Modeling Average Maintenance Behavior of Fleet of Airplanes using Fleet-MCS

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In aircrafts, fuselage inspections are performed regularly to remove large damages that threaten the safety of the structure. Recently, structural health monitoring techniques have been developed that uses sensors and actuators to detect damages on structures paving way for progressive inspection. The average maintenance hangar trips per airplane and the average number of panels replaced on it have a direct bearing on the cost of progressive inspection. The lifecycle of an airplane was modeled as blocks of damage propagation interspersed with inspection. The Paris model with random parameters is used to model damage growth, and detection probability during inspections is modeled by Palmberg expression. Conventionally. Monte Carlo Simulations delineate the process. In this paper, a fleet-MCS procedure is presented that predict the average behavior of a fleet of airplanes using simple analytical expressions. Fleet-MCS procedures reduce the high computational cost of Monte Carlo simulations in predicting the average fleet behavior while maintaining similar level of accuracy. Monte Carlo simulations involve random sampling and would require multiple simulations to predict the fleet average. Fleet-MCS procedure predicts the fleet average with a single run of the simulation reducing the computational burden. The fleet average from the regular MCS and the fleet-MCS has been compared in this paper and has been found to be in accordance with reasonable accuracy.

I. Introduction

TRADITIONALLY, aircraft structures have been designed using the concept of damage tolerance (Hoffman, 2009 [1]) in which the structures are designed to withstand small damage, and large damage is repaired through scheduled inspections and maintenance. This concept turned out to be more cost-effective than safe-life design because airplanes designed based on safe-life would be much heavier. In damage tolerance design, it is important to inspect the airplane regularly such that all damages that can possibly threaten the safety of the structure should be repaired.

Scheduling inspections requires a trade-off between the structural safety and lifecycle costs. For example, if an airplane is inspected every flight, the safety can significantly be improved but the cost of inspections could be very high, so only visual inspection by the pilot is conducted. The current practice is a thorough intrusive inspection, also called the 'C' type inspection at every 6,000 flights [2]. Kale et al. (2008)[3] showed that this interval is close to optimal for fuselage panels and that the lifecycle cost is reduced by 30% compared to the safe-life design, at the same level of safety. The long interval between manual inspections is due to high inspection costs and downtime.

Recently, structural health monitoring (SHM) systems have become available using on-board sensors and actuators. These systems can perform damage assessment as frequently as needed and work on condition based maintenance request leading to lower downtime and inspection costs (Boller (2000) [4]. Boller and Meyendorf (2008) [5] observed damage monitoring by SHM as a good tool to enhance inspection. However, the detectable damage size from most SHM devices is much larger than that of the manual inspections. The current technology of SHM allows to detect damage as low as 5 µm and to detect it at 20,000 cycles well before a panel's eventual failure at 46,000 cycles. (Papazian,et al (2009) [6]). These SHM devices can continuously monitor the damage growth and request for maintenance when damage grows beyond a threshold. In this process, many variables are involved to keep the airplane safe, such as the frequency of SHM inspections and the threshold for maintenance. In this paper, these variables are referred to as SHM parameters.

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Yang and Trapp (1975) [7] appear to have first formulated the problem of the determination of the optimum inspection frequency as a constrained minimization problem. They indicated that various variables including inspection frequency and inspection quality can be adjusted in such a way so as to minimize a pre-defined cost function. Hellevik et al. (1999)[8] optimized the pipeline thickness together with the inspection regime to minimize the total operational cost. Kleyner and Sandborn (2008) [9] minimize lifecycle cost for an automotive electronics application considering product reliability and warranty return cost. Mizutani and Fujimoto (1993) [10] presented a sequential minimization method which aims to find an optimal inspection strategy so that the total cost expected in the period between the present inspection and the next is a minimum. Kassapoglou (1997)[11] minimized the cost and weight for manufacturing of stiffened panels.

Kulkari and Achenbach (2007) [12] optimize inspection schedule by minimizing the total cost function. They model crack propagation using Paris law but by assuming constant material properties. In this paper, fatigue crack propagation in fuselage panels under repeated pressurizations is modeled using the Paris model with uncertain parameters. The Palmberg equation is used to model the probability of damage detection during the inspection process. Due to uncertainties of initial damage size and Paris model parameters, the damage sizes after a certain number of flights are randomly distributed. The inspection truncates the high tail portion of the distribution by detecting large damages and replacing the affected panels. Since it is extremely difficult to model the analytical distribution of damage sizes after propagation and replacement, Monte Carlo simulation (MCS) is employed for that purpose. The objective of the paper is to improve accuracy of lifecycle cost calculation with minimum computational effort. The objective is realized by modeling the average behavior of a fleet of airplanes to determine the average fleet cost.

The organization of the paper is as follows. In Section 2, the models for damage propagation, and probability of damage detection during inspection, have been presented. In Section 3, the process of detection and replacement has been explained. Section 4 presents the data used for illustration. Section 5 delineates the fleet-MCS procedure to model the average behavior of fleet of airplanes.

II. MODEL

A. Fatigue damage growth due to fuselage pressurization

A through thickness center crack in a fuselage panel of an airplane is termed as damage in this paper. The life of an airplane can be viewed as damage propagation cycles, interspersed with inspection and repair. The pressure difference between the interior and the exterior of the cabin during each flight is instrumental in propagating the damage. The damage propagation is modeled using the Paris model, which gives the rate of damage size growth as a function of damage half - size (a), pressure differential (p), thickness of fuselage panel (t), fuselage radius (r) and the material specific Paris parameters, C and m.

$$\frac{da}{dN} = C(\Delta K)^m, \qquad \text{………………} (1)$$

where $\Delta K = \frac{pr}{t}\sqrt{\pi a}$ is the range of stress intensity factor ………………. (2)

B. Inspection model

In a SHM–based maintenance assessment, the detection probability is modeled using the Palmberg equation given by,

$$P_d(a) = \frac{\left(2a / a_h \right)^\beta}{1 + \left(2a / a_h \right)^\beta} \qquad \text{……………….} (3)$$

The expression gives the probability of detecting a damage with size $2a$.In Eq (3), a_h is the damage size corresponding to 50% probability of deduction and β is the randomness parameter. The parameter a_h represents average capability of the inspection method, while β represents the variability in the process.

III. INSPECTION PROCESS MODEL

The inspection process monitors damage and replaces/repairs the fuselage panel when the damage reaches a critical size so as to threaten the safety of the airplane. In term of the distribution of crack sizes, the inspection process partially truncates the tail of the damage size distribution. The procedure is illustrated in Figure 1 for typical values. The red curve represents the damage size distribution when the airplane enters the maintenance hangar; i.e. before inspection. In the maintenance hangar, all damages detected and found to have a size greater than a_{rep} is replaced. Since the detection is also a random process, only a part of the tail of the damage size distribution is replaced. The partially truncated damage size distribution obtained after inspection is shown in green in Figure 1. This section discusses the inspection process considered in this paper

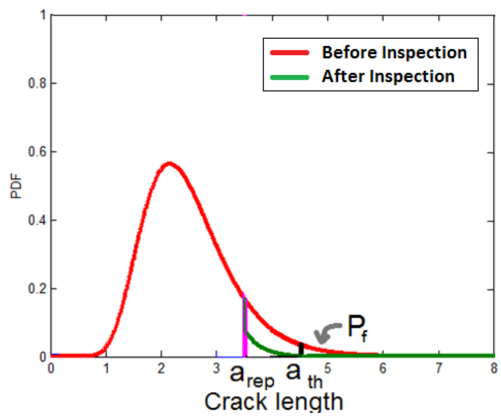


Figure 1: The effect of Inspection and Replacement process on crack length distributions

SHM-based damage assessment

SHM-based inspection uses on-board sensors and hence, can be performed frequently without hampering the operational schedule. Let N_{shm} denote the number of flights in between scheduled maintenance assessments using SHM. It is noted that the value of N_{shm} can be as low as one or two cycles and the maintenance assessment costs using SHM systems are quite negligible. The damage propagates in between the scheduled maintenance assessments due to repeated fuselage pressurization. During maintenance assessment, if damage is detected, and if its size found to be greater than a threshold value (a_{th}), the airplane is sent to a maintenance hangar. In the maintenance hangar, all the panels in the airplane are inspected by the on-board SHM equipment and those panels detected, and with damage size larger than another threshold value $a_{rep\text{-}shm}$, are replaced. The rationale for having another threshold, $a_{rep\text{-}shm}$ is to prevent sending the airplane back into the maintenance hangar at the next maintenance assessment.

Once inspected (and its damaged panels replaced), the airplane is brought back into service and repeated pressurization propagates damage for another N_{shm} flights till the next scheduled inspection / maintenance assessment. The flowchart in Figure 2 below depicts the maintenance scheduling procedure for SHM-based Inspection.

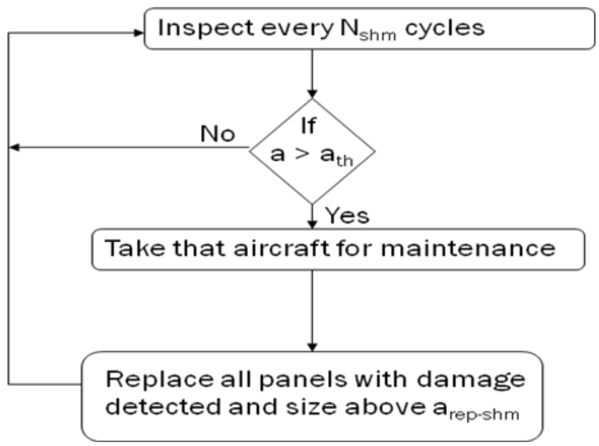


Figure 2: Flowchart depicting maintenance scheduling for SHM based Inspection

After inspection and maintenance, the tail portion of damage size distribution is partially truncated at $a_{rep\text{-shm}}$. For SHM-based inspection, the distribution beyond a_{th} corresponds to the percentage of panels with high damage size, missed due to randomness in the inspection process or to growth after the inspection. Hence, in Figure 1, the area beyond a_{th} for the green plot is the fraction of high damage panels missed during the inspection randomness and could fail in due time. The difference between area beyond a_{rep} for the green and red plots gives the fraction of panels replaced during that particular inspection.

It can be seen clearly that for SHM based inspections, the safety of the airplane depends on the frequency of inspections (N_{shm}), the threshold for sending the airplane for maintenance (a_{th}), and the threshold for replacement for highly damaged panel in the maintenance hangar ($a_{rep\text{-}shm}$). These variables have been collectively called the SHM parameters.

IV. DATA USED FOR ILLUSTRATION

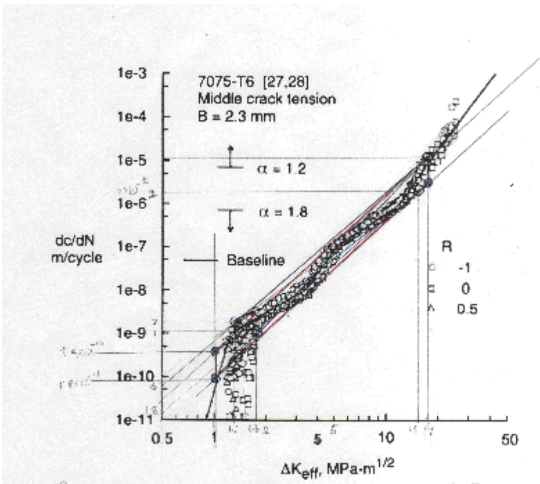
Aluminum alloy 7075 – T6 is considered as the material of the fuselage. All panels are of dimension 609.6mm × 609.6mm× 2.48mm and are assumed to possess a single crack at the center of the panel and subjected to tension. Table 2 shows the parameters used.

Table 2: Parameters and their values

Parameter	**Type**	**Value**
Initial damage size (a_0)	Random	*LN*(0.2, 0.07)
Pressure (p)	Random	*LN*(0.06,0.003)
Radius of fuselage (r)	Deterministic	3.25 m
Thickness of fuselage panel (t)	Deterministic	2.48 mm
Paris Law constant (C)	Random	*U*[5E-11, 5E-10]
Paris Law exponent (m)	Random	*U*[3, 4.3]
Palmberg parameter for SHM based inspection ($a_{h\text{-}shm}$)	Deterministic	5 mm
Palmberg parameter for SHM based inspection (β_{shm})	Deterministic	5.0

Newmann et al (1999) (Pg 113, Fig. 3)[13] shows the experimental data plot between the damage growth rate and the effective stress intensity factor for Al 7075 – T6 with a center crack in tension. The picture has been reproduced in Figure 3 below. The Paris law parameters *C* and *m* are estimated from the intercept and slope, respectively, of the region corresponding to stable damage propagation in the figure.
The data points in the region of stable damage propagation do not lie on a straight line in the log-log scale plot. Hence the region was visualized as bounded by a parallelogram with one edge parallel to the ordinate axes and the other edge parallel to the best fit straight line through the data points. The left edge of the parallelogram has a ΔK_{eff} value equal to one. As the region of the stable damage propagation can be bounded by a parallelogram, only the estimates of the bounds of the parameters, *C* and *m*, are obtained from the figure(Fig. 3, Newmann et al (1999)).

For the same reason, for a given value of intercept *C*, there is only a range of slope (*m*) values permissible. To parameterize the bounds, the left and right edges of the parallelogram were each assumed to be uniformly distributed. Each point on the left edge corresponds to a value of C chosen. For a given value of C chosen, there are only certain possible values of the slope, m. Figure 4 plots those permissible ranges of slope (*m*), for a given value of intercept (*C*). It can be clearly seen from Fig. 4 that the slope, and log(*C*) are negatively correlated; the correlation coefficient is found to be -0.8065.



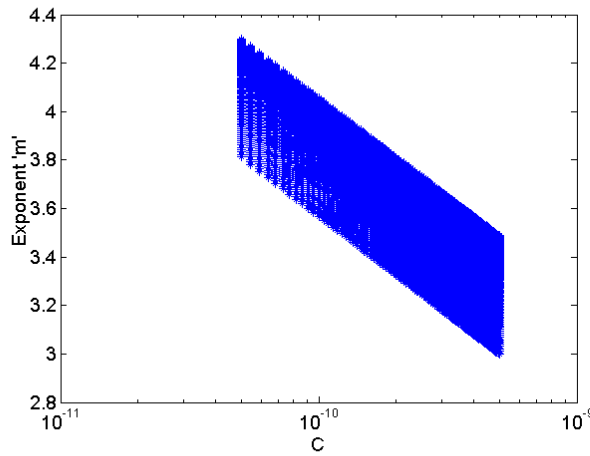


Figure 3: log-log plot of da/dN and ΔK for Al 7075- T6 Figure 4: Correlation between Paris model parameters

V. FLEET MONTE CARLO SIMULATIONS

With SHM based inspection, the maintenance is requested when damage is detected and found to have size greater than a_{th}. In the maintenance hangar, all panels of an airplane are inspected and the panels with damage detected and damage size found to be greater than $a_{rep\text{-}shm}$, are replaced.

A fleet of airplanes is assumed to comprise of 2000 airplanes and 500 fuselage panels / airplane. The damage size distribution is representative of the entire fleet of airplanes. At a given maintenance assessment, the cumulative distribution function (CDF) of the damage size distribution at a_{th} provides a measure of the fraction of panels in the fleet with damage detected and size > a_{th}. Let the number of such panels in a fleet be denoted, n_{th}. Similarly, at a given maintenance assessment, the CDF of the damage size distribution at $a_{rep\text{-}shm}$ provides a measure of the fraction of panels in the fleet with damage detected and size > $a_{rep\text{-}shm}$. Let the number of such panels be denoted, n_{rep}. It is difficult to find an analytical expression for the damage size distribution after N flights of propagation. In this paper, Monte Carlo simulations have been utilized to find the values of n_{th} and n_{rep}.

For instance, when n_{th} = 5 at a given maintenance assessment, these 5 highly damaged panels could lie on 5 different airplanes or they could be all part of a single airplane. Similarly, of the n_{rep} panels in the fleet with damage size > $a_{rep\text{-}shm}$, only those that lie on the airplanes that were sent for maintenance would be replaced. The number of airplanes sent to maintenance hangar and the number of panels replaced is usually determined by Monte Carlo simulations. In this regular MCS, all the 10^6 panels are randomly assigned to lie on 2000 airplanes. The number of airplanes to be sent to maintenance is decided based on the unique number of airplanes, the n_{th} panels lie. The number of panels replaced is based on the number of n_{rep} panels that lie on those airplanes sent for maintenance. As this regular MCS depends on randomly assigning panels to airplanes, the output is quite uncertain. To compute the average behavior of the fleet, the regular MCS process is simulated for six runs and average of the six simulations is computed.

As the frequency of maintenance assessment is increased, the simulations become computationally expensive. To reduce the computational burden, this paper proposes a method termed 'fleet-MCS' that use analytical expressions to capture the average behavior of the fleet. Fleet-MCS would require only run of the simulation and would prove to be much cheaper in capturing the average behavior of the fleet. It is noted that the average values of n_{th} and n_{rep} at each maintenance assessment has been obtained from the MCS and the average trips to maintenance and the average number of panels replaced / airplane have been calculated using the fleet-MCS expressions.

Average number of airplanes sent for maintenance:

In a fleet of 2000 airplanes, the average number of airplanes the n_{th} panels would lie on needs to be determined. Values of n_{th} were assumed to range between [0, 5000]. An upper limit of 5000 corresponds to 2.5 large cracks per airplane. Given the frequency of maintenance assessment for SHM systems, having 2.5 large cracks per airplane is an extreme case. A range of n_{th} values have been considered to obtain an analytical relation for the number of airplanes sent for maintenance. For each value of n_{th} chosen, a collection of n_{th} panels were randomly assigned to 2000 airplanes. The number of unique airplanes these collection of n_{th} panels lie on were counted. This process is repeated 100 times and the average of the number of unique airplanes is computed. This average will be the average number of airplanes sent to maintenance for a given n_{th}.
Figure 5 below plots the fraction of airplanes sent for maintenance as a function of ratio of n_{th} to number of airplanes. The band represents 2 times the standard deviations with respect to the mean value of n_{th}.

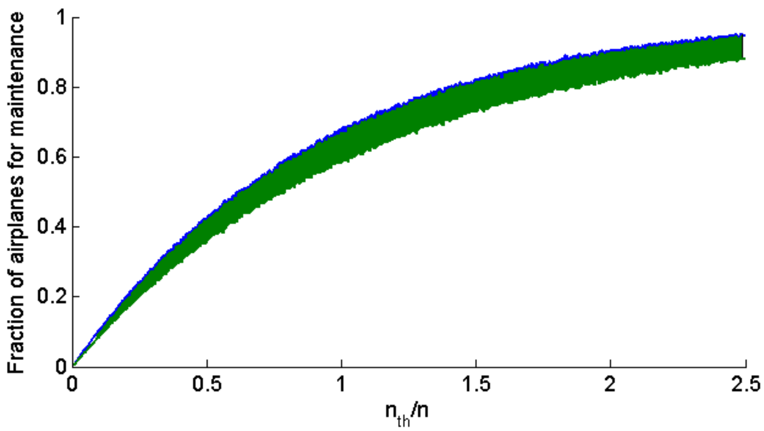


Figure 5: Variation of the fraction of airplanes sent to maintenance as a function of the ratio of n_{th} to airplanes

Another case with 200 airplanes and n_{th} ~ [0, 500] was considered plot between the ordinate and abscissa of Figure 5 resulted in an identical plot as Figure 5. The variation of the mean value of n_{th} was found to be modeled by $F(x) = 1 - e^{-x}$ as shown in Figure 6 below.

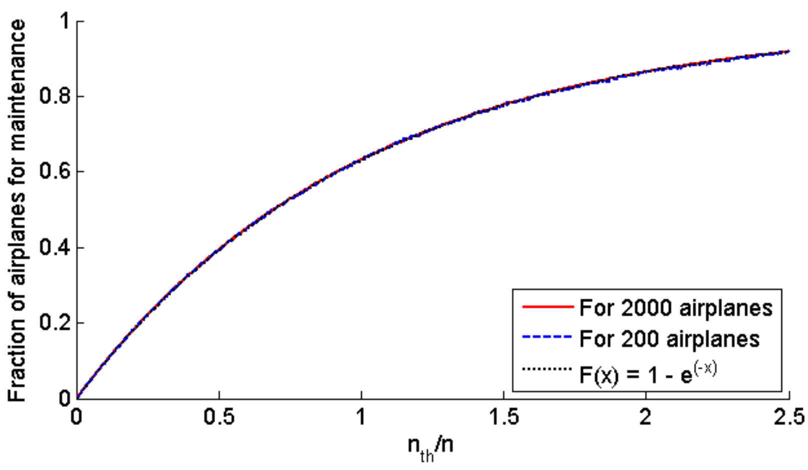


Figure 6: Variation of fraction of airplanes to maintenance as a function of the ratio of n_{th} to airplanes for different fleet of airplanes and modeling of the trend using a function

Based on Figure 6, the number of airplanes sent to maintenance (n_AM) can be determined from the following relation,

$n\_AM = [\ 1 - \exp(- n_{th}/n)] * n$ ………………….. (4)

Where, n is the size of the fleet.

Whenever an airplane goes into the maintenance hangar, it takes a while for it to go into the hangar again. The period where the airplanes is not considered as a maintenance hopeful is termed as "maintenance cool – off" period. This maintenance cool-off period has been assumed to be equal to 2000 flights. In Eqn (4), airplanes would simply the maintenance hopeful airplanes of the fleet.

For comparison, a fleet of 2000 airplanes and 500 panels / airplane were considered. The life of the airplane is considered as 50,000 flights and SHM based maintenance assessment is performed every 1,000 flights. A 2000 flight maintenance cool-off period has also been considered for the fleet MCS procedure. a_{th} = 40 mm and $a_{rep-shm}$ = 10mm were chosen. Table 3 below compares the results between Monte Carlo simulations (MCS) and fleet-MCS. In both these methods, the values of n_{th} and n_{rep} have been determined from MCS. In the regular MCS, the average number of maintenance trips per airplane and the average number of panels replaced per airplane are also determined using MCS, while in fleet-MCS method, analytical expressions have been used for the same Using the average values of n_{th} and n_{rep} values for those six simulations. Since the average values of n_{th} and n_{rep} have been used, the value of output predicted by fleet-MCS remains constant for different simulations. The measure of variability in the fleet-MCS values is modeled by Figure 5.

Table 3: Comparing the average number of maintenance hangar visits per airplane during 50,000 flights for MCS and fleet-MCS for 6 runs of simulation

Run	**Average trip per Airplanes for maintenance**	
	From fleet-MCS	**From regular MCS**
1	3.28	3.31(0.94)
2	3.28	3.32 (0.96)
3	3.28	3.30 (0.98)
4	3.28	3.32 (0.95)
5	3.28	3.33 (0.95)
6	3.28	3.27 (0.96)
Mean	**3.28**	**3.31 (0.96)**
SD		**0.02 (0.01)**

For each run of simulation, due to random assignment of panels to airplanes, some airplanes are sent to maintenance 6 times and some just a single time. Fleet average and standard deviation (in parenthesis) are tabulated for each run of simulation in Table 3. Table 3 shows considerable accordance between the fleet-MCS and average of the MCS predicted values. The regular MCS procedure has a lot of noise in its data and would require averaging out many runs of simulation to be considered reliable. This process is computationally very expensive. Fleet-MCS characterizes the average behavior of the fleet through simple analytical expressions thereby reducing the computational cost greatly.

<u>Average number of panels replaced:</u>

Damage size greater than a_{th} will also be greater than $a_{rep-shm}$. Hence, n_{th} is a subset of n_{rep}. In the maintenance hangar, all the high damage n_{th} panels will be replaced. The number of remaining n_{rep} panels ($n_{rep} - n_{th}$) that would be replaced in the hangar needs to be determined. Eqn (5) gives the average number of airplanes sent to maintenance hangar at a given maintenance assessment. The average number of remaining n_{rep} panels that would lie on these n_AM airplanes is calculated in the Eqn (6) below.

Average number of panels replaced (PR) = $n_{th} + (n_{rep} - n_{th}) * n\_AM / n$ ……………. (6)

Where, n is the number of airplanes.

The maintenance-cool off period provides an interesting issue on the value of 'n' to choose.

If 'n' is the total number of airplanes in the fleet , the assumption would be that all the n_{rep} panels are evenly distributed among all the airplanes in the fleet and the average number of panels replaced thus calculated will be under-predicted.

If 'n' is does not include the airplanes in the maintenance cool-off period, the assumption would be that there are no n_{rep} panels in those airplanes in the cool-off period. During the maintenance cool-off period the airplane still remains in service but won't be considered maintenance-hopeful. Hence, the airplanes in cool-off period will have a damage size greater than a_{rep}. This assumption is also flawed and the average number of panels thus calculated would be over-predicted.

Based on the discussion above, the value of 'n' has been calculated as the harmonic mean of total airplanes in fleet and the number of maintenance hopefuls at a given maintenance assessment.

Table 4 below compares the values of average maintenance hangar visits / airplane and the average number of panels replaced between the MCS and fleet-MCS process

Table 4: Comparing the average maintenance hangar visits per airplane and the average number of panels replaced per airplane between MCS and fleet-MCS processes for 6 runs of the simulation

Run	Average trip per Airplane for maintenance		Averge Panels replaced per airplane	
	Fleet-MCS	**MCS**	**Fleet-MCS**	**MCS**
1	3.28	3.31(0.94)	9.55	8.68 (3.22)
2	3.28	3.32 (0.96)	9.55	8.72 (3.19)
3	3.28	3.30 (0.98)	9.55	8.65 (3.24)
4	3.28	3.32 (0.95)	9.55	8.71 (3.13)
5	3.28	3.33 (0.95)	9.55	8.75 (3.21)
6	3.28	3.27 (0.96)	9.55	8.68 (3.18)
Mean	**3.28**	**3.31 (0.96)**	**9.55**	**8.70 (3.20)**
SD		**0.02 (0.01)**		**0.03 (0.03)**

Similarly, due to random assigning of panels to airplanes in regular MCS, the number of panels replaced per airplane is uncertain. An airplane could have simply only one panel until its lifetime, or it could have about 25 panels replaced in it during its lifetime. The average number of panels replaced and its standard deviation (in parenthesis) is tabulated in Table 4. The fleet-MCS slightly over-predicts the average values predicted from MCS. The method may need some refinement. With reasonable accuracy, we can conclude that the fleet-MCS expressions are able to model the average behavior of a fleet of airplanes with a fraction of the computational cost involved.

VI. CONCLUSIONS AND FUTURE WORK

- The paper focuses on modeling average behavior of a fleet of airplanes by reducing the computational cost
- Due to uncertainty in material properties and loading conditions, the damage size distribution at a maintenance assessment will be distributed.
- Since Monte Carlo simulations are generally used to model the process, the process is computationally expensive and also the values predicted are noisy.
- This paper focuses on modeling the average behavior of the fleet of airplanes in predicting the average maintenance trips and average number of panels replaced per airplane.
- The fleet MCS procedure is found to be computationally cheaper than **monte carlo** simulations while maintaining the same level of accuracy

- In future, the growth of damage size distribution would be modeled by direct integration procedure thereby giving consistent values of nth and nrep at each maintenance assessment.
- This would lead to consistent values of fleet behavior with high reliability and low computational cost.

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