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Being Conservative with a Limited Number of Structural Tests and Estimating Probability of Failure

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1. Abstract
Conventional aircraft structural design is followed by the building-block test process to detect failures in early stage due to manufacturing variability and errors in calculations. This more than fifty-year old process has been implemented heuristically without quantifying its contribution to the reliability

Safety-factor-based design uses the conservative failure stress with conservative load such as limit load with safety factor to design the structural element. We propose to estimate the probability of failure of the structural element to quantify safety of the designed structure. The probability of failure is probability that manufactured structure from the design is cannot sustain its design load. This paper demonstrates accuracy of the estimated probability of failure and the effects of the number of element tests. To illustrate the effects of the number of element tests, we calculate the average weight of structures designed with different number of element tests but have the same expected probability of failure for the same design load. For typical values of material variability and calculation errors we find that a design based on no element tests is 3.7% heavier for the same expected probability of failure than a design based on 3 element tests. A design with 5 element tests allows 1.3% additional weight reductions. In addition, we examine the dependence of the probability of failure on the load levels, for loads that exceed the design load.

2. Keywords: Safety-factor-based design, Probability of failure, The number of element tests.

3. Introduction
To achieve acceptable safety of aircraft structures, design of an airliner proceeds through the building block testing framework. The building block testing is multi-level testing, with each stage reducing the uncertainty in the design. Although the effect of the tests of each stage on safety is relatively well known, few papers have quantified the effect of the test to the aircraft structure. (e.g., Acar *et al*. [3] and [4]). In previous papers, the process was examined from the viewpoint of a researcher who is looking to estimate how much safety is obtained on average in the design of large number of structural components. Such quantification will be useful, for example, for a government regulatory agency, such as the FAA, who may want to examine how regulations govern design and testing affect airliner safety. A single analyst designing a single structural element would have a different perspective. The present paper looks at the process from the standpoint of an analyst estimating the probability of failure of a structural element, taking into account the effect of future element tests.

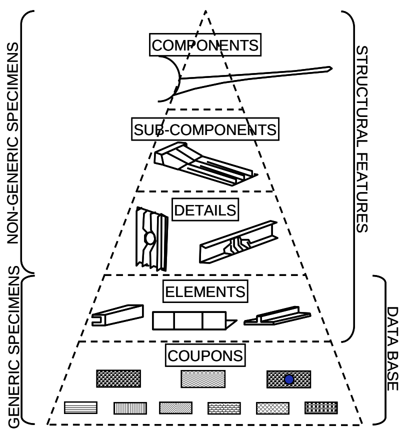


Figure 1: Building-block approach for testing aircraft structural components. At each stage, analytical models can be adjusted to account for discrepancies between the model and test and, thus, reduce the errors in the analytical model used to estimate the response at the next level.

The Federal Aviation Administration requires using A-basis (or B-basis) allowable strength that is below 99% (or 90%) of the tested strength with 95% confidence. The allowable is a conservative estimate of mean test result using knockdown factor. Also strength estimate of a structural element needs knockdown factor, it is obtained from element tests. The above processes are parts of the building block test process, which compensates for uncertainties from the material variability and estimation processes by using knockdown factors. There is a push to replace safety-factor-based design with probabilistic design (e.g, Lincoln [5] , Wirsching [6], [7], and Long *et al.*[8]). However, the latter cannot readily replace the former because current probabilistic design frameworks do not incorporate various uncertainty reduction measures (URMs), such as building-block tests, undertaken during the lifecycle. Therefore, it is important for probabilistic design to include the effects of URM on structural safety.

As a first step toward quantifying the effect of URMs, the present paper adopts the probabilistic design method into the safety-factor-based design processes. It allows estimating probability of failure of a design at the design stage (before the tests are performed). After the tests, the probability of failure is estimated again so that expectation of the effect of the tests can be made. The present paper focuses on quantifying the effect of the building block test at design stage using probabilistic approach. Also the effect of extreme loading on the structural element is illustrated.

There are several papers that investigate the effect of tests on structural safety. Guillemard [1]analyzed the role of tests in the successive stages of design, construction, and exploitation of a nuclear facility, Jiao and Moan [2] investigated the effect of proof tests on structural safety using Bayesian updating. They showed that the effect of the proof testing reduces the uncertainty in the strength of a structure; therefore it leads substantial reduction in probability of failure. Acar *et al.* [3] invests the effect of tests on reducing uncertainty in strength estimate using the Bayesian updating. Acar *et al.* [4] proposes a methodology that modeling the building block test which is multi-stage test procedure and showing uncertainty reduction effect of each test stage on probability of failure of the design in terms of the number of tests. Park *et al.* [10] models the building block test, however it applies fully probabilistic approach instead of knockdown factor to estimate strength of the structural element and analyze uncertainty reduction effect of the tests on the estimated strength in terms of the number of tests.

Here, the building block test procedure is modeled, structural element design process using the building block tests is simulated, probability of failure estimate is calculated and the estimate is compared to the true probability of failure for evaluating discrepancy between the estimate and the true probability. Also the effect of extreme load case is investigated using the model.

4. Structural Element Design and Test Procedure

In this section, the element design and test procedure are described. The procedure can be divided two stages: (i) the initial design stage based on coupon test results; (ii) and redesign based on element tests. We also consider the possibility that element test stage can be skipped at the discretion of the designer.

4.1. Structural Element Design and Test Procedure before Element Test

The design of an aircraft structural element is based on two test stages: coupon tests and element tests. Estimating strength of the structural element is required to design the structural element. Allowable strength is calculated based on coupon tests done under uni-axial loads. FAA regulations requires to use the B-basis (or A-basis) allowable that is calculated such that 90% of measured strengths from coupon tests should exceed the B-basis allowable with 95% confidence, which can be calculated by

$$\sigma_{c,\text{allow}} = \bar{\sigma}_{c,\text{test}} - k_B s_{c,\text{test}} \tag{1}$$

where $\bar{\sigma}_{c,\text{test}}$ and $s_{c,\text{test}}$ are, respectively, the mean and standard deviation of strength from n_c coupon tests. The tolerance limit factor, k_B, is a function of n_c calculated to ensure the 90% population with the 95% confidence requirement. Alternatively, the allowable strength at the coupon level can be defined using a knockdown factor, k_d , as

$$\sigma_{c,\text{allow}} = k_d \bar{\sigma}_{c,\text{test}} \tag{2}$$

In terms of the tolerance limit factor, the knockdown factor can be expressed as

$$k_d = 1 - k_B \frac{s_{c,\text{test}}}{\bar{\sigma}_{c,\text{test}}} \tag{3}$$

Because of the confidence requirement, the knockdown factor decreases as the number of coupons decreases; i.e., a large penalty is applied for a small number of coupons. The element allowable strength has to be estimated using failure theory (i.e. von Mises) since the structural element is under multi-axial stresses. Allowable strength of the element is function of the allowable strength of coupon as,

$$\sigma_{e,allow}^{calc} = f(\sigma_{c,allow}) \tag{4}$$

It is obtained from the failure envelope under the combined loads constructed from a failure theory using uni-axial strengths from coupon test. Based on the allowable, initial design of load carrying area is calculated as below

$$a_e^{calc} = \frac{1.5 p_{\lim}}{\sigma_{e,allow}^{calc}} \tag{5}$$

where p_{lim} is limit load of the structural element. The ultimate load is the safety factor times the limit load and initial design area is supposed to be sustainable the ultimate load.

4.2. Structural Element Design and Test Procedure after Element Test

After the initial design is completed, several nominally identical elements are made to perform the element test then measuring strength of those elements. Here we assume that the typical number is three element tests. Then we assume that the re-design proceeds based on the lowest strength of the elements, and applying the knockdown factor k_d to update element strength allowable.

$$\sigma_{e,allow}^{upd} = k_d \sigma_{e,test}^{lowest} \tag{6}$$

In practice, lowest element strength replaces the mean strength as Eq. (6). The element test, taking the lowest strength, is modeled using a probabilistic approach as equivalent to an additional knockdown factor to the mean test strength. If we simulate element test, taking the lowest element strength, and repeat the simulation infinite number of times, then the mean value of the lowest strengths to compare it to the true mean element strength will provide an explicit knockdown factor. The derivation and explanation of explicit knockdown factors can be found in the work of Acar et al.[3]. The Bayesian updating is used to obtain the mean strength and its benefit is availed in this study. Eq. (6) in common practice is replaced by

$$\sigma_{e,allow}^{upd} = k_d k_{\exp} \bar{\sigma}_{e,Bayes} \tag{7}$$

In Eq. (7), it is assumed that $\bar{\sigma}_{e,Bayes}$ is mean value of mean strength distribution of the structural element from the Bayesian updating. The most likely value of the distribution of the mean strength is the most probable true mean strength. However, the mean of the distribution gives a better estimate for the mean strength than the most likely value because it is less sensitive to poor test results.

The redesigned area is calculated using the updated element strength allowable.

$$A_e^{upd} = \frac{1.5 p_{\lim}}{\sigma_{e,allow}^{upd}} \tag{8}$$

5. Variabilities and Errors

5.1. Variability and Errors in coupon test

Coupon tests results display variability in strength; analyst needs conservative strength to compensate for the variability. To calculate the conservative strength, if the type of distribution of the strength and its parameters are known, desired conservativeness can be calculated accurately. However, with only finite number of samples, these parameters have errors.

In the present paper, it is assumed that coupon strength follows a normal distribution and then the distribution of error in estimating the mean and standard deviation of strength are can be calculated from the normalcy assumption.

5.2. Errors in failure theory

Coupon tests yield uni-axial strength, but the structural element is usually subject to combined loads, and a failure theory such as von Mises is used to predict its strength. There is error associated with the use of the failure theory and the sampling error mentioned earlier for the input to this theory from the coupon tests. These errors are combined in the present analysis.

5.3. Variability and Errors in element test

In order to reduce the effect of the combined error in prediction of the mean element strength, element tests are conducted, which also have variability, and consequently sampling has error. Because of the high cost of the element test, the number of these tests is very limited so that error in sampling of the element test is greater than the error in sampling of the coupon test. It is assumed that variability of the element test is independent to the variability of the coupon test.

It is assumed that the element strength is normally distributed around its mean with a coefficient of variation c_e.

6. Modeling Structural Element Design and Test Procedure Using Statistical Methods

6.1. Element Strength Estimation

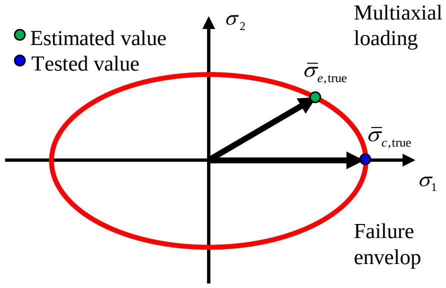
Since structural elements are in a multi-axial stress state, it is necessary to use the failure theory (e.g. von Mises), which converts the multi-axial stress into an equivalent stress. In order to simplify the following we make the assumption that the coupon test results for all strength components have the same coefficient of variation and are perfectly correlated. That is, for example, if the sampling error in the compressive strength is 10% we will have 10% sampling error in the shear strength. In that case, the true element strength can be represented as proportional to one representative coupon strength. The relation between coupon and element strengths can be represented by

$$\bar{\sigma}_{e,\mathrm{true}} = \alpha_{\mathrm{true}} \bar{\sigma}_{c,\mathrm{true}} \tag{9}$$

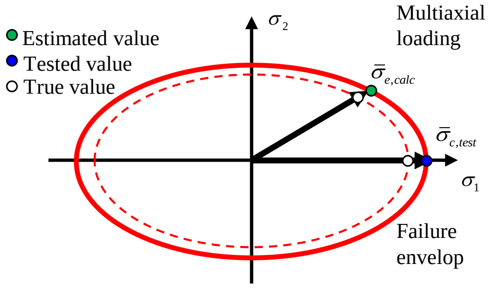
where α_{true} is the ratio between the uni-axial strength and the failure strength under multi-axial stress using a failure theory (e.g., von Mises theory for isotropic material or Tsai-Wu theory for composites) illustrated in Fig. 2. The subscript 'e' stands for structural elements. Since the failure theory is not perfect, the exact value of α_{true} is unknown (epistemic uncertainty). Instead, the following error model is used to represent the epistemic uncertainty in the failure theory:

$$\alpha_{\mathrm{true}} = (1 - e_{\alpha}) \alpha_{\mathrm{calc}} \tag{10}$$

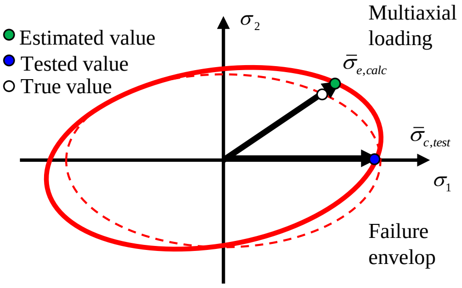
where α_{calc} is the ratio calculated from the failure theory, and e_{α} represents the error in the failure theory. In Fig. 2, showing two uncertainties combined in the estimated element strength, both are epistemic uncertainties.



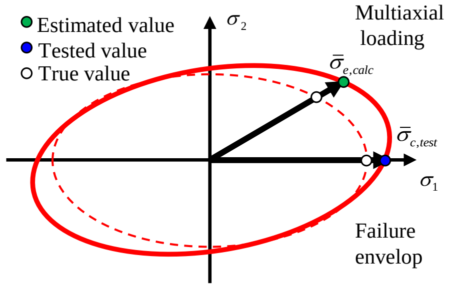
(a) Perfect failure theory and perfect sampling: The tested and estimated values are equal to the true values (the envelope is the true failure envelope, it will be shown as dashed line in the other figures)



(b) Sampling error due to coupon test :discrepancy between blue and empty circles, Solid line is computed failure envelope and dashed line is the true failure envelope. Error in failure envelope leads to error in estimatedelement failure strength (discrepancy between green and empty circle)



(c) Error only in failure theory: Solid line shows calculated failure envelope and dashed line shows true failure envelope



(d) Errors in both failure theory and sampling: The error in sampling (discrepancy between blue and empty circles) and error in failure theory are combined to an error in the estimated value (discrepancy between green and empty circles).

Figure. 2: Four cases of estimation element strength

The sign in Eq. (10) is chosen such that a positive error implies a conservative estimate. Without loss of generality, the case of $\alpha_{\mathrm{calc}} = 1$ is considered in this paper. In addition, even if the error might be a deterministic value, since the analyst does not know its value, it is considered as a random variable; i.e., both epistemic and aleatory uncertainties are modeled as random variables. Using the coupon test results, the mean failure strength of elements can be estimated by

$$\bar{\sigma}_{e,\mathrm{calc}} = \alpha_{\mathrm{calc}} \bar{\sigma}_{c,\mathrm{test}} \tag{11}$$

The calculated mean element failure strength includes two sources of epistemic uncertainty: one from the sampling error due to finite number of coupons and the other from the error in the failure theory. As an equivalent process of Eq. (4), the allowable strength of elements can be calculated as

$$\sigma_{e,\text{allow}}^{calc} = \alpha_{\text{calc}}\sigma_{c,\text{allow}} \quad (12)$$

The superscript 'calc' is used because it will be updated using element tests in the following subsection. Substituting Eqs. (2) and (11) into Eq. (12), the initial allowable strength of elements can also be expressed using the knockdown factor as

$$\sigma_{e,\text{allow}}^{calc} = k_d\bar{\sigma}_{e,\text{calc}} \quad (13)$$

6.2. Element Tests

Taking the lowest test result is equivalent to apply the explicit knockdown factor to the mean of the updated mean strength of the structural element. To obtain the updated allowable strength, we apply the knockdown factor k_d to it as in Eq. (7).

6.2.1 Bayesian Updating for the Mean Element Strength

This section shows how Bayesian updating is used to obtain the mean strength of the element. Uncertainty in the mean element strength is represented as a distribution In this paper, the updated distribution is called a possible true distribution and the random variable represented by the distribution is the possible true mean strength. In the previous section, mean of the possible true distribution of the mean strength $\bar{\sigma}_{e,Bayes}$ is used to obtain the allowable strength of the element in Eq. (7).

The calculated mean strength of the structural element is obtained using the calculated mean coupon test and the failure theory so that it its error comes from sampling and failure theory errors. The possible true distribution has to be convolution function of these two errors. However, in [10], we found that the sampling error has little effect, if the number of coupon tests exceeds 50. So that only error in the failure theory is considered in this paper.

It is assumed that the error in failure theory has a uniformly distributed between error bounds that represent our confidence about the calculation.

$$f^{init}\left(\bar{\sigma}_{e,Ptrue}\right) = \begin{cases} \dfrac{1}{2b_e\left(\sigma_{e,calc}\right)} & \text{if } \left|\dfrac{\bar{\sigma}_{e,Ptrue}}{\sigma_{e,calc}} - 1\right| \le b_e \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

where $\bar{\sigma}_{e,Ptrue}$ is possible true mean strength of structural element. This initial distribution $f^{init}\left(\bar{\sigma}_{e,Ptrue}\right)$ is a probability density function value and it can be interpreted as an initial prediction of the true mean of element strength, $\bar{\sigma}_{e,true}$ based on confidence in the failure theory. . Test results allow the analyst to reduce the uncertainty in the failure theory. The initial distribution is updated using the Bayesian technique with element tests.

$$f^{upd}\left(\bar{\sigma}_{e,Ptrue}\right) = \frac{f^{init}\left(\bar{\sigma}_{e,Ptrue}\right)\prod_{i=1}^{n_e} f_{i,test}\left(\bar{\sigma}_{e,Ptrue}\right)}{\int_{-\infty}^{\infty} f^{init}\left(\bar{\sigma}_{e,Ptrue}\right)\prod_{i=1}^{n_e} f_{i,test}\left(\bar{\sigma}_{e,Ptrue}\right)d\bar{\sigma}_{e,Ptrue}} \quad (15)$$

Likelihood function is defined as conditional probability density function and it represents likelihood of the true mean strength for given i^{th} sample $\sigma_{e,test}^{i}$.

$$f_{i,\text{test}}\left(\bar{\sigma}_{e,\text{Ptrue}}\right) = \text{Normal}\left(\sigma_{e,\text{test}}^{i} \mid \bar{\sigma}_{e,\text{Ptrue}}, \bar{\sigma}_{e,\text{Ptrue}}c_e\right) \quad (16)$$

where c_e is coefficient of variation of the strength of structural element. No element tests case, the random variable of the possible true mean strength $\bar{\Sigma}_{e,Ptrue}$ follows the initial distribution in Eq. (14). If the element tests are performed, the initial distribution is updated, $\bar{\Sigma}_{e,Ptrue}$ follows the updated distribution defined by $f_{i,\text{test}}\left(\bar{\sigma}_{e,\text{Ptrue}}\right)$.

6.3. Design of the Structural Element

To design the structural element, designers use conservative allowable stress and conservative design load. We use ultimate load that is composed of limit load and 1.5 factor of safety. Limit load is a maximum load per fleet lifetime. The present paper simplifies the design process as designing load carrying area under combined load as

$$a_e = \frac{1.5p_{\text{lim}}}{\sigma_{e,allow}} \quad (17)$$

where p_{lim} is the limit load and $\sigma_{e,allow}$ is the allowable element strength. Two options to design the load carrying area are considered here.

1) Using the calculated design allowable stress $\sigma_{e,allow}^{calc}$ in Eq. (4) as $\sigma_{e,allow} = \sigma_{e,allow}^{calc}$. It is a reasonable choice when the failure theory is very accurate, but it does not protect against unexpected large error.

2) Using the updated design allowable stress $\sigma_{e,allow}^{upd}$ in Eq. (7) as $\sigma_{e,allow} = \sigma_{e,allow}^{upd}$. This approach benefits from the additional information provided by element tests.

7. Calculating Probability of Failure of the Element Design

There are two types of probability of failures that estimated probability of failure and true probability of failure. Each design has its own true probability of failure that corresponds to the true value of the errors. The probability of failure can be calculated for each set of errors. Assuming that the errors have the assumed distribution, we can obtain a distribution of possible probabilities of failure. This is a single analyst's point of view. the analyst can estimate the probability of failure but the true probability of failure is unknown for the analyst.

7.1. Estimated Probability of Failure

To estimate the probability of failure, the possible true mean strength distribution from the Bayesian updating is used. For each value of the mean strength value $\bar{\sigma}_{e,Ptrue}$ we can calculate the corresponding probability of failure $pf_{e,Ptrue}$ using a conditional random variable on the mean strength distribution as

$$P\left(\Sigma_{e,Ptrue} \mid \bar{\Sigma}_{e,Ptrue} = \bar{\sigma}_{e,Ptrue} \le \frac{1.5 p_{\lim}}{a_e (1+V_m)} \right) = pf_{e,Ptrue} \tag{18}$$

It is assumed that the variability is measured accurately with little uncertainty and it follows uniform distribution with margin ±3%, also the element strength is normally distributed around its mean with a coefficient of variation c_e. $\bar{\Sigma}_{e,Ptrue}$ is a random variable, follows the possible true mean strength distribution, and mean of the distribution is random variable that follows the updated possible true mean strength distribution in Eq. (15), (e.g., probability density function of $\bar{\Sigma}_{e,Ptrue} = \bar{\sigma}_{e,Ptrue}$ is $f^{upd}(\bar{\sigma}_{e,Ptrue})$). That is, $\Sigma_{e,Ptrue} \mid \bar{\Sigma}_{e,Ptrue} = \bar{\sigma}_{e,Ptrue}$ is a random variable from a distribution that mean value of the distribution is equal to $\bar{\Sigma}_{e,Ptrue} = \bar{\sigma}_{e,Ptrue}$.

$$\Sigma_{e,Ptrue} \mid \bar{\Sigma}_{e,Ptrue} = \bar{\sigma}_{e,Ptrue} \sim Normal\left(\bar{\sigma}_{e,Ptrue}, \bar{\sigma}_{e,Ptrue} c_e \right) \tag{19}$$

The probability of failure has a form of a distribution because mean value of the mean strength distribution is a random variable due to variability in element tests.

$$P\left(\Sigma_{e,Ptrue} \mid \bar{\Sigma}_{e,Ptrue} \le \frac{1.5 p_{\lim}}{a_e (1+V_m)} \right) = PF_{e,Ptrue} \tag{20}$$

Since random variables $PF_{e,Ptrue}$ is strictly decreasing monotonically as function of $\bar{\Sigma}_{e,Ptrue}$, the probability that the random variable $PF_{e,Ptrue}$ is greater than a value $pf_{e,Ptrue}$ is equal to probability of the random variable $\bar{\Sigma}_{e,Ptrue}$ is less than a value μ for $PF_{e,Ptrue} = pf_{e,Ptrue}$ at $\bar{\Sigma}_{e,Ptrue} = \mu$.

$$P\left(PF_{e,Ptrue} > pf_{e,Ptrue} \right) = P\left(\bar{\Sigma}_{e,Ptrue} < \mu \right) \tag{21}$$

Using complementary law of probability, Eq. (21) can be expressed as

$$F_{Pf}(pf_{e,Ptrue}) = 1 - F_M(\mu) \tag{22}$$

$$F_{Pf}(pf_{e,Ptrue}) = P\left(PF_{e,Ptrue} < pf_{e,Ptrue} \right) \text{ and } F_M(\mu) = P\left(\bar{\Sigma}_{e,Ptrue} < \mu \right) \tag{23}$$

Using Eq. (18) and Eq. (22), the cumulative distribution function of the probability of failure can be calculated. Also probability density distribution is derived from the cumulative distribution and it is an estimated probability of failure distribution.

7.2. Estimated Probability of Failure for Arbitrary Load

The probability of failure discussed so far is the probability of failure of the element under ultimate load. The aircraft company may be also interested in the probability of failure under higher or lower loads. The probability of failure under limit load is close to the probability of the element failing in service (neglecting errors in the load calculation). The probability of failure for higher loads may be of interest if the mission of the aircraft changes, leading to higher loads. For the estimated probability of failure for given arbitrary load p_i were placed the $1.5 p_{\lim}$ term in Eq. (20) by p_i,.

$$P\left(\Sigma_{e,Ptrue} \mid \bar{\Sigma}_{e,Ptrue} \le \frac{P_{\mathrm{i}}}{A_e (1+V_m)} \right) = PF_{e,Ptrue} \tag{24}$$

7.3. True Probability of Failure

True probability of failure is defined as probability that stress on the manufactured structural element section is greater than actual strength of the structural element for the given load. The manufacturing variability is included to calculate the probability of failure and it is assumed that measurement of the manufacturing is very accurate. True probability of failure is calculated as

$$P\left(\Sigma_{e,true} \le \frac{1.5p_{\lim}}{a_e(1+V_m)}\right) = pf_{e,true} \tag{25}$$

where V_m is a random variable for manufacturing variability. Note that the true distribution of the strength of the structural element in Eq. (25) is unknown for analyst's view.

To keep the same mean probability of failure with 3 element cases, we adjust the allowable strength and calculate average weight of design according to the number of element tests. The additional knockdown factor is applied as

$$\sigma_{e,allow} = k_d k_{\exp} k_{add} \bar{\sigma}_{e,Bayes} \tag{26}$$

Also the true probability of failure as a function of load p_i is calculated in terms of the arbitrary load to see the true probability of failure at load p_i which is not the design load.

$$P\left(\Sigma_{e,true} \le \frac{p_i}{a_e(1+V_m)}\right) = pf_{e,true} \tag{27}$$

where the area is designed to be sustainable ultimate load as $a_e = 1.5p_{\lim} / \sigma_{e,allow}$.

8. Simulation Procedure

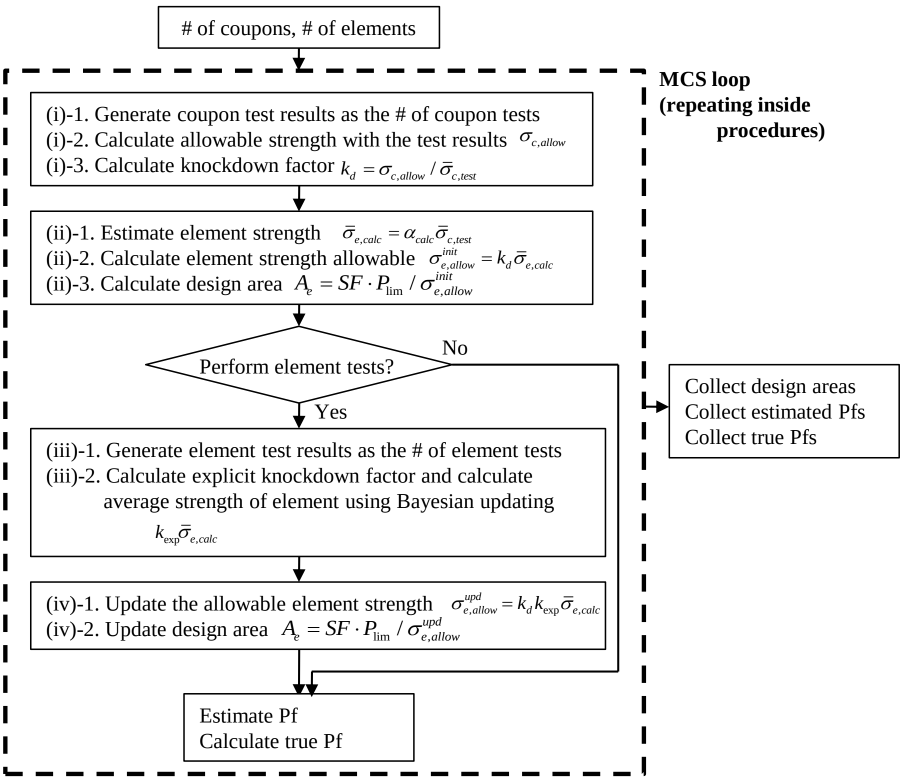


Figure. 3: Flowchart of simulation procedure

In this section, simulation for the design and test procedure is shown. The simulation is carried out from two points of view: an analyst's point of view corresponds to a single set of errors and test result; the FAA (or possibly aircraft company) point of view considers many analysts, each designing a different element with different test results. From the FAA point of view, the errors are no longer epistemic uncertainties, because they are truly randomly distributed between a large numbers of designers.

From an analyst's point of view, we simulate the design of a load carrying area tests, and the estimate of its probability of failure of the structure. Simulation of the design and test processes is largely composed of simulating four steps: (i).coupon test, (ii).initial design (iii).element test and (iv).updated design. Final outcomes of this simulation are designed area and the estimated probability of failure. All processes in dashed line box in Fig. 3 represent a single design process by a single analyst who decides whether to perform element tests or skip the tests.

For the FAA's point of view, the procedure in the dashed box of Figure 3 is repeated for many sets of test results corresponding to the errors.

9. Illustrative Example

In this section, a simple problem of design of a load carrying area design is used. The number of coupon tests is fixed to 50 tests and 2, 3, or 5 element tests are considered. The limit load p_{lim}=1, the safety factor of 1.5. Coefficient for the failure theory is α=1.0 as it includes error in estimation and the B-basis coefficient for 50 coupon tests is k_B=1.6455. The calculation of the explicit knockdown factor refers Acar *et.al.* [3]. k_{exp}=0.9018, =0.8776, and =0.8506 for the number of element tests 2,3,and 5, respectively. It implies that taking the lowest results of 5 is more conservative than the lowest results of 3 and 2. k_{exp}=1 is used for the no element test case.

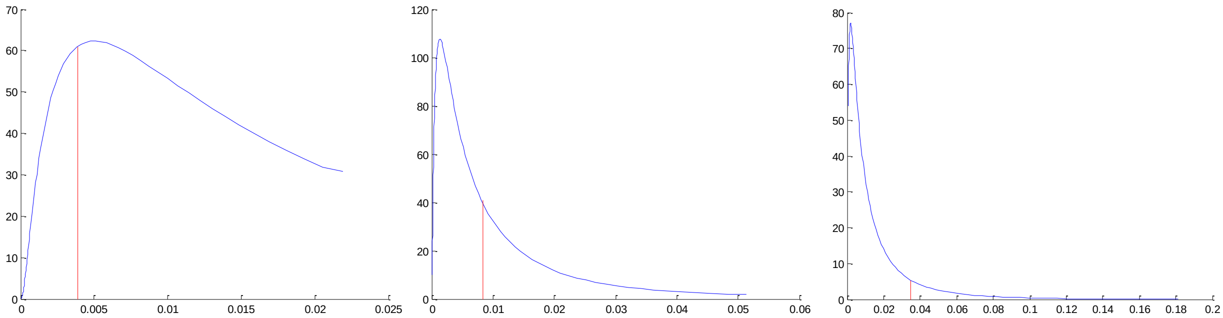
Table 1: Distributions of tests for simulation

Test	Distribution	Parameters
Coupon Test	Normal	μ=1.0 σ=0.07 (c_c =0.07)
Element Test	Normal	μ=0.95 σ=0.09 (c_e =0.0855)

9.1 Analyst's point of view

A single analyst has to design by given coupon test results and element test results, also he is obligate to use FAA rules such as using of B-basis or A-basis and then he estimate the probability of failure of the design. The analyst's view follows the simulation processes described in dashed box of Fig. 3 to design a load carrying area. The probability of failure is defined as probability that ultimate load exceeds capacity of the load carrying area (strength times area). The analyst estimates a distribution of the probability of failure; the analyst also has to decide how to translate this distribution to a single nominal probability of failure. In the present paper, we set mean of the distribution as the nominal probability of failure. In this case, the estimated probability of failure is compared to the true probability of failure. Note that the true probability of failure is unknown for the analyst.

Here, three typical situations are considered; conservative element tests, nearly no conservativeness, and un-conservative element tests.



(a) Conservative test results (b) Nearly no conservativeness test results (c) Un-conservative test results

Figure. 4: Distribution of the probability of failure for three typical cases
(50 coupon tests, 3 element tests: red line indicates true probability of failure and blue line indicates the probability of failure distribution of a single design)

From the three cases, the estimated probability of failure and the true probability of failure are calculated. The estimated probability of failure is given for both distribution and nominal value in Fig. 4 and Table 1, respectively. The nominal value is the mean value of the probability of failure distribution. From the Table 1, the mean probability of failure gives conservative estimate than the true probability of failure, except in the case of un-conservative test result. It is also observed that the probability of failure is quite sensitive to the design area. The mean of the coupon test varies slightly while the mean of the element test varies a lot and this phenomenon is caused by the number of the samples. The number of coupon tests is 50 so that standard deviation of the mean of the coupon tests is, $\sigma_{c,true}/\sqrt{50}$, 0.0099 whereas the standard deviation of the mean of the element tests is 0.5485. Here, the Bayesian method is used for effective mean to reduce uncertainty with the small samples. Efficiency of the Bayesian method to reduce uncertainty is already shown through comparing to the conventional method of taking the lowest result [9].

Table 2: Three cases of element test results

	Conservative test results (Fig. 4a)	Nearly no conservativeness test results (Fig. 4b)	Un-conservative test results (Fig. 4c)
True Pf	0.0033	0.0089	0.0347
Mean Pf	0.0101	0.0099	0.0168
Mean Coupon	1.0027	1.0104	0.9905
Std. Coupon	0.0673	0.0742	0.0629
Element test1	0.8914	0.9638	1.0714
Element test2	0.9271	0.9477	0.9780
Element test3	0.8484	0.9648	0.9907
Element Mean	0.8889	0.9588	1.0133
Design Area	2.0958	1.9839	1.8927

9.2 FAA's point of view

FAA governs the analysts by regulation such as B-basis value or A-basis value as strength allowable. In this point of view, we simulate 10000 analysts and their estimation of the probability of failures. The mean of the true probability of failure of the 10,000 analysts at the ultimate load with 3 element tests case is 0.011. Obviously, the probability of failure depends on the number of element tests. Using an additional knockdown factor to adjust conservativeness of the allowable strength of the structural element, the mean probability of failures of 10000 analysts can be controlled.

Table 3: True probability of failure and estimated probability of failure from analyst

	No element tests		2 element tests (k_{add} = 0.962)		3 element tests (k_{add} = 1)		5 element tests (k_{add} = 1.045)	
P_i / P_{lim}	**Mean of the True Pfs**	**Mean of the Nominal Pfs**	**Mean of the True Pfs**	**Mean of the Nominal Pfs**	**Mean of the True Pfs**	**Mean of the Nominal Pfs**	**Mean of the True Pfs**	**Mean of the Nominal Pfs**
1.0	1.0×10^{-7}	1.5×10^{-7}	1.1×10^{-7}	1.3×10^{-7}	1.3×10^{-7}	1.1×10^{-7}	1.3×10^{-7}	1.5×10^{-7}
1.35	0.0008	0.0010	0.0008	0.0009	0.0009	0.0010	0.0009	0.0011
1.4	0.0022	0.0024	0.0020	0.002	0.002	0.0025	0.0023	0.0027
1.45	0.0053	0.0053	0.0046	0.005	0.005	0.005	0.0054	0.0063
1.5	**0.011**	**0.009**	**0.011**	**0.011**	**0.011**	**0.011**	**0.011**	**0.013**
1.55	0.024	0.021	0.019	0.022	0.022	0.022	0.023	0.026
1.6	0.045	0.038	0.036	0.042	0.041	0.042	0.043	0.049
1.65	0.081	0.063	0.064	0.073	0.073	0.073	0.076	0.085

Table 3 provides the probability of failure with the external force P_i and the additional knockdown factor k_{add}. The additional knockdown factor adds more conservativeness to the design to decrease its probability of failure whereas its average weight is increased. The mean of the true probability of failures with 3 element tests with the ultimate load case is referred as a standard. Note that the mean of the true probability of failure is unknown in FAA's point of view; it is only used to see the true effect of the tests to achieved. The mean of the nominal probability of failures is known in FAA's point of view by gathering 10000 analysts' nominal probability of failures corresponding to their design. This result shows that the nominal probability of failure gives slightly conservative estimate compare to the true probability except no element tests.

The probability of failure increases exponentially as the external force P_i increases linearly. The ratio P_i/P_{lim} =1.5 case is equivalent to the certification test condition under the ultimate load. The ratio of 1.0 is reflecting flight condition with limit load P_{lim}. Under the limit load without variability, the probability of failure has 10^{-7} order of magnitude so that the probability of failure in actual flight is very low and structural failure is very rarely expected during flight.

Table 4 compares the average structural weights in terms of the number of element tests relative to the average weight with 3 elements and it calculated from the Table 3 results. The average weight of the design with no element tests is 3.7% heavier than the design with 3 element tests, 5 elements provide 1.3% lighter design which implies that importance of the element tests.

Table 4: Relative weight for the same probability of failure level

	No element tests	2 element tests (k_{add} = 0.962)	3 element tests (k_{add} = 1)	5 element tests (k_{add} = 1.045)
P_i / P_{lim}				
1.5	1.037 (+3.7%)	1.012 (+1.2%)	1	0.987 (-0.3%)

10. Conclusion

The current practice in aircraft structural design employs two methods for improving structural safety. Using knockdown factors and safety factors to compensate uncertainties in design or reducing the uncertainties in the design by performing structural tests. The former leads weight increase and the latter requires cost to perform tests. In this paper, the two effects have been quantified.

A method to estimate the probability of the failure is proposed using the Bayesian method and statistical method. It makes the analyst possible to provide the probability of failure distribution of the design as a function of load. According to the probability of failure, the analyst modifies the design that having the less probability of failure by increasing conservativeness in the design.

To achieve a design specification for the safety, two aspects are considered, the number of element tests and additional conservativeness for strength estimate. Also no element test case is considered to show importance of the structural tests. As the number of element tests increases, weight of the design decreases while the probability of failure is being kept. A design with 5 element tests allows 1.3% lighter design than a design with 3 element tests for the same expected true probability of failure. A design with 2 element tests requires 1.2% heavier design.

Especially sacrifice of the weight without element tests to keep the same level of the probability with element tests is substantial. No element case requires 3.7% heavier design to expect the same probability of failure with 3 element tests case.

In the present paper, the estimated probability of failure is compared to the true probability of failure to verify the accuracy of the estimated probability. The importance of tests on the structural design is shown in aspect of weight.

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