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BAYESIAN APPROACH FOR FATIGUE LIFE PREDICTION FROM FIELD DATA

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ABSTRACT
Due to uncertainty in design, manufacturing and operating processes, the initial prediction of a machine's useful life is often quite different from that of the actual machine. In this paper, we utilize the Bayesian technique to incorporate the field data with the initial predictions in order to improve the prediction. The field data is interpreted in terms of the probability of having defective hardware, and then the likelihood function is generated from the binomial distribution. Since the predictions incorporate field experience, as time progresses and more data becomes available the probabilistic predictions are continuously updated. This results in a continuous increase of confidence and accuracy of the prediction. The resulting distributions can then be used directly in risk analysis, maintenance scheduling, and financial forecasting by both manufacturers and operators of heavy-duty gas turbines. This presents a quantification of the real time risk for direct comparison with the volatility of the power market.

1. INTRODUCTION
Modern heavy-duty gas turbines are subject to numerous factors that have an influence on their lifetime, including but not limited to, environmental effects, maintenance processes, manufacturing processes, and material properties scatter.

Mücke, et al [1] used Monte Carlo Simulations to determine the distributions of burst strength, creep lifetime and low cycle fatigue of turbine blades accounting for scatter in aforementioned factors. Voigt, et al [2], Weiss, et al [3] used a combination of Monte Carlo Simulation and Response Surface Method to probabilistic assessment of turbine blade components.

Even with today's modern computing systems, it is infeasible to include all the relevant input variables into the analytical prediction, since all of the potential inputs are not known in the design phase. To account for the unknown variables, common practices use so called "safety factors" or statistical minimum properties in conjunction with the numerical method when evaluating lifetimes.

Due to these estimations, often analytical predictions are not in agreement with the engine fleet experience, which again may vary across the fleet. Often a gap exists in correlating the field data with the analytical predictions. Thus, there is an increased need to improve the analytical predictions using field data, which somehow represents the real status of a particular machine.

In addition, the analytical predictions often use damage parameters that are calculated in the laboratory environment. However, due to uncertainty in a particular batch of material process and due to variability in manufacturing process, the actual damage parameters of a particular machine can be different from that of the generic material. Even if the analytical model can cover wide range of distribution of damage parameters, it is still required to obtain more accurate information of the damage parameters of a particular machine. For this purpose, the field data can play an important role to reduce the distribution of the analytical prediction with actual information.

One of the important design criteria is the useful life of each part a machine. In most cases, computational models provide an initial estimate of the useful life. As the expected remaining useful life (RUL) of a machine changes according to the operating conditions and history, it is important to constantly update it based on the feedback from the field data. This is a time dependent iterative procedure, through which a more refined estimation can be made.

Many attempts have been made in literature to determine the RUL from the field data. Orchad et al [4] used particle filtering and learning strategies to predict the RUL of a defective component. Marahleh et al [5] predicted the creep life from test data, using Larson-Miller parameter. Park et al [6] used energy-based approach to predict constant amplitude multiaxial fatigue life. Guo et al [7] performed the reliability analysis for wind turbines using maximum likelihood function, incorporating test data.

The Bayesian theorem [8] has its roots in the conditional probability and has been used as a novel method to update distributions of life with additional test/field data. Acar et al [9] showed the conservativeness of Bayesian update. Guerin et al [10] used Bayesian update to update the life of rubber like boot seal material in automobiles. Cross et al [11] used Bayesian technique to update probabilistic structural life models with maintenance data. Gogu et al [12] performed two-dimensional Bayesian update to update the joint PDF of material constants simultaneously.

In this paper, the Bayesian theorem is utilized to incorporate field data with analytical predictions of fatigue life distribution. The analytical predictions are obtained either from numerical models or laboratory tests. The field data, although they are noisy, invariably portray environmental factors, measurement errors, and loading conditions, or in short, reality. The RULs of several turbine components under low-cycle fatigue and oxidation damage are estimated in terms of the probability of defects. Since the predictions incorporate field experience, as time progresses and more data are available, the probabilistic prediction is continuously updated. This results in a continuous increase of confidence and accuracy of the prediction. The resulting distributions can then be used directly in risk analysis, maintenance scheduling, and financial forecasting by both manufacturers and operators of heavy-duty gas turbines. This presents a quantification of the real time risk for direct comparison with the volatility of the power market.

2. BAYSIAN INFERENCE FOR FATIGUE LIFE DISTRIBUTION

In this section, Bayesian inference is explained in the view of updating distribution of fatigue life using test data. The Bayesian theorem is first presented in a general form, followed by a specific expression for estimating the distribution of fatigue life.

2.1. Bayes' Theorem

Bayesian inference estimates the degree of belief in a hypothesis based on collected evidence. Bayes [8] formulated the degree of belief using the identity in conditional probability:

$$P(X \cap Y) = P(X \mid Y)P(Y) = P(Y \mid X)P(X) \tag{1}$$

where $P(X|Y)$ is the conditional probability of X given Y. In the case of estimating the probability of fatigue life using test data, the conditional probability of event X (i.e., fatigue life) when the probability of test Y is available can be written as

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)} \tag{2}$$

where $P(X|Y)$ is the posterior probability of fatigue life X for given test Y, $P(Y|X)$ is called the likelihood function or the probability to get test Y for given fatigue life X. In Bayesian inference, $P(X)$ is called the prior probability, and $P(Y)$ is the marginal probability of Y and acts as a normalizing constant. The above equation can be used to improve the knowledge of $P(X)$ when additional information $P(Y)$ is available.

The Bayes' theorem in Eq. (2) can be extended to continuous probability distribution with probability density function (PDF), which is more appropriate for the purpose of the present paper. Let f_X be a PDF of fatigue life X. If the test measures a fatigue life Y, it is also a random variable, whose PDF is denoted by f_Y. Then, the joint PDF of X and Y can be written in terms of f_X and f_Y, as

$$f_{XY}(x, y) = f_X(x \mid Y = y) f_Y(y) = f_Y(y \mid X = x) f_X(x) \tag{3}$$

When X and Y are independent, the joint PDF can be written as $f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$ and Bayesian inference cannot be used to improve the probabilistic distribution of $f_X(x)$. Using the above identity, the original Bayes' theorem can be extended to the PDF (Papoulis [13]) as

$$f_X(x \mid Y = y) = \frac{f_Y(y \mid X = x) f_X(x)}{f_Y(y)} \tag{4}$$

Note that it is trivial to show that the integral of $f_X(x|Y = y)$ is one by using the following property of marginal PDF:

$$f_Y(y) = \int_{-\infty}^{\infty} f_Y(y \mid X = \xi) f_X(\xi) \mathrm{d}\xi \tag{5}$$

Thus, the denominator in Eq. (4) can be considered as a normalizing constant. By comparing Eq. (4) with Eq. (2), $f_X(x \mid Y = y)$ is the posterior PDF of fatigue life X given test $Y = y$, and $f_Y(y \mid X = x)$ is the likelihood function or the probability density value of test Y given fatigue life $X = x$.

When the analytical expressions of the likelihood function, $f_Y(y \mid X = x)$, and the prior PDF, $f_X(x)$, are available, the posterior PDF in Eq. (4) can be obtained through simple calculation. In practical applications, however, they may not be in the standard analytical form. In such a case, the entire range of X is discretized first, and the values of PDF at discrete points need to be calculated. This can be a computationally intensive process especially when X is a random vector because the calculation should be performed at grid points of multi-dimensional mesh. For the calculation of likelihood function, at each discrete point of $X = x$, the PDF f_Y is calculated first and then the likelihood is the value of PDF f_Y at $Y = y$. When the uncertainty structure of Y is known, this can be a straightforward process. Otherwise, Monte Carlo simulation (MCS) can be performed first to estimate the cumulative distribution function (CDF), and then the PDF can be obtained by differentiating the CDF. Again, this process can be computationally intensive because the posterior PDF is calculated at discrete grid points, and MCS is required at each grid point.

When multiple, independent tests are available, Bayesian inference can be applied either iteratively or all at once. When N number of tests are available; i.e., $\mathbf{y} = \{y_1, y_2, \ldots, y_N\}$, the Bayes' theorem in Eq. (4) can be modified to

$$f_X(x \mid Y = \mathbf{y}) = \frac{1}{K}\prod_{i=1}^{N}\left[f_Y(y_i \mid X = x)\right] f_X(x) \tag{6}$$

where K is a normalizing constant. In the above expression, it is possible that the likelihood functions of individual tests are multiplied together to build the total likelihood function, which is then multiplied by the prior PDF followed by normalization to yield the posterior PDF. On the other hand, one-by-one update formula for Bayes' theorem can be written in the recursive form as

$$f_X^{(i)}(x \mid Y = y_i) = \frac{1}{K_i} f_Y(y_i \mid X = x) f_X^{(i-1)}(x), \quad i = 1, \ldots, N \tag{7}$$

where K_i is a normalizing constant at i-th update and $f_X^{(i-1)}(x)$ is the PDF of X, updated using up to $(i-1)$th tests. In the above update formula, $f_X^{(0)}(x)$ is the initial prior PDF, and the posterior PDF becomes a prior PDF for the next update.

In the view of Eqs. (6) and (7), it is possible to have two interesting observations. Firstly, the Bayes' theorem becomes identical to the maximum likelihood estimate when there is no prior information; i.e., $f_X(x)$ = constant. Secondly, the prior PDF can be applied either at first or at last. For example, it is possible to update the posterior distribution without prior information and then to apply the prior PDF after the last update.

An important advantage of Bayes' theorem over other parameter identification methods, such as the least square method and maximum likelihood estimate, is its capability of estimating the uncertainty structure of the identified parameters. These uncertainty structures depend on that of the prior distribution and likelihood function. Accordingly, the accuracy of posterior distribution is directly related to that of likelihood and prior distribution. Thus, the uncertainty in posterior distribution must be interpreted in that context.

2.2. Application to Fatigue Life Estimation

In deriving the Bayes' theorem in the previous section, it requires two sets of information: a prior PDF and a likelihood function. In estimating fatigue life, the prior distribution can be obtained from numerical models and laboratory tests. Since they can be performed multiple times with different input parameters that represent various uncertainties, it is possible to evaluate the distribution of fatigue life, which can be served as a prior PDF of fatigue life.

On the other hand, the field data cannot be obtained in the laboratory environment. In this section, it is presented how to use the field data in calculating the likelihood function. When a gas turbine engine is built and installed in the field, the maintenance/repair reports include the history of the number of parts that were defective and replaced at specific operating cycles. Although these data are not obtained under the controlled laboratory environment, they represent reality with various effects of uncertainties in environmental factors, measurement errors, and loading conditions. Thus, it is desirable to use these data to update the fatigue life of the specific machine using Bayes' theorem.

The standard approach to applying Bayes' theorem is to use the field data to build the likelihood function, which is basically the same PDF form with the fatigue life. However, different from specimen-level tests, the field data cannot be repeated multiple times to construct a distribution. Only one data point exists for specific operating operation cycles. Thus, the original formulation of Bayes' theorem needs to be modified. First, instead of updating the PDF of fatigue life, it is assumed that the distribution type of fatigue life is known in advance. This can be a big assumption, but it is possible that different types of distribution are assumed and the most conservative type can be chosen. Once the distribution type is selected, then it is necessary to identify distribution parameters. For example, in the case of normal distribution, the mean (μ) and standard deviation (σ) need to be identified. In this paper, these distribution parameters are assumed to be uncertain and Bayes' theorem is used to update their distribution; i.e., the joint PDF of mean and standard deviation will be updated. In this case, the vector of random variables is defined as $\mathbf{x} = \{\mu, \sigma\}$, and the joint PDF $f_{\mathbf{X}}$ is updated using Bayes' theorem. Initially, it is assumed that the mean and standard deviation are uncorrelated.

A field data set consists of number of hours of operation until inspection (N_f), and the number of defective blades (r) out of the total number of blades (n). Thus, the field data are represented by $y = \{N_f, n, r\}$. Then, the likelihood function is the PDF f_Y for given $\mathbf{x} = \{\mu, \sigma\}$. Since the field data is given at fixed N_f and n, f_Y can be represented in terms of r. Unfortunately, the number of defective blades cannot be a continuous number; it is an integer. Thus, the likelihood function f_Y can be represented using the following probability mass function:

$$f_Y\left(y \mid \mathbf{X} = \{\mu, \sigma\}\right) = \frac{n!}{r!(n-r)!}(P)^r(1-P)^{n-r} \tag{8}$$

where P is the probability of defects at given N_f for given $\mathbf{x} = \{\mu, \sigma\}$. Since the distribution of fatigue life is given as a function of $\mathbf{x}$, the probability of defects can be calculated by

$$P(\mu, \sigma) = \int_0^{N_f} f_{\text{life}}(t; \mu, \sigma)\, dt \tag{9}$$

where, f_{life} is the PDF of fatigue life distribution. The probability of mass function in Eq. (8) is a binomial distribution, which models the probability distribution of having 'r' defects out of 'n' samples with defect probability of P. Figure 1 illustrates the relation in Eq. (9).

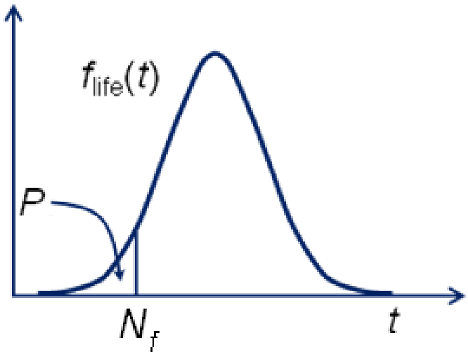


Figure 1: Probability of defects calculation from life distribution

The procedure of Bayesian update is as follows. First, the range of mean and standard deviation are divided by 500×500 grids. At each grid point (i.e., each value of μ and σ), the value of likelihood function is calculated. This will construct the discrete likelihood function for Bayes' theorem. Then, Eq. (7) is employed to update the joint PDF of mean and standard deviation. Once the joint PDF is updated, the distribution of life is estimated using the mean values of the joint PDF:

$$f_{\text{life}}(t) = N(\mu_\mu, \mu_\sigma) \tag{10}$$

where μ_μ and μ_σ are the mean values of mean and standard deviation, respectively.

3. ANALYTICAL EXAMPLE

In this section, the properties of Bayes' theorem are studied using analytical examples. In particular, the effects of noise and bias of data on the final distribution are discussed in detail. The data here is simulated to demonstrate the new analytical technique. This data is then perturbed in order to study the effects of noise and bias errors on the algorithm.

3.1 Field Data Governed by a Distribution

The first study is to test the accuracy of updated distribution using the Bayes' theorem. Table 1 shows an example of field data that are used in this section. The total number of blades is n = 100. The field data are generated from a known distribution, $B \sim N(12000, 2000)$, and the objective is to test if the updated distribution recovers the distribution B. For the prior distribution, the mean is uniformly distributed in the interval of [0, 30000], while the standard deviation is also uniformly distributed in the interval of [0, 4500]. Using the four sets of field data in Table 1, the posterior joint PDF are calculated using Bayes' theorem. After obtaining the posterior joint PDF, the mean values are used to predict the distribution of life. Figure 2 compares the updated distribution of fatigue life with the distribution B, along with the predicted probability of defects from field data. It is clear that both distributions match each other quite well. When the field data are governed by a particular distribution, Bayes' theorem can reproduce the distribution well when more than 3 sets of data are available.

Table 1: Sample field data for inspected turbine blades

Engine	Operation Hours	No. of Defective Blades
1	9,437	10
2	10,317	20
3	10,951	30
4	11,493	40

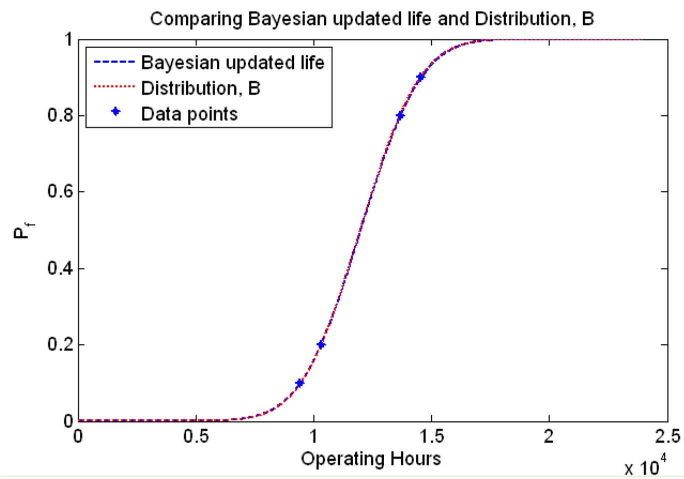


Figure 2: Comparison between the updated life distribution using Bayes' theorem and the original distribution

3.2 Effect of Bias

In practice, the field data are often accompanied by noise and bias. The former is caused by variability in measurement environment, while the latter represents systematic departure, such as calibration error. The difference is that the former is random, while the latter is deterministic, although its value is unknown. In some cases, a positive bias is consciously applied to remain conservative. This section analyzes the effect of bias on the updated life distribution using three sets of sample data shown in Table 2. The data are generated from the distribution $B \sim N(12000, 2000)$. All data sets are symmetrically located about the 50% probability of defects location.

The bias is given in terms of the number of defective blades. For example, bias = 5 means that the number of defective blades is five more than the nominal numbers. The bias is imposed all data in the same set. The bias affects the updated distribution of mean and standard deviation. Figure 3 shows the variations of the mean of mean and the mean of standard deviation due to different magnitudes of bias. All data are normalized by 12,000 hours. It is found that bias has a minimal effect on the standard deviation. A positive bias leads to a conservative estimate of the mean of the updated life distribution.

Table 2: Sample field data sets for bias study

Set	Engine	Operation Hours	No. of Defective Blades
1	1	9,437	10
	2	12,000	50
	3	14,563	90
2	1	10,317	20
	2	12,000	50
	3	13,683	80
3	1	10,951	30
	2	12,000	50
	3	13,049	70

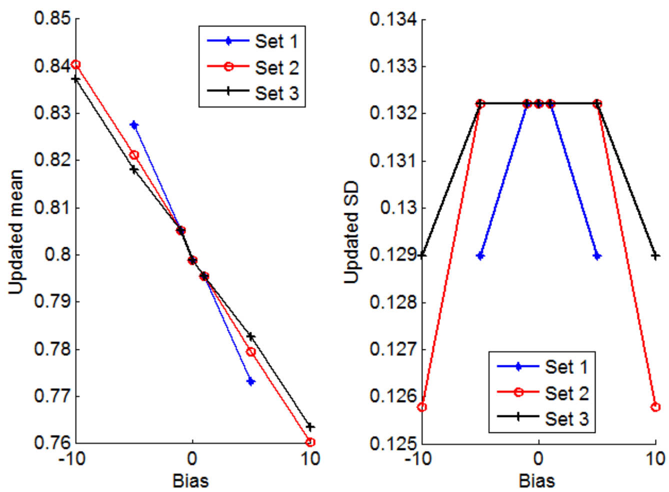


Figure 3: Variation of mean and standard deviation due to bias in data

3.3 Effect of Noise

In general, the field data contains variability caused by environmental and operational conditions. This variability is random and is called a noise. As opposed to bias, the noise is induced on individual field data. In this section, the nominal field data in Table 2 is perturbed by 10% to study the effect of noise on the updated distribution. When 10 blades need to be removed from service, (Engine 1 in Set 1, $P_f = 0.1$), 10% noise means the field data show the removal from service of 11 blades. On the other hand, when 90 blades are defective (Engine 3 in Set 1, $P_f = 0.9$), -10% noise means the field data show that 81 blades are defective. By perturbing each data individually, the updated joint PDF is calculated first. Then, the mean of mean and the mean of standard deviation are plotted against the nominal value of probability of defects, which is shown in Figure 4. The solid horizontal lines are the updated mean and standard deviation without having noise. The trends for updated mean and standard deviation follow a mirror image pattern for negative and positive noises. A positive noise gives a conservative estimate of the updated mean, while a negative noise gives an unconservative estimate. In addition, the higher the probability of a defective component is, the greater the effect of noise on the updated mean. This can be explained from the fact that 10% noise is interpreted as one more defective blade when $P_f = 0.1$, while it means nine defective blades when $P_f = 0.9$.

The variation of standard deviation with noise follows an interesting pattern. The noise at $P_f = 0.5$ does not affect the updated standard deviation. The effect on standard deviation depends on the location of the field data on distribution and also on the other field data present in the set.

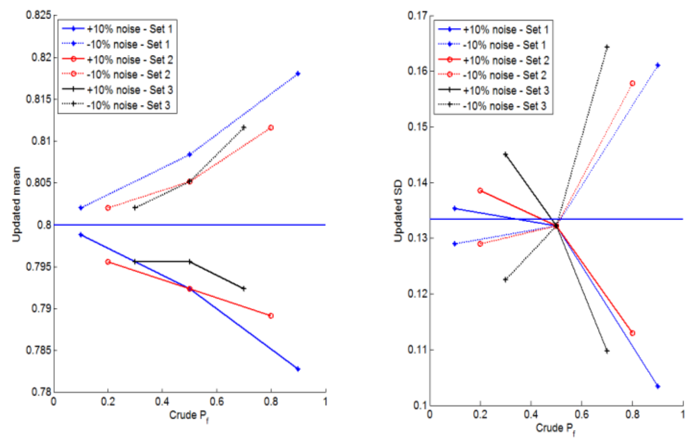


Figure 4: Effect of ±10% noise on three sets of field data

Instead of having a constant fraction of noise, the effect of noise caused by a constant number of defective blades is shown in Figure 5. The effect of constant noise on the updated mean is almost negligible. At the point of symmetry for all 3 field data sets, there is no effect of noise on the updated standard deviation. The trend of negative and positive noise on updated standard deviation mirrors itself at this point of symmetry.

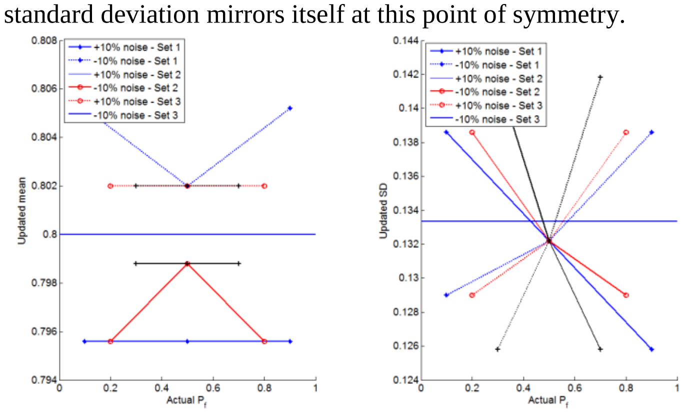


Figure 5: Effect of constant noise on 3 sets of field data

4. UPDATE OF LOW-CYCLE FATIGUE LIFE

In this section, the Bayes' theorem is utilized to update the distribution of low-cycle fatigue life of gas turbine engines using actual field data. In this particular type of engine, the total number of blades is n = 40. A total of 13 engines are inspected at specific operation hours and the numbers of defective blades are recorded as shown in Table 3.

From the Monte Carlo simulation of analytical model, the normalized mean and standard deviation of fatigue life are estimated by 1.0 hour and 0.154 hours, respectively. Since these values are uncertain, we assume that the mean and standard deviation are normally distributed with the coefficient of variance equals 50%. Thus, the assumed initial distributions for the mean and standard deviation are summarized in Table 4.

Table 3: Field data for low-cycle fatigue defects of turbine blades

Engine	Date	Operation Hours (Normalized)	No. of Defective Blades
1	Jan-04	0.836	2
2	Mar-04	0.604	1
3	Mar-04	0.290	1
4	Jul-04	1.770	0
5	Sep-04	2.321	12
6	Sep-04	2.254	3
7	Oct-04	1.162	6
8	Nov-04	0.281	0
9	Feb-05	5.053	13
10	Mar-05	0.707	0
11	Apr-05	1.652	0
12	Nov-06	1.265	10
13	Mar-06	3.615	18

Table 4: Initial distribution of the distribution parameters

	Mean	Standard deviation
Mean (life)	1.0	0.5
Standard deviation (life)	0.154	0.077

Figure 6 shows an example of likelihood function. It is noted that the mean and standard deviation are strongly correlated. This makes sense because in order to have the same value of probability of defects, it is necessary to have either large mean with small standard deviation or small mean with large standard deviation.

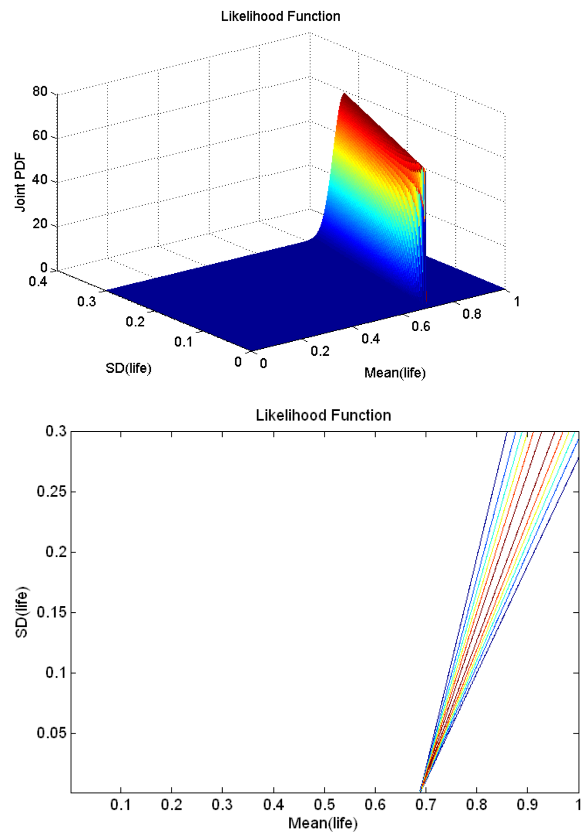
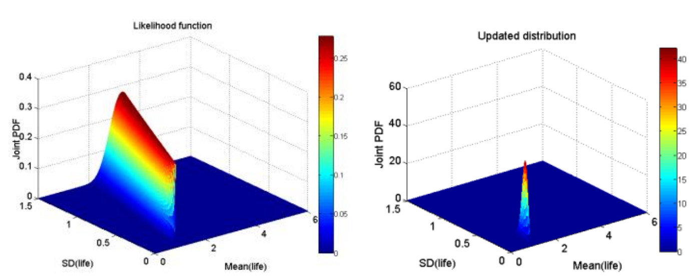


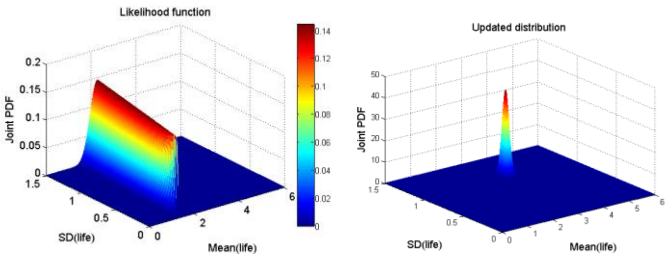
Figure 6: Contour plots of the likelihood function for the first field data

Using all thirteen field data in Table 3, the Bayesian update is performed to update the joint PDF of mean and standard deviation. After finishing updates, the mean values of the joint PDF are used to predict the distribution of fatigue life. Figure 7 shows a snap shot of likelihood functions and distribution of the joint PDF after update. It is clear that the distribution narrows down significantly as more updates are performed.

Once the updates are finished, the mean values of the joint PDF are used to estimate the distribution of the fatigue life. Figure 8 shows the updated distribution of fatigue life along with the predicted probability of defects from the field data. It is noted that both mean and standard deviation progressively increase. This means that the analytical prediction is overly conservative.



(a) First field data



(b) Twelfth field data

Figure 7: Likelihood functions and updated distributions

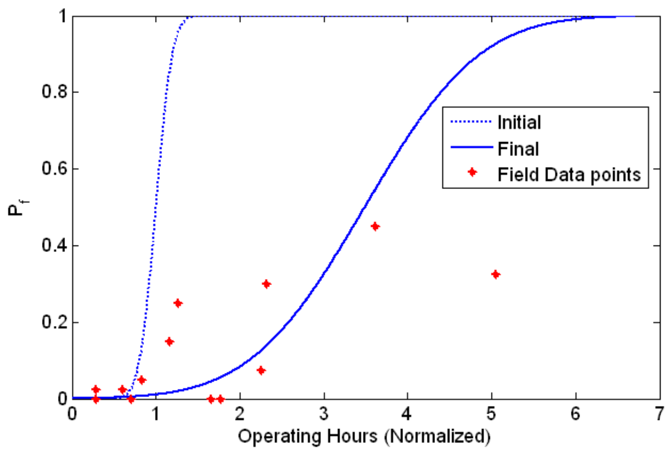


Figure 8: Updated distribution of fatigue life

5. UPDATE OF OXIDATION DAMAGE

For oxidation damage, the initial life prediction is estimated using 500 finite element analyses and the response surface method. The thermo-mechanical analysis of 60,000 quadratic elements with 180,000 nodes takes about 2.5 hours for Windows-based desktop computer. By applying Monte Carlo simulation on the response surface, the initial distribution of the oxidation damage life is obtained. Based on the distribution, it is assumed that the oxidation damage life is normally distributed with normalized mean of 1.0 and normalized standard deviation of 0.196 operating hours.

As oxidation takes a long time to set in, the number of field data is scarce. Four sets of field data, tabulated in Table 5 are used to update the joint PDF of mean and standard deviation of life. The joint PDF of mean and standard deviation of life after update with four sets of field data are shown in Figure 9.

The means of the Joint PDF is used to construct the distribution of life after each update. The initial and updated distributions of life are plotted in Figure 10.

As shown in Figure 10, all four field data show consistent behavior. All the field data points look to be governed by a particular distribution, and the updated distribution of life seems to model that distribution quite closely. However, the initial distribution is over predicted the distribution. In this case, the finite element analysis turns out to be unconservative. This can happen especially when the mathematical model does not take into account important uncertain parameters.

Table 5: Field data for oxidation damage

Engine	Hours	No. Parts	Scrapped	Reconditioned
1	0.464	33	3	30
2	0.605	32	6	26
3	0.685	32	9	23
4	0.953	32	25	7

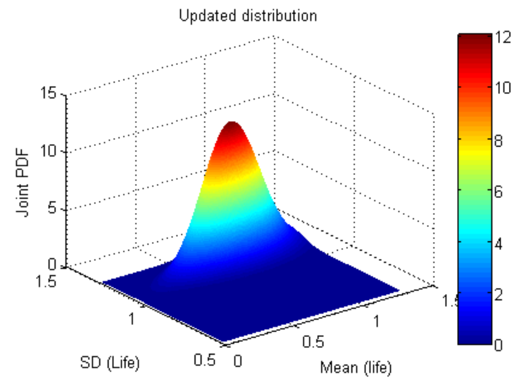


Figure 9: Updated Joint PDF of Mean and standard deviation of life

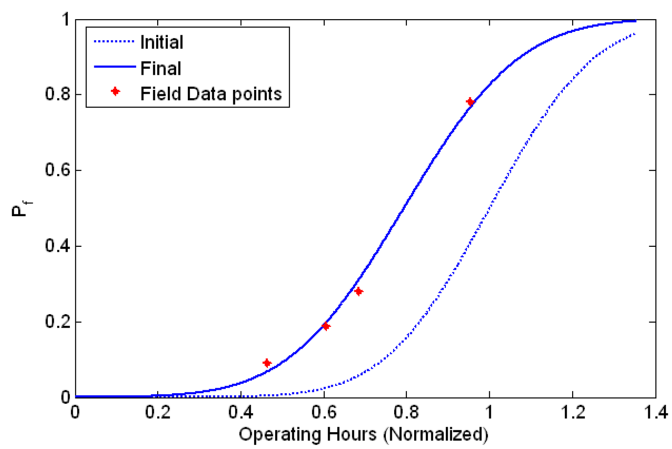


Figure 10: Initial and final distributions of life

To analyze the effect of assumption on distribution type, a lognormal distribution and a Weibull distribution are assumed on life, and the results are compared with the normally distributed life case in Figure 11. It can be readily seen that normal distribution of life predicts a more conservative estimate. The 1% probability of damage for the normal distribution life is 0.280 hours, while it is 0.376 hours for log-normal distribution life and 0.341 hours for Weibull distribution life.

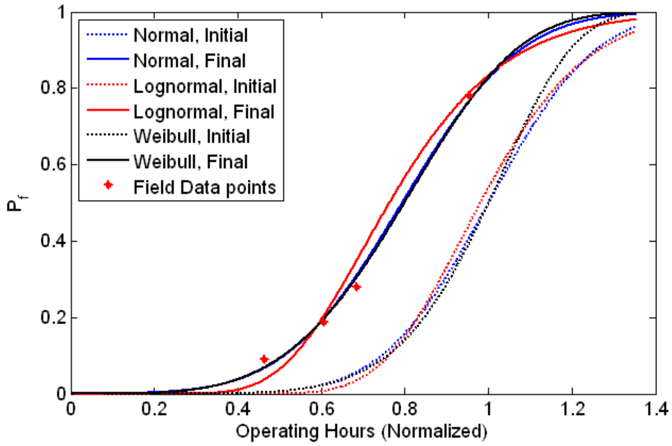


Figure 11: Comparison between types of distributions

6. DEVELOPMENT OF GRAPHICAL USER INTERFACE

The proposed method of updating posterior distribution using field data is a relatively simple process in which it only requires prior distribution and counting the number of defective samples during scheduled inspection. In order to make this method available to the field engineers, a graphical user interface (GUI) is developed that can collect required information and provide posterior distribution of fatigue life. In addition, the developed tool can perform parameter study with respect to parameters in the prior distribution. This tool can automatically incorporate a spreadsheet of field data and an analytical probabilistic analysis into an updated or posterior lifetime distribution. These distributions can directly be used for risk analysis, lifetime extensions, inventory control, calibration of analysis methods, and design optimization, with continuous improvements of life estimates as more field experience becomes available. Figure 12 shows a snapshot of the developed GUI.

The prior distribution can be defined in three different ways: two-point method, Monte Carlo simulation method, and multiple CDF points. This information is converted into the distribution type and parameters for prior distribution. The current implementation supports three different types of distributions: normal, lognormal, and Weibull. The field data needs to be prepared in the spreadsheet format. In this implementation, Alstom's field summary sheet (FSS) format is used. In general, the inspection spreadsheet includes various information, and thus, the GUI extracts necessary information from the spreadsheet.

Once prior distribution and field data are selected, the tool calculates the posterior distribution using Bayes' theorem. The results can be displayed in various forms, such as the cumulative distribution function (CDF) of fatigue life at each update, the joint PDF of mean and standard deviation at each update. The results can also be displayed in tabulated format. This tool shows how the predicted life distribution evolves according to each field data.

The tool is further extended to performing parameter study so that it can be used as a design tool. The What-If module can estimate a new posterior distribution when the prior distribution is changed due to design improvement.

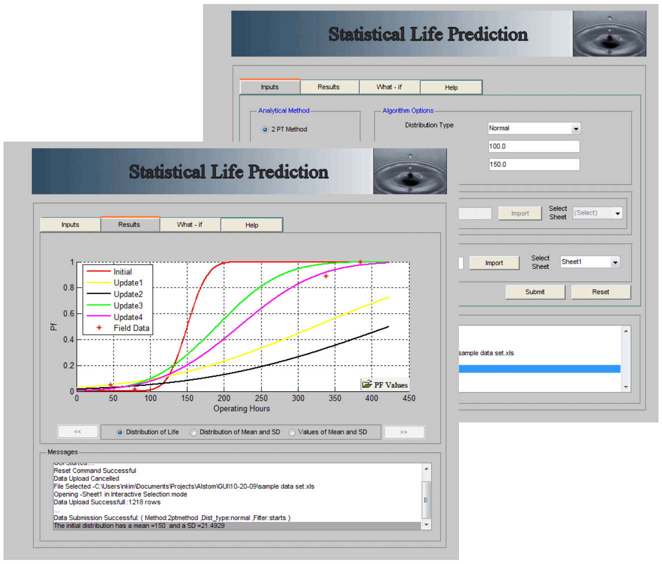


Figure 12: Graphical user interface for statistical life prediction using Bayes' theorem and field data

7. CONCLUSIONS

In this paper, we present a Bayesian updating technique to incorporate the analytical prediction with field data. In the case of low-cycle fatigue, the initial prediction is overly conservative and the updated distribution using field data shows much higher expected life. Bayesian update can also be utilized as a method to fit the distribution of field data when no prior knowledge of the damage mechanism is available. The sensitivity of the Bayesian update to the number of defective blades reported during an inspection has been analyzed. Reporting conservative values of number of defective blades would still lead to under-conservative life distribution, if the reported value is at a high crude probability of failure region.

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