Design Sensitivity Analysis and Optimization of Nonlinear Transient Dynamics

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Introduction

➢ **Shape Design Sensitivity Analysis of Nonlinear Structure**

- Finite Deformation Elastoplasticity Using Multiplicative Decomposition of the Deformation Gradient
- Classical Return Mapping Algorithm Is Preserved Using Principal Kirchhoff Stress and Logarithmic Strain
- Design Sensitivity Equation Is Obtained at the Initial Domain and Then Transformed into the Current Domain to Recover the Updated Lagrangian Form
- Path-Dependency Comes from the Intermediate Configuration and Plastic Evolution Variables
- Exact Tangent Operator Yields Iteration-Free DSA

Introduction *cont.*

- **Design Sensitivity Analysis of Structural Dynamics**
  - Newmark Family Implicit Time Integration Is Used to Solve 2nd-Order Design Sensitivity Differential Equation with Homogeneous Initial Conditions
  - Design Sensitivity Equation Solves for the Material Derivative of Acceleration at Converged Time Steps

Time Integration ⟶ Material Derivative of Displacement

  - Sensitivity Equation Is More Efficient for the Implicit Time Integration Method Than the Explicit Method Compared to the Cost of Response Analysis

Multiplicative Elastoplasticity

- Kinematics (Deformation Gradient)

$$\mathbf{F}(\mathbf{X}) = \mathbf{F}^{e}(\mathbf{X})\mathbf{F}^{p}(\mathbf{X})$$

- Principal Logarithmic Strain

$$\mathbf{b}^{e} = \mathbf{F}^{e}\mathbf{F}^{e^{T}} = \sum_{i=1}^{3} \nu_i^2 \, \mathbf{m}^{i} \qquad \mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \log(\nu_1) \\ \log(\nu_2) \\ \log(\nu_3) \end{bmatrix}$$

- Kirchhoff Stress

$$\boldsymbol{\tau} = \sum_{i=1}^{3} \tau_i^p \, \mathbf{m}^{i} \qquad \mathbf{m}^{i} = \mathbf{n}^{i} \otimes \mathbf{n}^{i}$$

$\mathbf{n}^i$: Principal Vector

τ_i^p: Principal Stress

Isotropic Material Assumption

Constitutive Relation

- Trial Elastic Principal Stress

$$(\boldsymbol{\tau}^p)^{tr} = \mathbf{c}^e \mathbf{e}^{tr} \qquad \mathbf{c}^e = (\lambda + \tfrac{2}{3}\mu)\mathbf{1}\otimes\mathbf{1} + 2\mu\mathbf{I}_{dev}$$

All incremental deformation is assumed to be elastic

- Yield Function

$$f(\boldsymbol{\eta}, e^p) = \|\boldsymbol{\eta}\| - \sqrt{\tfrac{2}{3}}\kappa(e^p)$$

$$= \|\boldsymbol{\eta}^{tr}\| - (2\mu + H_\alpha(e^p))\gamma - \sqrt{\tfrac{2}{3}}\kappa(e^p) = 0$$

$$\boldsymbol{\eta} = dev(\boldsymbol{\tau}^p) - \boldsymbol{\alpha}$$

- Return Mapping Algorithm

$$\boldsymbol{\tau}^p_{n+1} = (\boldsymbol{\tau}^p)^{tr} - 2\mu\gamma\mathbf{N}$$

$$\boldsymbol{\alpha}_{n+1} = \boldsymbol{\alpha}_n + \gamma H_\alpha(e^p)\mathbf{N}$$

$$e^p_{n+1} = e^p_n + \sqrt{\tfrac{2}{3}}\gamma$$

Linearization and Tangent Stiffness

- Tangent Stiffness Tensor

$$\mathbf{c} = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^{\text{alg}} \mathbf{m}^i \otimes \mathbf{m}^j + 2\sum_{i=1}^{3} \tau_i^p \mathbf{c}_{\text{trial}}^i$$

$$\mathbf{c}^{\text{alg}} = \frac{\partial \boldsymbol{\tau}^p}{\partial \mathbf{e}^{tr}} = \mathbf{c}^e - 4\mu^2 A \mathbf{N} \otimes \mathbf{N} - \frac{4\mu^2\gamma}{\left\|\boldsymbol{\eta}^{tr}\right\|}[\mathbf{I}_{dev} - \mathbf{N} \otimes \mathbf{N}]$$

$\mathbf{c}^{\text{alg}}$ is the same as classical elastoplasticity
in principal stress/strain space

- Linearization

$$a(\mathbf{z}, \overline{\mathbf{z}}) = \int_{\Omega} \tau_{ij} \varepsilon_{ij}(\overline{\mathbf{z}})\, d\Omega$$

Structural Energy Form

$$a^*(\mathbf{z}; \Delta\mathbf{z}, \overline{\mathbf{z}}) \equiv \int_{\Omega} \Big(\varepsilon_{ij}(\overline{\mathbf{z}}) c_{ijkl} \varepsilon_{kl}(\Delta\mathbf{z}) + \tau_{ij} \eta_{ij}(\Delta\mathbf{z}, \overline{\mathbf{z}}) \Big)\, d\Omega$$

Updated Lagrangian Formulation

Intermediate Configuration

- Update Intermediate Configuration

$$\mathbf{F}_n^{e^{tr}} = \mathbf{F}_n \mathbf{F}_{n-1}^{p^{-1}} = \sum_{i=1}^{3} \exp(e_i^{tr})\mathbf{m}^i$$

$$\mathbf{f}^p = \sum_{i=1}^{3} \exp(-\gamma N_i)\mathbf{m}^i$$

Incremental Plastic Deformation Gradient

$$\mathbf{F}_n^e = \mathbf{f}^p \mathbf{F}_n^{e^{tr}}$$

$$\mathbf{F}_n^p = \mathbf{F}_n^{e^{-1}} \mathbf{F}_n$$

It is assumed that the incremental plastic spin is arbitrary

→ Continuous intermediate configuration

Variational Formulation of Structural Dynamics

- Weak Form

$$d(\mathbf{z}_{,tt},\bar{\mathbf{z}}) + a(\mathbf{z},\bar{\mathbf{z}}) = \ell(\bar{\mathbf{z}}), \qquad \forall \bar{\mathbf{z}} \in Z$$

$$d(\mathbf{z}_{,tt},\bar{\mathbf{z}}) = \int_{\Omega} \rho \bar{\mathbf{z}}^T \mathbf{z}_{,tt} d\Omega$$ Kinetic Energy Form

$$\ell(\bar{\mathbf{z}}) = \int_{\Omega} \bar{\mathbf{z}}^T \mathbf{f}^b \, d\Omega + \int_{\Gamma^h} \bar{\mathbf{z}}^T \mathbf{f}^h \, d\Gamma$$ Load Linear Form

- Variational Equation in Structural Domain
- 2nd-Order Differential Equation in Time Domain

- Initial Conditions

$$\mathbf{z}(\mathbf{x},0) = \mathbf{z}^0(\mathbf{x}) \qquad \mathbf{x} \in \Omega$$

$$\mathbf{z}_{,t}(\mathbf{x},0) = \mathbf{z}^0_{,t}(\mathbf{x}) \qquad \mathbf{x} \in \Omega$$

Implicit Time Integration

- Newmark Method

  – Predictor

$$ {}^{n}\mathbf{z}_{,t}^{pr} = {}^{n-1}\mathbf{z}_{,t} + (1-\gamma)\Delta t \; {}^{n-1}\mathbf{z}_{,tt} $$

$$ {}^{n}\mathbf{z}^{pr} = {}^{n-1}\mathbf{z} + \Delta t \; {}^{n-1}\mathbf{z}_{,t} + (\tfrac{1}{2} - \beta)\Delta t^{2} \; {}^{n-1}\mathbf{z}_{,tt} $$

  – Corrector

$$ {}^{n}\mathbf{z}_{,t} = {}^{n}\mathbf{z}_{,t}^{pr} + \gamma\Delta t \; {}^{n}\mathbf{z}_{,tt} $$

$$ {}^{n}\mathbf{z} = {}^{n}\mathbf{z}^{pr} + \beta\Delta t^{2} \; {}^{n}\mathbf{z}_{,tt} $$

β, γ: Newmark Parameters

Implicit Time Integration *cont.*

- Linearization [$a_\Omega(\mathbf{z},\bar{\mathbf{z}})$ Is Nonlinear w.r.t. $\mathbf{z}$]

n : Time t_n
k : Iteration Counter

$$d(\Delta\mathbf{z}_{,tt}^{k+1},\bar{\mathbf{z}}) + a^*({}^n\mathbf{z}^k;\Delta\mathbf{z}^{k+1},\bar{\mathbf{z}})$$
$$= \ell(\bar{\mathbf{z}}) - a({}^n\mathbf{z}^k,\bar{\mathbf{z}}) - d({}^n\mathbf{z}_{,tt}^k,\bar{\mathbf{z}}), \qquad \forall\bar{\mathbf{z}}\in Z$$

- Acceleration Form

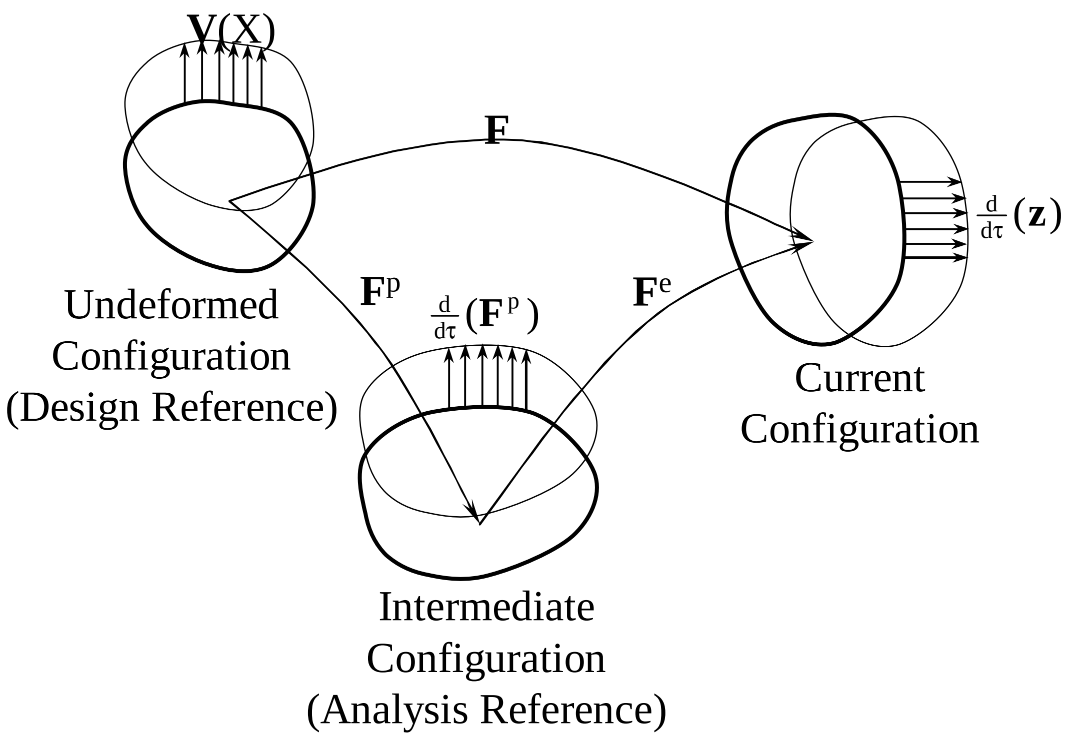
$$\Delta\mathbf{z}^{k+1} = \beta\Delta t^2\Delta\mathbf{z}_{,tt}^{k+1}$$

$$d(\Delta\mathbf{z}_{,tt}^{k+1},\bar{\mathbf{z}}) + \beta\Delta t^2 a^*({}^n\mathbf{z}^k;\Delta\mathbf{z}_{,tt}^{k+1},\bar{\mathbf{z}})$$
$$= \ell(\bar{\mathbf{z}}) - a({}^n\mathbf{z}^k,\bar{\mathbf{z}}) - d({}^n\mathbf{z}_{,tt}^k,\bar{\mathbf{z}}), \qquad \forall\bar{\mathbf{z}}\in Z$$

- Update Kinematic Variables

$${}^n\mathbf{z}^{k+1} = {}^n\mathbf{z}^k + \Delta\mathbf{z}^{k+1}$$
$${}^n\mathbf{z}^0 = {}^n\mathbf{z}^{pr}$$

Finite Deformation DSA



- Updated Lagrangian Formulation
- Finite Deformation Elastoplasticity
- No Need to Update Velocity Fields
- Updating Sensitivity Information of Intermediate Configuration and Plastic Variables

Finite Deformation DSA *cont.*

- Material Derivative

$$\dot{\mathbf{z}} = \tfrac{d}{d\tau}(\mathbf{z})$$

$$= \lim_{\tau \to 0} \frac{1}{\tau}\left[\mathbf{z}_\tau(\mathbf{X} + \tau\mathbf{V}) - \mathbf{z}(\mathbf{X})\right]$$

$$= \mathbf{z}' + \nabla_0 \mathbf{z} \mathbf{V}$$

$\mathbf{V}(\mathbf{X})$: Design Velocity Field

- Material Derivative of Structural Energy Form

$$\tfrac{d}{d\tau}[a(\mathbf{z}, \bar{\mathbf{z}})] = a^*(\mathbf{z}; \dot{\mathbf{z}}, \bar{\mathbf{z}}) + a'_V(\mathbf{z}, \bar{\mathbf{z}})$$

Explicitly Dependent Terms on **V**(**X**)
Path-Dependent Terms

Implicitly Dependent Terms
Same As Structural Linearization

Finite Deformation DSA *cont.*

- Fictitious load

$$a'_V(\mathbf{z},\overline{\mathbf{z}}) = \int_\Omega \left(\varepsilon_{ij}(\overline{\mathbf{z}}) c_{ijkl} \varepsilon^P_{kl}(\mathbf{z}) + \tau_{ij}\eta^P_{ij}(\mathbf{z},\overline{\mathbf{z}}) + \tau^{fic}_{ij}\varepsilon_{ij}(\overline{\mathbf{z}}) \right) d\Omega$$
$$+ \int_\Omega \left(\varepsilon_{ij}(\overline{\mathbf{z}}) c_{ijkl} \varepsilon^V_{kl}(\mathbf{z}) + \tau_{ij}\eta^V_{ij}(\mathbf{z},\overline{\mathbf{z}}) + \tau_{ij}\varepsilon_{ij}(\overline{\mathbf{z}}) div\mathbf{V} \right) d\Omega$$

- Explicitly Dependent Terms

$$\boldsymbol{\varepsilon}^V(\mathbf{z}) = -sym(\nabla_0 \mathbf{z} \nabla_n \mathbf{V})$$

$$\boldsymbol{\eta}^V(\mathbf{z},\overline{\mathbf{z}}) = -sym(\nabla_n \overline{\mathbf{z}}^T \nabla_0 \mathbf{z} \nabla_n \mathbf{V}) - sym(\nabla_0 \overline{\mathbf{z}} \nabla_n \mathbf{V})$$

- Path-Dependent Terms

$$\boldsymbol{\varepsilon}^P(\mathbf{z}) = -sym(\mathbf{G})$$

$$\boldsymbol{\eta}^p(\mathbf{z},\overline{\mathbf{z}}) = -sym(\nabla_n \overline{\mathbf{z}}^T \mathbf{G}) \qquad \mathbf{G} = \mathbf{F}^e \tfrac{d}{d\tau}(\mathbf{F}^p)\mathbf{F}^{-1}$$

$$\boldsymbol{\tau}^{fic} = \sum_{i=1}^{3} \left[\frac{\partial \tau^p_i}{\partial \boldsymbol{\alpha}} \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \frac{\partial \tau^p_i}{\partial \hat{e}^p} \tfrac{d}{d\tau}(e^p_n) \right] \mathbf{m}^i$$

DSA for Structural Dynamics

- Kinetic Energy

$$\frac{d}{d\tau}[d(\mathbf{z}_{,tt},\overline{\mathbf{z}})] = \int_\Omega \rho \overline{\mathbf{z}}^T \dot{\mathbf{z}}_{,tt}\, d\Omega + \int_\Omega \rho \overline{\mathbf{z}}^T \mathbf{z}_{,tt} div\mathbf{V}\, d\Omega$$

$$\equiv d(\dot{\mathbf{z}}_{,tt},\overline{\mathbf{z}}) + d'_V(\mathbf{z}_{,tt},\overline{\mathbf{z}})$$

- Design Sensitivity Equation

$$\frac{d}{d\tau}[d({}^n\mathbf{z}_{,tt},\overline{\mathbf{z}})] + \frac{d}{d\tau}[a({}^n\mathbf{z},\overline{\mathbf{z}})] = \frac{d}{d\tau}[\ell(\overline{\mathbf{z}})], \quad \forall \overline{\mathbf{z}} \in Z$$

$$d({}^n\dot{\mathbf{z}}_{,tt},\overline{\mathbf{z}}) + a^*({}^n\mathbf{z};{}^n\dot{\mathbf{z}},\overline{\mathbf{z}})$$

$$= \ell'_V(\overline{\mathbf{z}}) - a'_V({}^n\mathbf{z},\overline{\mathbf{z}}) - d'_V({}^n\mathbf{z}_{,tt},\overline{\mathbf{z}}), \qquad \forall \overline{\mathbf{z}} \in Z$$

– Initial Conditions (Homogeneous)

$$\dot{\mathbf{z}}(\mathbf{x},0) = \mathbf{0} \qquad \mathbf{x} \in \Omega$$

$$\dot{\mathbf{z}}_{,t}(\mathbf{x},0) = \mathbf{0} \qquad \mathbf{x} \in \Omega$$

DSA for Structural Dynamics *cont.*

- Predictor

$$ {}^{n}\dot{\mathbf{z}}_{,t}^{pr} = {}^{n-1}\dot{\mathbf{z}}_{,t} + (1-\gamma)\Delta t\ {}^{n-1}\dot{\mathbf{z}}_{,tt} $$

$$ {}^{n}\dot{\mathbf{z}}^{pr} = {}^{n-1}\dot{\mathbf{z}} + \Delta t\ {}^{n-1}\dot{\mathbf{z}}_{,t} + (\tfrac{1}{2}-\beta)\Delta t^{2}\ {}^{n-1}\dot{\mathbf{z}}_{,tt} $$

- Corrector

$$ {}^{n}\dot{\mathbf{z}}_{,t} = {}^{n}\dot{\mathbf{z}}_{,t}^{pr} + \gamma\Delta t\ {}^{n}\dot{\mathbf{z}}_{,tt} $$

$$ {}^{n}\dot{\mathbf{z}} = {}^{n}\dot{\mathbf{z}}^{pr} + \beta\Delta t^{2}\ {}^{n}\dot{\mathbf{z}}_{,tt} $$

- Acceleration Form DSA

Sensitivity Equation Is Linear and Solves for Total Acceleration

$$ \begin{aligned} & d({}^{n}\dot{\mathbf{z}}_{,tt}, \bar{\mathbf{z}}) + \beta\Delta t^{2} a^{*}({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}_{,tt}, \bar{\mathbf{z}}) \\ & \quad = \ell'_{V}(\bar{\mathbf{z}}) - a'_{V}({}^{n}\mathbf{z}, \bar{\mathbf{z}}) \\ & \quad - d'_{V}({}^{n}\mathbf{z}_{,tt}, \bar{\mathbf{z}}) - a^{*}({}^{n}\mathbf{z}; {}^{n}\dot{\mathbf{z}}^{pr}, \bar{\mathbf{z}}), \qquad \forall \bar{\mathbf{z}} \in Z \end{aligned} $$

Update Path-Dependent Variables

- Updating Plastic Variables

$$\tfrac{d}{d\tau}(\boldsymbol{\alpha}_{n+1}) = \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \left(H_\alpha + \sqrt{\tfrac{2}{3}}H'_\alpha\gamma\right)\tfrac{d}{d\tau}(\gamma)\mathbf{N} + H_\alpha\gamma\tfrac{d}{d\tau}(\mathbf{N})$$

$$\tfrac{d}{d\tau}(e^p_{n+1}) = \tfrac{d}{d\tau}(e^p_n) + \sqrt{\tfrac{2}{3}}\tfrac{d}{d\tau}(\gamma)$$

$$\tfrac{d}{d\tau}(\mathbf{N}) = \frac{1}{\|\boldsymbol{\eta}^{tr}\|}[\mathbf{I}_{dev} - \mathbf{N}\otimes\mathbf{N}]\left[2\mu\tfrac{d}{d\tau}(\mathbf{e}^{tr}) - \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n)\right]$$

$$\tfrac{d}{d\tau}(\gamma) = A\mathbf{N}^T\left[2\mu\tfrac{d}{d\tau}(\mathbf{e}^{tr}) - \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n)\right] - A\kappa'\tfrac{d}{d\tau}(e^p_n)$$

- Updating Intermediate Configuration

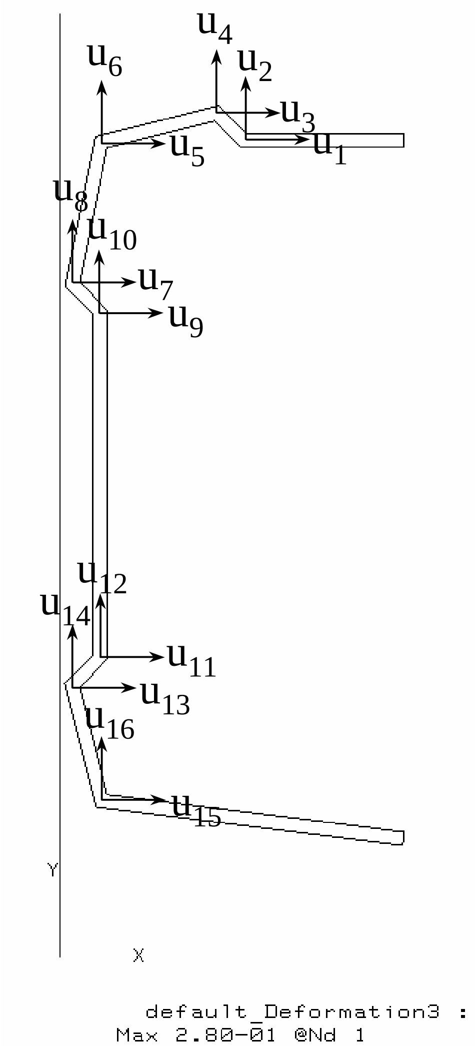
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}) = \tfrac{d}{d\tau}(\mathbf{I} + \nabla_0\mathbf{z}) = \nabla_0\dot{\mathbf{z}} - \nabla_0\mathbf{z}\nabla_0\mathbf{V}$$

$$\tfrac{d}{d\tau}(\mathbf{F}^p_{n+1}) = \tfrac{d}{d\tau}({\mathbf{F}^e_{n+1}}^{-1})\mathbf{F}_{n+1} + {\mathbf{F}^e_{n+1}}^{-1}\tfrac{d}{d\tau}(\mathbf{F}_{n+1})$$

$$\tfrac{d}{d\tau}(\mathbf{F}^e_{n+1}) = \tfrac{d}{d\tau}(\mathbf{f}^p)\mathbf{F}^{e^{tr}}_{n+1} + \mathbf{f}^p\tfrac{d}{d\tau}(\mathbf{F}^{e^{tr}}_{n+1})$$

$$\tfrac{d}{d\tau}({\mathbf{F}^e_{n+1}}^{-1}) = -{\mathbf{F}^e_{n+1}}^{-1}\tfrac{d}{d\tau}(\mathbf{F}^e_{n+1}){\mathbf{F}^e_{n+1}}^{-1}$$

Bumper Impact Problem



Analysis	Meshfree Method
Density	ρ = 7,800 kg/m^3
Initial Velocity	v_0 = 8.05 km/hr
Analysis Time	t = 0 ~ 10 msec
Time Increment	Δt = 0.1 msec
Mounting Displ.	d = 2.8 cm
Thickness	h = 0.5 cm
Contact Penalty No.	w_n = 1,000
Friction Coeff.	μ_f = 0.4
Lame's Constants	λ = 110.8 GPa
	μ = 80.2 GPa
Plastic Hardening	H = 1.1 GPa
	Isotropic Hardening
Initial Yield Stress	σ_Y = 500 MPa
Newmark Parameters	γ = 0.26
	β = 0.5

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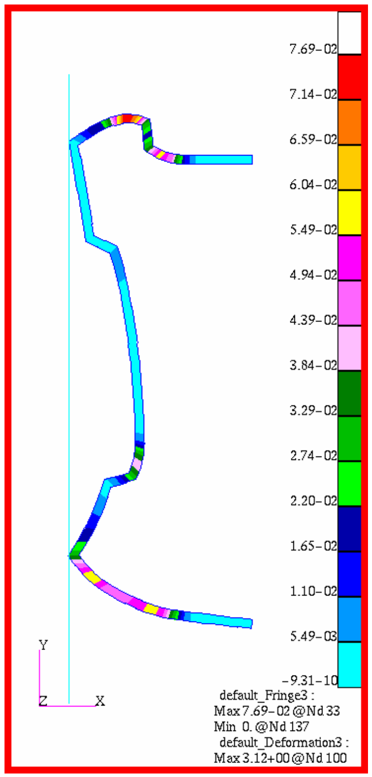
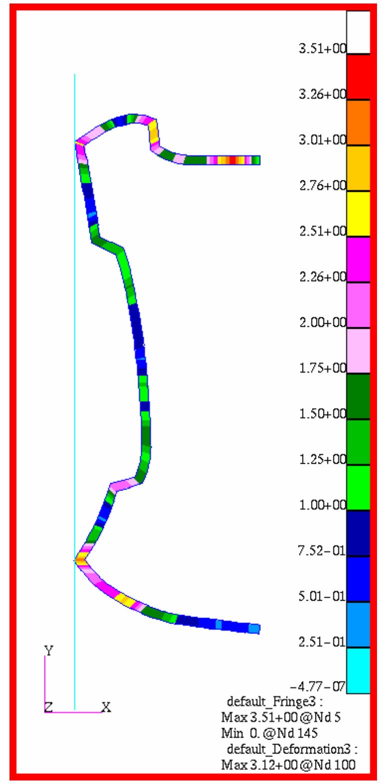
Analysis Results



Effective Plastic Strain

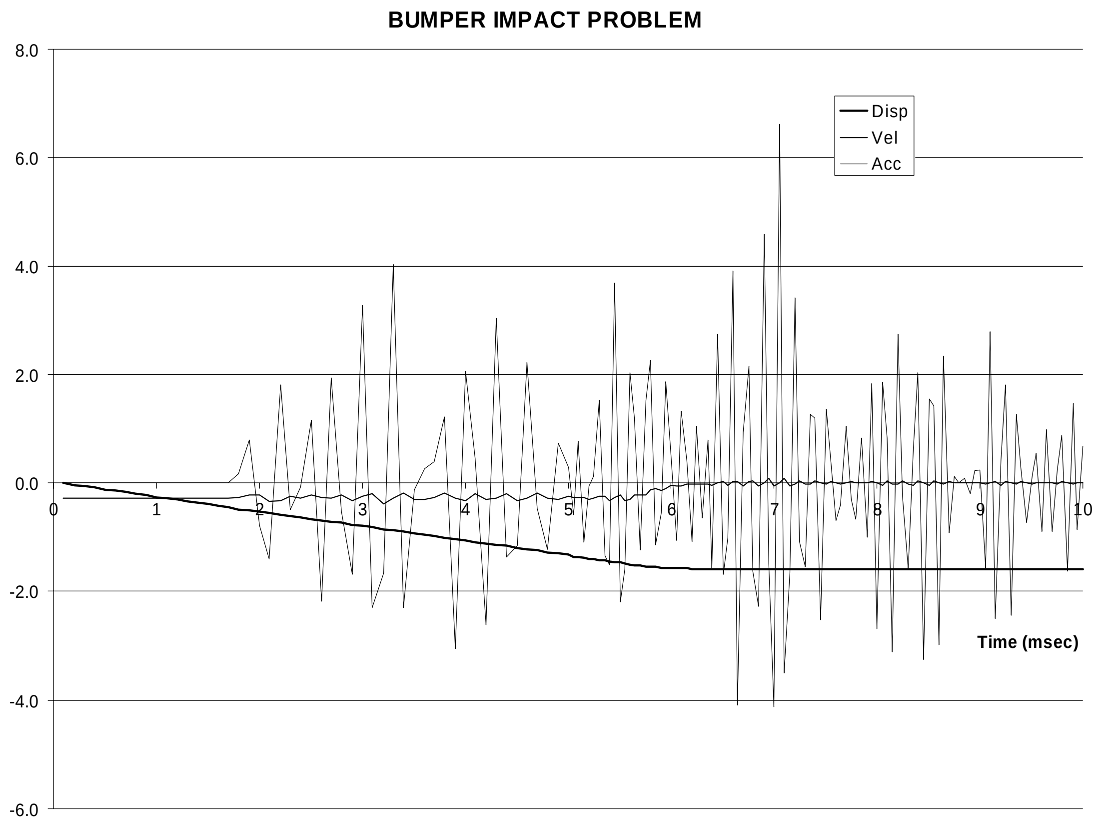


Von Mises Stress

Response Analysis
1,600 sec

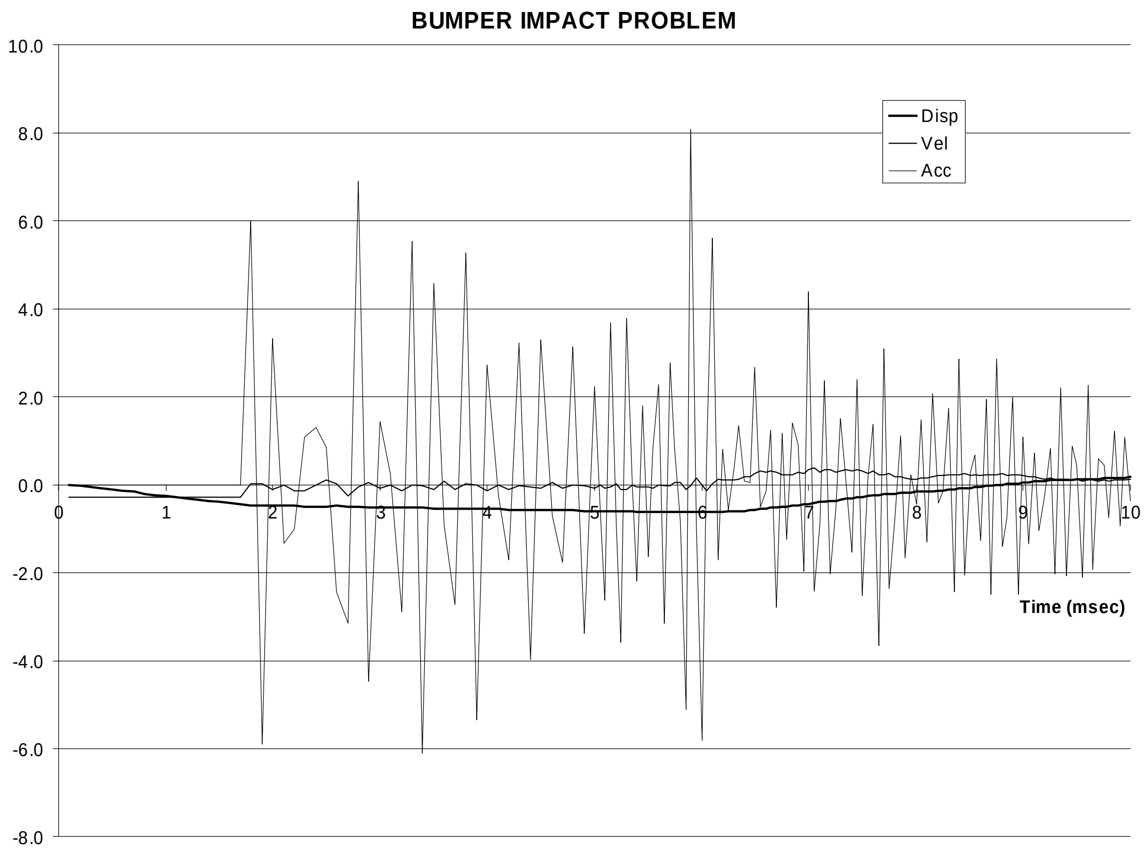
Sensitivity Analysis
853 / 16 sec

Time History



Time History of Node 24

Time History *cont.*



Time History of Node 39

Sensitivity Results and Optimization Problem

Performance (Ψ)		ΔΨ	Ψ′	(ΔΨ / Ψ′)×100%
u_2				
e^p_{15}	.653533E-01	-.754098E-07	-.754105E-07	100.00
e^p_{65}	.618309E-01	.313715E-07	.313668E-07	100.02
e^p_{29}	.460146E-01	.441192E-07	.441162E-07	100.01
z_{x39}	.175053E+00	.790973E-05	.791092E-05	99.98
F_{Cx100}	.128266E+01	-.657499E-06	-.657074E-06	100.06
u_4				
e^p_{15}	.653533E-01	.268699E-06	.268712E-06	100.00
e^p_{65}	.618309E-01	-.843101E-09	-.863924E-09	97.59
e^p_{29}	.460146E-01	.123988E-06	.123993E-06	100.00
z_{x39}	.175053E+00	-.847749E-05	-.847586E-05	100.02
F_{Cx100}	.128266E+01	.410724E-07	.407515E-07	100.79
u_6				
e^p_{15}	.653533E-01	-.317362E-06	-.317349E-06	100.00
e^p_{65}	.618309E-01	-.640031E-07	-.640159E-07	99.98
e^p_{29}	.460146E-01	-.163051E-06	-.163051E-06	100.00
z_{x39}	.175053E+00	-.190521E-05	-.190392E-05	100.07
F_{Cx100}	.128266E+01	.473040E-06	.472876E-06	100.03
u_8				
e^p_{15}	.653533E-01	.888094E-08	.890589E-08	99.72
e^p_{65}	.618309E-01	.355128E-07	.354794E-07	100.09
e^p_{29}	.460146E-01	-.981276E-08	-.981572E-08	99.97
z_{x39}	.175053E+00	-.239706E-05	-.239333E-05	100.16
F_{Cx100}	.128266E+01	-.184457E-06	-.183954E-06	100.27
u_{10}				
e^p_{15}	.653533E-01	-.642594E-08	-.643542E-08	99.85
e^p_{65}	.618309E-01	-.151580E-07	-.151527E-07	100.03
e^p_{29}	.460146E-01	.172663E-07	.172698E-07	99.98
z_{x39}	.175053E+00	-.154011E-05	-.154125E-05	99.93
F_{Cx100}	.128266E+01	-.134372E-06	-.134701E-06	99.76
u_{12}				
e^p_{15}	.653533E-01	.107017E-07	.107147E-07	99.88
e^p_{65}	.618309E-01	-.928369E-07	-.928496E-07	99.99
e^p_{29}	.460146E-01	-.163080E-06	-.163083E-06	100.00
z_{x39}	.175053E+00	-.982943E-07	-.957423E-07	102.67
F_{Cx100}	.128266E+01	-.120596E-05	-.120568E-05	100.02

Design Optimization Problem Definition

MIN *Area*

S.T.

$e^p{}_6(0.07) \le 0.04$ $\quad e^p{}_7(0.02) \le 0.04$

$e^p{}_{12}(0.02) \le 0.04$ $\quad e^p{}_{13}(0.02) \le 0.04$

$e^p{}_{14}(0.02) \le 0.04$ $\quad e^p{}_{15}(0.07) \le 0.04$

$e^p{}_{16}(0.09) \le 0.04$ $\quad e^p{}_{17}(0.05) \le 0.04$

$e^p{}_{28}(0.04) \le 0.04$ $\quad e^p{}_{29}(0.05) \le 0.04$

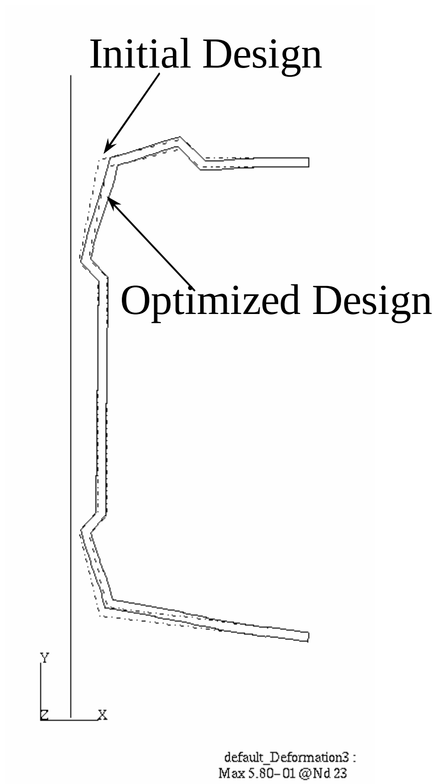
$e^p{}_{45}(0.01) \le 0.04$ $\quad e^p{}_{46}(0.01) \le 0.04$

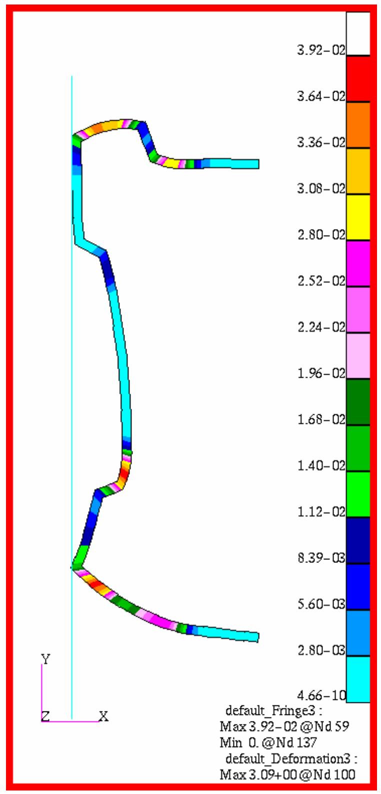
$e^p{}_{65}(0.06) \le 0.04$ $\quad e^p{}_{66}(0.04) \le 0.04$

$e^p{}_{67}(0.02) \le 0.04$ $\quad F_{Cx}(2.0) \ge 2.0$

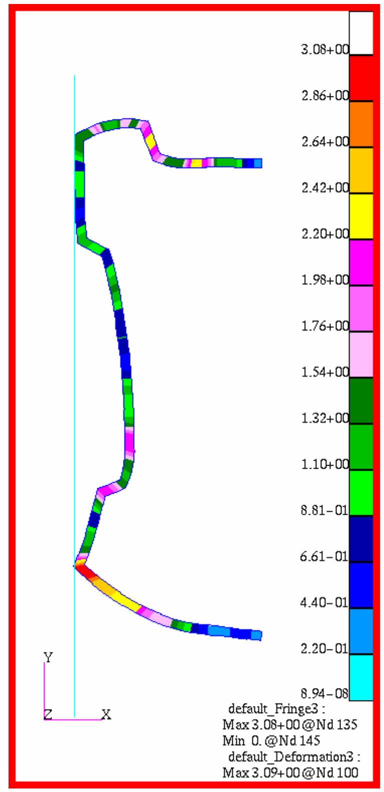
$-1.0 \le u_i \le 1.0 \quad i = 1,16$

Optimization Results



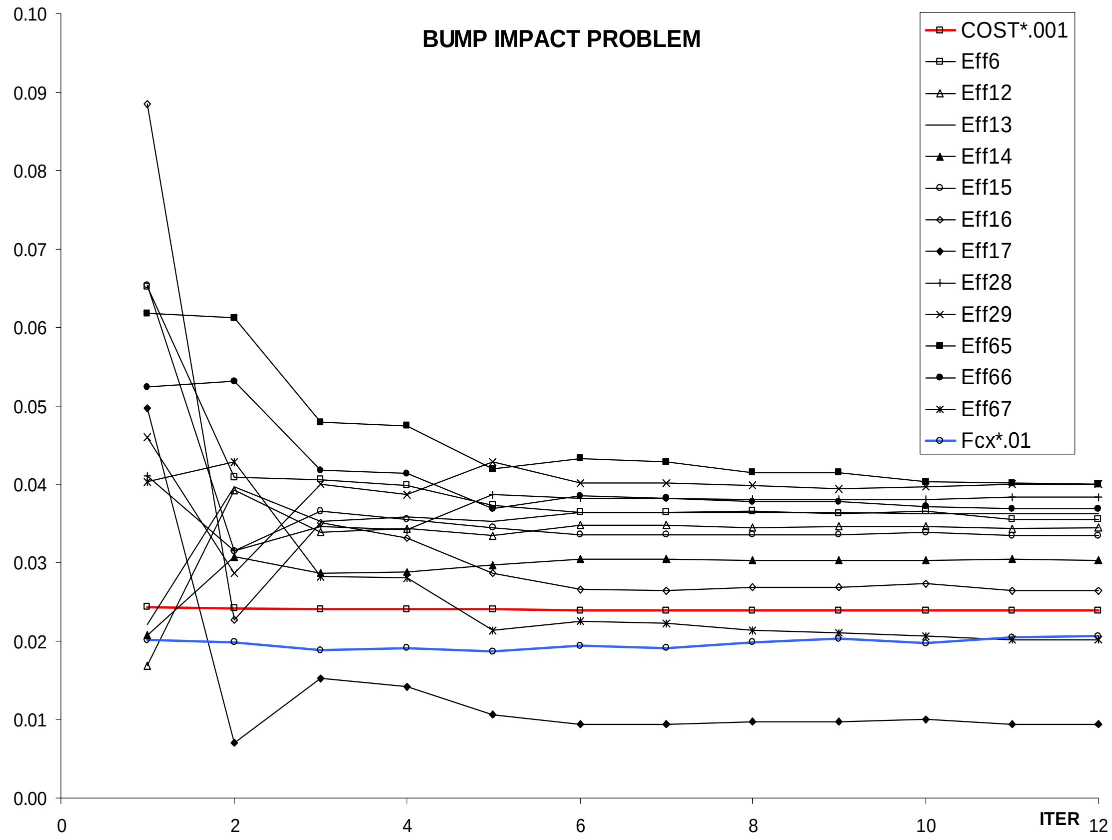


Effective Plastic Strain



Von Mises Stress

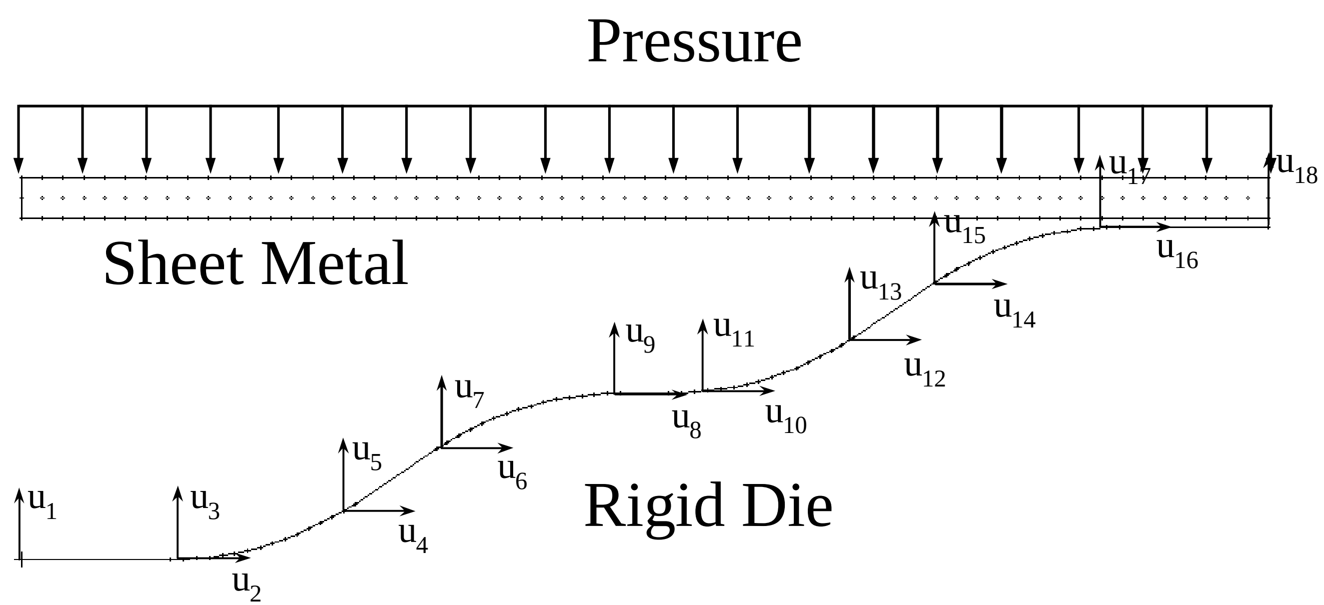
Optimization History



Response Analysis : 30
Sensitivity Analysis : 12

Pressurized Sheet Metal Stamping

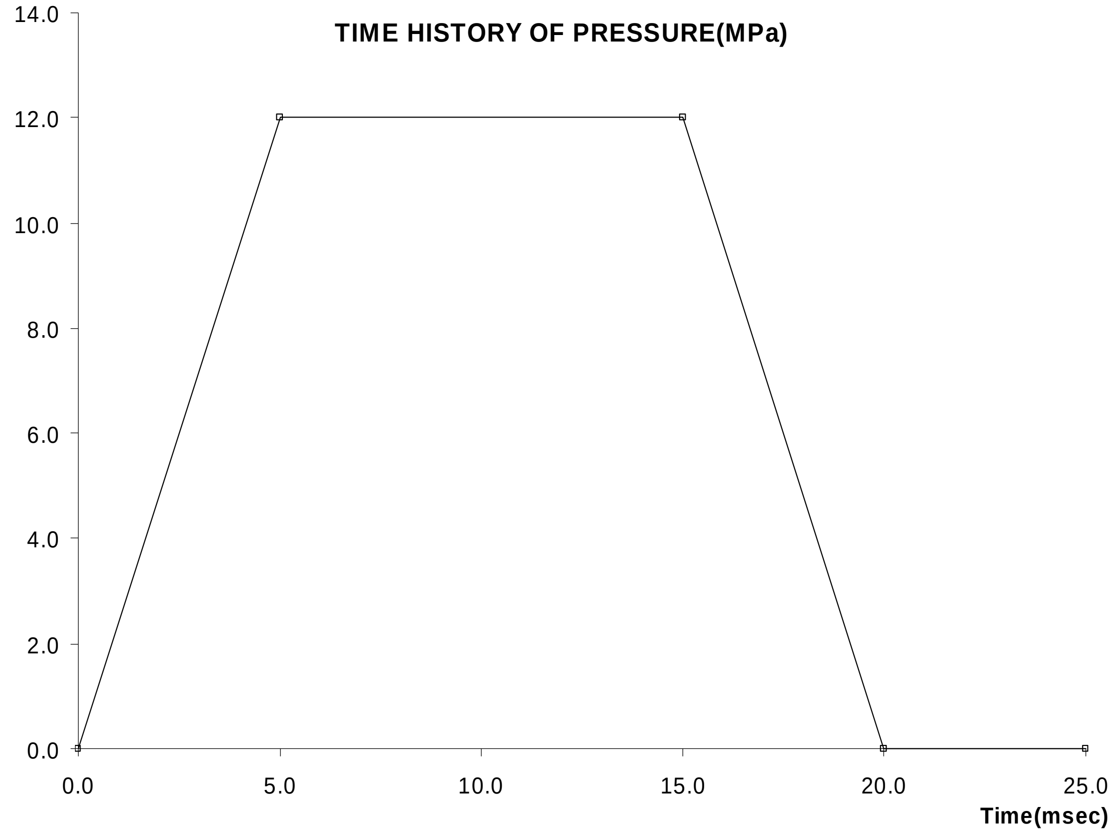
Initial Geometry and Design Parameters



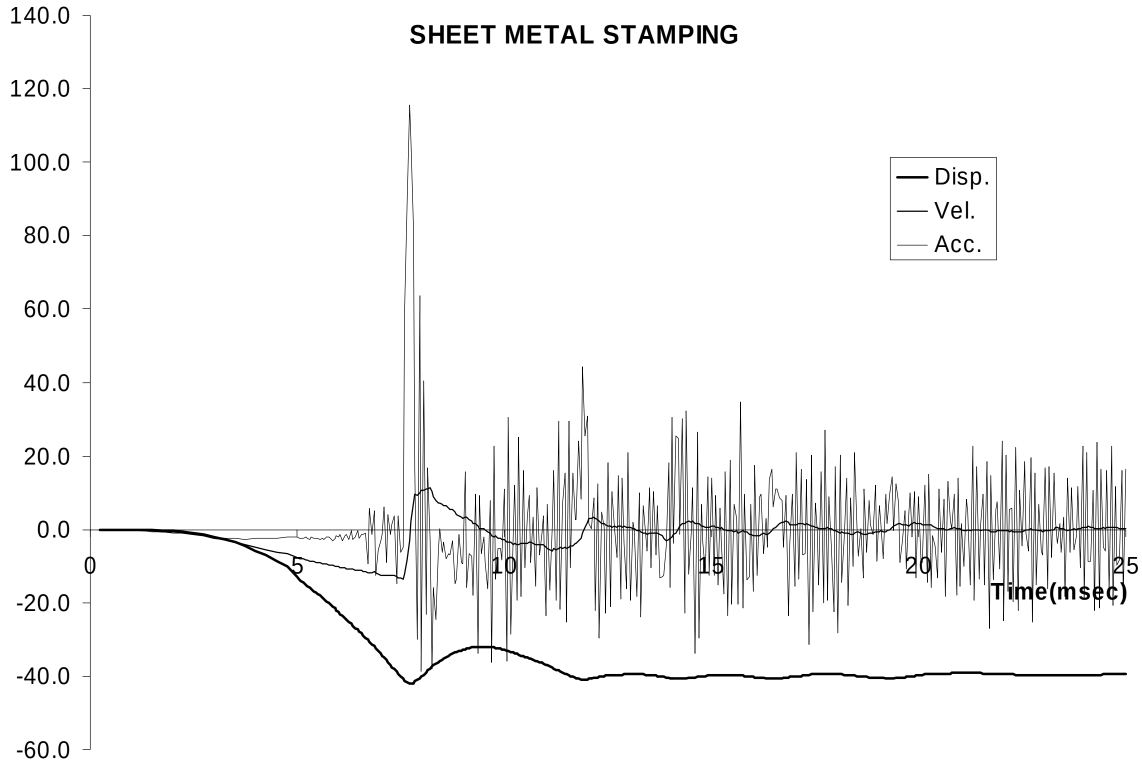
Die Shape DSA and Optimization

Kim *et al*. Comp. Mech. 25 (2000) 157-168

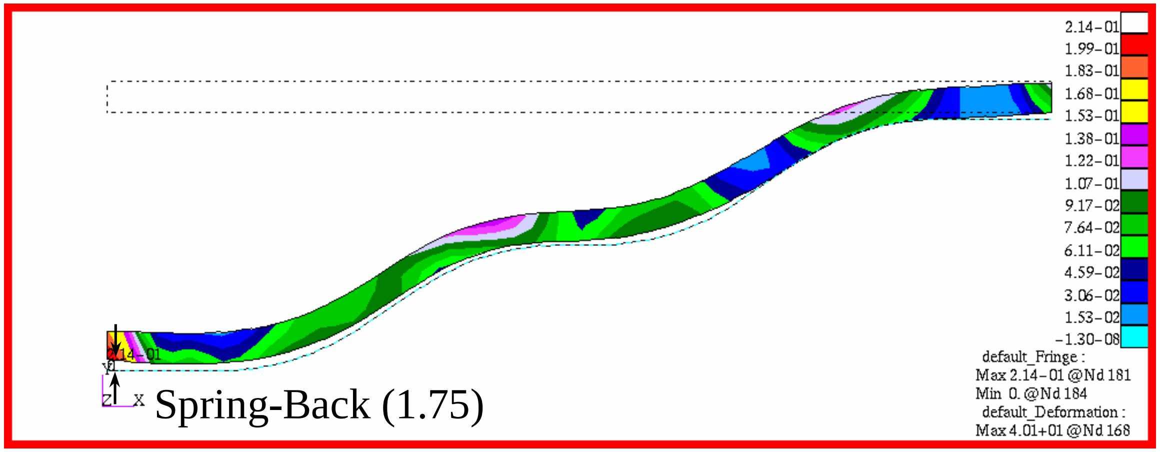
Pressure Load Time History



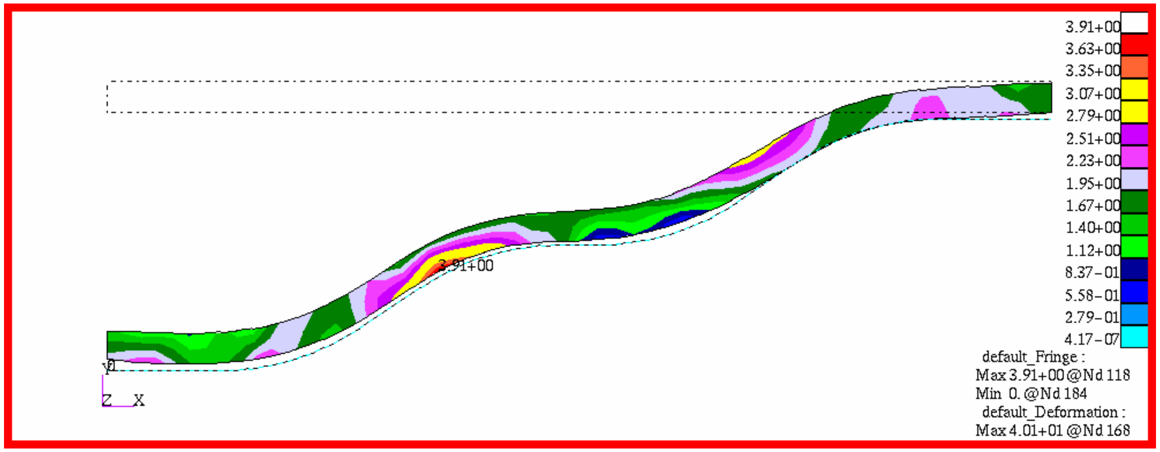
Time History of Deformation



Analysis Results



Effective
Plastic Strain



Von Mises
Stress

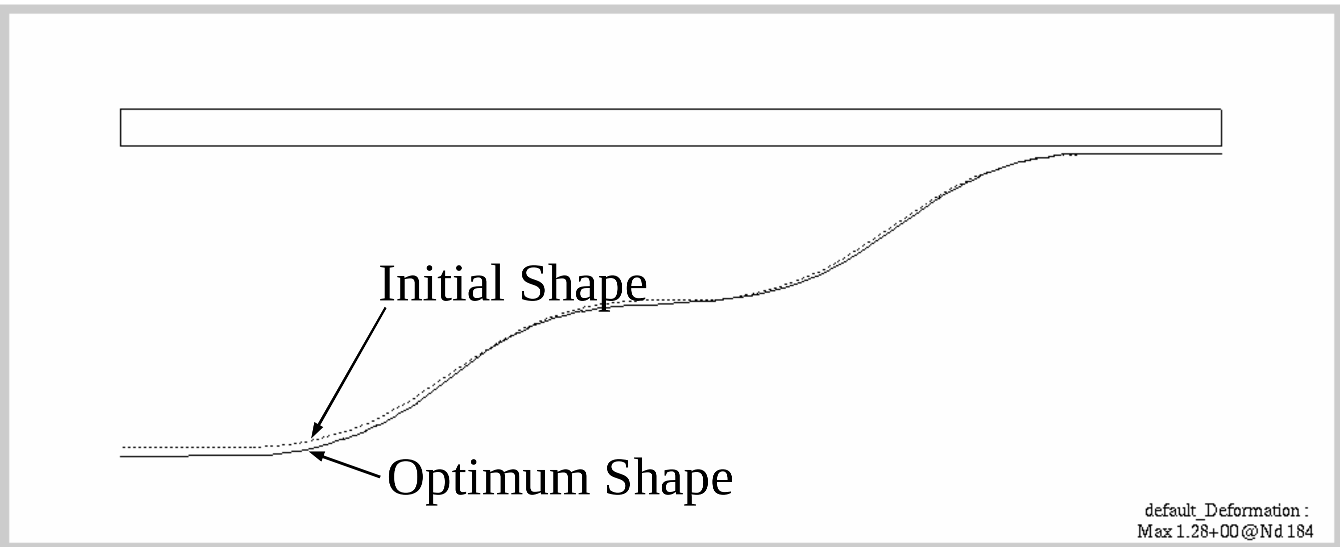
Design Optimization

$$MIN \quad G = \int_{\Gamma} \left\| \pi(\mathbf{x}) - \mathbf{x} \right\|^2 \ d\Gamma$$

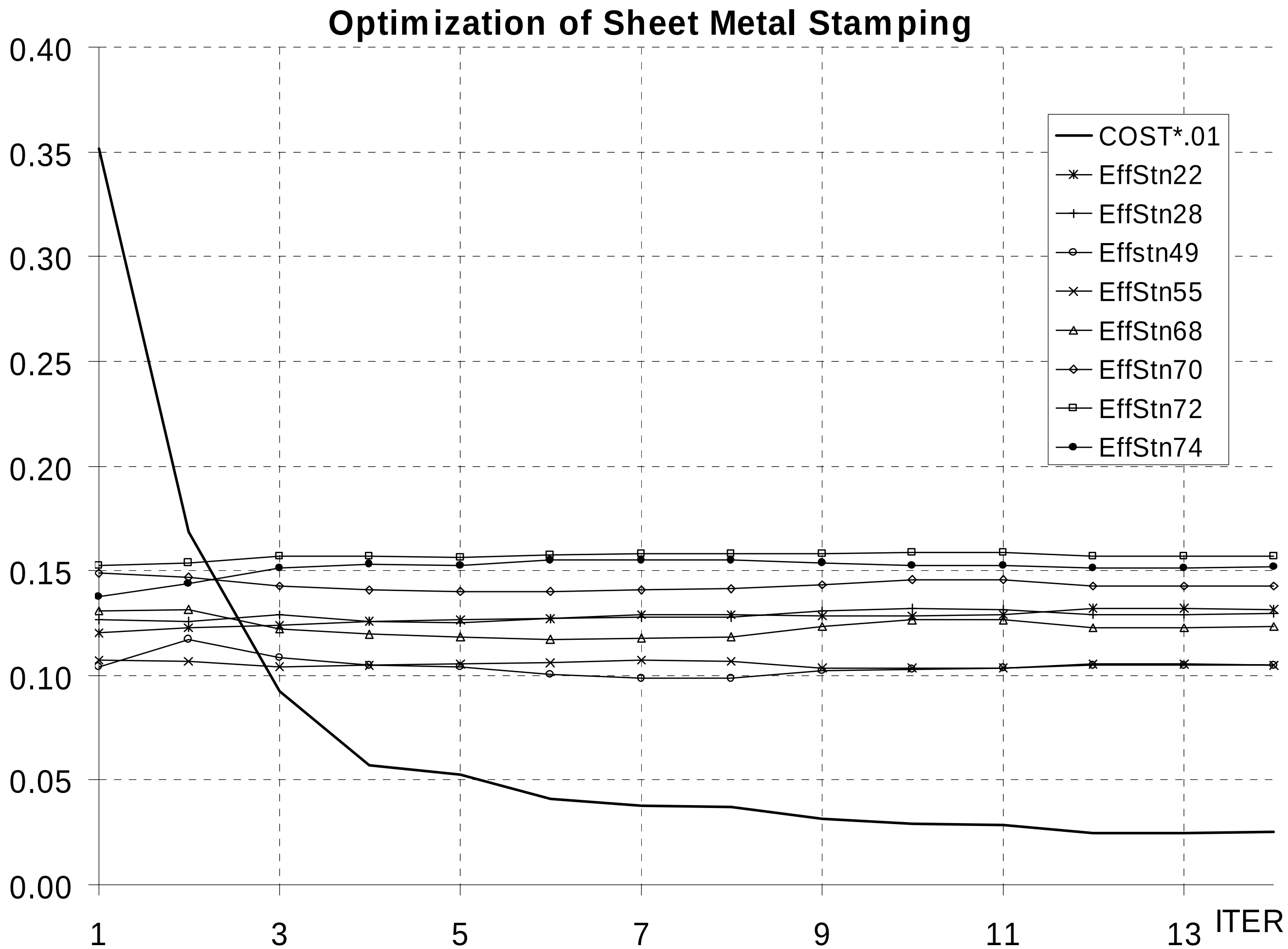
$$S.T. \quad e^{p}{}_{i} \le 0.16 \qquad i = 22, 28, 49, 55, 68, 70, 72, 74$$

$$-3.0 \le u_j \le 3.0 \ j = 1, \cdots, 18$$

Response Analysis : 12,018 sec
Sensitivity Analysis : 3,215/18 sec



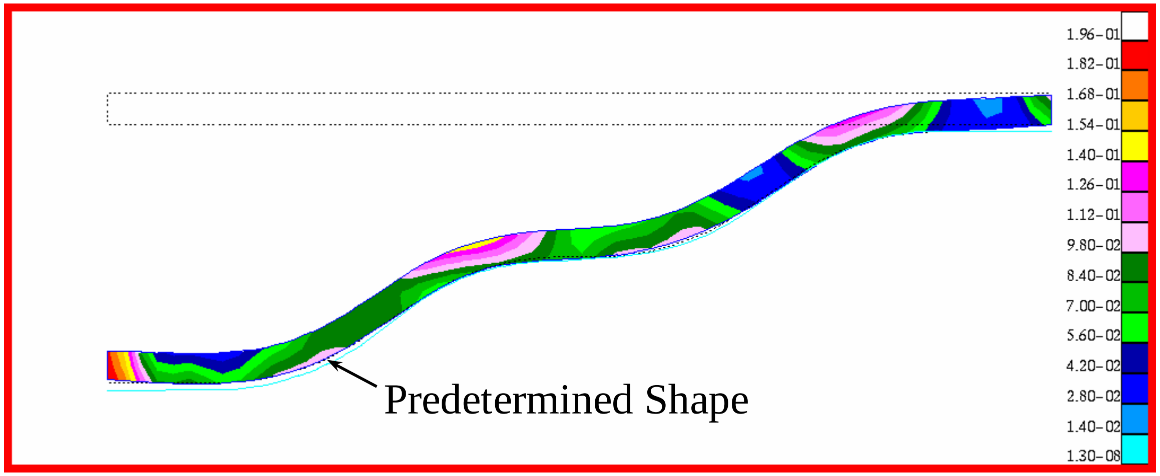
Optimization History



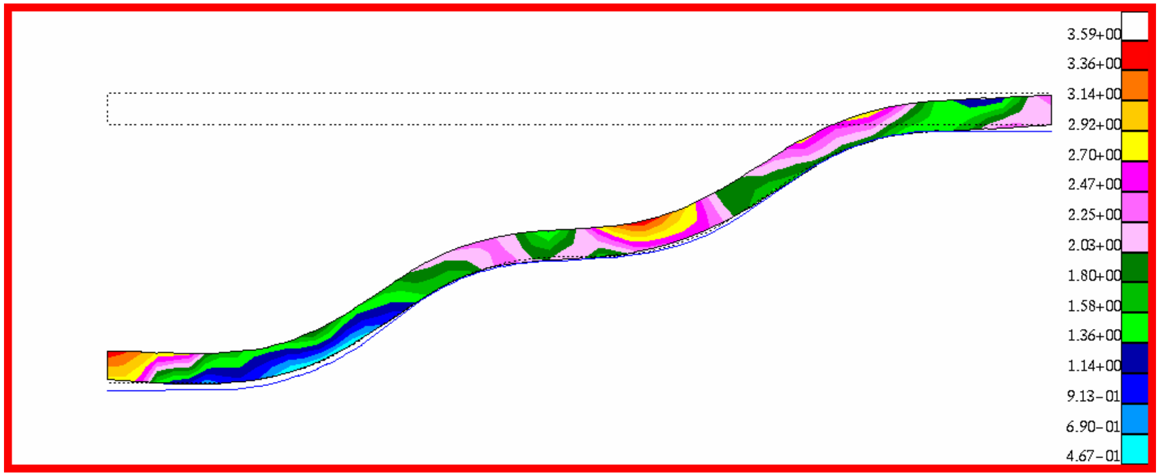
Response Analysis : 41

Sensitivity Analysis : 14

Optimization Results



Effective Plastic Strain



Von Mises Stress

Conclusions

- An Accurate and Efficient Shape DSA and Optimization of Structural Transient Dynamics is Proposed.
- Finite Deformation Elastoplastic Material and Frictional Contact Condition Are Considered in DSA
- Design Sensitivity Equation Is Solved at Each Converged Time Step without Iteration Using the Same Tangent Stiffness Matrix from Analysis
- Sensitivity Equation Is More Efficient for the Implicit Time Integration Method Than the Explicit Method Compared to the Cost of Response Analysis