6th China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems
June 22–25, 2010, Kyoto, Japan

Design Optimization for Wear in Pin Joints of Backhoe Systems

Nam Ho Kim[1], Saad Mukras[2]

[1]University of Florida, Gainesville, Florida, USA, nkim@ufl.edu
[2]Qassim University, Buraydah, Saudi Arabia, mukras@qec.edu.sa

Abstract
The design of a backhoe system which experiences wear at various joints of the system is presented. The backhoe system is designed so that the frequency of maintenance due to wear at the joints is reduced. This is achieved by redesigning the main joints of the system such that the maximum allowable wear for all joints is attain in approximately the same number of operating cycles. This would allow for all the concerned joints to be refurbished in a single maintenance session rather than requiring various sessions for the repair of each joint. The design problem is restated as an optimization problem that involves minimizing the difference in the number of cycles required by the various wearing joints of the systems to reach the maximum allowable wear. The size of the joint (joint depth) together with the joint component wear coefficient are taken as the design variables. The immense computational burden associated with wear analysis, is managed through the use of surrogate models to represent the relationship between the design variables and the number cycles required to attain the maximum allowable wear. Subsequently a surrogate base optimization is performed in order to determine the optimum design.

Keywords: *Three to five keywords should be typed here in italic separated by commas.*

1. Introduction

Wear at the joints of mechanical systems results from the contact and relative motion of the joint components. It is therefore inevitable that wear will be experienced during system operation. The consequences of wear, in general, are undesirable; in most cases leading to degradation in the performance of the system and in some cases, failure of the system. It is therefore no surprise that time and resources have been dedicated to study the phenomenon of wear. In one such area of study, effort has been placed to determine procedures that can be used to predict wear in simple cases such as two body sliding wear. Early studies in this area employed a linear wear model, known as the Archard's wear model [1], to estimate worn geometry based on initial contact conditions. The total wear for a predetermined number of cycles of operation (and thus the final geometry) were determined through linear extrapolations of the initial wear. The assumption that the geometry and contact pressure did not evolve as a function of the wear formed the basis for these extrapolations. While this procedure provided ballpark estimates of the wear, they were found to procedure erroneous results [2,3] especially at large operating cycles.

A need for more accurate wear predictions gave rise to the iterative wear prediction procedures where the contact geometry and the contact pressure are allowed to evolve gradually with the wear. Wear at increments of the total cycles are estimated using the Archard's wear law and accumulated to the desired cycles. Geometry updates, to reflect worn material, are also conducted at successive iterations. The procedure provided a more realistic representation of the wear process and correspondingly more accurate prediction of the wear as has been reported in a number of studies [4-9]. Consequently, the procedure has found use in mechanical applications such as wear prediction in gears [10-12], cam-follower [13-15], as well as in medical application involving wear prediction in the knee and hip joints [16-18].

The iterative prediction procedure can serve as a useful tool for manufacturing companies, allowing them to estimate the useful life of a product and determining the terms of warranties as well as preparation of schedules for maintenance. In addition, these procedures can play a crucial role in the design process by allowing the designer to incorporate wear early in the design stage. In this paper a demonstration of how the wear prediction procedure can be incorporated in the design of a backhoe system is presented. In the first part of the paper, a discussion of the iterative prediction procedure will be presented. Thereafter a brief description of the backhoe system will be presented followed with a description of the design problem and how it is restated as an optimization problem. Next, a discussion on how to manage computational costs associated with wear prediction through the use of surrogate models will be presented. Finally, the optimization results will be presented together with observations and concluding remarks.

2. Wear Analysis

In this work, the iterative wear prediction procedure based on the Archrd's wear model was used. The procedure has been used in numerous studies [2-18] and has been found to yield reasonable estimates of the wear.

Archard's wear model estimates wear based on information about the contact condition (contact pressure and sliding distance) and tribological data (generally expressed as a wear coefficient reflecting information about the materials in contact and the operating conditions). Archard's wear model can be expressed in various forms depending on the intended application. In the present work, a useful form of the model is:

$$\frac{h}{s} = kp \,, \tag{1}$$

where s is the relative sliding between the concerned bodies in contact, h is the wear depth, p is the contact pressure resulting from the contact, and k is a wear coefficient. When two bodies made of different materials are in contact, each body will have a separate wear coefficient. For wear analysis, Eq. (1) can conveniently be expressed as a dynamic process leading to the following differential equation:

$$\frac{dh}{ds} = kp(s) \,. \tag{2}$$

Through a temporal discretization of the relative motion between the bodies in contact, an estimate of the wear can be determined as follows:

$$h_i = h_{i-1} + kp_i \Delta s_i \,. \tag{3}$$

In Eq. (3), Δs_i is the incremental siding distance, p_i and h_i are the contact pressure and wear depth at the ith cycle, and h_{i-1} is the wear depth at the previous cycle. It is clear from Eq. (3) that once the contact pressure and incremental sliding distance at each iteration as well as the wear coefficient are known, the total wear can be determined. The wear coefficient is determined through experiments [4,19,20] whereas the contact pressure is determined using numerical techniques such as the Winkler surface [6,16] model or finite element analysis (FEA) [4,7-9,18]. In this work, the commercial finite element software, ANSYS, was used to compute the contact pressure as well as the sliding distance.

As was mentioned earlier, geometry updates are required in order to provide accurate estimates of the wear. In this work, geometry update is achieved by moving the contact surface in the direction of the surface normal by an amount equal to the estimated wear at each iteration. The result is a gradual evolution of the contact surface. It should be mentioned that the geometry evolution is extremely gradual (incremental wear depth during one cycle of motion is usually in the microscopic range). As a result the computation cost to estimate wear of appreciable quantity is quite high. While a number of techniques have been proposed to alleviate this problem, a popular technique (adopted in this work) is the use of extrapolation. This involves calculating the incremental wear depth for a representative cycle and then extrapolating the wear depth over a preselected number of cycles. In order to incorporate extrapolation in Eq. (3), it is written as:

$$h_i = h_{i-1} + kA_E p_i \Delta s_i \,, \tag{4}$$

where A_E is the extrapolation factor. Figure 1 shows a flow chart for incremental wear simulation

Selecting an appropriate value for the extrapolation can be a challenging task owing to is effect on the accuracy and stability of the simulation. While a large value of the extrapolation may seem to speed up the simulation, an exceedingly large value will compromise the accuracy of the simulation since the geometry is not allowed to evolve gradually. On the other hand, using an overly conservative (small) value will result in a less than optimum use of resources. A more comprehensive study on the use of extrapolation as well as a detailed discussion on the wear prediction procedure is available in the literature [9].

3. Problem Definition

The wear prediction procedure, discussed in the previous section, can be used to incorporate wear in system design in order to develop better designs. As was mentioned earlier, the backhoe system will be used to demonstrate one such example. In this section, a brief description of the system and the design problem is presented.

3.1. Backhoe System

The backhoe system, shown in Figure 2, is a construction equipment generally used to dig trenches or scoop dirt from trenches. It consists of three main components; a bucket, a dipper and a boom. These components are interconnected by three main revolute joints (shown as joints 1-3 in Figure 2) which connect the bucket to the dipper, the dipper to the boom and the boom to the main body. The joints are assumed to consist of a pin and bushing similar to the one shown in Figure 3. Hydraulic actuators are attached to the components at various locations which enables the operator to move the bucket in the desired direction.

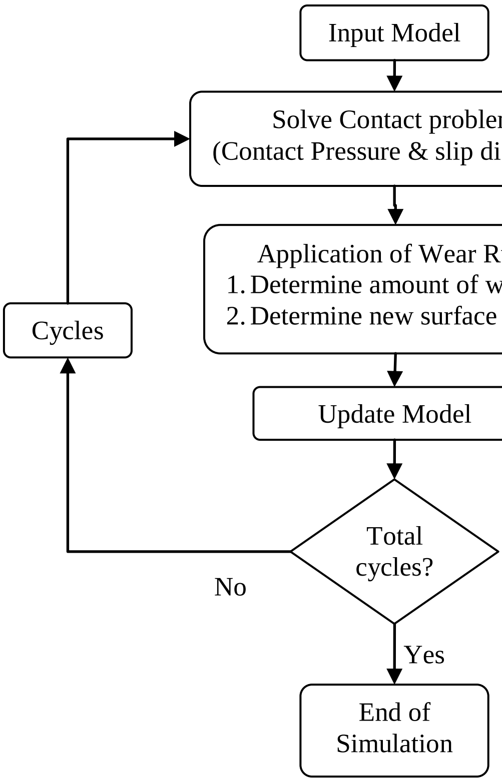

Figure 1: Incremental wear simulation flow chart.

For the backhoe system the three revolute joints shown in Figure 2 have been determined to be the main joints of the system and will be considered for redesign. Due to the contact and relative motion of the joint components, wear is expected to occur in all the joints. If one cycle is defined as scooping and dropping off the dirt using the bucket then the wear depth at any of the joints is a function of 1) the number of cycles of operation, 2) the joint reaction forces, 3) the material used to construct the joint components (pin and bushing), 4) the size of the joint components (depth of the joint components) and 5) the operating conditions at the joint. For the current backhoe system, the joint forces, material properties, joint dimensions and wear coefficients are listed in Table 1 and Table 2.

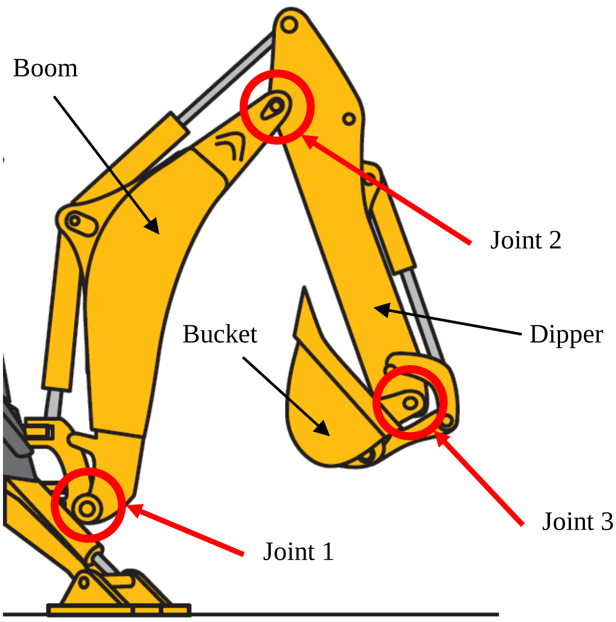

Figure 2: Backhoe system with three main revolute joints.

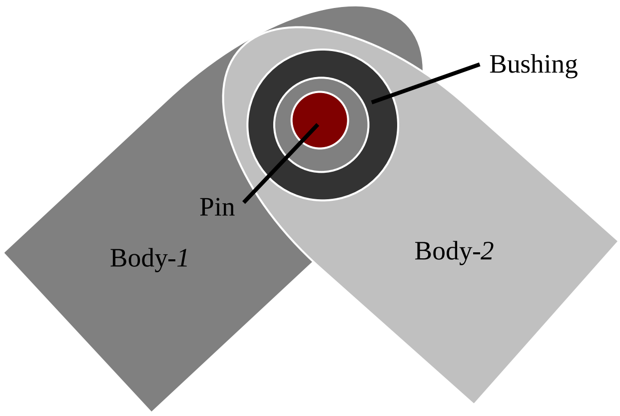

Figure 3: Revolute joint between body-1 and body-2.

Table 1: Dimension and mass parameter for slider-crank mechanism

	Modulus (GPa)	Poisson's	Wear Coefficient (mm^3/Nm)	Joint Force (N)
Joint 1	206.8	0.29	$1.0\text{x}10^{-4}$	36000
Joint 2	206.8	0.29	$1.0\text{x}10^{-4}$	28000
Joint 3	206.8	0.29	$1.0\text{x}10^{-4}$	25000

Table 2: Material properties and dimensions for the joint components

	Bushing Inner Diameter (mm)	Bushing Depth (mm)
Joint 1	75.44	20
Joint 2	76.11	20
Joint 3	55.19	18

3.2. Design Problem

In conventional design of systems such as the backhoe, it is unlikely that the maximum allowed wear at all the joints will simultaneously occur at the same number of operating cycle. As a consequence, when the maximum allowable wear occurs on one of the joints the use of the system will have to be halted and the joint has to be repaired or replaced. The system can then be operated until the maximum allowable wear occurs on a second joint. The system operation is once again halted and the second joint is repaired. In the case of the backhoe system, repair of all the main joints would require that the system be out of operation in three occasions and thus leading to longer downtimes.

One design requirement could be to design the joints of the backhoe system such that the maximum allowable wear on all the joints occur at the same (or approximately the same) number of operation cycles. This would allow for all the joints to be replaced on a single maintenance session as opposed to three. The result of this design is that the frequency of maintenance is reduced.

The design requirement, outlined in the previous paragraph, can be restated as an optimization problem as follows; determine the bushing dimension (bushing depth) and the corresponding wear coefficient for the backhoe joints (joints 1-3) so that the maximum allowable wear depth for each component occurs at approximately the same number of cycles. This can be obtained by minimizing the sum of the difference in cycles required to reach the maximum allowable wear depth for the three joints. Mathematically this can be expressed as:

$$\begin{aligned} \text{Minimize}: \; f = & \left(cyc_1(h_1,k_1) - cyc_2(h_2,k_2) \right)^2 + \left(cyc_1(h_1,k_1) - cyc_2(h_3,k_3) \right)^2 \\ & + \left(cyc_2(h_2,k_2) - cyc_3(h_3,k_3) \right)^2 \\ \text{Such that}: \; & h_{LB} \le h_1,h_2,h_3 \le h_{UB} \quad \& \quad k_{LB} \le k_1,k_2,k_3 \le k_{UB} \end{aligned} \tag{5}$$

where f, is the objective function that depends on the dimensions of the bushing (h_1, h_2, h_3) and the wear coefficient of the bushing components (k_1, k_2, k_3). It is assumed that the joint components (pin and bushing) are made of the same material so that wear at any of the joints is sufficiently described by a single wear coefficient. In Eq.(5), cyc_1, cyc_2 and cyc_3 are the number of operating cycles required for wear to accumulate to the maximum allowable amount for *Joint-1*, *Joint-2* and *Joint-3*, respectively. It is clear from Eq.(5), that the design variables are the bushing dimension and the wear coefficients for the three joints. These variables are constrained between their corresponding upper bounds ($h_{UB}\ k_{UB}$) and lower bounds ($h_{LB}\ k_{UB}$). A discussion of the solution of the optimization problem will be discussed in the following section.

4. Solution of Design Problem

Upon careful examination of Eq.(5), one will realize that the objective function is extremely expensive to evaluate. This is because determining a single value of cyc_1, cyc_2 or cyc_3 for any combination of the joint dimension and wear coefficient, requires a complete wear analysis involving several thousand cycles. Since numerous evaluation of the objective function is required during optimization, the current implicit form of the objective function will render the optimization problem extremely costly. In order to solve this problem, a surrogate based optimization procedure was used. This procedure will be discussed in this section.

4.1. Surrogate Based Optimization

Instead of conducting the optimization using the current implicit form of the objective function, also called the high fidelity model/function, a surrogate based optimization approach [21-26] can be employed. This approach entails replacing the high-fidelity models using approximate surrogate models, such as the response surface approximations [27-29], support vector regression [30 31], and kriging [32 33]. The surrogates are constructed using data drawn from the high-fidelity models through a process termed as design of experiments (DOE) [34-36].

The surrogates offer fast approximations of the high-fidelity objective functions at other locations in the design space. The consequence is that the speed of optimization is substantially increased. This reduction of computational costs is, however achieved at the expense of accuracy. There are techniques such as the trust region approach [26 37], in which construction of the surrogate is focused in regions of possible optima based on previous optimization results, which can be used to improve the surrogate based optimization results.

Despite the various possible surrogate models available, for this work, the objective function will be replaced by a second order (quadratic) response surface approximation. This will be based on second order response surface approximations constructed for cyc_1, cyc_2 or cyc_3 (see Eq.(5)). Furthermore the design points (for the DOE) are selected by a space filling technique called Latin Hypercube Sampling (LHS) [38]. For convenience the surrogate Matlab toolbox developed by Viana [39] was used to generate the DOE as well as the surrogate models.

4.2. Solution Using Surrogate Based Optimization

For the current backhoe system, the maximum allowable wear depth for the three joints was set at 2mm. The design range for the bushing depth (for all three joints) was set between 15mm and 30mm, whereas the range for the wear coefficient was set between 0.5e-8 $mm^3/(Nmm)$ and 1.5e-8 $mm^3/(Nmm)$. These specifications are summarized in Table 3. The design space is defined as any combination of the two variables within the two ranges. Using the LHS, design points were selected within the design space. Since the design space for all three joints was identical, the same design location were used in the construction of surrogate models for cyc_1, cyc_2 and cyc_3 (appearing in Eq.(5)). A plot of these design points is shown in Figure 4. In this plot the design space is boxlike with both variables (bushing depth and wear coefficient) normalized between 0 and 1. It is emphasized that this DOE was used to construct the surrogate models for cyc_1, cyc_2 and cyc_3 in Eq.(5).

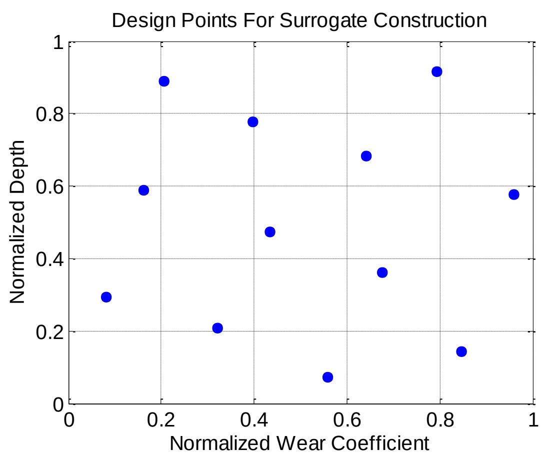

Figure 4: Design points used to construct the surrogate model.

Table 3: Material properties and dimensions for the joint components

	Wear coeffcient ($mm^3/(Nmm)$	Bushing Depth (mm)
Joint 1	0.5 e-8 – 1.5e-8	15.00 - 30.00
Joint 2	0.5 e-8 – 1.5e-8	15.00 - 30.00
Joint 3	0.5 e-8 – 1.5e-8	15.00 - 30.00

In Figure 5 - Figure 7, the response of cyc_1, cyc_2 and cyc_3 generated using the constructed surrogate models for cyc_1, cyc_2 and cyc_3 are shown. It can be inferred from the three plots that as the bushing depth increases, cyc_1, cyc_2 and cyc_3 also increase. This is reasonable since a longer bushing depth would mean that the contact pressure responsible for wear is distributed over a longer length. As a result, the rate of change of the wear depth would reduce and cyc_1, cyc_2 and cyc_3 would increase. It can also be inferred from the figures that as the wear coefficient increases, cyc_1, cyc_2 and cyc_3 decreases. This observation is accurate since for a larger wear coefficient more wear would be expected. Consequently less number of cycles would be required to attain the maximum allowable wear.

Once the explicit surrogates models for cyc_1, cyc_2 and cyc_3 are available, the optimization problem in Eq.(5) can easily be solved. For this problem a standard optimizer available in the Matlab software was used to conduct the optimization. The results from the optimization are shown in Table 4. The combination of bushing depth and the wear coefficient (for the three joints) necessary to allow the wear at the three joint to simultaneously attain the maximum allowable wear is shown in the table. It is however expected that the results are not dead accurate since they were obtained using surrogate models which provide only approximates for the high-fidelity model. In Table 5, a comparison of the results generated using the high-fidelity model and the surrogate model at the optimum solution, is presented. A maximum error of 5.7% for the cycles required to attain the maximum allowable wear, corresponding to joint-1, is observed in the comparison. These differences are attributed to the approximate nature of the surrogate model. While this error is reasonably small for this study, more accurate results can be generated using techniques such as the trust region approach [26,37].

In Table 6, the wear results for the original backhoe system are presented. Listed in the 4th column is the number of cycles of operation required to reach the maximum allowable wear (in this case 2mm). It is clear from the table that the maximum allowable wear would not be attained in the same number of cycles. As a result three maintenance sessions would have to be scheduled in order to repair the joints leading to longer down times.

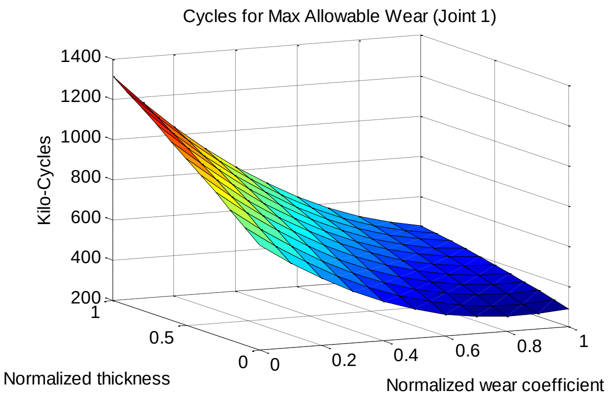

Figure 5: Response generated using the surrogate for cyc_1.

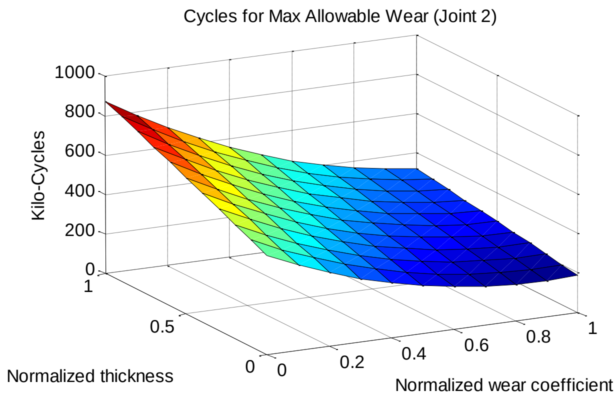

Figure 6: Response generated using the surrogate for cyc_2.

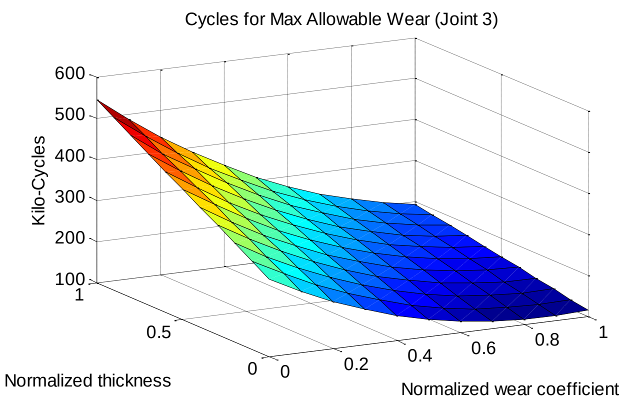

Figure 7: Response generated using the surrogate for cyc_3.

Table 4: Solution of optimization problem (Eq. (5))

	Bushing Depth (mm)	Wear coeffcient (mm^3/(Nmm)	Cycles at Optimum
Joint 1	15.73	1.28e-8	293730
Joint 2	19.10	1.05e-8	293730
Joint 3	26.78	0.93e-8	293730

Table 5: Comparison of results between surrogate and high-fidelity model

	Cycles requred to reach max. wear depth (at optimum)		
	Surrogate	High-fidelity model	Error
Joint 1	293730	311700	5.7%
Joint 2	293730	309030	5.0%
Joint 3	293730	296610	0.9%

Table 6: Wear results for initial backhoe design.

	Wear Coefficeint(mm^3/(Nmm)	Bushing Depth (mm)	Cycles
Joint 1	1.0e-8	20	483498
Joint 2	1.0e-8	20	328459
Joint 3	1.0e-8	18	174946

5. Summary and Concluding Remarks

In this work, by considering wear, a backhoe system is redesigned so as to allow the maximum allowable wear depth at three main joints of the system to be attained simultaneously. This design problem was restated as an optimization problem that would involve minimizing the difference in the number of operation cycles (for the three joints) required to attain the maximum wear depth. To combat the high computational costs of optimizing the system, the high-fidelity objective function was replaced by a quadratic response surface approximation. A fast solution was then obtained from the approximation.

The design of the backhoe system is an illustration of how wear prediction procedures can be integrated into the design of mechanical systems. It is the belief of the authors that by incorporating wear in the design of mechanical systems, better designs, such as the redesigned backhoe system, can be developed.

Acknowledgements

The authors gratefully acknowledge support by the National Science Foundation (DMI-0600375) and Deere & Company.

Nomenclature

A	: Contact area
A_E	: Extrapolation factor
E	: Young's modulus
F_N	: Normal force in the contact interface
h	: Wear depth
k	: Dimensioned wear coefficient
p	: Contact pressure
s	: Sliding distance
t	: Time
υ	: Poisons ratio

References

1. Archard, J.F., 1953, "Contact and rubbing of flat surfaces," Journal of Applied Physics, **24,** pp. 981–988.
2. Sawyer, W.G., 2001, "Wear Predictions for a Simple-Cam Including the Coupled Evolution of Wear and Load," Lubrication Engineering, pp. 31–36.
3. Blanchet, T.A., 1997, "The Interaction of Wear and Dynamics of a Simple Mechanism," Journal of Tribology, **119**, pp. 597–599.
4. Kim, N.H., Won, D., Burris, D., Holtkamp, B., Gessel, G.R., Swanson, P., Sawyer, W.G., 2005, "Finite element analysis and experiments of metal/metal wear in oscillatory contacts," Wear, **258**, pp 1787–1793.
5. Oqvist, M., 2001, "Numerical simulations of mild wear using updated geometry with different step size approaches," Wear, **249**, pp. 6–11.
6. Podra, P., and Andersson, S., 1997, "Wear simulation with the Winkler surface model," Wear, **207**, pp. 79–85.

7. Podra, P., and Andersson, S., 1999, “Finite element analysis wear simulation of a conical spinning contact considering surface topography,” Wear, **224**, pp. 13–21.
8. Podra, P., and Andersson, S., 1999, “Simulating sliding wear with finite element method,” Tribology International, **32**, pp. 71–81l.
9. Mukras, S., Kim, N.H., Sawyer, W.G., Jackson, D.B., Bergquist, L.W., 2009, “Numerical integration schemes and parallel computation for wear prediction using finite element method,” Wear, **266** pp. 822–831.
10. Flodin, A., and Andersson, S., 1997, “Simulation of Mild Wear in Spur Gears,” Wear, **207**, pp. 16–23.
11. Flodin, A., and Andersson, S., 2001, “A simplified model for wear prediction in helical gears, Wear, **249**, pp 285–292.
12. Brauer, J., and Andersson, S., 2003, “Simulation of Wear in Gears with Flank Interference-a Mixed FE and Analytical Approach,” Wear, **254**, pp 1216–1232.
13. Hugnell, A., and Andersson, S., 1994, “Simulating follower wear in a cam-follower contact,” Wear, **179**, pp. 101–107.
14. Hugnell, A.B.-J., Björklund, S., and Andersson, S., 1996, “Simulation of the Mild Wear in a Cam-Follower Contact with Follower Rotation,” Wear, **199**, pp. 202–210.
15. Nayak, N., Lakshminarayanan, P.A., Babu, M.K.G., and Dani, A.D., 2006, “Predictions of cam follower wear in diesel engines,” Wear, **260**, pp. 181–192.
16. Fregly, B.J., Sawyer, W.G., Harman, M. K., and S. Banks, A., 2005, “Computational wear prediction of a total knee replacement from in vivo kinematics,” Journal of Biomechanics, **38**, pp. 305–314.
17. Maxian,T.A., Brown, T.D., Pedersen, D.R., and Callaghan, J.J., 1996,“A Sliding-Distance Coupled Finite Element Formulation for polyethylene wear in total hip arthroplasty,” Journal of Biomechanics, **29**, pp. 687–692.
18. Bevill, S.L., Bevill, G.R., Penmetsa, J.R., Petrella, A.J., and Rullkoetter, P.J., 2005, “Finite Element Simulation of Early Creep and Wear in Total Hip Arthroplasty,” Journal of Biomechanics, **38**, pp. 2365–2374.
19. Schmitz, T.L., Action, J.E., Burris, D.L., Ziegert, J.C., Sawyer, W.G., 2004, “Wear-Rate Uncertainty Analysis,” ASME Journal of Tribology, **126**(4), pp. 802–808.
20. Yang, L.J, 2005, “A test methodology for determination of wear coefficient,” Wear, **259** 1453-1461.
21. Queipo, N.V., Haftka, R.T., Shyy,W., Goel,T., Vaidyanathan, R., Tucker, P.K., 2005, “Surrogate-based analysis and optimization,” Progress in Aerospace Science, **41**, pp 1–28.
22. Mack, Y., Goel, T., Shyy, W., Haftka, R.T., Queipo, N.V., 2005, “Multiple surrogates for the shape optimization of bluff body-facilitated mixing,” Proceedings of the 43rd AIAA aerospace sciences meeting and exhibit, Reno, NV. Paper no.–0333, AIAA.
23. Booker, A.J., Dennis Jr, J.E., Frank, P.D., Serafini, D.B., Torczon V. and Trosset, M.W., 1999, “A rigorous framework for optimization of expensive functions by surrogates,” Structural Optimization, **17**(1), pp. 1–13.
24. Audet, C., Dennis, Jr.,J.E., Moore, D.W., Booker, A.J, Frank ,P.D., 2000, “A surrogate-model-based method for constrained optimization,” 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisiplanary Analysis and Optimization, Long Beach, CA, Paper no, AIAA-2000-4891.
25. Jansson, T., Nilsson, L., Redhe, M., 2003, “Using surrogate models and response surfaces in structural optimization – with application to crashworthiness design and sheet metal forming,” Structural Multidisciplinary Optimization, **25**, pp. 129–140.
26. Mack, Y., Goel, T., Shyy, W., Haftka, R., 2007, “Surrogate Model-Based Optimization Framework: A Case Study in Aerospace Design,” Studies in Computational Intelligence **51**, pp. 323–342.
27. Roux, W.J., Stander, N., Haftka, R.T., 1996, “Response surface approximations for structural optimization,” AIAA/NASA/ISSMO, Symposium on Multidisciplinary Analysis and Optimization, Reston,VA.
28. Venter, G., Haftka, R.T., Starnes Jr, J.H., 1998“Construction of Response Surface Approximations for Design Optimization,” AIAA **36**(12), pp. 2242–2249.
29. Papila, M., Haftka , R.T., 2000, “Response surface approximations: noise, error, repair and modeling errors,” AIAA **38**(12), pp. 2336–2343.
30. Smola, A.J., Schölkopf, B., 2004, “A tutorial on support vector regression,” Statistics and computing, **14**, pp. 199–222.
31. Girosi, F., 1998, “An equivalence between sparse approximation and support vector machines,” Neural Computing, **10**, pp.1455–1480.
32. Martin, J.D., Simpson, T.W., 2004, “On the use of kriging models to approximate deterministic computer models,” ASME IDET Conferences and Computers and Information in Engineering Conference Salt Lake City, Utah.
33. Simpson, T.W., Mauery, T.M., Korte, J.J., Mistree, F., 1998, “Comparison of response surface and kriging models for multidisciplinary design optimization,” AIAA, pp. 1–11.
34. Simpson, T.W., Lin, D.K.J., Chen, W., 2001, “Sampling strategies for computer experiments: design and analysis,” International Journal of Reliability and Applications; **2**(3):209–240.
35. Giunta, A.A., Wojtkiewicz, S.F., Eldred, M.S., 2003, “Overview of modern design of experiments methods for computational simulations,” Forty-first AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV, 6–9 January;

AIAA-2003-0649.
36. Goel, T., Haftka, R.T., Shyy, W., Watson, L.T., 2008, “Pitfalls of using a single criterion for selecting experimental designs,” Int. J. Numer. Meth. Engng; **75**:127–155.
37. Alexandrov, N.M., 1996, “A trust region framework for managing approximation models in engineering optimization,” AIAA/NASA/USAF multidisciplinary analysis & optimization symposium, Bellevue, WA,. Paper no. 96–4102.
38. Mckay, M.D., Beckman, R.J., Conover, W.J., 1979 “A comparison of three methods for selecting values of input variables in the analysis of output from a computer code,” Technometrics, **21**,pp. 239–245.
39. Viana, F.A.C., 2009, "Surrogates Toolbox User’s Guide," Users manual, http://fchegury.googlepages.com.