DESIGN OPTIMIZATION USING MESHFREE METHOD

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INTRODUCTION

⟹ Meshfree Discretization

- Reproducing Kernel Particle Method Is Used
- Direct Transformation/Mixed Transformation/Boundary Singular Kernel Methods for Essential B. C.
- Stable Solution for Large Shape Changing Problem
- Accurate Result for Finite Deformation Problem

⟹ Frictional Contact Analysis

- Continuum-Based Contact Formulation
- Penalty Regularization of Variational Inequality
- Regularized Coulomb Friction Model

⟹ DSA of Frictional Contact Problem

- DSA Variational Inequality Is Approximated Using the Same Penalty Method
- Die Shape Change Is Considered by Perturbing Rigid Surface
- Path Dependent Sensitivity Results for Frictional Problem

INTRODUCTION cont.

➡ Structural Analysis of Elastoplasticity

- Finite Deformation Elastoplasticity Using Multiplicative Decomposition of Deformation Gradient
- Return Mapping Algorithm in Principal Stress Space
- Stress Is Computed Using Hyper-Elasticity w.r.t. Stress-Free Intermediate Configuration
- Exact Linearization Is Required for Quadratic Convergence of Analysis and Accuracy of DSA

➡ Structural Design Sensitivity Analysis (DSA)

- Material Derivative Approach Is Used for Shape DSA
- Updated Lagrangian Formulation Is Used for Elastoplasticity
- Shape Function of RKPM Depends on Shape Design
- Direct Differentiation Method Is Used to Solve Displacement Sensitivity
- DSA Equation Is Solved at Each Converged Configuration without Iteration
- Material Derivative of Intermediate Configuration Is Updated at Each Load Step Instead of Stress in Conventional Method

REPRODUCING KERNEL PARTICLE METHOD

Reproduced Displacement Function

$$z^R(x) = \int_\Omega C(x; y-x)\,\phi_a(y-x)\,z(y)\,dy$$

$$\begin{cases} \phi_a(y-x) > 0 & \text{if } |y-x| < a \\ \phi_a(y-x) = 0 & \text{otherwise} \end{cases}$$

$z^R(x) \to z(x)$ as $a \to 0$ Dirac Delta Measure

Correction Function

$$C(x; y-x) = \mathbf{q}(x)^T \mathbf{H}(y-x)$$

$$\mathbf{H}(y-x)^T = [1, (y-x), (y-x)^2, \cdots, (y-x)^n]$$

$$\mathbf{q}(x)^T = [q_0(x), q_1(x), \cdots, q_n(x)]$$

n-th Order Completeness Requirement (Reproducing Condition)

$$z^R(x) = \int_\Omega C(x; y-x)\,\phi_a(y-x)\,z(y)\,dy$$

$$= \overline{m}_0(x) z(x) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \overline{m}_n(x) \frac{d^n z(x)}{dx^n}$$

$\overline{m}_0(x) = 1$ $\overline{m}_k(x) = 0$ $k = 1, \ldots, n$

RKPM cont.

Reproducing Condition

$$\mathbf{M}(x)\,\mathbf{q}(x) = \mathbf{H}(0) \qquad \mathbf{H}(0)^T = [1, 0, ..., 0]$$

$$\mathbf{M}(x) = \begin{bmatrix} m_0(x) & m_1(x) & ... & m_n(x) \\ m_1(x) & m_2(x) & ... & m_{n+1}(x) \\ . & . & ... & . \\ m_n(x) & m_{n+1}(x) & ... & m_{2n}(x) \end{bmatrix}$$

$$C(x; y-x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \mathbf{H}(y-x)$$

$$z^R(x) = \mathbf{H}(0)^T \mathbf{M}(x)^{-1} \int_\Omega \mathbf{H}(y-x)\phi_a(y-x) z(y)\, dy$$

$$z^R(x) = \sum_{I=1}^{NP} C(x; x_I - x)\phi_a(x_I - x) z_I \Delta x_I = \sum_{I=1}^{NP} \Phi_I(x) d_I$$

RKPM cont.

- Shape Function $\Phi_I(x_I)$ Depends on Current Coordinate Whereas FEA Shape Functions Depend on Coordinate of the Reference Geometry
- Does Not Satisfy Kronecker Delta Property: $\Phi_I(x_J) \neq \delta_{IJ}$
- Lagrange Multiplier Method for Essential B.C.

$$\Pi = U - \int_{\Gamma_D} \lambda^T (z - \zeta)\, d\Gamma$$

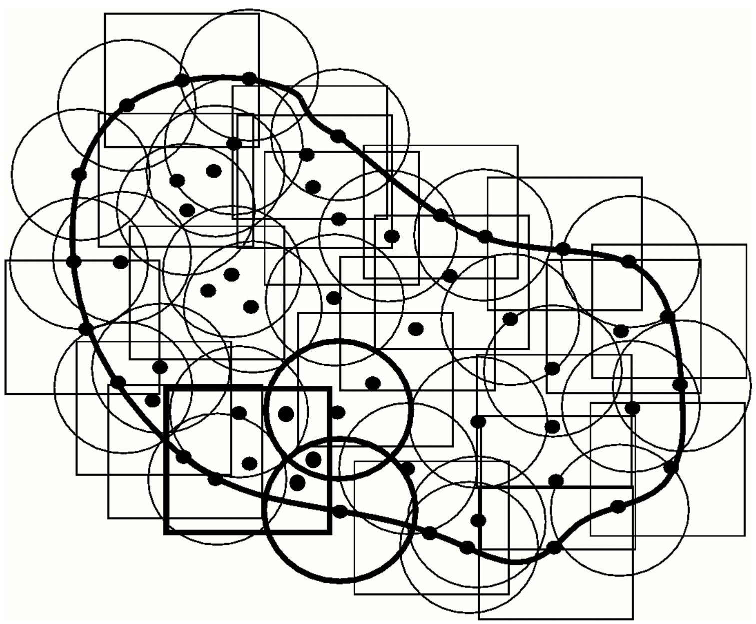
  – First-order variation is

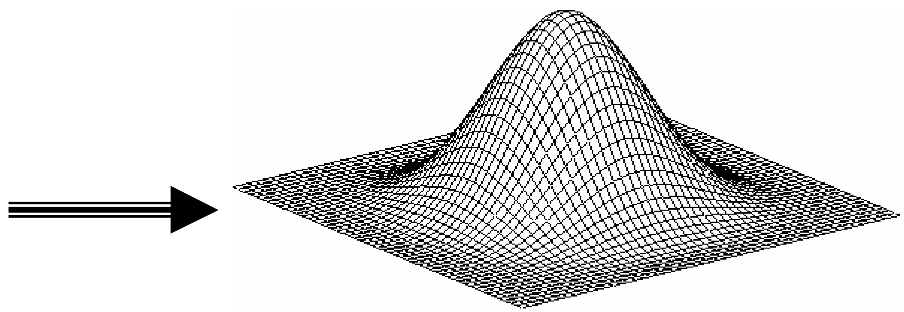
$$\bar{\Pi} = \bar{U} - \int_{\Gamma_D} \lambda^T \bar{z}\, d\Gamma - \int_{\Gamma_D} \bar{\lambda}^T (z - \zeta)\, d\Gamma$$

- Direct Transformation Method, Mixed Transformation Method, and Singular Kernel Methods Are Available.

DOMAIN DISCRETIZATION

Meshfree Discretization





Meshfree Shape Function

MESHFREE METHOD

Advantages

Construction of Shape/Interpolation Function in Global Level

Mesh Independent Solution Accuracy Control

Versatile hp-Adaptivity

A Remedy to Mesh Distortion in Shape Optimization

Accurate Solution to Large Deformation Problem

Disadvantages

Difficulties in Imposing Essential Boundary Conditions

Expensive Computational Cost

Larger Bandwidth of Stiffness Matrix Than FEM

NONLINEAR STRUCTURAL ANALYSIS

- Nonlinear Variational Equation
  (Updated Lagrangian Formulation)

$$a_\Omega({}^n\mathbf{z},\bar{\mathbf{z}}) + b_\Gamma({}^n\mathbf{z},\bar{\mathbf{z}}) = \ell_\Omega(\bar{\mathbf{z}}), \quad \forall\bar{\mathbf{z}}\in Z$$

$$a_\Omega(\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega \boldsymbol{\tau} : \bar{\boldsymbol{\varepsilon}}\, d\Omega$$ Structural Energy Form

$$b_\Gamma(\mathbf{z},\bar{\mathbf{z}})$$ Contact Variational Form

$$\ell_\Omega(\bar{\mathbf{z}}) = \int_\Omega \bar{\mathbf{z}}^T\mathbf{f}^B\, d\Omega + \int_{\Gamma^S} \bar{\mathbf{z}}^T\mathbf{f}^S\, d\Gamma$$ Load Linear Form

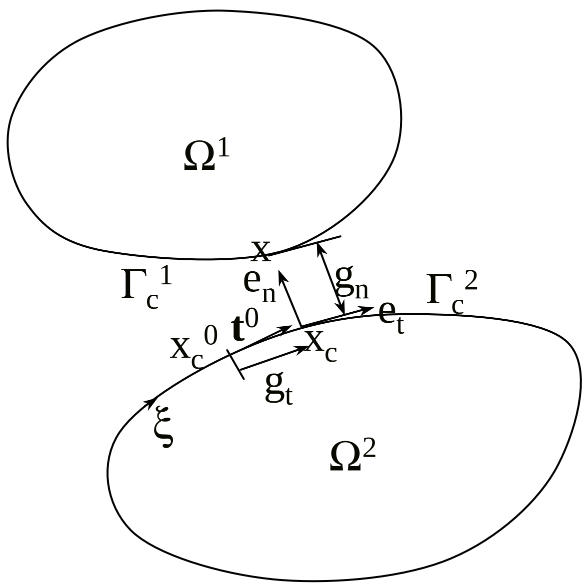
- Linearization

$$a^*_\Omega({}^n\mathbf{z}^k;\Delta\mathbf{z}^{k+1},\bar{\mathbf{z}}) + b^*_\Gamma({}^n\mathbf{z}^k;\Delta\mathbf{z}^{k+1},\bar{\mathbf{z}})$$
$$= \ell_\Omega(\bar{\mathbf{z}}) - a_\Omega({}^n\mathbf{z}^k,\bar{\mathbf{z}}) - b_\Gamma({}^n\mathbf{z}^k,\bar{\mathbf{z}}), \qquad \forall\bar{\mathbf{z}}\in Z$$

$$a^*_\Omega(\mathbf{z};\Delta\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega [\bar{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}(\Delta\mathbf{z}) + \boldsymbol{\tau} : \boldsymbol{\eta}(\Delta\mathbf{z},\bar{\mathbf{z}})]\, d\Omega$$

$$\mathbf{c} = \frac{\partial\boldsymbol{\tau}}{\partial\boldsymbol{\varepsilon}} = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^{\mathrm{alg}}\mathbf{m}^i \otimes \mathbf{m}^j + 2\sum_{i=1}^{3}\tau_i^p\hat{\mathbf{c}}^i$$

FRICTIONAL CONTACT PROBLEM



Impenetration Condition

$$g_n \equiv (\mathbf{x}-\mathbf{x}_c(\xi_c))^T \mathbf{e}_n(\xi_c) \geq 0, \;\; \mathbf{x} \in \Gamma_c^1, \mathbf{x}_c \in \Gamma_c^2$$

Tangential Slip Function

$$g_t \equiv \left\| \mathbf{t}^0 \right\| (\xi_c - \xi_c^0)$$

Contact Consistency Condition

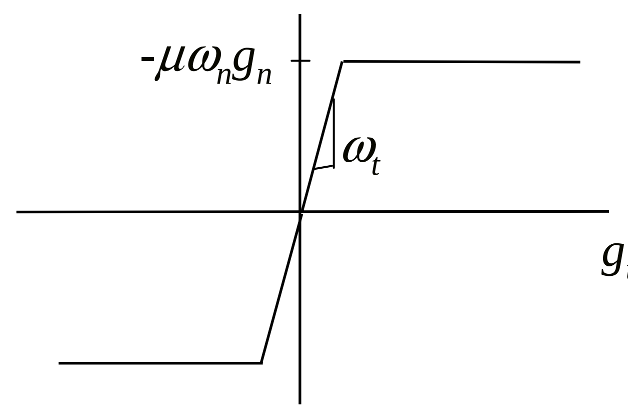
$$\varphi(\xi_c) = (\mathbf{x}-\mathbf{x}_c(\xi_c))^T \mathbf{e}_t(\xi_c) = 0$$

Contact Penalty Function

$$P = \frac{1}{2}\omega_n \int_{\Gamma_C} g_n^{\ 2}\, d\Gamma + \frac{1}{2}\omega_t \int_{\Gamma_C} g_t^{\ 2}\, d\Gamma$$

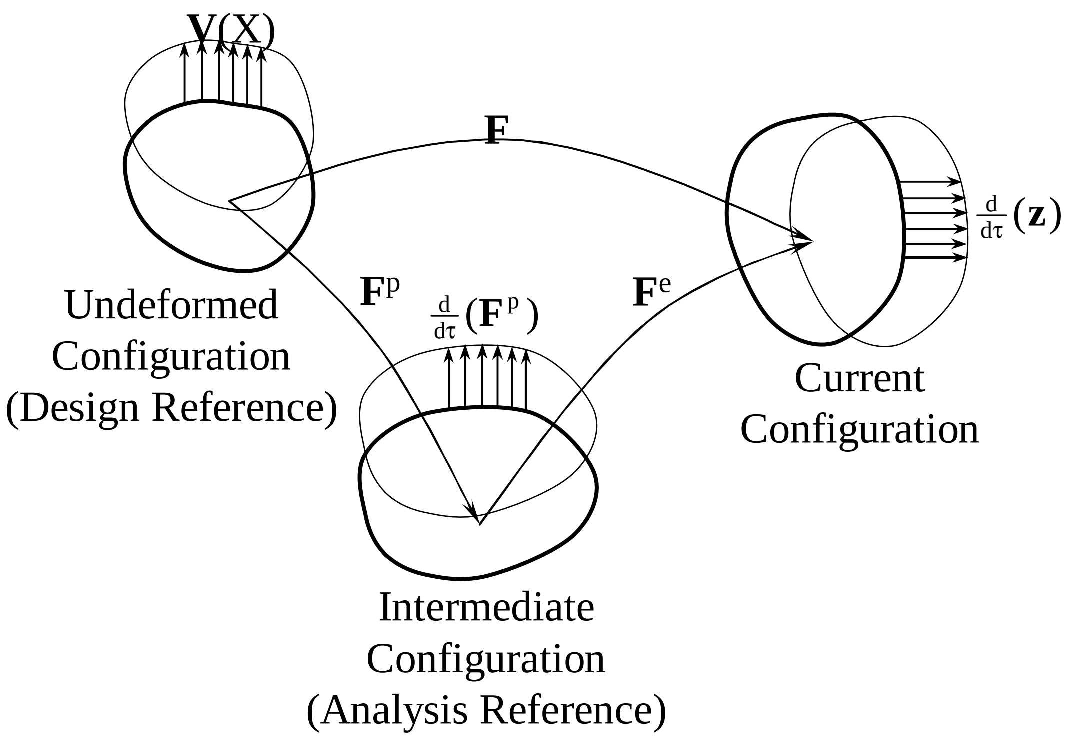
Contact Variational Form

$$b_\Gamma(\mathbf{z},\overline{\mathbf{z}}) = \omega_n \int_{\Gamma_C} g_n \overline{g}_n\, d\Gamma + \omega_t \int_{\Gamma_C} g_t \overline{g}_t\, d\Gamma S$$



Modified Coulomb Friction Model

DESIGN SENSITIVITY ANALYSIS



- Updated Lagrangian Formulation
- Finite Deformation Elastoplasticity
- No Need to Update Velocity Fields
- Updating Sensitivity Information of Intermediate Configuration and Plastic Variables

FINITE DEFORMATION DSA

Material Derivative of Variational Equation

$$\frac{d}{d\tau}\left[a_{\Omega_\tau}({}^n\mathbf{z}_\tau,\overline{\mathbf{z}}_\tau)\right]_{\tau=0}+\frac{d}{d\tau}\left[b_{\Gamma_\tau}({}^n\mathbf{z}_\tau,\overline{\mathbf{z}}_\tau)\right]_{\tau=0}=\frac{d}{d\tau}\left[\ell_{\Omega_\tau}(\overline{\mathbf{z}}_\tau)\right]_{\tau=0},\ \forall\overline{\mathbf{z}}_\tau\in Z_\tau$$

Design Sensitivity Equation

$$a^*_\Omega({}^n\mathbf{z};{}^n\dot{\mathbf{z}},\overline{\mathbf{z}})+b^*_\Gamma({}^n\mathbf{z};{}^n\dot{\mathbf{z}},\overline{\mathbf{z}})=\ell'_V(\overline{\mathbf{z}})-a'_V({}^n\mathbf{z},\overline{\mathbf{z}})-b'_V({}^n\mathbf{z},\overline{\mathbf{z}})$$

Remarks:

– The same tangent operator is used as analysis -- Need accurate computation of tangent operator
– Direct Differentiation -- DSA needs to be carried out at each converged load step
– Update sensitivity information: intermediate configuration (analysis reference), plastic internal variables, and frictional effect
– Total form of sensitivity equation
– No iteration is required to solve the sensitivity equation

FINITE DEFORMATION DSA cont.

Fictitious load

$$a'_V(\mathbf{z},\bar{\mathbf{z}}) = \int_\Omega \left(\bar{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}_V(\mathbf{z}) + \bar{\boldsymbol{\varepsilon}} : \mathbf{c} : \boldsymbol{\varepsilon}_P(\mathbf{z}) + \boldsymbol{\tau}^{fic} : \bar{\boldsymbol{\varepsilon}}\right) d\Omega$$
$$+ \int_\Omega \left(\boldsymbol{\tau} : \boldsymbol{\eta}_V(\mathbf{z},\bar{\mathbf{z}}) + \boldsymbol{\tau} : \boldsymbol{\eta}_P(\mathbf{z},\bar{\mathbf{z}}) + \boldsymbol{\tau} : \bar{\boldsymbol{\varepsilon}} div\mathbf{V}\right) d\Omega$$

$$\boldsymbol{\varepsilon}_V(\mathbf{z}) = -sym(\nabla_0 \mathbf{z} \nabla_n \mathbf{V})$$
$$\boldsymbol{\eta}_V(\mathbf{z},\bar{\mathbf{z}}) = -sym\left(\nabla_n \bar{\mathbf{z}}^T \nabla_0 \mathbf{z} \nabla_n \mathbf{V}\right) - sym(\nabla_0 \bar{\mathbf{z}} \nabla_n \mathbf{V})$$
$$\boldsymbol{\varepsilon}_P(\mathbf{z}) = -sym(\mathbf{G})$$
$$\boldsymbol{\eta}_P(\mathbf{z},\bar{\mathbf{z}}) = -sym(\nabla_n \bar{\mathbf{z}}^T \mathbf{G})$$

$$\boldsymbol{\tau}^{fic} = \sum_{i=1}^{3} \left[\frac{\partial \tau_i^p}{\partial \boldsymbol{\alpha}} \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \frac{\partial \tau_i^p}{\partial \hat{e}^p} \tfrac{d}{d\tau}(e_n^p) \right] \mathbf{m}^i$$

$$\mathbf{G} = \mathbf{F}^e \tfrac{d}{d\tau}(\mathbf{F}^p) \mathbf{F}^{-1}$$

Updating path-dependent terms

$$\tfrac{d}{d\tau}(\boldsymbol{\alpha}_{n+1}) = \tfrac{d}{d\tau}(\boldsymbol{\alpha}_n) + \left(H_\alpha + \sqrt{\tfrac{2}{3}} H'_\alpha \gamma\right) \tfrac{d}{d\tau}(\gamma) \mathbf{N} + H_\alpha \gamma \tfrac{d}{d\tau}(\mathbf{N})$$
$$\tfrac{d}{d\tau}(e_{n+1}^p) = \tfrac{d}{d\tau}(e_n^p) + \sqrt{\tfrac{2}{3}} \tfrac{d}{d\tau}(\gamma)$$
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^p) = \tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\,-1}) \mathbf{F}_{n+1} + \mathbf{F}_{n+1}^{e\,-1} \tfrac{d}{d\tau}(\mathbf{F}_{n+1})$$
$$\tfrac{d}{d\tau}(\mathbf{F}_{n+1}^e) = \tfrac{d}{d\tau}(\mathbf{f}^p) \mathbf{F}_{n+1}^{e\,tr} + \mathbf{f}^p \tfrac{d}{d\tau}(\mathbf{F}_{n+1}^{e\,tr})$$
$$\mathbf{f}^p = \sum_{j=1}^{3} \exp(-\gamma N_j) \mathbf{m}^j$$

Incremental Plastic Deformation Gradient

CONTACT DSA

- **Material Derivative of Contact Variational Form**

$$\frac{d}{d\tau}\Big[b_{\Gamma_\tau}(\mathbf{z}_\tau,\overline{\mathbf{z}}_\tau)\Big] = b_\Gamma^*(\mathbf{z};\dot{\mathbf{z}},\overline{\mathbf{z}}) + b_V'(\mathbf{z},\overline{\mathbf{z}})$$

$$b_V'(\mathbf{z},\overline{\mathbf{z}}) = b_N'(\mathbf{z},\overline{\mathbf{z}}) + b_T'(\mathbf{z},\overline{\mathbf{z}})$$

- **Normal Contact Fictitious Load Form for DSA**

$$\begin{aligned} b_N'(\mathbf{z},\overline{\mathbf{z}}) = {} & \omega_n \int_{\Gamma_C} (\overline{\mathbf{z}}-\overline{\mathbf{z}}_c)^T \mathbf{e}_n\mathbf{e}_n^{\,T}(\mathbf{V}-\mathbf{V}_c)\,d\Gamma - \omega_n \int_{\Gamma_C} \big(\alpha g_n/c\big)(\overline{\mathbf{z}}-\overline{\mathbf{z}}_c)^T \mathbf{e}_t\mathbf{e}_t^{\,T}(\mathbf{V}-\mathbf{V}_c)\,d\Gamma \\ & -\omega_n \int_{\Gamma_C} \big(g_n\|\mathbf{t}\|/c\big)(\overline{\mathbf{z}}-\overline{\mathbf{z}}_c)^T \mathbf{e}_t\mathbf{e}_n^{\,T}\mathbf{V}_{c,\xi}\,d\Gamma - \omega_n \int_{\Gamma_C} \big(g_n\|\mathbf{t}\|/c\big)\overline{\mathbf{z}}_{c,\xi}^{\,T}\mathbf{e}_n\mathbf{e}_t^{\,T}(\mathbf{V}-\mathbf{V}_c)\,d\Gamma \\ & -\omega_n \int_{\Gamma_C} \big(g_n^{\,2}/c\big)\overline{\mathbf{z}}_{c,\xi}^{\,T}\mathbf{e}_n\mathbf{e}_n^{\,T}\mathbf{V}_{c,\xi}\,d\Gamma + \omega_n \int_{\Gamma_C} \kappa g_n(\overline{\mathbf{z}}-\overline{\mathbf{z}}_c)^T \mathbf{e}_n(\mathbf{V}^T\mathbf{n})\,d\Gamma \end{aligned}$$

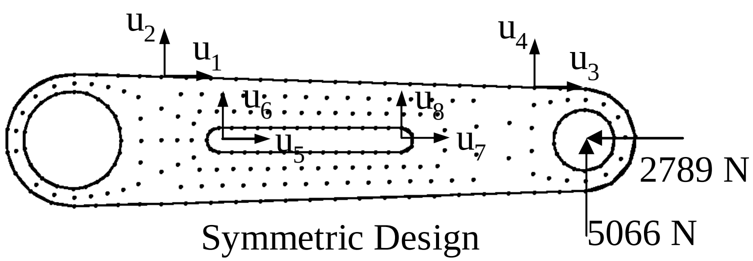
CONTACT DSA cont.

- **Tangential Stick Fictitious Load Form for DSA**

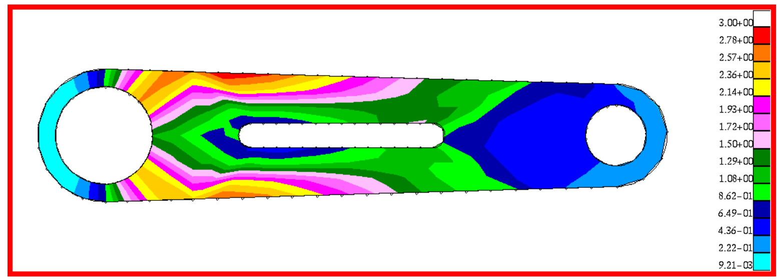
$$
\begin{aligned}
& b_T'({}^n\mathbf{z},\overline{\mathbf{z}}) = b_T^*({}^n\mathbf{z};\mathbf{V},\overline{\mathbf{z}}) \\
& +\omega_t \int_{\Gamma_C} {}^n\left(2g_t \left\|\mathbf{t}\right\| / c\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left({}^n\nu(2\,{}^{n-1}\beta\,{}^n g_t - \left\|{}^{n-1}\mathbf{t}\right\|^2\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V} + {}^{n-1}\dot{\mathbf{z}} - \mathbf{V}_c - {}^{n-1}\dot{\mathbf{z}}_c)\, d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^{n-1}\beta\,{}^n g_n\,{}^n g_t \left(\left\|{}^{n-1}\mathbf{t}\right\| + \left\|{}^n\mathbf{t}\right\|\right) / {}^n c\,{}^{n-1}c\right](\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{n-1}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& -\omega_t \int_{\Gamma_C} \left[{}^n g_n \left\|{}^n\mathbf{t}\right\| \left\|{}^{n-1}\mathbf{t}\right\|^2 / {}^n c\,{}^{n-1}c\right](\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\, {}^{t-\Delta t}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\quad d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^n g_n \left\|{}^{n-1}\mathbf{t}\right\| \left(2\,{}^{n-1}\beta\,{}^n g_t - \left\|{}^{n-1}\mathbf{t}\right\|^2\right) / {}^n c\,{}^{n-1}c\right]\overline{\mathbf{z}}_{c,\xi}^{\,T}\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_t^{\,T}(\mathbf{V} + {}^{n-1}\dot{\mathbf{z}} - \mathbf{V}_c - {}^{n-1}\dot{\mathbf{z}}_c)\, d\Gamma \\
& +\omega_t \int_{\Gamma_C} \left[{}^n g_n\,{}^{n-1}g_n \left(2\,{}^{n-1}\beta\,{}^n g_t - \left\|{}^{n-1}\mathbf{t}\right\|^2\right) / {}^n c\,{}^{n-1}c\right]\overline{\mathbf{z}}_{c,\xi}^{\,T}\, {}^n\mathbf{e}_n\, {}^{n-1}\mathbf{e}_n^{\,T}(\mathbf{V}_{c,\xi} + {}^{n-1}\dot{\mathbf{z}}_{c,\xi})\, d\Gamma \\
& +\omega_t \int_{\Gamma_C} \kappa\left\{{}^n\nu\,{}^n g_t (\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t + \left({}^n g_n\,{}^n g_t \left\|{}^{n-1}\mathbf{t}\right\| / c\right)(\overline{\mathbf{z}}-\overline{\mathbf{z}})^T\, {}^n\mathbf{e}_t\right\}(\mathbf{V}^T\mathbf{n})\, d\Gamma
\end{aligned}
$$

TORQUE ARM OPTIMIZATION

Initial Design & Design Parameters



Stress Contour



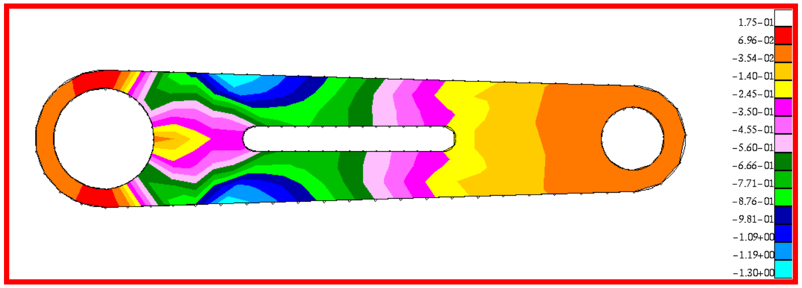
Minimize mass

Subject to $\sigma_{max} \leq 800$ MPa

- Automatic Node Generation Capability Is Used
- 239 RKPM Particles (478 DOF)
- Thickness = 0.3 cm
- Steel with E = 207 GPa and $\nu = 0.3$
- Used *Design Sensitivity and Optimization (DSO) Tool* for Design Parameterization and Design Velocity Computation
- Meshfree Analysis = 5.19 sec.; DSA per DV = 0.57 sec on HP Exemplar S-Class

DESIGN SENSITIVITY RESULTS

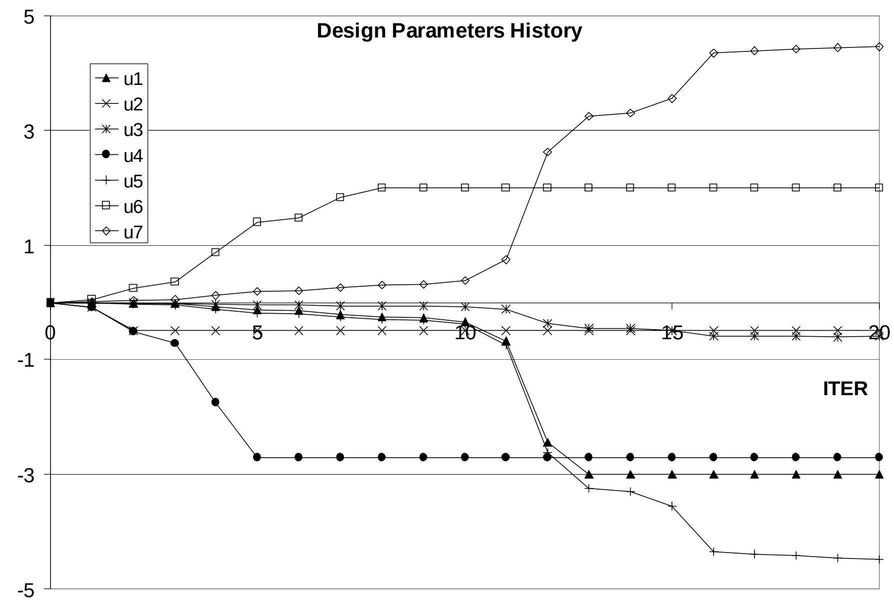
Performance(ψ)		$\Delta\psi$	ψ'	$\Delta\psi/\psi' \times 100$
u1				
Area	.374606E+03	.103614E-05	.103615E-05	100.00
S82	.305009E+01	-.628917E-07	-.628906E-07	100.00
S85	.295060E+01	-.177360E-08	-.177217E-08	100.08
S88	.295177E+01	-.888293E-07	-.888278E-07	100.00
S91	.276433E+01	-.112452E-06	-.112451E-06	100.00
S97	.236361E+01	-.777825E-07	-.777813E-07	100.00
S136	.210658E+01	-.159904E-06	-.159907E-06	100.00
S133	.218336E+01	.336654E-07	.336665E-07	100.00
S100	.216967E+01	-.676240E-07	-.676229E-07	100.00
u3				
Area	.374606E+03	.101184E-05	.101185E-05	100.00
S82	.305009E+01	-.780835E-09	-.781769E-09	99.88
S85	.295060E+01	.146740E-09	.146784E-09	99.97
S88	.295177E+01	-.587522E-08	-.587479E-08	100.01
S91	.276433E+01	-.193873E-07	-.193869E-07	100.00
S97	.236361E+01	-.393579E-07	-.393573E-07	100.00
S136	.210658E+01	-.388212E-09	-.388864E-09	99.83
S133	.218336E+01	.415960E-09	.415245E-09	100.17
S100	.216967E+01	-.597876E-07	-.597868E-07	100.00
u5				
Area	.374606E+03	.200000E-05	.200000E-05	100.00
S82	.305009E+01	-.257664E-07	-.257638E-07	100.01
S85	.295060E+01	-.775315E-07	-.775303E-07	100.00
S88	.295177E+01	.894471E-08	.894603E-08	99.99
S91	.276433E+01	.385595E-07	.385607E-07	100.00
S97	.236361E+01	.108496E-07	.108508E-07	99.99
S136	.210658E+01	.353638E-07	.353624E-07	100.00
S133	.218336E+01	-.565799E-07	-.565810E-07	100.00
S100	.216967E+01	.974523E-08	.974644E-08	99.99



von Mises Stress Sensitivity w.r.t. u_3

Accuracy of DSA
Compared with FDM

OPTIMIZATION HISTORY

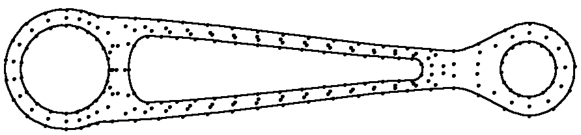


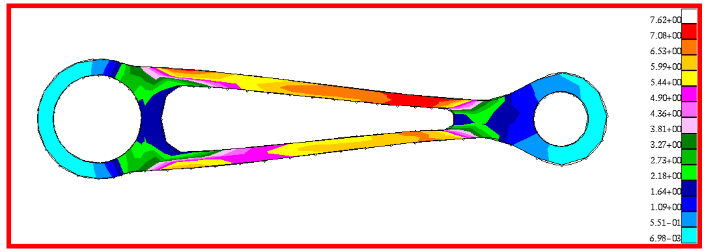
Cost Function

Design Parameters

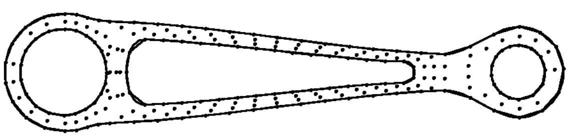
OPTIMIZATION RESULTS

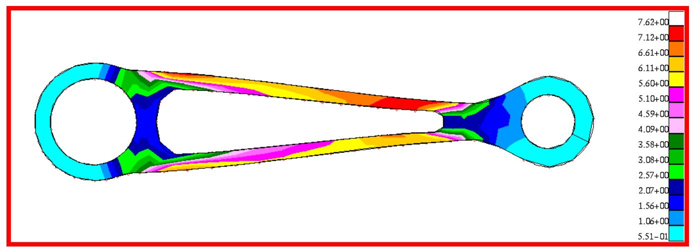
Weight Reduction: 0.878kg $\rightarrow$ 0.421 kg (48%)





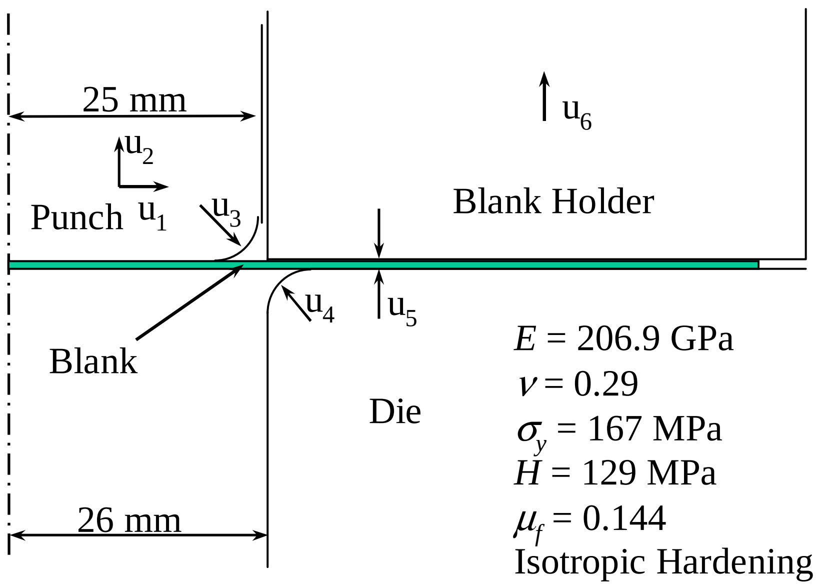
FEA Took 45 Iterations with 8 Remodelings (AIAA Paper by J. Bennett & M. Botkin), whereas Meshfree w/o Any Remodeling Took 20 Iterations

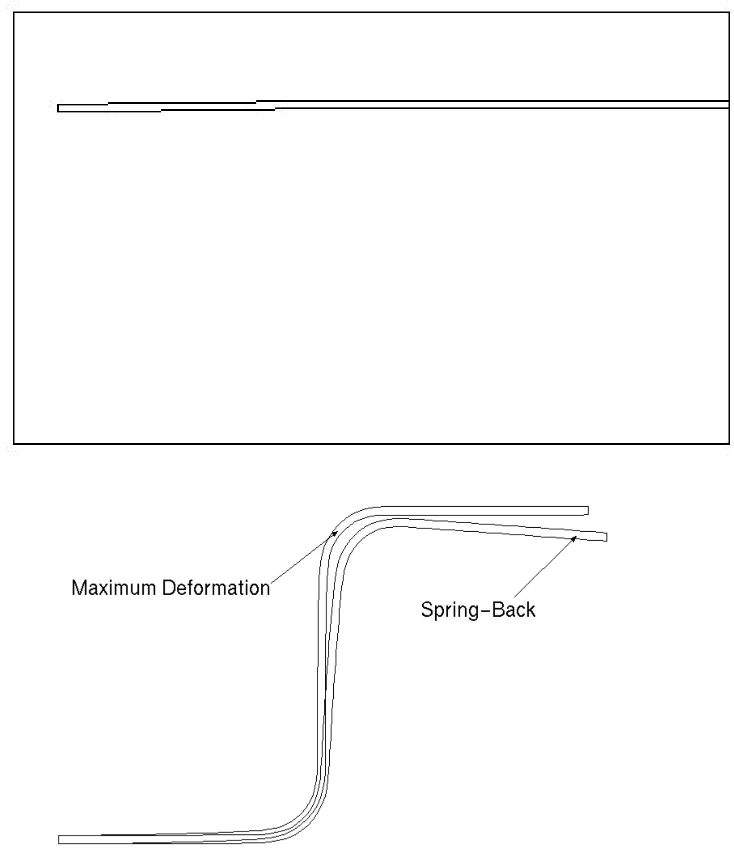




Confirmed Analysis Results After Meshfree Remodeling at the Optimum Design

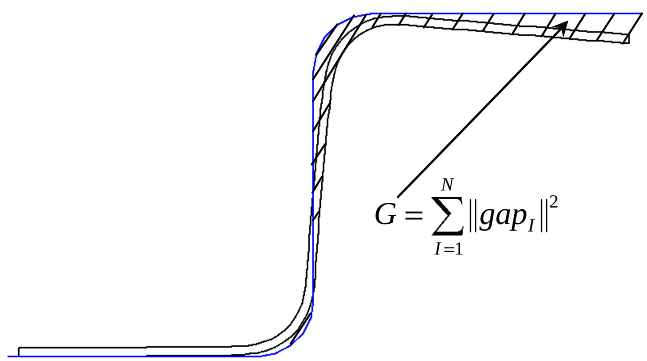
DEEPDRAWING PROCESS OPTIMIZATION





DSA AND OPTIMIZATION

Performance (Ψ)		$\Delta\Psi$	$\Psi'\Delta\tau$	RATIO(%)
u_1				
e^p_{31}	.189554+1	-.147351-6	-.147312-6	100.03
e^p_{32}	.160840+1	-.458731-6	-.458733-6	100.00
e^p_{33}	.113010+1	-.268919-6	-.268949-6	99.99
e^p_{34}	.812794+0	-.217743-6	-.217764-6	99.99
e^p_{35}	.568991+0	-.144977-6	-.144996-6	99.99
e^p_{36}	.383581+0	-.802032-7	-.802123-7	99.99
G	.510092+2	-.159478-4	-.159503-4	99.98
u_2				
e^p_{31}	.189554+1	-.727109-7	-.726800-7	100.04
e^p_{32}	.160840+1	.764579-7	.764732-7	99.98
e^p_{33}	.113010+1	.118855-6	.118849-6	100.01
e^p_{34}	.812794+0	.829857-7	.829822-7	100.00
e^p_{35}	.568991+0	.700201-7	.700157-7	100.01
e^p_{36}	.383581+0	.345176-7	.345177-7	100.00
G	.510092+2	.494067-4	.494016-4	100.01



Optimization Problem

$$\text{minimize} \quad G = \sum (P(\mathbf{x}_i) - \mathbf{x}_i)^2$$

$$\text{subject to} \quad e^p \le 0.2$$

$$t_i \ge 0.6$$

$$-0.1 \le u_1 \le 0.1$$

$$-0.01 \le u_2 \le 0.1$$

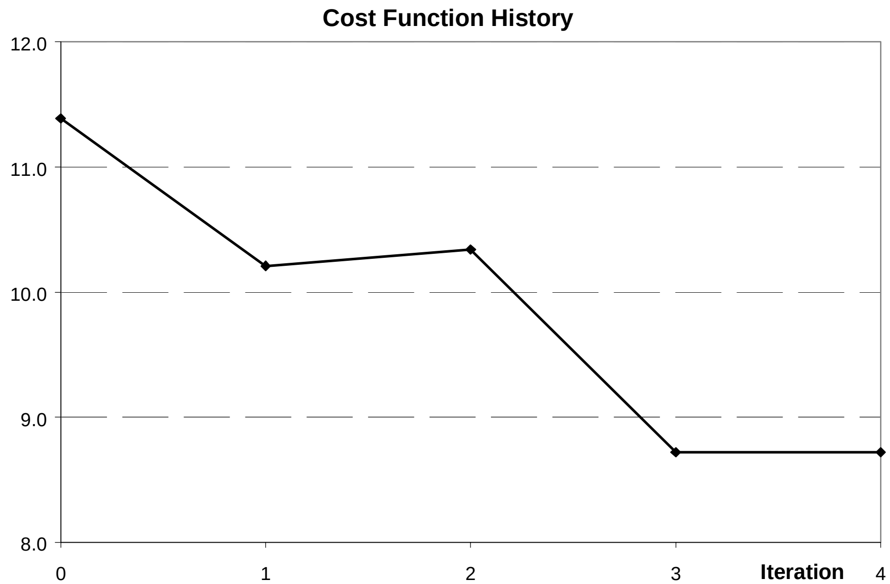
$$-1.1 \le u_3 \le 1.1$$

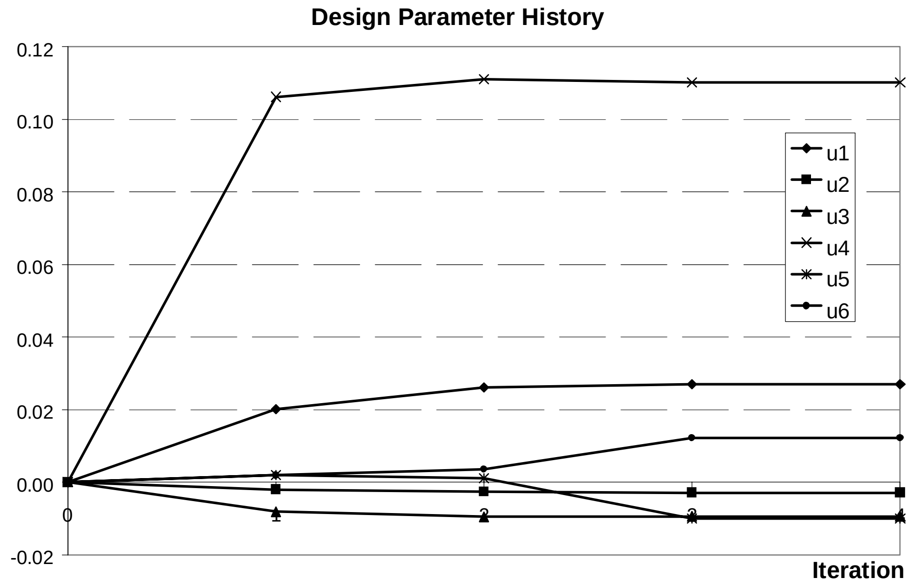
$$-1.1 \le u_4 \le 1.1$$

$$-0.01 \le u_5 \le 0.01$$

$$-0.02 \le u_6 \le 0.1$$

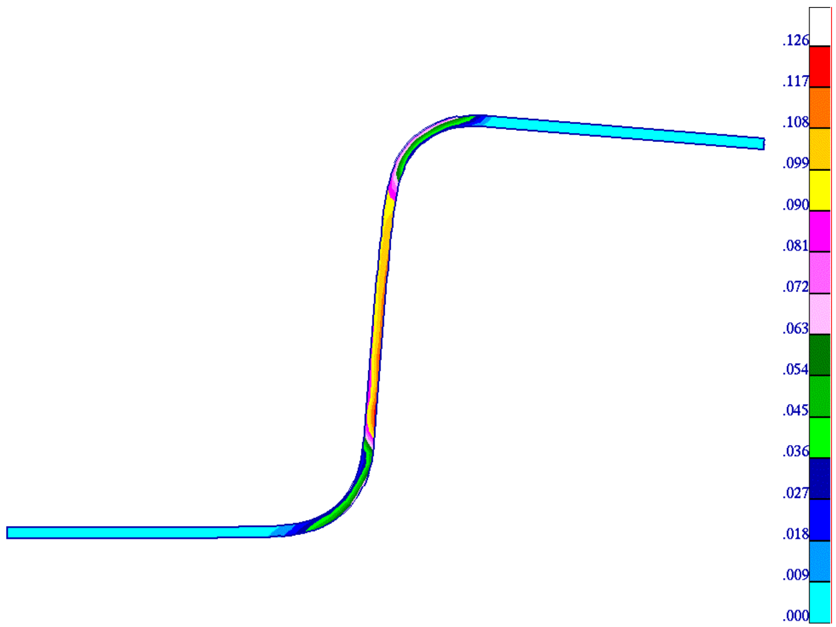
OPTIMIZATION HISTORY



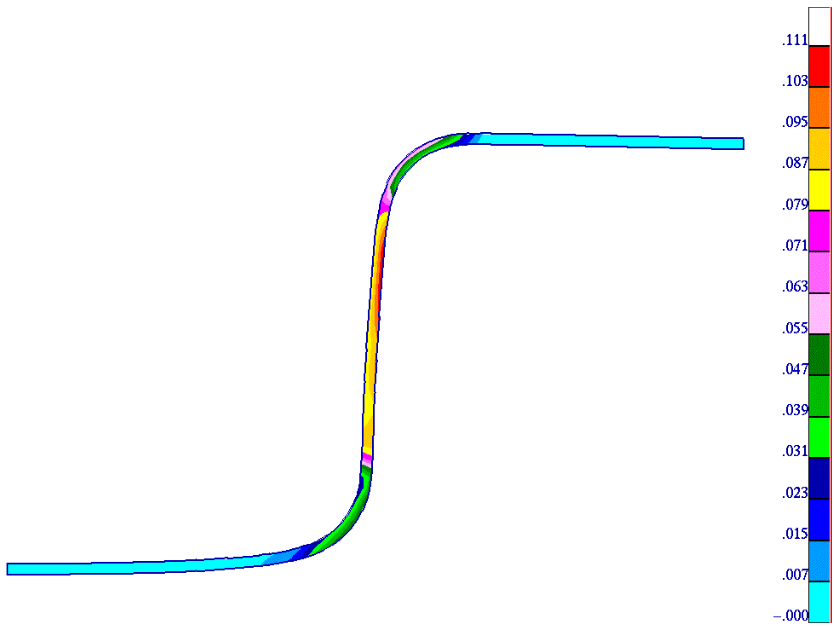


No. of Design Iteration : 4
No. of Structural Analysis : 9

OPTIMIZATION RESULT

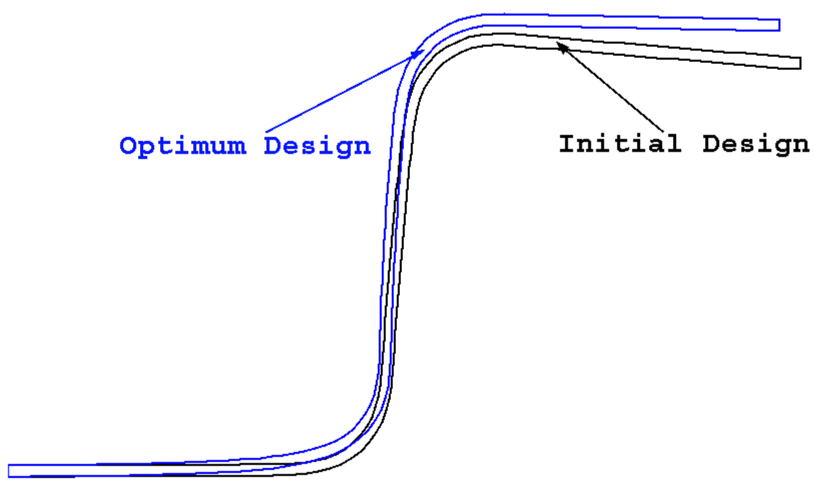


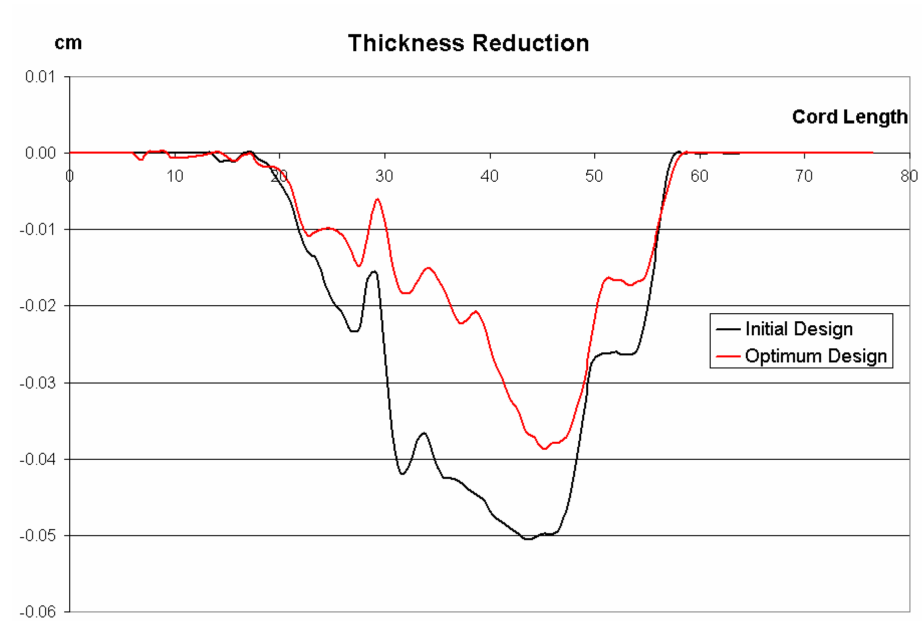
Plastic Strain (Initial Design)
Maximum = 0.126



Plastic Strain (Optimum Design)
Maximum = 0.111

OPTIMIZATION RESULTS cont.





Smaller Thickness Variation Due to Smaller Plastic Strain

SUMMARY

- Meshfree Method Is Effective in Shape Optimization
- Developed Accurate and Efficient Shape DSA Method for Finite Deformation Elastoplasticity Using Multiplicative Decomposition of Deformation Gradient
- Die Shape Design Sensitivity Formulation Is Developed for the Frictional Contact Problem
- Deepdrawing Optimization Is Successfully Carried out to Reduce the Springback Amount
- Very Accurate DSA Results Made Optimization Problem Converged in a Small Number of Iterations

FUTURE PLANS

- Develop Configuration Design Sensitivity Formulation for Meshfree Shell Structure
- Current Gauss Integration Method Will Be Replaced by Stabilized Conforming Nodal Integration for DSA
- Develop DSA of Contact Formulation Using Meshfree Smooth Contact Surface Representation
- Integrate Automatic HP-Adaptivity Into Shape Design Optimization Process