parametrically variable program that can account for all types of loading conditions and changes in geometry that a design engineer could expect. The final section of this paper will investigate the effects on non-ideal loading conditions.

NON-IDEAL LOADING CONDITIONS

With any mechanical assembly there are certain tolerances which are applied to each part of the assembly. As the different parts are assembled these tolerances can interact to affect how the total tolerances of the entire assembly are maintained. For a spur gear and pinion idealized into 2D there are a limited amount of non-ideal loading conditions which can develop. The type of non-ideal loading condition that will be discussed is the amount of axial separation between the gear and pinion. Due to the tolerances in the shafts of the gear and pinion the axial separation between the two can vary by as much as .02" if the tolerance on the gear and pinion shafts is within ±.01"[7]. The effect this has on the bending stress and contact pressure will be evaluated.

As the distance between the gear and pinion is increased the bending stress and contact pressure will increase. This is due directly to the distance between the gear and pinion. With any type of assembly the stress at a point due to an applied force is related to how far away that force is applied. As the length between the point of application and the point of interest increases the bending moment will increase as well. Due to this effect the bending stress at the root of the spur gear will increase.

The bending stress at an arbitrary node at the base of the spur gear will be investigated. At nominal or ideal conditions the separation between the gear and pinion should be 88.9 mm. Due to the tolerance stack up mentioned previously the amount of separation was varied by as much as .02" or .51mm. The effect of axial separation on the bending stress is illustrated in Figure 5.

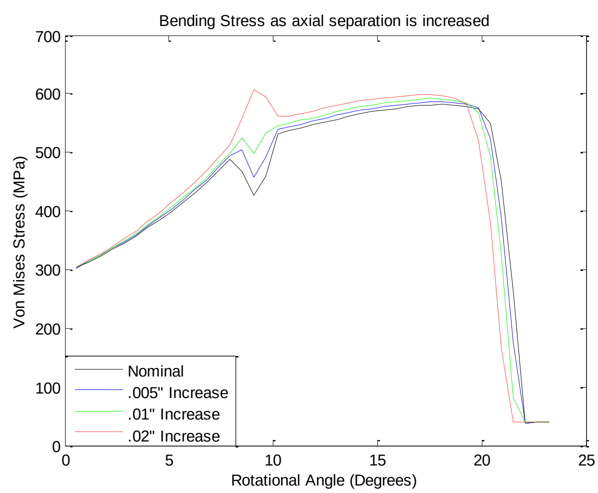

Figure 5: Effect of axial separation on bending stress

Figure 5 begins with the nominal clearance location given by the solid black lines. Three axial separations are considered: .005", .01", and .02". The corresponding increase in the bending stress can be seen. Over the majority of the plot the bending stress has increased. The maximum values have an increase of 4.4% over the nominal configuration. An important aspect of this plot is how the increase in the axial separation changes the load sharing capability of the gear and pinion. Instead of the bending stress decreasing due to three teeth being in contact, the bending stress increases. The increase in axial separation eliminates the three teeth in contact and the bending stress rises due to the higher and higher load solely on one pinion tooth. Once contact on the final pinion tooth is encountered the bending stress finally begins to decrease. The difference in values at this critical point can be as much as a 41.9% increase in the bending stress.

Table 6: Bending stress comparison at the increase axial separation

Nominal distance	Non-ideal distance	Difference (%)
581.4 MPa	606.8 MPa	4.4%

Table 6 gives the data that was used in calculating the percent difference between the nominal and non-ideal loading maximum stress. This data implies that as normal tolerances and assembly practices are utilized the tolerance stack up can very easily lead to the type of situation shown above. Engineers must be able to plan for these kinds of conditions in the design and assembly of their gears. This increase in the bending stress will cause more wear and fatigue on the root of the gear. Over time an increase in these parameters will cause the gear to fail before its intended lifetime.

Next, a similar approach was taken for the contact stress between the gear and pinion. For this contact stress analysis three nodes were chosen along the contact profile of the gear and pinion. This way, a much smoother plot could be developed to explore what is happening as the axial separation is increased. The following gives the results.

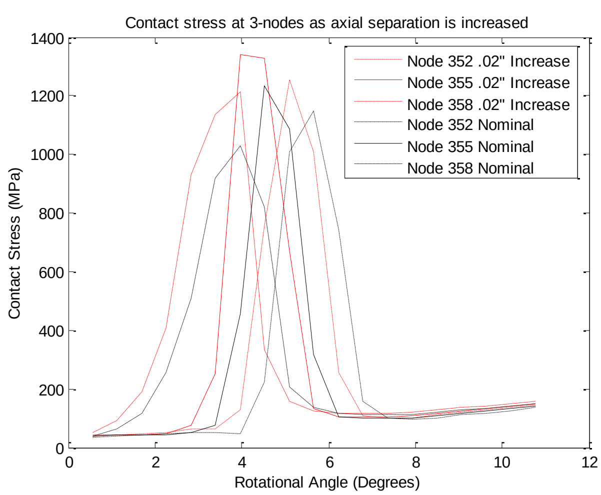

Figure 6: Contact stress at the increased axial separation

From Figure 6 the relationship between axial separation and the inherent increase in the contact stress is established. The first portion of this plot is the baseline values for the nominal clearance. The data is gathered from three nodes chosen along the pinion tooth profile. These nodes were chosen so that their location corresponds with the pitch circle of the pinion. Recall that the pitch circle is the optimal point for gear and pinion meshing. The nominal clearance values are plotted as the three separate solid black lines. For each node the contact stress is recorded as the gear rotates. The full rotation that was used for the bending stress is not needed and the corresponding values are truncated. Next, the axial separation was increased by .02". This resulted in the data given by the solid red lines. For each of the three nodes the contact stress has increased. The maximum increase occurred at the first node and was found to be 17.9% greater. Table 7 relates the data for this increase.

Table 7: Non-ideal loading conditions on contact stress

Nominal distance	Non-ideal distance	Difference (%)
1,029.4 MPa	1,213.8 MPa	17.9%

Because of this increase engineers must take into account the effect that the axial separation has on the contact stress. It has been shown that the amount of wear is directly proportional to the contact pressure [8]. If certain areas of the gear and pinion are experiencing higher contact stress levels the wear will increase. Over billions of cycles the increase in contact stress will become significant enough to increase the wear past that which is predicted into the safety factors. This wear will ultimately cause the destruction of the gear train.

CONCLUSION

A method to properly develop the gear and pinion geometries has been developed parametrically in ANSYS. With the help of this program the implementation of many different geometrical configurations can easily be obtained. The finite element method is a good means to solve this problem for a spur gear model.

By implementing the necessary parameters into the gear code it is possible to simulate real world gears. The AGMA bending equation and the Hertzian contact equations are the basis for which engineers design gears to this day [1,9]. By comparing the analytical results obtained from these equations to the numerical analysis results under the exact same configurations the accuracy of the numerical analysis can be verified. The correlation between the values was shown to be within .46% for the bending stress and 4% for the contact stress. These values correspond very well to the analytical solutions which confirm the validity of the program.

Once the validity of the program was verified the effects of the non-ideal loading conditions were taken into account. Because engineers specify the tolerances to within the assembly is deemed acceptable the range of these values should be investigated. As the axial separation was increased both the bending stress and the contact pressures increased. With a 0.57% increase in the axial separation between the gear and pinion the bending stress increased by 4.4%. Also, with a 0.57% increase in the axial separation between the gear and pinion the contact stress increased by 17.9%. The increase in axial separation also caused the positive effects of load sharing to be decreased. The load sharing capability was reduced from three teeth to two, and a difference of 49% in the predicted bending stress was shown. This increase in bending stress is substantial and would greatly reduce the factor of safety due to the decrease of fatigue life.

From the conclusions drawn through this research a number of important points were developed. When an engineer uses the Hertzian contact stress equation to solve for the maximum values that he or she expects the gear to be subjected to they must realize that the configuration of the gear plays an important role. As the gear and pinion rotate through their contact areas the contact stress can increase by large amounts. Although these values may only exist for a short amount of time their effects on the wear of the tooth can be pronounced. This increased wear can account for the failure of a gear before its predicted life cycle. To properly engineer around this problem the solution can take one of two forms. The type of material and hardening techniques can be improved so as to obtain a better resistance to wear. Along with this type of solution comes an increased production cost of the gear. Another solution would be to increase the size and thicknesses of the gears being used. This would lead to more material and a heavier final product.

In addition to the increase in contact stress the effects that non-ideal loading present cannot be ignored. The tolerance stack up will yield end products that have discrepancies in their axial separation on the same level that was presented in this research. A tool which can accurately and quickly determine how these non-ideal conditions affect the wear of the tooth will be a very useful commodity. By understanding the processes that occur under these conditions a better and more thorough design of the spur gear can be obtained.

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