Bayesian Approach for Parameter Estimation in the Structural Analysis and Prognosis

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ABSTRACT

In this study, a Bayesian framework is outlined for the parameter estimation that arises during the uncertainty quantification in the numerical simulation as well as in the prognosis of the structural performance. In the framework, the parameters are estimated in the form of posterior distribution conditional on the provided data. Several case studies that implement the estimation are presented to illustrate the concept. First one is an inverse estimation, in which the unknown input parameters are inversely estimated based on a finite number of measured response data. Next one is a metamodel uncertainty problem that arises when the original response function is approximated by a metamodel using a finite set of response values. Third and fourth one are a prognostics problem, in which the unknown parameters of the degradation model are estimated based on the monitored data. During the numerical implementation, Markov Chain Monte Carlo (MCMC) method is employed, which is a modern computational technique for the efficient and straightforward estimation of parameters. Once the samples are obtained, one can proceed to the posterior predictive inference on the response at the unobserved points or at the future time in the form of confidence interval.

Key Words *: Bayesian framework, inverse estimation, metamodel uncertainty, prognostics and health management (PHM), Markov Chain Monte Carlo (MCMC).*

1. INTRODUCTION

There are many circumstances that require parameter estimation either in the structural analysis at the design stage or in the health management of the existing structures. At the design stage, material parameters, which significantly affect the validity of the analyses, need to be estimated based on the direct or indirect measurements. During the costly structural analysis, metamodel is often introduced to save computational cost, in which the associated parameters, e.g., regression coefficients, are estimated using a finite set of response data. In the health management, degradation parameters of underlying physical model in the deteriorating structures are estimated using the monitoring data over times for the prognostics of remaining useful life (RUL).

Common practice for parameter estimation is that the parameters are measured directly by appropriate lab tests or inversely estimated from the data taken by the field inspection. Whichever they are, these parameters can't be determined in a deterministic manner but be treated with stochastic way due to the various uncertainties arising from the measurements, inherent material defects and manufacturing processes. In the simplest terms, the parameters are determined via classical regression technique using the given data. In view of uncertainty management, however, the Bayesian approach is recently gaining considerable attention, which provides a logical framework for inference of the parameters in light of the observed data while incorporating the prior knowledge. As more data are collected, the process is repeated with updated prior, and the parameters converge toward the real values providing more confidence.

In this study, Bayesian framework is outlined in the context of parameter estimation, in which the unknown parameters are estimated in the form of posterior distribution which is proportional to the likelihood of the observed data multiplied by the prior distribution. During the numerical implementation, Markov Chain Monte Carlo (MCMC) method (Andrieu et al., 2003) is employed, which is a modern computational technique for the efficient and straightforward estimation of parameters, which draw samples of parameters having complex distribution. Once the samples are obtained, one can proceed to the posterior predictive inference on the response at the unobserved points or at the future time in the form of confidence interval.

Four case studies are presented to illustrate the concept of the Bayesian application and the power of MCMC technique. First one is an inverse estimation, in which the unknown input parameters are inversely estimated based on a finite number of response data. A mathematical problem is addressed, followed by a more practical example, which is a parameter identification of creep behavior of a solder joint in microelectronics package under a thermal cycle. The estimated parameters can be used with more reliability to predict the response at the unobserved points or for a new design. Next one is a metamodel uncertainty problem that arises when the original response function is approximated by a metamodel using a finite set of response values. A mathematical problem is addressed to exemplify the concept. Third and fourth one are a prognostics problem, in which the unknown parameters of the degradation model are estimated based on the monitored data. Progressive wear in a revolute joint in motion and crack growth in an aircraft fuselage are addressed to illustrate the concept. Using the determined parameters, the degradation behavior at the future time as well as the RUL is predicted, which plays very useful roles in the condition based maintenance (CBM).

2. BAYESIAN INFERENCE TECHNIQUE FOR PARAMETER ESTIMATION

For the Bayesian parameter estimation, Bayes' rule is used (Bayes, 1763):

$$p(\boldsymbol{\theta}|\mathbf{y}) \propto L(\mathbf{y}|\boldsymbol{\theta})\, p(\boldsymbol{\theta}) \tag{1}$$

where $L(\mathbf{y}|\boldsymbol{\theta})$ is the likelihood of observed data $\mathbf{y}$ conditional on the given parameters $\boldsymbol{\theta}$, $p(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta}|\mathbf{y})$ is the posterior distribution of $\boldsymbol{\theta}$ conditional on $\mathbf{y}$. The equation states that our degree of belief on the parameter $\boldsymbol{\theta}$ is expressed as posterior PDF in light of the given data $\mathbf{y}$. As more data are provided, the posterior distribution is again used as a prior at the next step, and the values are updated to more confident information. This is called Bayesian updating. The procedure to obtain posterior distribution $p(\boldsymbol{\theta}|\mathbf{y})$ consists of proper definition of probability distribution for the likelihood and prior respectively.

Once the expression for posterior PDF is available, one can proceed to sample from the PDF. MCMC simulation is an effective solution to this end since it easily produces posterior distribution with any complexity including no closed form expressions. As an example of MCMC, in Figure 1 is shown the sampling result of fictitious PDF given as (Andrieu et al., 2003)

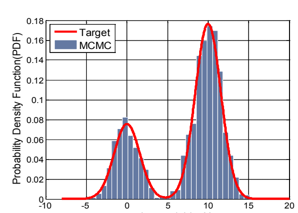
$$p(x) \propto 0.3\exp\left(-0.2x^2\right) + 0.7\exp\left(-0.2(x-10)^2\right) \tag{2}$$

With only 5,000 iterations, the sampling result follows the distribution quite well.

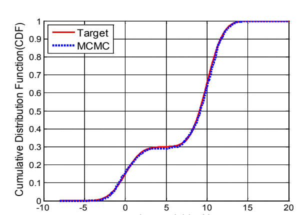
Once the samples of $\boldsymbol{\theta}$ are obtained, one can obtain the predictive distribution of the response function at new points using the formula:

$$p(\tilde{\mathbf{y}}|\mathbf{y}) = \int L(\tilde{\mathbf{y}}|\boldsymbol{\theta})\, p(\boldsymbol{\theta}|\mathbf{y})\, d\boldsymbol{\theta} \tag{3}$$

where the symbol $\sim$ represents the prediction, $p(\tilde{\mathbf{y}}|\mathbf{y})$ is the predictive distribution of $\tilde{\mathbf{y}}$ conditional on $\mathbf{y}$, $p(\boldsymbol{\theta}|\mathbf{y})$ is the posterior distribution of $\boldsymbol{\theta}$ from Eq. (1), and $L(\tilde{\mathbf{y}}|\boldsymbol{\theta})$ is the likelihood to obtain $\tilde{\mathbf{y}}$ conditional on $\boldsymbol{\theta}$. Although the expression is in the form of integration, $\tilde{\mathbf{y}}$ distribution are obtained in practice by drawing samples from $L(\tilde{\mathbf{y}}|\boldsymbol{\theta})$, given each value of drawn samples $\boldsymbol{\theta}$.



a) PDF



b) CDF

Figure 1: Results of MCMC simulation

3. INVERSE PARAMETER ESTIMATION

In this section, inverse parameter estimation is addressed that determines inversely some unknown parameters in the input variables by using a finite number of observed response data. During the procedure, the inherent (aleatory) uncertainty associated with the measurements and statistical (epistemic) uncertainty due to the limited data are accounted for using the Bayesian approach.

The experimental observation of the response function is assumed as

$$y^e(\mathbf{x}) = y^c(\mathbf{x} \mid \mathbf{t}) + \varepsilon \tag{4}$$

where y^e is the experimental measurement y^c is the computer model, ε is the white noise error with $N(0,\sigma^2)$, $\mathbf{x}$ is the explanatory variable, and $\mathbf{t}$ is the unknown parameters to be calibrated conditional on the observed data y^e. Suppose we have n experimental observation $\mathbf{y}_e = \left[y_1{}^e, ..., y_n{}^e\right]'$ at a set of points $D^e = \left[\mathbf{x}_1{}^e, ..., \mathbf{x}_n{}^e\right]'$. Then the likelihood of the observation error at the points is defined as

$$\begin{aligned} &L(\mathbf{y}^e \mid \mathbf{t}, \sigma) \\ &\propto \sigma^{-n} \exp\left\{ -\frac{1}{2\sigma^2} \left(\mathbf{y}^e - y^c(\mathbf{x}^e, \mathbf{t})\right)' \left(\mathbf{y} - y^c(\mathbf{x}^e, \mathbf{t})\right) \right\} \end{aligned} \tag{5}$$

In this case, the unknowns to be estimated are the parameters $\mathbf{t}$ and error variance σ^2. Multiplying prior PDF's for these parameters, which are assumed non-informative in this case, one obtains the posterior PDF of the parameters.

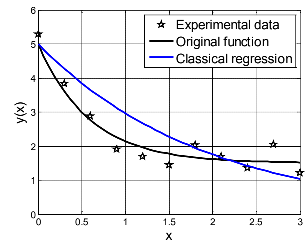
As a first example, a math model is considered as follows.

$$y(x) = 1.5 + 3.5\exp(-1.7x);\; x \in [0,3] \tag{6}$$

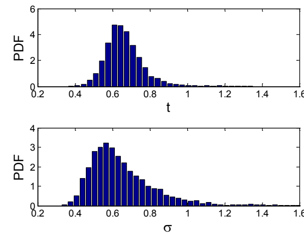
Assume that we have measured data for this model with the error $\varepsilon \sim N(0, 0.6)$ which are given as black stars in Figure 2 (a). Suppose we have no prior knowledge on this model, the response function is assumed to be expressed as

$$y^c(x \mid t) = 5\exp(-tx) \tag{7}$$

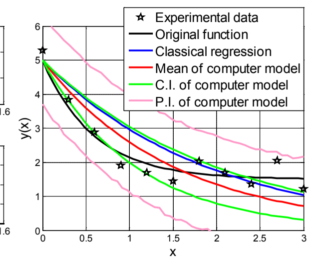
where t is the unknown parameter to be estimated conditional on the observed data $\mathbf{y}^e$. After running MCMC, unknown parameters t, σ where σ denotes the standard deviation of the measurement error are obtained in the PDF form as in Figure 2 (b). Using the parameters, one can predict the response function values at an arbitrary point in the probabilistic manner, and the results are given in a confidence band as in Figure 2 (c).



a) classical regression



b) posterior distributions of unknown parameters



c) confidence bands of predictive response function

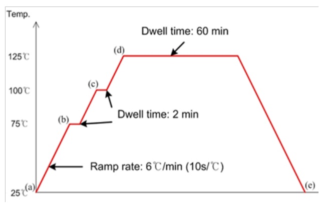
Figure 2: Results of mathematical example

Next example is to inversely estimate creep parameters of the solder joint in a microelectronics package conditional on the observed data under a thermal cycle which is given by Figure 3 (a). A special specimen (Pollack, 2003) as shown in Figure 3 (b) is used to measure the deformation via Moiré interferometry, of which a case at 125℃ is given in Figure 3 (c). Finite element model shown in Figure 3 (d) is created to conduct viscoplastic analysis. Material properties are given in Table 1. Anand model (Anand, 1985) is employed for characterizing the solder material, which is expressed by 9 parameters:

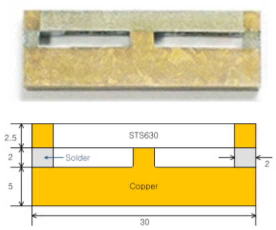
$$\begin{aligned} \dot{\varepsilon}_p &= A\exp\left(-\frac{Q}{RT}\right)\left[\sinh\left(\xi\frac{\sigma}{s}\right)\right]^{1/m} \\ \dot{s} &= \left\{h_0\left|1-\frac{s}{s^*}\right|^a \cdot sign\left(1-\frac{s}{s^*}\right)\right\}\cdot\dot{\varepsilon}_p \\ s^* &= \hat{s}\left[\frac{\dot{\varepsilon}_p}{A}\exp\left(\frac{Q}{RT}\right)\right]^n \end{aligned} \tag{8}$$

ANOVA is carried out to identify the most influential ones over the range of the parameters, from which $S_0, Q/R, \xi$ and m are found whereas the others are presumed insignificant. These parameters as well as σ of the measurement error are the unknowns to be determined. Since the finite element analysis is computationally expensive, a response surface employing fourth order polynomial in terms of the temperature and the four Anand parameters is constructed, and replaced into the original y^c. After

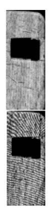
MCMC, their posterior PDFs are obtained by the likelihood of three data – displacements at A, B, and strain at C. From their posterior PDFs, the predictive distribution of the displacements as well as the strain at the same points are obtained in the form of confidence bands as shown in Figure 4.



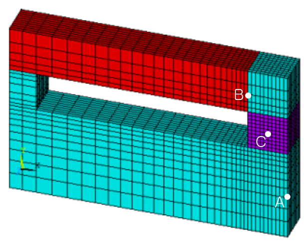
a) thermal cycle



b) special specimen



c) Moiré fringe

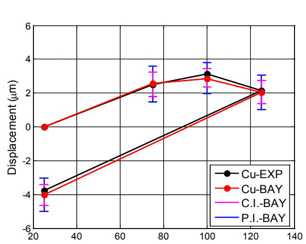


d) finite element model

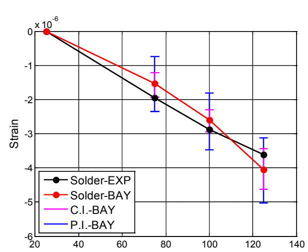
Figure 3: Illustrations of creep parameters estimation

Table 1: Elastic properties used for the steel, copper and solder

	Elastic Modulus(GPa)	Poisson's ratio	CTE (ppm/°C)
STS630	275	0.22	5.3
Copper	130	0.344	17.8
Sn37Pb	32	0.38	24.7



a) displacement



b) strain

Figure 4: Confidence bands of predictive response model

4. METAMODEL UNCERTAINTY PROBLEM

Metamodel uncertainty arises when the original response function is approximated by a metamodel using a finite set of response values. In this section, Kriging metamodel is considered which is popular in the design analysis of the computer experiments. Due to the limited data, however, there is statistical uncertainty with the response in between the points.

Let us try to approximate a function using a finite number of responses $\mathbf{y} = [y_1, ..., y_m]'$ at a set of DOE points $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_m]'$ with m number. For this purpose, Gaussian random function is introduced as follows.

$$\hat{y}(\mathbf{x}) = \mathbf{f}(\mathbf{x})\boldsymbol{\beta} + Z(\mathbf{x}) \tag{9}$$

where

$$Z \sim N(0\mathbf{I}_m, \sigma^2\mathbf{R}),\ \mathbf{R} = R(\mathbf{x}_i, \mathbf{x}_j),\ i, j = 1, ..., m \tag{10}$$

In the equation, ^ denotes the surrogate representation, $\mathbf{f}(\mathbf{x})\boldsymbol{\beta}$ is the normal linear model, $\mathbf{f} = [f_1, ..., f_k]$ and $\boldsymbol{\beta} = [\beta_1, ..., \beta_k]'$ are k number of the trial functions and associated parameters, respectively, Z is a Gaussian stochastic process with zero mean and variance σ^2, $\mathbf{I}_m$ is the $m \times m$ identity matrix, and R is a correlation function between $\mathbf{x}_i$ and $\mathbf{x}_j$ which is represented by

$$R(\mathbf{x}_i, \mathbf{x}_j) = \exp\left\{-\left(\frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{h}\right)^2\right\} \tag{11}$$

where h is a correlation parameter that controls the degree of smoothness of the function. If the h gets higher, the model becomes smoother, but the singularity is encountered in the correlation matrix if it is too high. In most Kriging studies, h is treated as a constant that is determined by the method of maximum likelihood estimate (MLE). However, MLE method is not only computationally expensive which requires additional optimization process, but also the quality of the obtained parameter is questionable. In this study, h is considered as an unknown parameter as well to avoid this. Assuming the computed outputs follow multivariate normal distribution, the joint posterior distribution of the parameters is given by

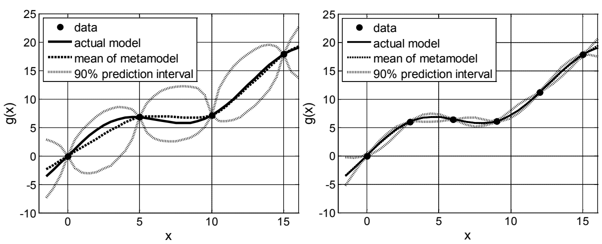
$$\boldsymbol{\beta}, \sigma^2, h \mid \mathbf{y} \propto$$

$$(\sigma^2)^{-\frac{m+2}{2}} \left|\mathbf{R}_{(\mathbf{x})}\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2\sigma^2}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta})' \mathbf{R}_{(\mathbf{x})}^{-1} (\mathbf{y} - \mathbf{F}\boldsymbol{\beta})\right) \tag{12}$$

where $\mathbf{F}=\left[\mathbf{f}\left(\mathbf{x}_1\right),...,\mathbf{f}\left(\mathbf{x}_m\right)\right]'$ is $m\times k$ dimensional matrix of trial functions at the DOE points. In the equation, the non-informative priors are assumed for the unknown parameters, i.e., $f\left(\boldsymbol{\beta},\sigma^2,h\right)\propto\sigma^{-2}$. Consider a mathematical function (O'Hagan, 2006)

$$g\left(X\right)=X+3\sin\left(X/2\right) \tag{13}$$

With the trial functions $\mathbf{f}=\left[1,x\right]$, the posterior distributions of the unknown parameters β_1,β_2, σ, and h are obtained using MCMC. 90% prediction intervals due to the metamodel uncertainty are shown in Figure 5 (a) and (b) for the case of 4 points and 6 points respectively. It is observed that the uncertainty is reduced as more data are provided.



a) the case of 4 points b) the case of 6 points

Figure 5: Confidence bands of predictive response function due to the metamodel uncertainty

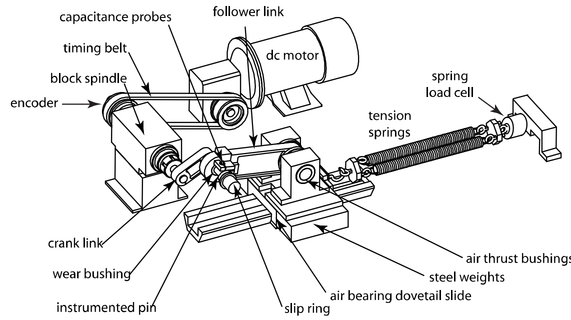
5. PROGNOSTICS PROBLEM

Prognostics is to predict the remaining useful life (RUL) of a structural parts due to the degradation by measuring and monitoring the associated response during the use of the part. Physics based degradation model should be available for this, in which the associated parameters determines the exact form of degradation behavior. Using the Bayesian approach, the parameters are obtained in the posterior PDF conditional on the measured responses. Then the RUL can be estimated in the form of predictive distribution, which is very important information for the condition based maintenance. In this section, progressive wear in a revolute joint in motion and crack growth in an aircraft fuselage are addressed to illustrate the concept.

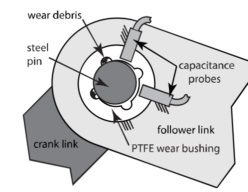
5.1 Progressive Wear

Wear is a gradual removal of material from contacting surfaces in relative motion, which eventually causes failure. Prediction of wear volume and its service life is important in this regard. Among various wear models, Archard's wear model (Archard, 1953) is the most widely used one which is given by

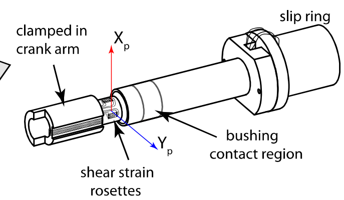
$$k=\frac{V}{\int_0^s F_n(s)\mathrm{d}s} \tag{14}$$



a) the layout of slider-crank mechanism



b) capacitance probes measure the location of the pin from fixed locations on the follower link



c) instrumented steel pin load cell for measuring joint force

Figure 6: Illustrations of wear measurement in a revolute joint

where k is the wear coefficient, V is the wear volume up to current moment, F_n is the applied normal force, and s the slip distance. In order to estimate the unknown wear coefficient k, a crank-slider test apparatus (Mauntler et al., 2007) is used as shown in Figure 6, in which the revolute joint is connected by a pin with high hardness whereas the bushing at the follower link is worn out. Capacitance probes are inserted into the follower link as in the figure to measure the pin displacements, from which the wear volume can be calculated. Forces are measured via a load cell built into the pin. The measured wear volumes and forces are given at 6 sets of number of cycles as given in Table 2. Denoting the set of wear volume data as $\mathbf{V}$, the likelihood of the volume for is assumed to follow normal distribution:

$$L\left(\mathbf{V}\,|\,k,\sigma\right)\sim N\left(\mu,\sigma\right) \tag{15}$$

where μ and σ are the mean and standard deviation of the wear volume. Recalling the eq.(7), the mean wear volume is given by

$$\mu=kC\sum_{i=1}^{n}F_{n_i}\Delta s_i \tag{16}$$

where C is the number of cycles. This equation means that the mean is only a function of k while the other terms are given from the measurement. Then the unknown parameters are k and σ which should be estimated conditional on the observed wear volume $\mathbf{V}$.

Employing non-informative prior for the parameter k, the posterior PDF's of k and σ are obtained using the MCMC. As a validation study, the wear volume at the future 6th cycle is predicted using the coefficient estimated using the first 5 set of data, and compared with the realized data. The result is in Figure 7, which shows excellent prediction of wear volume.

Table 2: Wear coefficient calculation

Cycles	Force	Volume	Slip distance	$k \times 10^4$
1	64.41	1.59	0.06	4134.80
100	62.80	2.57	5.99	68.25
1000	63.17	8.10	59.85	21.44
5000	64.77	24.48	299.24	12.63
10000	62.65	46.21	598.47	12.32
20585	59.96	93.90	1232.00	12.71

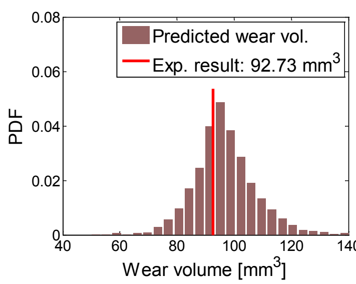


Figure 7: Result of wear volume prediction

5.2 Crack Growth in Aircraft Fuselage

Prognosis of crack growth is addressed for an aircraft fuselage panel in which a through-the-thickness crack grows through cycles of pressurization and depressurization (Coppe et al, 2010). A simple damage growth model, Paris model, is employed to this end (Paris, 1999):

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{17}$$

where a is the crack size, N the number of cycles, da/dN the crack growth rate, and ΔK the range of stress intensity factor. In this equation, the unknown parameters to be estimated are C and m. Accurate estimation of these parameters is important in the prediction of RUL of the panel in terms of the condition based maintenance. Based on the Figure 8 for the fuselage panel and the crack, the crack size a_N after N cycles of fatigue loading grown from the initial size a_0 is expressed as:

$$a_N = \left[NC\left(1-\frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_0^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \tag{18}$$

where $\Delta\sigma = (\Delta p)r/t$ denotes the range of hoop stress due to the pressure differential. The panel is regarded as failure when a_N reaches a critical crack size a_C which is given by:

$$a_c = \left(\frac{K_{IC}}{\Delta\sigma\sqrt{\pi}}\right)^2 \tag{19}$$

where K_{IC} is the fracture toughness of the panel. Based on the information given in Table 3, a_c is given by 46.3 mm in this study.

The objective is to identify the two parameters C and m using the inspected crack sizes over intervals of cycles. Since no actual data are available, the crack growth data are made by fictitious simulation by including the effect of bias and noise of the measurement. Using the true values of the parameters $C_{true} = 1.5\times10^{-10}$ and $m_{true} = 3.8$, 25 data are generated at every 100 flights beginning from the initial crack size $a_0 = 10mm$ up to 2500 flights, which is given in Table 4. In the data, the bias and the noise are assumed as 1 mm and $U(-1.5,1.5)mm$ respectively.

The posterior distribution of the C and m is determined by combining the likelihood to observe the actual crack size with the error following iid normal distribution and the prior PDFs given by Table 3. Using the MCMC for this distribution, the 30000 samples of C and m are drawn. These are used to obtain the predictive distribution of the crack growth as given in Figure 9. In the figure, the black curve is the true model and the symbol • denotes the measured crack size by the simulation. At the early stage like Figure 9 (a) which underwent 500 cycles, the predictive band is very wide due to the few number of data. The lower bound of the cycles with 90% CI to reach the critical level is found at about 1400 cycles, which means that the corresponding RUL is approximately 1400-500 = 900 cycles. As can be seen in subsequent figures, the band gets narrower as more data are added, and the curve becomes closer to the true model.

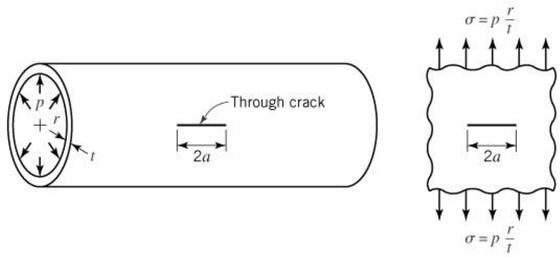


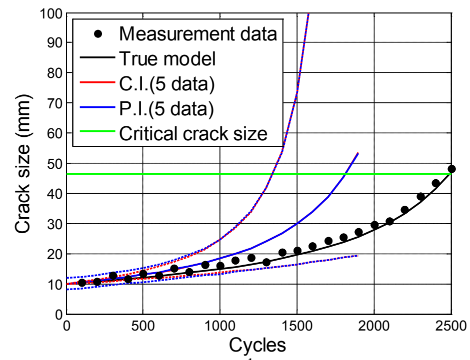
Figure 8: Through the thickness crack illustration

Table 3: Geometry, loading and fracture parameters of 7075-T651 Aluminum alloy

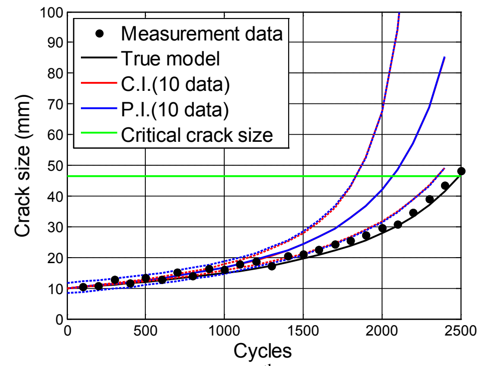
Pressure, Δ*p* (MPa)	0.06
K_{IC} (MPa·m$^{1/2}$)	30
Fuselage radius, *r* (m)	3.25
Fuselage thichness, *t* (m)	0.00248
Paris law exponent, *m*	U(3.3,4.3)
Damage parameter, C	U(5 x 10^{-11},5 x 10^{-10})

Table 4: The synthetic measurement data

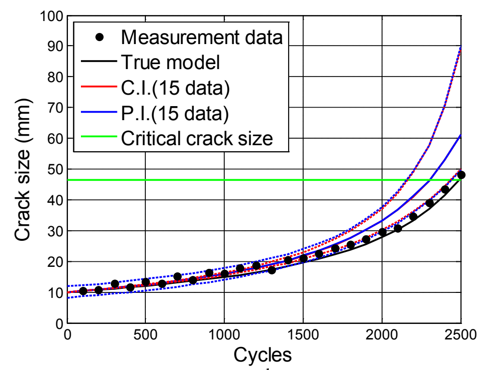
cycles	meas. crack size (mm)	cycles	meas. crack size (mm)	cycles	meas. crack size (mm)
0	10.00	900	16.44	1800	25.48
100	10.59	1000	16.17	1900	27.27
200	10.63	1100	17.93	2000	29.78
300	12.70	1200	18.78	2100	30.86
400	11.60	1300	17.23	2200	34.71
500	13.49	1400	20.59	2300	39.15
600	12.69	1500	21.15	2400	43.47
700	15.17	1600	22.68	2500	48.13
800	13.87	1700	24.29	a_c	46.30



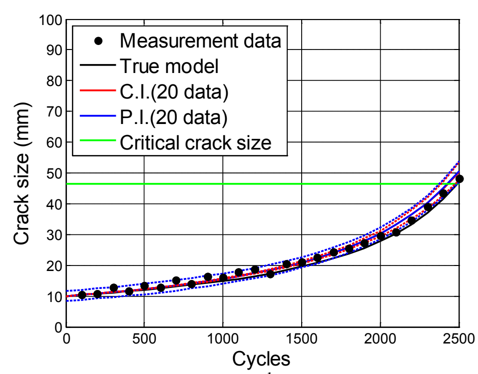
a) prediction based on upto 5th data



b) prediction based on upto 10th data



c) prediction based on upto 15th data



d) prediction based on upto 20th data

Figure 9: Estimated crack size

6. CONCLUSION

A Bayesian framework is addressed for the parameter estimation that arises during the analysis and prognosis of the structural performance. In the framework, the degree of belief on the parameters is expressed via a posterior probability distribution in light of the observed data combined with the prior knowledge. The distribution is obtained using the MCMC simulation, which is used to obtain the posterior predictive distribution, hence, the predictive bounds, of the performance at the unobserved points or at the future time. As more data are added, the bounds may be reduced, giving more confidence to the estimation. As a result of the case studies, the Bayesian approach is proved to be useful means for the uncertainty quantification of the unknown parameters in the finite element analysis of electronics package. The method is also found useful in the prognostics problems of the progressive wear in the revolute joint in motion and the crack growth in an aircraft fuselage.

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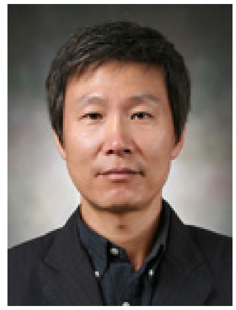

Jooho Choi received the B.S. degree of mechanical engineering from Hanyang University in 1981, the M.S. degree and Ph.D. degree of mechanical engineering from Korea Advanced Institute of Science and Technology (KAIST) in 1983 and 1987, respectively. During the year 1988, he worked as a Postdoctoral Fellow at University of Iowa. He joined the School of Aerospace and Mechanical Engineering at Korea Aerospace University, Korea, in 1997 and is now Professor. During 1988-96, he worked with Samsung Corning, Suwon, Korea, where he applied CAD/CAM/CAE techniques for TV glasses development and production. He has served as a board member of the Computational Structural Engineering Institute of Korea since 2001, and is now serving as a vice-president at the society. Since 2006, he has been serving as the editor in the field of materials and design engineering in the Journal of Mechanical Science and Technology. He has served as a program manager or advisory member in a number of governmental committees such as Korea Science and Engineering Foundation and Ministry of Commerce, Industry and Energy. He received Silver Prize Award in the 4th Samsung Group CAD and Super Computer User's Conference in 1993 and Excellent Paper Award from the Journal of Computational Structural Engineering Institute of Korea in 2002. He has numerous publications including 20 peer-reviewed journal articles, 40 conference proceedings in his research history. His research is now funded by a number of institutions including BK21 scholarship program and basic research project from Korea Research Foundation. He is now a member of several professional organizations including KSME, KSAS, SME, ASME and AIAA. His current research is focused on the reliability analysis, design for life-time reliability, and prognosis and health management.