“Special Session: Nondeterministic Approaches to Structural Health Management – Diagnostics and Prognostics”

Improving Diagnosis from Past Prognosis in Structural Health Monitoring

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Abstract

This paper presents a method of improving the accuracy of current diagnosis by using the prediction from previous inspection results. A major drawback of structural health monitoring (SHM) systems is that the uncertainty in inspection results is large compared to that of manual inspection. However, unlike manual inspection, SHM can take frequent measurements and trace crack growth. By taking advantage of this fact, higher accuracy about current crack size can be achieved. First, using the previous SHM measurements and the crack propagation model, we predict statistical distribution of crack sizes at the next SHM inspection cycle. Then, this predicted distribution is combined with the SHM measurement at the next cycle by using the Bayesian approach for more precise measurement. That is, the propagated distribution from the previous inspection is used as a prior and the variability at the current inspection is used to build the likelihood function. The uncertainty in measurements is modeled by a lognormal distribution, and several different prognostic approaches are considered for the construction of prior distribution. Results for through-the-thickness crack in a plate show substantial improvements in accuracy.

Keywords: Structural Health Monitoring (SHM), Damage prognosis, Size prediction model, Bayesian approach, Inspection error

I. Introduction

The importance of proper maintenance is a key issue for all mechanical components in an airplane. Current manual inspection procedures are very accurate in that sub-millimeters cracks can often be detected during the inspection [1]. However, since it is very costly and time consuming, there is great interest in the development of structural health monitoring (SHM) using embedded sensors as an alternative for manual inspection [2-5].

The biggest advantage of SHM is that the preventive maintenance can be replaced by condition-based maintenance by following crack propagation, thereby substantially increasing the average time intervals between maintenances [6]. However, due to the lack of accuracy, current SHM-based inspection does not provide accurate estimates of crack sizes to allow acceptable prediction of remaining useful life of the structure. Many damage identification techniques are being developed in search of accurate estimation of current crack size [7-12], and this technology is still in an emerging stage.

However, the fact that SHM can trace a crack as it grows provides an opportunity for improving the accuracy of current estimation as well as prognosis. Under regular inspection cycles, previous inspection results may be used to predict the crack size at the next inspection (prognosis). There are a number of approaches to predict the behavior of a crack and estimate the remaining useful life of a structure [14-16]. Most of them predict the behavior from the current state, and the information is usually discarded and we make a new prediction when new measurement is available. We call it the information from past prognosis.

The objective of this paper is to describe how we can make diagnosis more accurate by employing past prognosis information to construct prior distributions for crack size at the curent time. This distribution is then used to improve diagnosis. We compare the accuracy of various prognostic methods used to improve diagnosis.. Section 2 describes through-the-thckness crack propagation model and a model of noisy inspection measurements. Section 3 describes a Bayesian approach for prognosis. Section 4 shows the improvement of diagnosis using the suggested approach. Since we have many options to perform prognosis, we compare several different cases to perform prognosis and constructing the prior distribution for each case. Section 5 provide concluding remarks.

II. Modeling

A through-the-thickness center crack in a fuselage panel is considered as damage in this paper. Since the pressurization is the major loading factor for a fuselage, the example can be considered as a low-cycle fatigue. Paris' law is used to model the true crack behavior during the simulation (Eq. 1).

$$\frac{da}{dN} = C\Delta K^m \tag{1}$$

where a is the half crack size, N is the number of loading cycles (treated as a real number), $\Delta K = \Delta\sigma\sqrt{\pi a}$ is the range of stress intensity factor and C and m are crack propagation parameters related to material properties and geometry.

We also assume that the material has inherent defects, or micro cracks within the structure where the crack initiates. With the parameters given in Table 1, the true crack propagation is modeled by the solution of the Paris' law as shown in Eq. 2 and Figure 1. Note that *a(N)* shown in Eq. 2 is the half crack size.

$$a(N) = \left[NC\left(1 - \frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_i^{1-m/2} \right]^{\frac{2}{2-m}} \tag{2}$$

Table 1. Parameters related to simulated crack propagation.

Parameters	Value
Initial flaw size, a_i	1 mm
Minimum detectable crack size	6 mm
Maximum allowable crack size	50 mm
Crack propagation parameter, *C*	2.0×10^{-10}
Crack propagation parameter, *m*	3.7
Applied stress $\Delta\sigma$	78.63 MPa
SHM Inspection interval	50 cycles
First detection	34,300 cycles
Total life	42,300 cycles

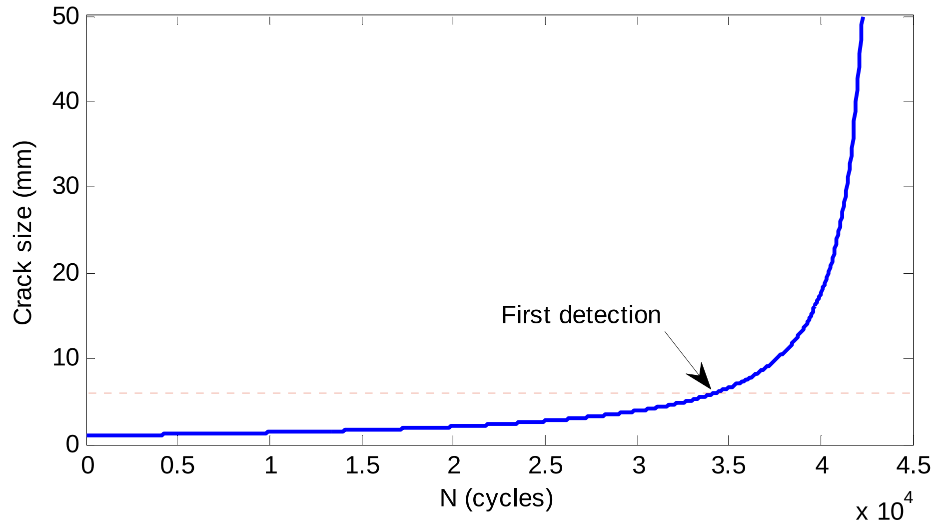


Figure 1. Crack propagation. We assumed that the crack is first detected when it is bigger than 6mm.

The minimum detectable crack size of SHM is larger than the manual inspection. In ultrasonic testing, the minimum size is about a half of the wavelength of the excitation wave [13]. Since there are practical limitations of the excitation frequency that we can use, actual value of minimum detectable crack size that we can use is around 5 ~ 10 mm. We assumed that the minimum detectable crack size to be 6 mm for this research. After the detection returns positive signal, the crack will be monitored until it is large enough to trigger maintenance. Here we assume that this is done when the crack size is 50 mm.

Uncertainty in the detected crack size varies widely by inspection methods, data processing techniques, and associated noise with the procedure. In general, it is known that the uncertainty in the detected size is larger when the crack is large [9-13], so this paper models the ratio between true crack size and inspected crack size by a lognormal distribution which has mean value 1 and coefficient of variation 0.1 (Eq 3).

$$a_{inspected} / a_{true} \sim LogN(-0.005, 0.1^2) \tag{3}$$

Here we assume that the measurements at different times are uncorrelated, and we neglect bias in the measurement. Using this distribution, one sample of a series of detected crack size at each inspection event is shown in figure 2. Although the general trend follows the true crack growth (green line), individual measured crack sizes are often quite different from the true one. The objective is to improve the measured crack size information by using predicted crack size information from the previous measurements.

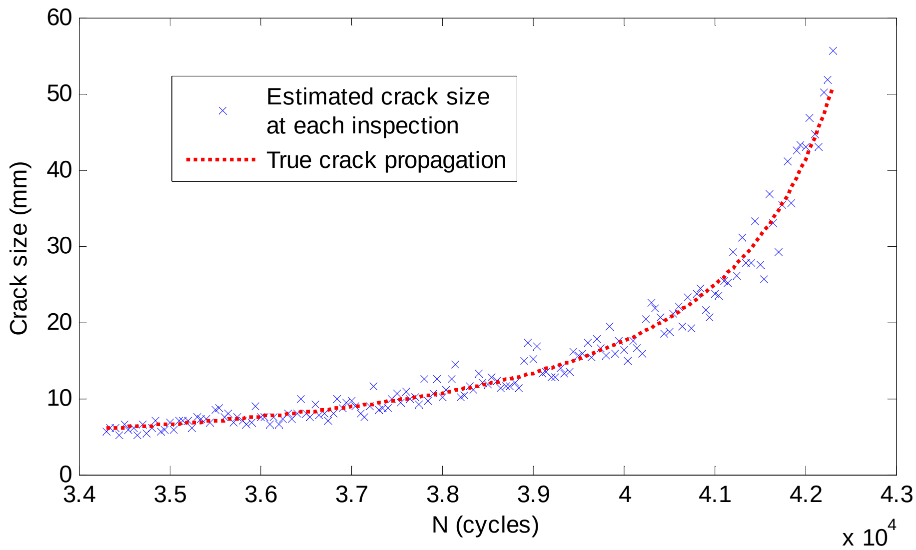


Figure 2. Sample of detected estimated crack sizes (based on Eq. 3)

III. Improved diagnosis by using Bayesian approach

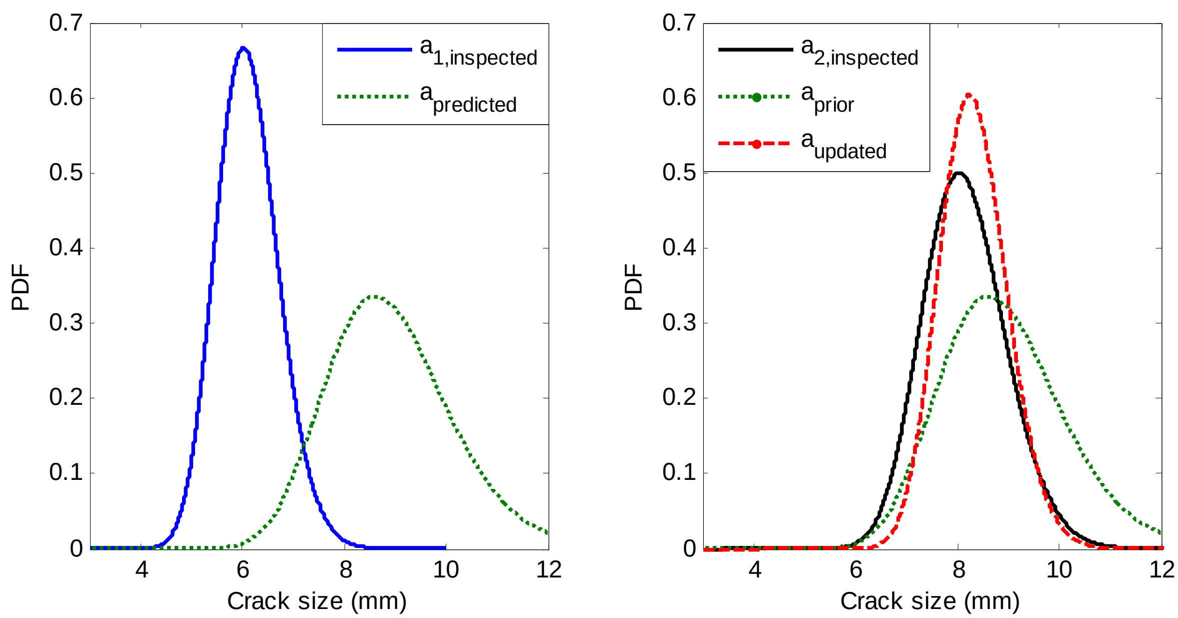
Bayes' rule for estimation of current crack size based on the inspection measurementis:

$$f_a(a_{poss} \mid a = a_m) = \frac{f(a_{poss}) l(a_m \mid a = a_{poss})}{\int_0^\infty f(a_{poss}) l(a_m \mid a = a_{poss}) da} \tag{4}$$

where a_m is the measured crack size from the inspection, and a_{poss} is a possible crack length. The likelihood function $l(a_m \mid a = a_{poss})$ is the probability density that we will have a_m given that the true crack size being a_{poss}. Here, the prior distribution $f(a_{poss})$ is based on prognosis using previous inspections. Using Eq. 4, we can improve the accuracy of current crack size estimation, and the posterior distribution $f_a(a_{poss} \mid a = a_m)$ is used for prognosis for the next inspection cycle by propagating it with Eq. 2.

Figure 3 illustrates this procedure using a large step (ΔN=2500) for clearer demonstration. In the actual simulation, we assume that the inspection interval is 50 cycles. In

Figure 3, the first detection result is 6 mm with 10% uncertainty, and the distribution of crack size follows the solid curve in Figure 3(a). We used Monte Carlo simulation (MCS) to perform the calculations. First, a set of random crack sizes following the posterior distribution is generated. Here, since no previous prognosis is available. This is the measurement uncertainty, Eq. 3. Then each crack sizeis propagated by ΔN (Eq.2) to obtain the dotted curve, which is the prognosis for crack sizes after 2500 additional cycles (dotted curve).



(a) (b)

Figure 3. Simulation procedure (a) Measurement probability distributionj and predicted probability distribution of crack size after 2500 cycles (b) Predicted distribution is used as a prior for the next inspection. Posterior distribution is calculated by combining current inspection result and the prior distribution.

At the next inspection, the measured crack size is 8 mm, and the uncertainty in measurement is interpreted by the likelihood function as in Eq (5).

$$l(a_m \mid a = a_{poss}) = LN(a_m; \ln(a), 0.1) \tag{5}$$

Where LN indicates the probability density function of a log-normal distribution. This explains that the likelihood is equivalent to the probability density that we will have our current measurement a_m if the true crack size is a for a range of values of a. By combining the likelihood with the prior distribution, we have an updated distribution about current crack size as in Figure 3(b).

IV. Constructing prior distribution using different prognosis

Prognosis is a process of predicting the future behavior, which adds uncertainties to the initial measurement uncertainty. There are many approaches for predicting crack propagation from current measurements. To examine the effect of the prognosis model, we selected four cases of possible prognosis. First, we select a case of perfect prognosis when we know the exact crack propagation model with accurate parameters. Second case is when our prognosis model is accurate, but the parameters are uncertain . The third case is the same as the second case, but we use a least square fit instead of Monte Carlo simulation. Finally, we model a case where we have a simplistic failure prediction model, not based on any physics, but fitting a quartic polynomial to past measurements and extrapolating it. Perfect prognosis

We first discuss a case where we have a confidence in our failure prediction model, which is Paris law with parameters given in Table 1. In this example, the only uncertainty is the uncertainty of measurements The inspection is performed every 50 cycles after the first detection. The probability distribution function (pdf) for the possible true crack size is constructed and updated at every inspection cycle. The evolution of the posterior pdf of crack sizes is shown in Figure 4.

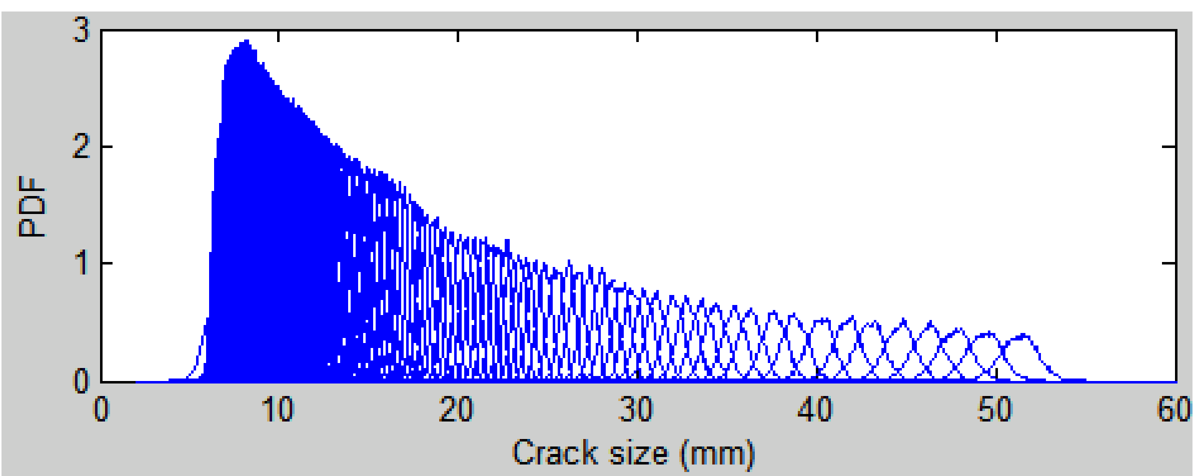


Figure 4. Probability distribution function of crack size updated at every 50 cycles with perfect prognosis.

When the crack grows, the standard deviation increases as well. Also, the crack propagates faster as the crack size increases. As a result, we can observe a wider posterior distribution when the crack is large. The estimated crack size at each inspection is the modal value of the corresponding pdf curve. The estimated crack size along the inspection cycle is shown in Figure 5.

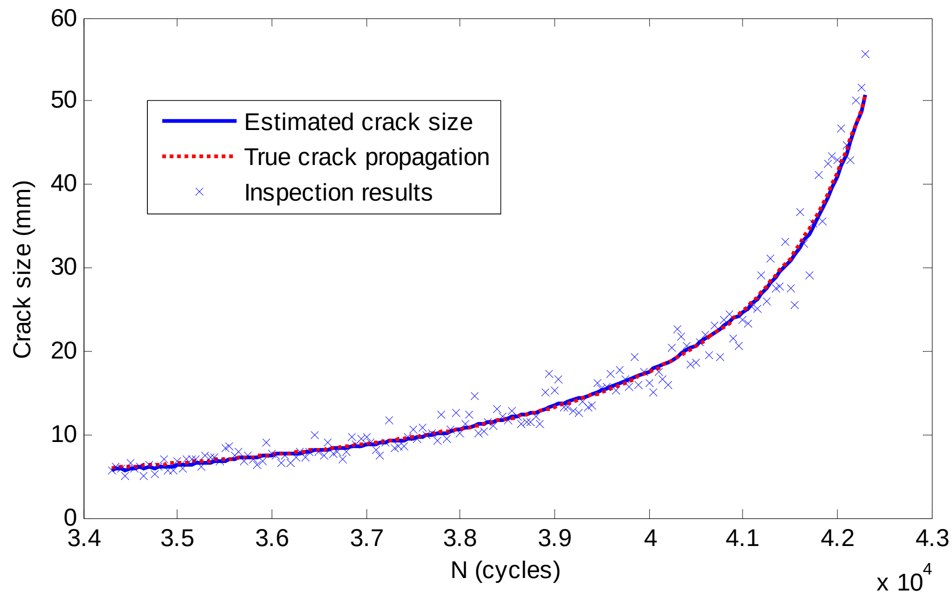


Figure 5. Crack size found by Bayesian update with perfect prognosis

Although the final crack size distribution was wide in figure 4, the estimated crack size is fairly close to the true crack size curve.

1. Uncertainty in crack propagation parameters

The distribution of the Paris law parameters m and C can be fairly wide to begin with, but we assume that it has been narrowed based on crack propagation in the present or similar panels (e.g. using the method of [17]) to

$$m \sim N(3.7,\ 0.1^2)$$
$$\log C \sim N(\log(2\times10^{-10}), 0.2^2) \qquad (7)$$

For this prognosis model, MCS is also used to evaluate the prior distribution with random values of C and m. The change in the updated distribution indicating possible crack size is shown in Figure 6.

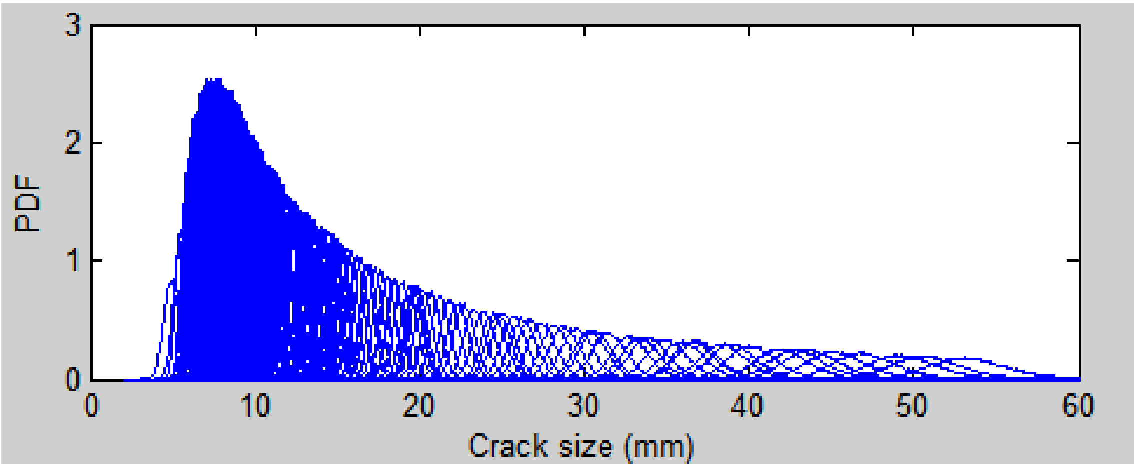


Figure 6. Probability distribution function of crack size updated at every 50 cycles with uncertain parameters C and m.

In this case, the resulting posterior distribution is much wider than the previous case due to the increased uncertainty in the prediction. The estimated crack size at each inspection is the modal value of the corresponding pdf curve. The estimated crack size along the inspection cycle is shown in Figure 7. We can see that these modal values still provide accurate estimates of crack sizes.

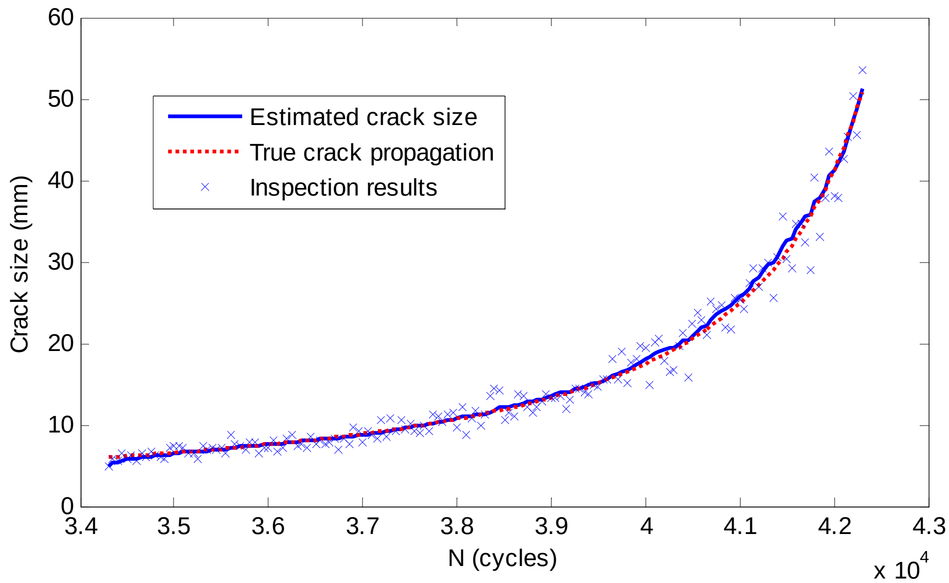


Figure 7. Crack size found by Bayesian update with uncertain C and m

2. Prognosis using least squares

Probably the simplest approach to do the prognosis, is using a least square fit the initial crack size and the parameters in Paris law based on the solution of Eq. (1) given as

$$a(N) = \left[NC\left(1 - \frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_i^{1-m/2} \right]^{\frac{2}{2-m}} \tag{8}$$

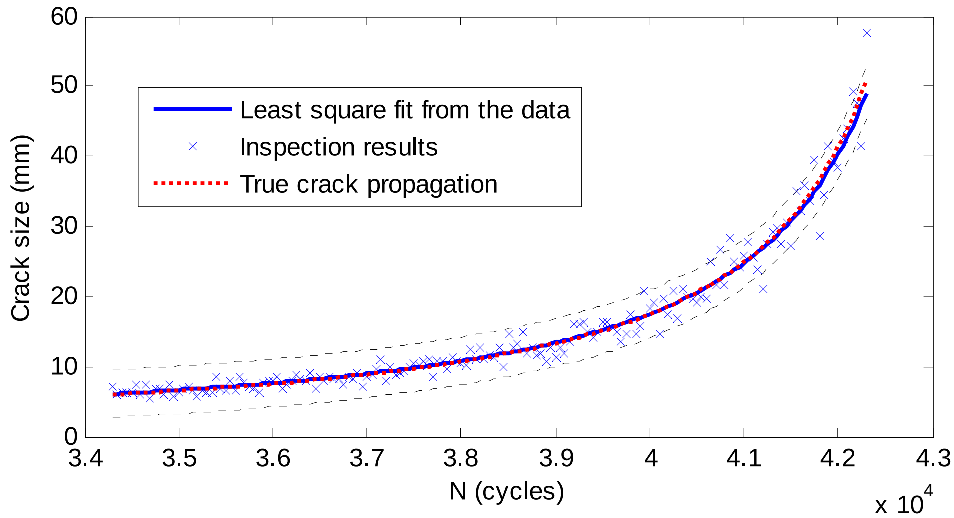


Figure 8. Approximation of the inspection data by Least squares fit of Paris law with 95% prediction bounds.

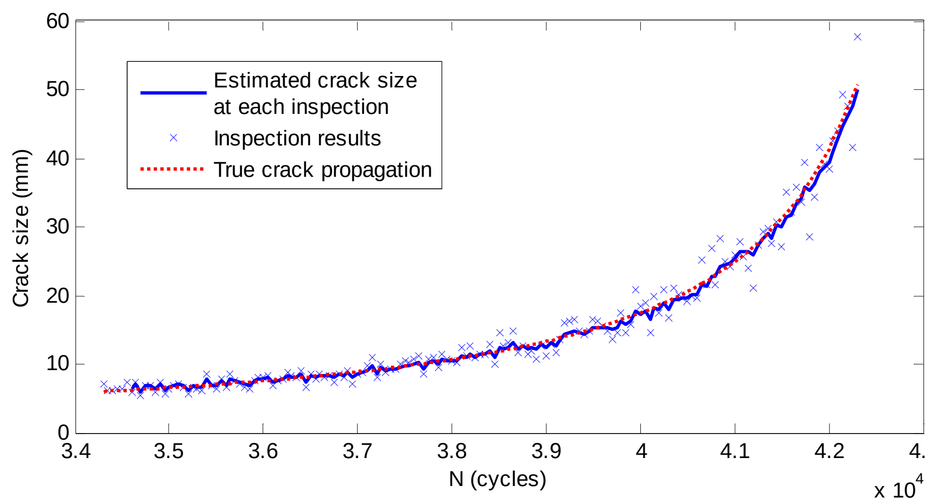


Figure 9. Estimated crack size by least square fitting of Paris law at each inspection cycle

Here we assume that we know C and the uncertainty is only in the intial crack size and the parameter m. Figure 8 shows results of the least-squares fit a long with 95% prediction bounds. Figure 9 shows how the raw predictions of the least squares fit are improved by the proposed approach. At each inspection, the prior distribution is constructed using data up to that point, and combined with current inspection result to estimate the crack size. By constructing the prior distribution using the prediction bounds, we can update the current measurement with the given prior distribution. The procedure is explained with Figure 10, which shows also the probability distributions at the last measurements.

Note that the abscissa and ordinate are switched for demonstration, the updated distribution is estimated by combining the prediction (a_{prior}) and current measurement ($a_{161,inspected}$) using Bayesian approach as shown in Figure 3(b).

The suggested Bayesian approach has an advantage over the raw least square fit which uses each data with equal weight. Figure 10 indicates that the current measurement affects more than previous measurements the estimated crack size, and makes us aware of the current crack size so that the up to the date information cannot be missed during SHM.

3. Prognosis using least squares (data-driven)

If we assume a case where we use no physical basis for predicting the propagation of crack, the simplest approach is to extrapolate crack growth by fitting the measurement history. Here we emply a 4th order polynomial

$$a(N) = a_4 N^4 + a_3 N^3 + a_2 N^2 + a_1 N + a_0 \tag{9}$$

The procedure is exactly the same as explained in the previous case, except that the curve fitting is done based on the 4th order polynomial instead of Paris' law. A single example with the same set of data as Section 4.3 is shown in Figure 11. Figure 11 also shows the prediction bounds for the curve fitting.

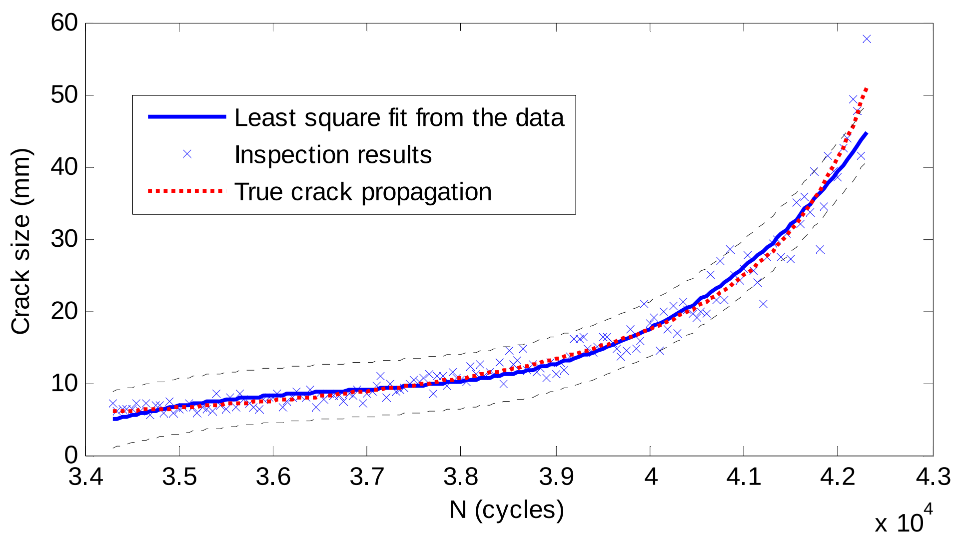


Figure 11. Approximation of the inspection data by 4th order polynomial with 95% prediction bounds

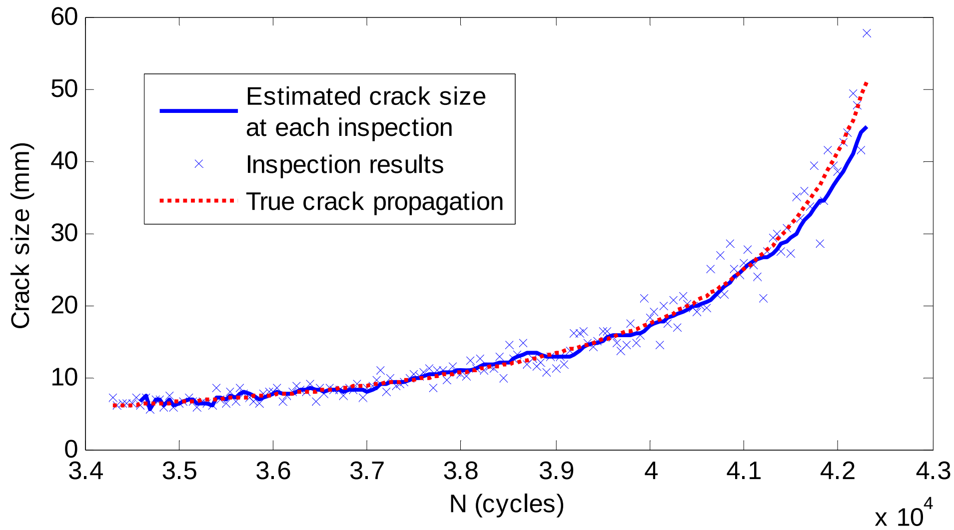


Figure 12. Estimated crack size by least square fitting using a quartic polynomial at each inspection cycle

Figure 12 shows the estimated crack size at each inspection occasion using suggested approach. We can see that the suggested approach underestimates the true crack size. To check this effect, we have run multiple cases to estimate the accuracy of our estimation.

To summarize the impact on diagnosis caused by past prognosis, we have run 10000 Monte Carlo Simulation with inspection uncertainty, and Table 2 summarizes the result.

Table 2. Comparison of accuracy with 10000 MCS

	Estimated value (Mode, most probable value)	Standard deviation of estimation
Single inspection	50.8 mm	5.08 mm
1. Perfect prognosis	50.8 mm	0.98 mm
2. With uncertainty of parameters	50.8 mm	1.48 mm
3. Least square fit of Paris law prognosis	50.6 mm	1.55 mm
4. Data-driven (quartic polynomial)	45.9 mm	1.29 mm

All cases employing past prognosis significantly reduce the standard deviation of the estimation. We have a noticeable difference in the estimated value for the data-driven case caused by the application of approximate model. However, we can take advantage in the predicted confidence interval even for that case. Next chapter discusses the advantage of using this approach further.

Implication to replacement time

The focus of this work is to reduce the standard deviation, which eventually leads us to have a narrower confidence interval for our diagnosis. This can reduce the cost of replacement while avoiding the aircraft from severe cracks at the same time. The biggest advantage of accurate measurements is helping the decision to determine a particular panel is significantly damaged or not. If we do not want to put an aircraft in service with a crack bigger than 50 mm, a usual criterion for replacing panel is when the measurement reading reaches 40 mm. For this example, the replacement time for each case is shown in Figure 13.

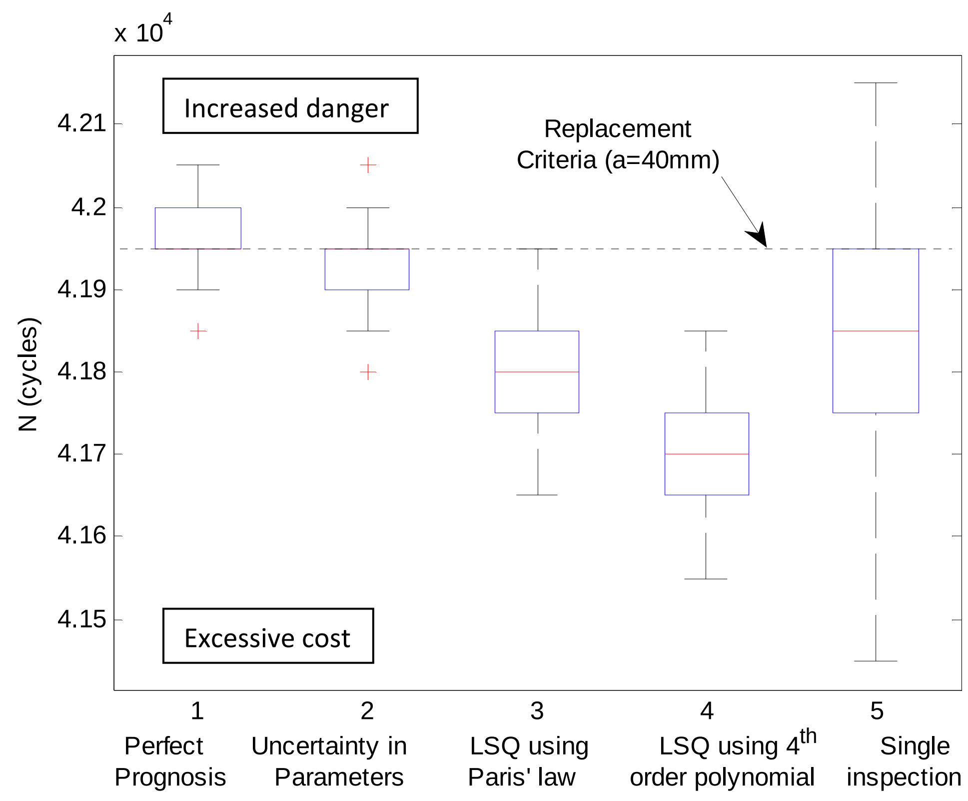


Figure 13. Estimated replacement time for each examples discussed in this paper

Since the exact number of cycles for replacement under given criterion is 41950, the single inspection is quite safe if we compare the mean replacement time. However, because of the misinterpretation of data, the possibility that the crack propagates over 40 mm without notice is quite big. On the other hand, the suggested Bayesian approach decreases the excessive cost by assessing the replacement time more accurately. Also, the danger of underestimating the crack size is lower compared to a single inspection. (Note that the inspection interval is 50 cycles for this example, so the error is usually within one inspection cycle.) Even when we did the prognosis using least square fit by a quartic function, the replacement time is very conservative compared to the case when we rely on a single inspection.

V. Concluding remarks

Prognosis is the process of estimating remaining useful life by predicting crack propagation, but it can be applied to future diagnosis to improve overall accuracy. This paper explains how we can benefit from the result of the past prognosis for accuracy of current estimation. By Bayesian approach, we were able to reduce the standard deviation of current estimate by 80%. This means that the inspection result is less affected from the large inspection error of SHM.

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