Bayesian approach for fatigue life prediction from field inspection

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ABSTRACT: In the design considering fatigue life of mechanical components, uncertainties arising from the materials and manufacturing processes should be taken into account for ensuring reliability. In this paper, the approach based on the Bayesian technique is proposed, which incorporates the field failure data with the prior knowledge to obtain the posterior distribution of the unknown parameters of the fatigue life. Posterior predictive distributions and associated values are estimated afterwards, which represents the degree of our belief of the life conditional on the observed data. As more data are provided, the values will be updated to more confident information. In order to obtain the posterior distribution, Markov Chain Monte Carlo (MCMC) technique is employed.

Keywords*: Fatigue life, Prior distribution, Posterior distribution, Bayesian approach, Markov Chain Monte Carlo Technique, Field Inspection.*

1. INTRODUCTION

Performance of mechanical components undergoes a change by uncertainties such as environmental effects, dimensional tolerances, loading conditions, material properties and maintenance processes. Fatigue life of the components in particular are significantly influenced even by small changes. In the design for fatigue life, it is not feasible to consider all the uncertainties of the relevant variables, since most of them are not characterized in the design phase. Analytical prediction of fatigue life is therefore, often not in agreement with the field data. Common practice in the design is then to apply proper "safety factor" when evaluating fatigue life. This approach, however, causes overdesign or risk of design, since it relies on the designer's experience. Recently, for more reliable life prediction, the study using field data have been undertaken (Marahleh et al., 2006). Field data can be helpful in predicting fatigue life that has uncertainties due to the unknown potential inputs. This approach can be dealt with Bayesian technique which incorporates the field failure data with the prior knowledge to obtain the posterior distribution of the unknown parameters of the fatigue life (Kim et al., 2009). As more data are provided, the values will be updated to more confident information. In this paper, Markov Chain Monte Carlo (MCMC) technique is employed as an efficient means to draw samples of given distribution (Andrieu et al., 2003). Consequently, the posterior distribution of the unknown parameters of the fatigue life is obtained in light of the field data collected from the inspection of turbine blades. Subsequently, fatigue life of turbine blades is predicted a posteriori based on the drawn samples.

2. BAYESIAN TECHNIQUE FOR LIFETIME PREDICTION

Bayesian technique is employed to update lifetime prediction using analytical model and field data, which is based on Bayes' rule and defined as (Gelman et al., 2004) :

$$p(\mu,\sigma|\mathbf{D}) \propto L(\mathbf{D}|\mu,\sigma)\,p(\mu,\sigma) \tag{1}$$

where $L(\mathbf{D}|\mu,\sigma)$ is the likelihood of observed data $\mathbf{D}(=t,y,n)$ conditional on the given parameters μ,σ, $p(\mu,\sigma)$ is the prior distribution of μ,σ, and $p(\mu,\sigma|\mathbf{D})$ is posterior distribution of μ,σ conditional on the $\mathbf{D}$. The procedure to obtain posterior distribution $p(\mu,\sigma|\mathbf{D})$ is outlined as follows.

In Eq.(1), likelihood is just a multiplication of each binomial PDF, given as

$$L(\mathbf{D}|\mu,\sigma) = \prod_{i=1}^{N} \mathrm{Bin}\left(y_i | n_i, p_{f_i}\right) \tag{2}$$

where $p_{f_i} = \int_0^{t_i} f_{\mathrm{life}}(t|\mu,\sigma)\,dt$, and N,t_i,y_i,n_i are given in the Table 1. Here, $f_{\mathrm{life}}(t|\mu,\sigma)$ denotes PDF of life of turbine blade with two parameters μ,σ, which can be assumed as normal, lognormal or weibull distribution, depending on the nature of failure data. The prior PDF is assumed as

$$p(\mu,\sigma^2) = N(\mu_0,\mu_0/2)\,N(\sigma_0,\sigma_0/2) \quad \text{where} \quad \mu_0 = 5169,\ \sigma_0 = 794 \tag{3}$$

Consequently, the posterior PDF is obtained by multiplicatying Eq.(2) and Eq.(3).

Table 1. Field data for inspected turbine blades

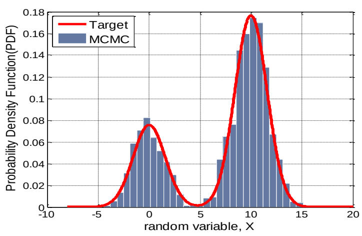
Engine	**Hours(t_i)**	**Failed(y_i)/Total(n_i)**	**Engine**	**Hours(t_i)**	**Failed(y_i)/Total(n_i)**
1	4321	2/40	8	1456	0/40
2	3125	1/40	9	26123	13/40
3	1500	1/40	10	3654	0/40
4	9152	0/40	11	8541	0/40
5	12000	12/40	12	6542	10/40
6	11654	3/40	13(N)	18687	18/40
7	6011	6/40			

3. MCMC SIMULATION

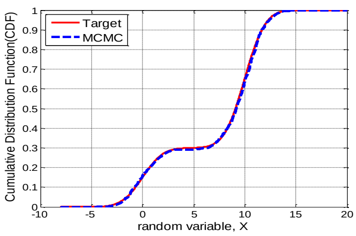
Once the expression for posterior PDF is available, one can proceed to sample from the PDF. Primitive way is to compute the values at a grid of points after identifying the effective range, and sample by inverse cdf method. The method, however, has several drawbacks such as the difficulty finding correct location and scale of the grid points, spacing of the grid, and so on. MCMC simulation is an effective solution in this case (Andrieu et al., 2003). As an example of MCMC, in Fig. 1 is shown the sampling result of fictitious PDF given as

$$p(x) \propto 0.3\exp\left(-0.2x^2\right) + 0.7\exp\left(-0.2(x-10)^2\right) \tag{4}$$

With only 5,000 iterations, the sampling result follows the distribution quite well.



(a) PDF

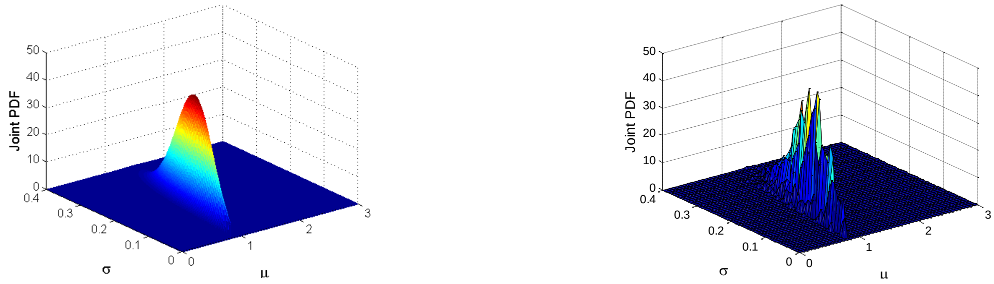


(b) CDF

Fig. 1 Results of MCMC simulation

4. POSTERIOR DISTRIBUTION USING MCMC

The joint posterior PDF of the unknown parameters of the fatigue life using only the first data is shown in Fig. 2, which represents the degree of belief on the concerned parameters in the form of PDF. The joint posterior PDF using grid method as well as MCMC sampling are shown in Fig. 2(a) and Fig. 2(b), respectively. In Table 2, statistical moments by the two methods are compared. The results agree closely with each other. The joint PDF's are updated as more data are added. Contour plots of the parameters at several stages of observed data are shown in Fig. 3. The location and range of μ,σ moves and narrows down, converges to a certain point as more data are added.



(a) using grid method (500×500 grid) (b) using MCMC (10^4 iterations)

Fig. 2 Joint posterior PDF of μ,σ

Table 2. Statistical moments by the two methods

	Eμ	Eσ	E$\mu\mu$	E$\sigma\sigma$	E$\mu\sigma$
Grid	1.1476	0.1854	0.0160	0.0043	-0.0000
MCMC	1.1498	0.1857	0.0178	0.0048	0.0000

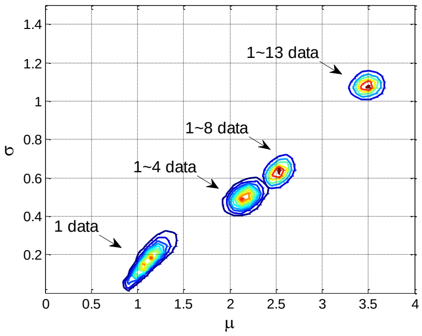


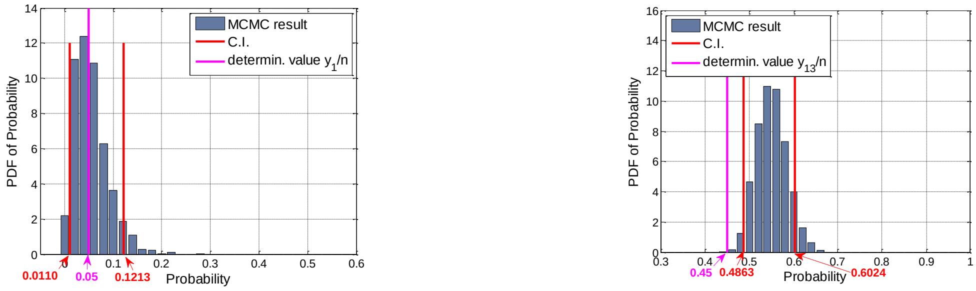
Fig. 3 Contour plots with increasing data

5. POSTERIOR PREDICTIVE DISTRIBUTION

The drawn samples of the parameters obtained in section 4 are used for predicting the failure probability. Assuming the PDF of life is normal distribution, one can evaluate failure probability at a lifetime given each set of samples of the parameters. The predicted distribution of failure probability at t_1=4321 with first data and t_{13}=18687 with 13 data are shown in Fig. 4(a) and Fig. 4(b), respectively. The distribution of failure probability comes from the randomness of μ,σ, which was originally caused by the insufficiency of the observed data. In view of safety, we should choose the upper bound of 90% confidence interval as the failure probability, which is 0.1213 and 0.6024, respectively. In this manner, one can construct confidence bounds of CDF which are given in Fig. 5. In the first data case, the gap is wider whereas it is narrow with whole 13 data. This suggests that as more data are used, higher reliabilities with smaller interval are gained. Obviously, we should choose the red curve, which is the 95% upper bound, for safety.

6. CONCLUSIONS

In this paper, a Bayesian updating technique is presented, which incorporates the statistical prediction with field data. By using MCMC simulation, samples of μ, σ are drawn effectively, which are parameters of the fatigue life distribution. After getting samples for joint posterior PDF of μ, σ, the fatigue life prediction results are obtained. From the results, failure probability at a certain lifetime is obtained as a probability distribution and associated confidence bounds, which have arisen from the insufficiency of data. As the number of observed data increases, the variance of the probability and the width of the confidence bounds are reduced, and reliability is improved.



(a) using only the first data

(b) using whole 13 data

Fig. 4 PDF of failure probability

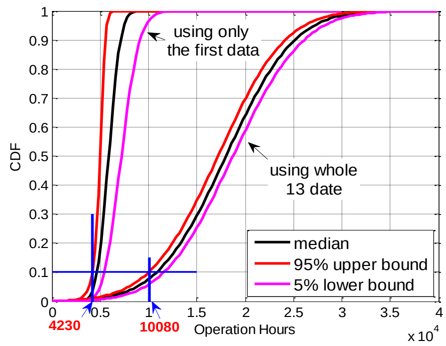


Fig. 5 CDF of Fatigue life

ACKNOWLEDGMENTS

This research was supported by Basic Science Research Program Through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2008-02-010 and 2009-0081438)

REFERENCES

Andrieu, C., de Freitas, N., Doucet, A., and Jordan, M. 2003. An Introduction to MCMC for Machine Learning. *Machine Learning* 50: 5–43.

Gelman, A., Carlim, J.B., Strern, H.S., and Rubin, D.B. 2004. *Bayesian Data Analysis*. New York: CHAPMAN & HALL/CRC, Inc.

Kim, N.H., Pattabhiraman, S., and Houck Ⅲ L.A. 2009. Bayesian Technique for Incorporation Field Experience into Analytical Model for Life Predictions. *ASME International Mechanical Engineering Congress and Exposition*.

Marahleh, G., Kheder, A.R.I., and Hamad, H.F. 2006. Creep-Life Prediction of Service-Exposed Turbine Blades. *Materials Science* 42 4: 49-53.