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DRAFT: EQUIVALENT DAMAGE GROWTH PARAMETERS USING A SIMPLIFIED MODEL

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ABSTRACT

Most damage growth models require accurate stress intensity factor as well as model parameters for predicting damage growth. Depending on geometries and loading conditions, these models become complicated with additional model parameters. This paper shows that a simple model, such as the Paris model, can be used for complex geometries by compensating the error in stress intensity factor with the equivalent model parameters that are different from the true ones. Actual damage growth is simulated using the extended finite element method to model the effects of crack location and geometry on the relationship between crack size and stress intensity factor. The detection process of crack using structural health monitoring systems is modeled by adding random noise and a deterministic bias. The equivalent model parameters are then identified using the least-square-filtered Bayesian method, from which the remaining useful life is estimated. Using three examples, it is shown that the RUL estimates are accurate even when an inaccurate stress intensity factor is used.

INTRODUCTION

For last two decades, the structural health monitoring (SHM) technology has been significantly developed such that it is feasible to not only detect damage but also to characterize the significance of damage [1,2]. In the case of structural damage due to cracks, SHM systems can now continuously monitor the growth of cracks during the lifecycle of an aircraft. When the monitoring results are incorporated with crack growth models, it is possible to predict the future behavior of cracks, which is called model-based prognosis [3,4]. This is very valuable information in terms of providing safety of aircraft and estimating appropriate maintenance schedules.

Although the model-based prognosis can be a powerful technology, it has a drawback that many physical models are limited to simple conditions. For example, the original Paris model [5] describes the rate of crack growth as a function of stress intensity factor. However, it is limited to a center crack in an infinite plate under mode I loading condition because the stress intensity factor is a complicated function of applied loading, boundary conditions, crack position, and geometry. There is an analytical equation available for the stress intensity factor for an infinite plate with a through-the-thickness center crack, and correction factors for taking into account for finite plate size or edge cracks [6,7]. These analytical equations are typically not available for complex engineering systems. Often numerical techniques, such as finite element analysis, are used to calculate accurate stress intensity factor [8]. This can cause a significant computational difficulty because the statistical nature of prognosis requires evaluations of stress intensity factor for numerous damage sizes.

The objective of this paper is to demonstrate that in model-based prognosis, one can use simple models to predict the remaining useful life even if the model differs from the true behavior. This is accomplished through the identification of an equivalent damage growth parameter that compensates for the difference between the simple model and the true stress intensity factor. An important question that is explored in this paper is if a simple, analytical stress intensity factor can be used for arbitrary crack geometries for the purpose of prognosis. The key concept in this paper is that the original Paris model can be considered as an extrapolation tool. Thus, even if the actual crack growth behavior is different from the one obtained with the analytical stress intensity factor, Bayesian inference can identify equivalent damage growth parameters, different from the true ones, such that the model accurately predicts future damage growth behavior.

The damage growth is simulated using the extended finite element method (XFEM) for calculating "true" stress intensity factors, and the Paris model is used to grow the crack. XFEM [9] allows for discontinuities to be modeled independently of the finite element mesh, which avoids costly remeshing as the crack grows. The stress intensity factors which are the driving force for crack growth are calculated within the XFEM framework using the domain form of the contour integrals [10].

In practice, the actual damage sizes are measured using SHM systems in which on-board sensors and actuators are used to detect damage location and size. In this paper, instead of using actual measurement data, synthetic data are generated using random noise and deterministic bias. First, true values of the Paris model parameters are assumed. Then, the true crack will grow according to the given model parameters, prescribed operating, and loading conditions by XFEM simulations. Thus, the true crack size at every measurement time is known. With the true crack size, the remaining useful life is defined when the crack size reaches the critical crack size, which is a function of material, operating, and loading conditions. It is assumed that the measurement instruments may have a deterministic bias and random noise. These bias and noise are added to the true crack sizes, which are denoted as synthetic measured crack sizes. Then, these data are used to predict the equivalent damage growth parameters and thus the remaining useful life. In this way, it is possible to evaluate the accuracy of prognosis method.

Of the many methods available for parameter identification, the least-square-filtered Bayesian method (LSFB) [11] is used to identify damage growth parameters. This method applies nonlinear least-square method to the measurement data, so that the magnitude of noise can be reduced, followed by Bayesian inference [12] to identify a probability distribution for model parameters. The identified distribution of damage growth parameters can then be used to predict the distribution of RUL.

The paper is organized into the following sections. In Section 2, the crack growth model is introduced along with the notion of equivalent model parameter. In Section 3, the model for measurement uncertainty that is used in this paper is explained. The least-square-filtered Bayesian method is summarized in Section 4. Three numerical examples with increasing difference between the simple and true stress intensity factor models are presented in Section 5, followed by concluding remarks and future work in Section 6.

CRACK GROWTH MODEL

A crack in a plate can grow due to repeated application of stress. For example, a crack in a fuselage panel of aircraft can grow due to repeated pressurizations. In this paper, the original Paris model [5] is used to predict the crack growth in a plate. In this model, the range of stress intensity factor ΔK is the main factor driving the crack growth with two parameters, C and m, as

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{1}$$

where a is the characteristic crack size and N is the number of fatigue loading cycles. The range of stress intensity factor is calculated by the difference between maximum and minimum stress intensity factors; i.e., $\Delta K = K_{\max} - K_{\min}$. Although the number of cycles is an integer, it is considered a real number as the crack grows over a great number of cycles. The two parameters, C and m, are usually estimated from experiments. When a log-log scale plot is made for the growth rate versus the stress intensity factor, the slope corresponds to m, while the y-intercept at $\Delta K = 1$ corresponds to C.

It is well known that the original Paris model is good for a through-the-thickness center crack in an infinite plate under Mode I fatigue loading condition. In such a case, the stress intensity factor can be calculated as

$$\Delta K = \Delta\sigma\sqrt{\pi a} \tag{2}$$

where $\Delta\sigma$ is the range of applied nominal stress; i.e., stress far from the crack tip. By substituting Eqn. (2) into Eqn. (1), the differential equation can be solved for the crack size as a function of the number of cycles N as

$$a_N = \left[NC\left(1-\frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_0^{1-\frac{m}{2}} \right]^{\frac{2}{2-m}} \tag{3}$$

where a_0 is the initial crack size. Note that the initial crack size does not have to be the size of initial micro-crack in the pristine plate. When a structural health monitoring (SHM) system is used, a_0 can be the size of crack that is initially detected. Then, Eqn. (3) can be used to predict the crack size a_N after N cycles starting from a crack with size a_0, assuming that the parameters, C and m, are known.

Considering Eqn. (3) can calculate crack size for a given cycle number, it is also possible to calculate the required number of cycles for a crack to grow to a certain size. Especially, it is important to estimate how many cycles remain before failure. In general, a critical crack size, a_C, is defined in which the crack grows rapidly and becomes unstable. Then, starting from the current crack size (let us say that it is a_N), the remaining cycles until the crack grows to the critical crack size can be calculated by

$$N_f = \frac{a_C^{1-\frac{m}{2}} - a_N^{1-\frac{m}{2}}}{C\left(1-\frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)} \tag{4}$$

In SHM, Eqn. (4) can be used to predict the remaining useful life (RUL) before the crack needs to be repaired. Again, the prediction process requires the two Paris parameters.

When the plate is not infinite and the crack is not located at the center of the plate, the original Paris model needs to be modified. In general, the accuracy of Eqn. (2) depends on geometrical effects, boundary conditions, crack shape, and crack location. A more general expression [6,7] of the range of stress intensity factor can be written as

$$\Delta K' = Y\Delta K \tag{5}$$

where Y is the correction factor, given as the ratio of the true stress intensity factor to the value predicted by Eqn. (2). The correction factor depends on geometry of crack and plate and the loading conditions. Examples of the dependence of the correction factor on the crack size are shown in Figure 1 for a center crack in an infinite plate, a center crack in a finite plate, and an edge crack in a finite plate [6,7]. Many advanced models are also available that can consider the effect of crack tip plasticity as well as the effect of crack closure [13-15]. The advanced models normally come with more parameters that need to be identified.

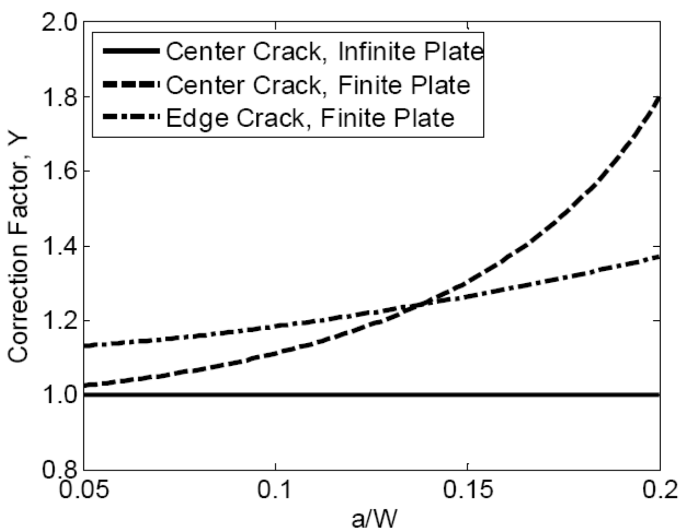


Figure 1. COMPARISON OF CORRECTION FACTORS FOR SEVERAL PLATE GEOMETRIES AND CRACK SIZES FOR A PLATE WIDTH OF 200 MM.

By comparing Eqns. (1), (2), and (5), an interesting, but critically important observation can be made. For example, it is possible to use the range of stress intensity factor in Eqn. (2) instead of the one in Eqn. (5) for the crack in a finite plate; i.e., it is possible to move the correction factor into the two Paris parameters. In such a case, the crack growth model in Eqn. (1) can be modified as

$$\frac{da}{dN} = C\left(\Delta K'\right)^{m} \approx C'\left(\Delta K\right)^{m'} \tag{6}$$

where C' and m' are 'equivalent' Paris parameters for using the stress intensity factor in Eqn. (2). Of course, the above relation does not exactly satisfy for all possible ranges of stress intensity factor, but it can make a good approximation when the variation of stress intensity factor is not significant. In the viewpoint of Eqn. (6), it is possible to interpret the two Paris parameters as curve-fitting parameters, not material properties. This observation is consistent with the fact that the Paris parameters become different when the road ratio $R = K_{\min} / K_{\max}$ varies even if ΔK is same [16]. The critical advantage of this viewpoint is that instead of making more advanced models for crack growth for SHM prognosis, the simple model in Eqn. (6) can be used as long as the equivalent parameters can be identified. In the numerical example section, this aspect will be tested using various examples.

For complex geometries with combined loadings, analytical expressions as given in Eqns. (2) and (5) may not be sufficient. For example, when a crack growth changes its path, the analytical growth rate equations cannot predict the correct path and growth of the crack. In such a case, a numerical method can be used to calculate the stress intensity factor as well as the direction of crack growth. In this paper, the extended finite element method (XFEM) is used to calculate the stress intensity factor ΔK for complex geometry and loadings, and Eqn. (6) is used to numerically integrate the crack size as a function of the number of cycles.

MEASUREMENT UNCERTAINTY MODEL

The crack growth model in the previous section can be a powerful tool in providing safety of the system and predicting maintenance schedules. However, the usefulness of the method depends on the accuracy of the parameters. For example, a 10% error in the exponent, m, can cause more than 100% difference in the predicted RUL from Eqn. (4). Therefore, it is critical to accurately estimate these parameters. However, challenges exist when these parameters are measured from laboratory experiments. Firstly, the variability in different batches of materials is too large to make useful predictions. Secondly, different loading and boundary conditions of practical panels affect these parameters. The premise of SHM is that frequently measured crack sizes can be used to identify 'panel-specific' damage growth parameters under given loading and boundary conditions, which is the main purpose of this paper. Then, these parameters can be used to predict the RUL before which the crack should be repaired.

Since no airplanes are equipped with SHM systems yet, synthetic data are used in this paper; i.e., crack sizes are simulated and assumed to have been measured by SHM. In general, the crack sizes measured from SHM systems include the effect of bias and noise of sensor signals. The former is deterministic and represents a systematic departure caused by calibration error, while the latter is random and represents a noise in the signal. The synthetic measurement data are generated by (a) assuming that the true parameters, m_{true} and C_{true}, are known, (b) calculating the true crack growth, a_N^{true}, using the crack growth model in the previous section for a given N, and (c) adding a deterministic bias and random noise.

Let a_N^{true} be the true half crack size at cycle N, b the bias, and v the noise. The measured half crack size a_N^{meas} is then generated from

$$2a_N^{\text{meas}} = 2a_N^{\text{true}} + b + v \qquad (7)$$

For subsequent measurements the bias b remains constant, while the noise v is assumed to vary uniformly between $-V$ and $+V$; i.e., $v \sim \text{U}[-V;+V]$ where U represents a uniform distribution between lower and upper bounds. It is noted that the current assumption on bias and noise is for convenience. In general, it is possible that the bias can vary as a function of time, and the noise can have different distribution types. Under the given models of bias and noise, the measured half crack sizes are uniformly distributed as

$$a_N^{\text{meas}} \sim \text{U}\left[a_N^{\text{true}} + \frac{b}{2} - \frac{V}{2}; a_N^{\text{true}} + \frac{b}{2} + \frac{V}{2} \right]. \qquad (8)$$

Once the synthetic measurement data are generated, the true half crack size a_N^{true} is not used any further, nor are the true values of parameters, m_{true} and C_{true}. The questions that need to be addressed are (a) if it is possible to accurately estimate the two parameters using the synthetic data that have bias and noise, and (b) what would happen if different geometry or boundary conditions are used for the actual panel. Figure 2 shows the measured crack sizes at every 100 cycles with a deterministic bias of $b = -2.0\,\text{mm}$ and the bounds of random noise $V = 1.0\text{mm}$, which is used in the numerical examples in Section 5. Note that due to the bias, the trend of data is shifted from the true crack growth curve.

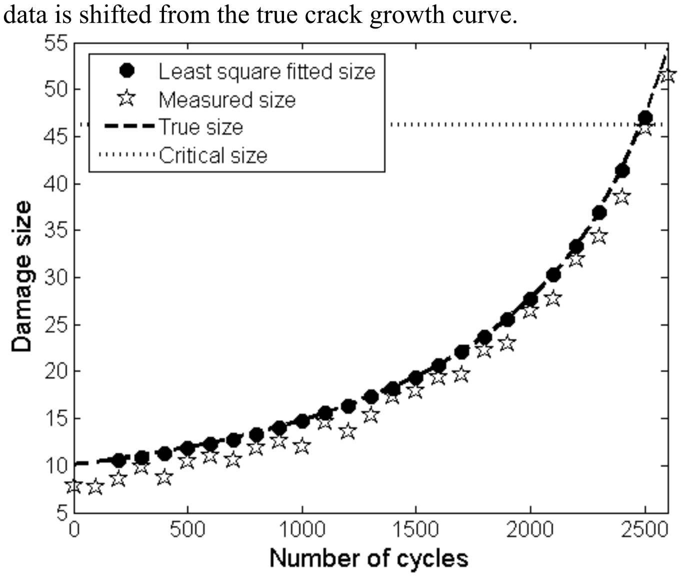


Figure 2. MEASURED CRACK SIZE AT EVERY 100 CYCLES WITH NOISE AND BIAS.

LEAST-SQUARE-FILTERED BAYESIAN (LSFB) METHOD

The least square method [17] and Bayesian inference [18] are often used for identifying unknown model parameters from data. The former minimizes errors between data and model predictions, while the latter progressively improves the knowledge on the parameters using data, starting from the initial knowledge. It is generally observed that the least square method is powerful in identifying deterministic parameters and also has a capability of reducing noise in the data. On the other hand, the Bayesian inference is customized to incorporate the initial knowledge of the parameters and statistically identifies model parameters. In this section, the advantages from both methods will be utilized to identify model parameters using SHM data.

In estimating the RUL, the statistical information of parameters is important because a conservative estimate is required for the maintenance schedule. Coppe et al.[11] used Bayesian inference to estimate the distribution of Paris model parameters, from which the distribution of the RUL is estimated. It was shown that the Bayesian inference was sensitive to the level of noise—the convergence of model parameters was slow when the measured crack size data have large noises. Therefore, the main contribution of this paper is to reduce the noise in the data using the least square method, followed by Bayesian inference to statistically identify model parameters. In the mean time, some deterministic parameters, such as bias and initial crack size can also be identified from the least square method. This proposed method is thus named the 'least-square-filtered' Bayesian (LSFB) method.

Filtering data using least square method

For the convenience of explanation, it is assume that the true value of parameter $C = C_{\text{true}}$ is known with its true value, while the exponent m is unknown and needs to be identified. For the case that both parameters are unknown, readers are referred to Coppe et al. [19]. Therefore, unknown variables are the Paris model exponent m, bias b, and initial crack size a_0. As mentioned before, a_0 is the true crack size at the cycle at which it is detected for the first time. Due to noise and bias, the true value of a_0 is also unknown.

The LSFB method processes information collected at every cycle using the least square method in order to reduce the noise, and identify the bias b and initial crack size a_0. The least square problem is expressed as

$$\underset{a_0, m, b}{\text{Minimize}} \sum_i \left(a_i^{\text{meas}} - \frac{b}{2} - a_i(m, a_0) \right)^2 \qquad (9)$$

where a_i^{meas} are the synthetic measured crack sizes with bias and noise model to simulate measurement data. Since the SHM process is time-dependent, the above least square method should use measured data up to the current time. Therefore, initially the identified results may not be accurate because only a limited number of data are used, but the accuracy will be improved as more data are available in the later time. In addition, the initial data may not provide valuable information because the crack growth rate is very small at this stage. As the crack starts growing fast, the process will identify parameters more accurately.

In this paper, the LSFB method utilized ΔK calculated from Eqn. (2), and an equivalent value of m is identified resulting in the same solution to Eqn. (1) as if the true ΔK were used. The values of a_0, m and b identified from Eqn. (9) are then used to generate a new estimate of the damage size at the current cycle using Eqn. (3); they are referred to as filtered data. These filtered data are then used in Bayesian inference in order to narrow down the distribution of m and obtain a more accurate prediction. The identified a_0 and b are considered to be deterministic. Only uncertainty in m is considered in the Bayesian inference.

Bayesian inference for parameter identification

Bayesian inference is based on the Bayes' theorem on conditional probability[20]. It is used to obtain the updated (also called posterior) probability of a random variable by using new information. In this paper, it is used to improve the statistical distribution of unknown parameter m using SHM measured crack size a (indeed, this is the same as a_N^{meas} in Eqn. (8)). Therefore, the Bayes' theorem is extended to the continuous probability distribution with probability density function (PDF), which is more appropriate for the purpose of the present paper. Let $f_X(m)$ be a PDF of Paris model parameter $X = m$. The measured crack size $Y = a$ is also random due to the noise, whose PDF is denoted by $f_Y(a)$. Then, the joint PDF of X and Y can be written in terms of f_X and f_Y, as

$$f_{XY}(m,a) = f_X(m \mid Y = a) f_Y(a) = f_Y(a \mid X = m) f_X(m) \quad (10)$$

When X and Y are independent, the joint PDF can be written as $f_{XY}(m,a) = f_X(m) \cdot f_Y(a)$ and Bayesian inference cannot be used to improve $f_X(m)$. Using the above identity, the original Bayes' theorem can be extended to the PDF form as [21,22]

$$f_X(m \mid Y = a) = \frac{f_Y(a \mid X = m) f_X(m)}{f_Y(a)} \quad (11)$$

Since the integral of $f_X(m \mid Y = a)$ should be one, the denominator in Eqn. (11) can be considered as a normalizing constant. In Eqn. (11), $f_X(m \mid Y = a)$ is the posterior PDF of Paris model parameter given measured crack size $Y = a$, and $f_Y(a \mid X = m)$ is the likelihood function or the PDF value of obtaining the measured crack size a for a given parameter value of $X = m$.

When the analytical expressions of the likelihood function, $f_Y(a \mid X = m)$, and the prior PDF, $f_X(m)$, are available, the posterior PDF in Eqn. (11) can be obtained through simple calculation. The likelihood function is designed to integrate the information obtained from SHM measurement to the knowledge about the distribution of m. Instead of assuming an analytical form of the likelihood function, uncertainty in measured crack sizes is propagated and estimated using the Monte Carlo simulation (MCS). Although this process is computationally expensive, it will provide accurate information for the posterior distribution. The derivation of the likelihood function can be found in Coppe et al. [4].

When multiple, independent measurements are available, Bayesian inference can be applied either iteratively or all at once. When K number of measured data are available; i.e., $\mathbf{a} = \{a_1, a_2, \ldots, a_K\}$, the Bayes' theorem in Eqn. (11) can be modified to

$$f_X(m \mid Y = \mathbf{a}) = \frac{1}{V} \prod_{i=1}^{K} \left[f_Y(a_i \mid X = m) \right] f_X(m) \quad (12)$$

where V is a normalizing constant to satisfy the PDF property. In the above expression, it is possible that the likelihood functions of individual measurements are multiplied together to build the total likelihood function, which is then multiplied by the prior PDF followed by normalization to yield the posterior PDF. On the other hand, the one-by-one update formula for Bayes' theorem can be written in the recursive form as

$$f_X^{(i)}(m \mid Y = a_i) = \frac{1}{V_i} f_Y(a_i \mid X = m) f_X^{(i-1)}(m) \quad (13)$$

where V_i is a normalizing constant at i-th update and $f_X^{(i-1)}(m)$ is the PDF of m, updated using up to $(i-1)$th measurements. In the above update formula, $f_X^{(0)}(m)$ is the initial prior PDF, and the posterior PDF becomes a prior PDF for the next update.

An important advantage of Bayes' theorem over other parameter identification methods, such as the least square method and maximum likelihood estimate, is its capability to estimate the uncertainty structure of the identified parameters. These uncertainty structures depend on that of the prior distribution and likelihood function. Accordingly, the accuracy of posterior distribution is directly related to that of likelihood and prior distribution. Thus, the uncertainty in posterior distribution must be interpreted in that context.

Although the least square process uses all data that are measured at every cycle, the Bayesian inference uses data only every 100 cycles. This is partly because the Bayesian inference process is computationally expensive and it works well when the crack growth is large.

Conservative prediction of remaining useful life

Once the distribution of m is identified at a given cycle N, it can be used to calculate the distribution of RUL N_f using Eqn. (4). The distribution of RUL represents the possibility of remaining cycles before the crack size becomes the critical one. In this paper, the critical crack size is defined as when the half crack size become a_c = 24 mm. This is different from the conventional definition of critical crack size, which depends on the fracture toughness. Rather, it is a threshold of crack size that an airline company may want to repair. Therefore, the threshold should be less than the critical crack size.

Since the updated distribution of m does not follow any analytical distribution, MCS is used to estimate the distribution of RUL. In addition, the measured crack size is also randomly distributed according to Eqn. (8). From the RUL distribution, the 5^{th} percentile is used as a conservative estimate of RUL. Therefore, the estimated RUL will be less than the true RUL with a 95% confidence.

An important advantage of using synthetic data is that it allows predicting the statistical characteristic of predicted results. In this paper, random noises are added to the true crack sizes. Due to this randomness, different RUL distributions are expected if another set of synthetic data are used. The same will happen during actual experiments. Therefore, in order to cover actual experimental variability, the process of identifying damage parameters and predicting the conservative RUL is repeated 100 times with different sets of synthetic data. In the numerical examples, 68% confidence interval of 5^{th} percentiles is plotted, which corresponds to mean ± one standard deviation.

NUMERICAL EXAMPLES

In this section, three numerical examples are presented in the order of increasing difference between the true and assumed stress intensity factor model. For all examples, an aluminum 7075 square plate with dimension of 0.2 m × 0.2 m and thickness of 2.48 mm is used with Young's modulus E = 71.7 GPa, Poisson's ratio $\nu = 0.33$, and Paris model parameters C_{true} = $1.5\text{x}10^{-10}$ and m_{true} = 3.8. Mode I fatigue loading is applied to the plate with the range of stress $\Delta\sigma = 78.6$ MPa at R = 0, which corresponds to the case of fuselage pressurization loading. The relatively large initial crack size a_0 = 10 mm is chosen because many SHM sensors cannot detect small cracks. In addition, there is no significant crack growth when the size is small. This size of crack is still too small to threaten the safety of aircraft.

'True' crack growth data were calculated using XFEM simulations, which were performed on a structured mesh of square linear quadrilateral elements with characteristic length of 1 mm. Each cycle of fatigue crack growth was modeled until the half crack size reaches a threshold size of 24.0mm; i.e., the crack will be repaired beyond this size. Synthetic measurement data are then generated by adding a deterministic bias and random noise to the true crack size according to Eqn. (8). The crack size at each iteration was then used to identify the equivalent Paris model exponent through the use of the least-square-filtered Bayesian (LSFB) method with the simplified stress intensity formula in Eqn. (2). Lastly, the RUL is estimated as the number of remaining cycles that the current crack size reaches the threshold one.

Center crack in a finite plate

The first example considered is that of a center crack in a finite plate as shown in Figure 3A. Only the right half of the plate was modeled with XFEM through the use of symmetry. The corresponding curve of the correction factor Y for the center crack in a finite plate is given in Figure 3B. In this example, it is clear that the effect of the correction factor is less than 5%. Therefore, it is expected that the identification of damage growth parameter will be close to the true one.

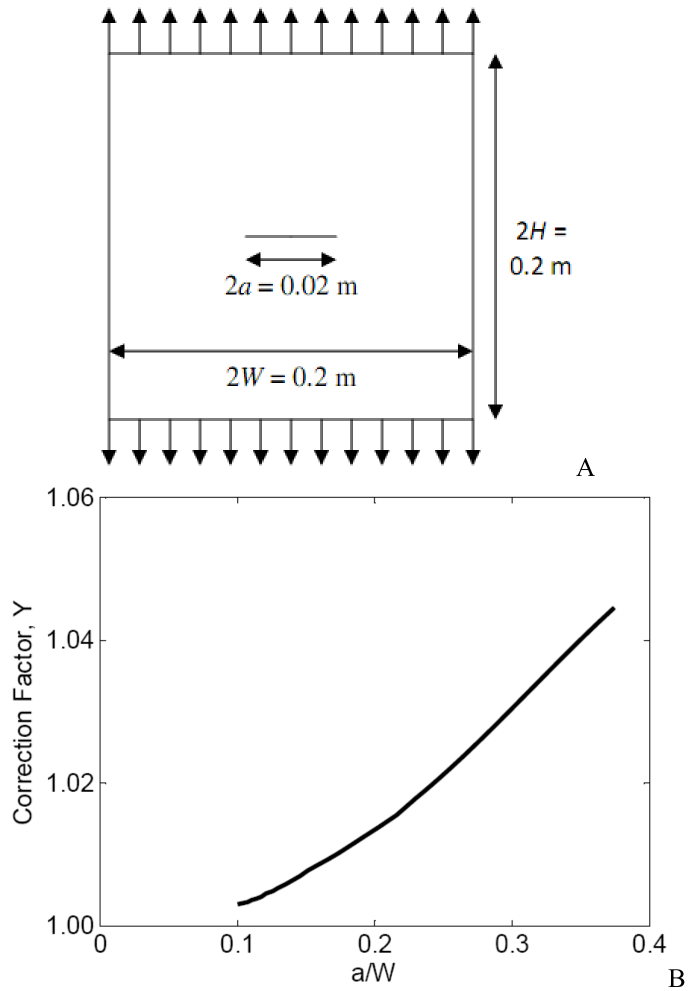
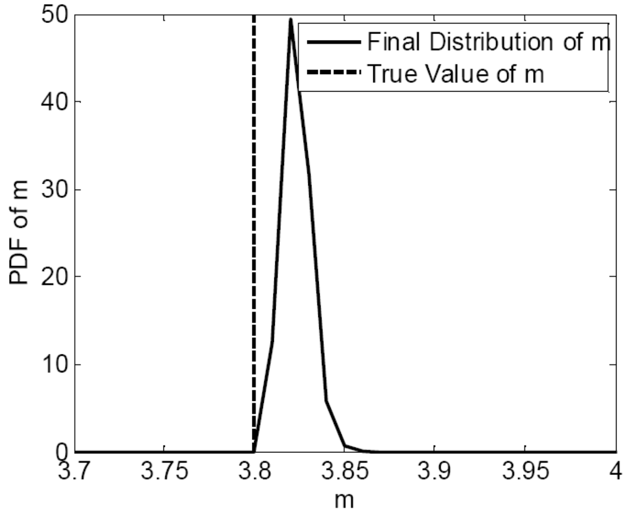


Figure 3. THE CENTER CRACK IN A FINITE PLATE MOMDEL AND THE CORRECTION FACTOR. (A) INITIAL CRACK GEOMETRY AND LOADING CONDITION, (B) CORRECTION FACTOR AS A FUNCTION OF CRACK SIZE.

The crack growth went up to 2,100 cycles, and the crack size at 2,200 cycles becomes larger than the threshold size. Figure 4A shows the updated distribution of m using LSFB at 2,100 cycles. The initial distribution was assumed to be uniformly distributed; i.e., m-U[3.3;4.3]. The standard deviation at the final update turns out to be about 0.01, which is significantly reduced from the initial value of 0.29. For comparison, the true value, m_{true}, is also shown as a vertical dashed line. Although the true crack grows according to the range of stress intensity factor in Eqn. (5), the LSFB assumes that it is given in Eqn. (2). The maximum likelihood value, 3.82, is slightly overestimates the true one in order to compensate for the error in ΔK. This is expected because the correction factor is slightly larger than one, which makes the crack to grow faster than it would if it was in an infinite plate. Figure 4B shows the distribution of crack size obtained using the identified parameter *m*. The bounds of the final distribution of *m* (e.g. 3.80 and 3.86 in Figure 4A) are used as input into Eqn. (3) to calculate crack lengths to the cycle of the last LSFB analysis. These two *a*-*N* curves are the upper and lower bounds of the grey region in Figure 4B which corresponds to the possible crack growth curves corresponding to the final distribution of *m*. For comparison, the true crack sizes calculated from XFEM is plotted with a solid curve. It is noted that the true crack sizes (black line) fall within the bounds of the LSFB identification (gray region).



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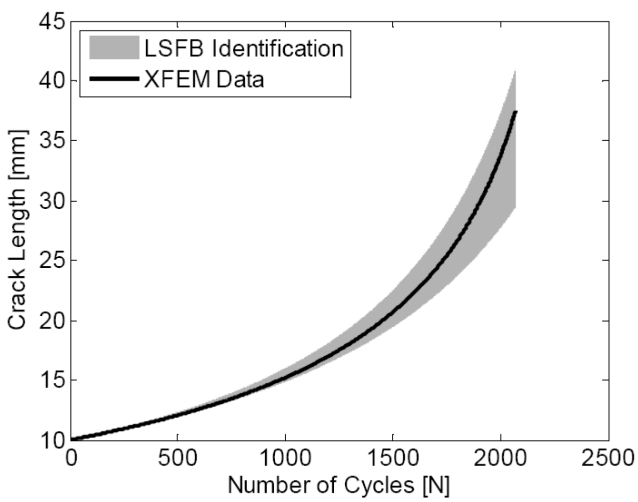

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Figure 4. COMPARISON OF XFEM CRACK GROWTH DATA WITH CRACK GROWTH PREDICTED FROM LSFB ANALYSIS FOR CENTER CRACK IN A FINITE PLATE. (A) UPDATED PDF OF PARAMETER M AT CYCLE 2,100, (B) DISTRIBUTION OF IDENTIFIED CRACK SIZE (68% CONFIDENCE INTERVAL).

Figure 5(a) shows the 5th-percentile conservative estimates of RUL. The black solid line represents the true RUL—it starts with 1,700 because the crack will grow to the threshold size after 1,700 cycles from the first detection. A single life is consumed at every cycle. The true RUL is calculated using the range of stress intensity factor in Eqn. (5). In the same plot, the dashed line is the estimated RUL when the correction factor is assumed to be $Y = 1$; i.e., the range of stress intensity factor is calculated using Eqn. (2). Due to the slight underestimate of correction factor, the dashed RUL is also slightly higher than the true one, which is an unconservative estimate. However, both RULs eventually meet toward the end of life. Both lines use the information of true parameter m_{true}. The gray area in Figure 5(a) represents the 68% confidence interval (mean ± standard deviation) of the estimated RUL using LSFB method. It can be observed that the estimate of RUL converges to the true RUL from the conservative side. It is note that even if Eqn. (2) is used, the error in the stress intensity factor is compensated by identifying equivalent parameter m that is slightly larger than the true one.

Figure 5(b) shows the error between the maximum likelihood estimation of the estimated RUL distribution, $N_f^{\max}$, and the true RUL, N_f^{true} defined as

$$\text{error} = N_f^{\max} - N_f^{\text{true}} \tag{14}$$

The positive error means an unconservative estimate. It can be observed that the estimate is very conservative at the beginning (because the distribution of m is initially wide), and it then becomes unconservative but with a smaller amplitude. Therefore, the maximum likelihood is consistently unconservative, but the 5th-percentile is always conservative. It is also noted that the maximum error in the unconservative side is less than 100 cycles, which is the inspection interval.

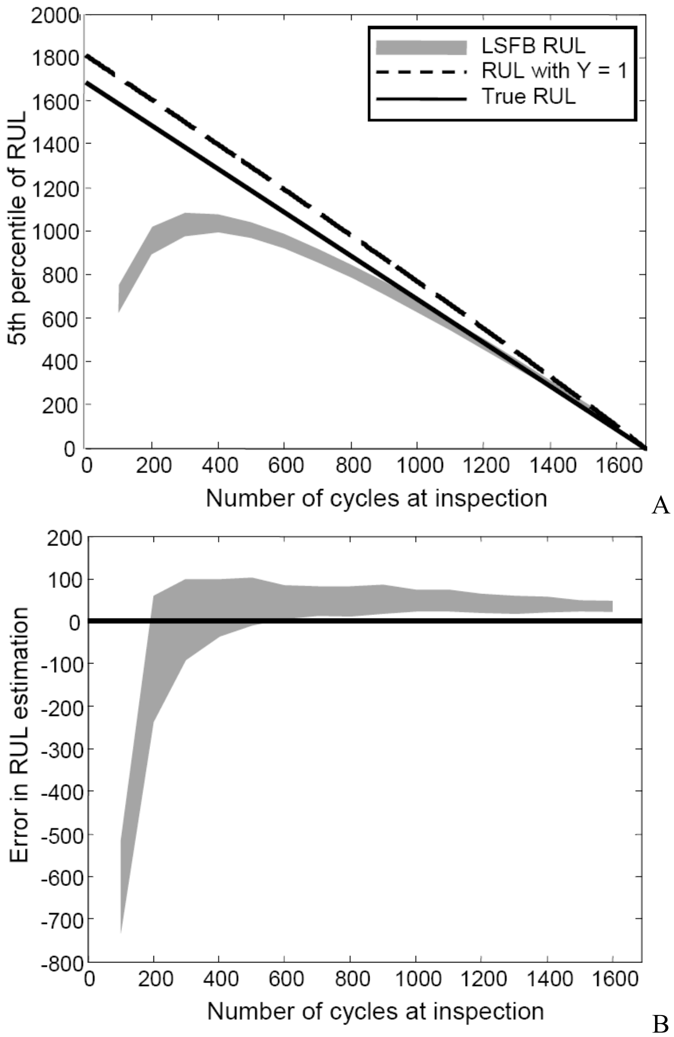

Figure 5. ESTIMATED RUL AND ERROR FOR A CENTER CRACK IN A FINITE PLATE. (A) 68% CONFIDENCE INTERVAL OF 5TH-PERCENTILE CONSERVATIVE RUL ESTIMATES, (B) 68% CONFIDENCE INTERVAL OF ERROR BETWEEN THE TRUE RUL AND THE MAXIMUM LIKELIHOOD OF THE ESTIMATED RUL DISTRIBUTION.

Edge crack in a finite plate

Next, an edge crack in a finite plate is considered as shown in Figure 6(a). For this case the boundary conditions were fixing the lower right hand corner and allowing the top right corner to only move in the vertical direction. It was found that the threshold crack size was reached at 1,018 cycles. Therefore, the LSFB was applied only 10 times (one at every 100 cycles). The correction factor corresponding to the finite effect which this edge crack represented is given in Figure 6(b). In this case, the correction factor can contribute up to 35% to the stress intensity factor. Therefore, it is expected that the equivalent damage growth parameter m will overestimate the true one in proportion.

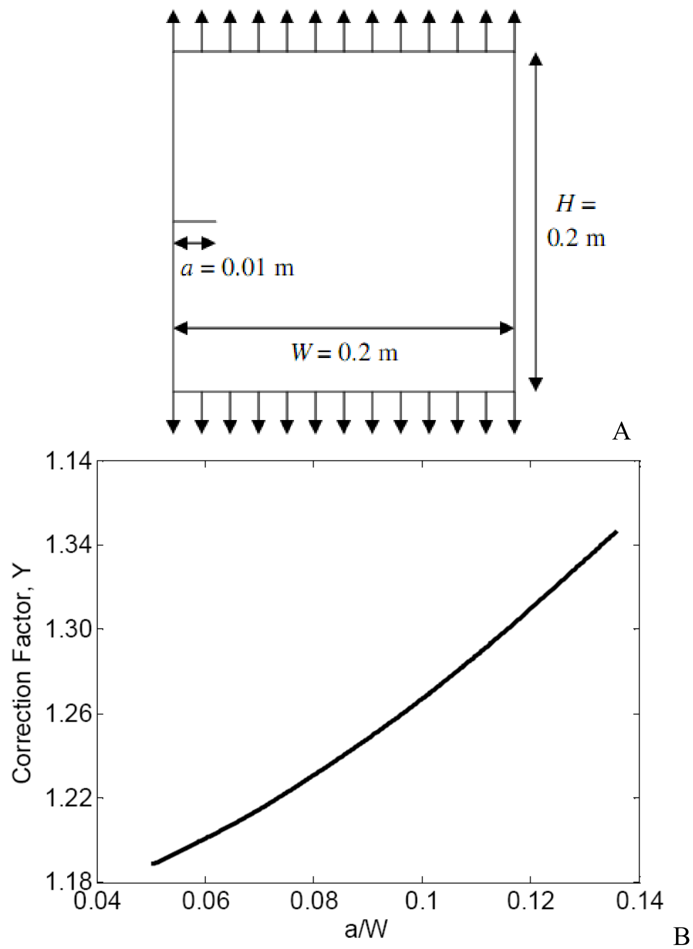

Figure 6. THE EDGE CRACK IN A FINITE PLATE MODEL AND THE CORRECTION FACTOR. (A) INITIAL CRACK GEOMETRY, (B) CORRECTION FACTOR AS A FUNCTION OF CRACK SIZE.

Figure 7(a) shows the updated distribution of m using LSFB at 1,000 cycles after the first detection. As expected, it compensates for the error in ΔK by overestimating m by 5.5%. It can be observed that the larger the error in ΔK compared to the center crack case increases the overestimation of m to compensate for it. The effect of correction factor is moved to the damage growth parameter. The standard deviation of the final distribution of m is about 0.03, which is wider than the case of a center crack in an infinite plate. This is probably caused by the increased difference between the actual and assumed models for the stress intensity factor.

As the LSFB analysis results in a final distribution of m, the predicted crack sizes with this distribution are plotted and compared directly to the XFEM data in Figure 7(b). The XFEM data fall within the bounds of the LSFB identification.

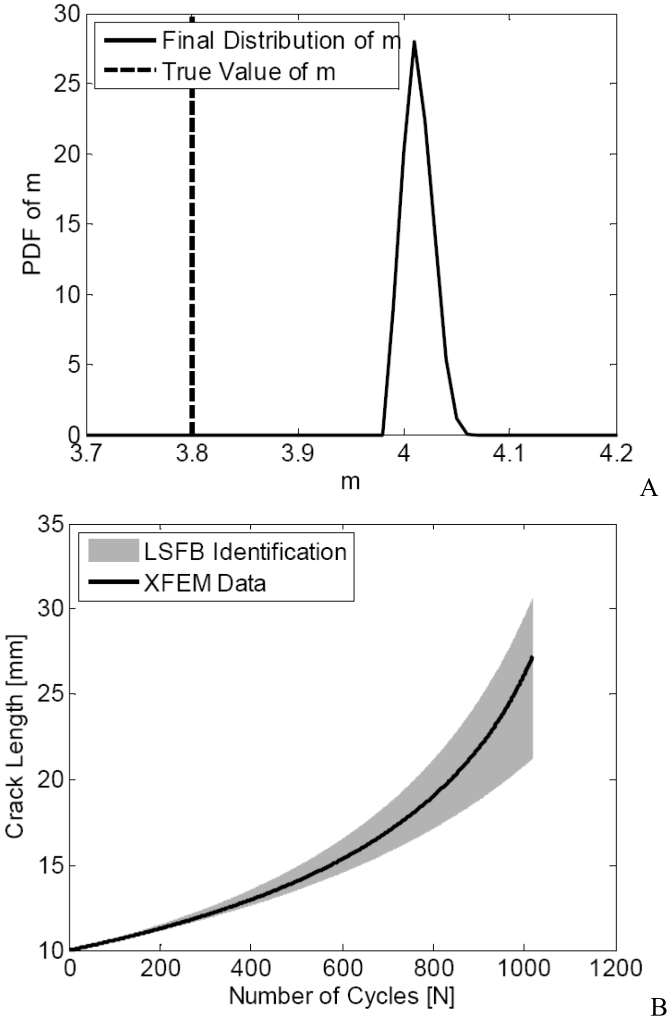

Figure 7. COMPARISON OF XFEM CRACK GROWTH DATA WITH CRACK GROWTH PREDICTED FROM LSFB ANALYSIS FOR EDGE CRACK. (A) UPDATED PDF OF PARAMETER M AT CYCLE 1,000, (B) DISTRIBUTION OF IDENTIFIED CRACK SIZE (68% CONFIDENCE INTERVAL).

Figure 8(a) shows the 5th-percentile conservative estimates of RUL, similar to Figure 5(a). In this case, there was a large difference between the true RUL and the RUL with $Y = 1$ assumption. The error of up to 35% in the correction factor leads to an overestimation in the RUL of almost 100%. However, both RULs eventually managed to meet toward the end of life. Even if the RUL with $Y = 1$ leads to a large overestimation, the 68% confidence interval of the conservative estimated RUL using LSFB method (gray area) stays in the conservative side and converges to the true RUL. Again, the large error in the correction factor has successfully been compensated by identifying equivalent parameter m that is about 5.5% larger than the true one.

Figure 8(b) shows the error between the maximum likelihood of the estimated distribution of the RUL and the true RUL. As observed previously, LSFB leads to a somewhat unconservative estimate of the RUL if the maximum likelihood data is used, but it converges to the true value fairly accurately.

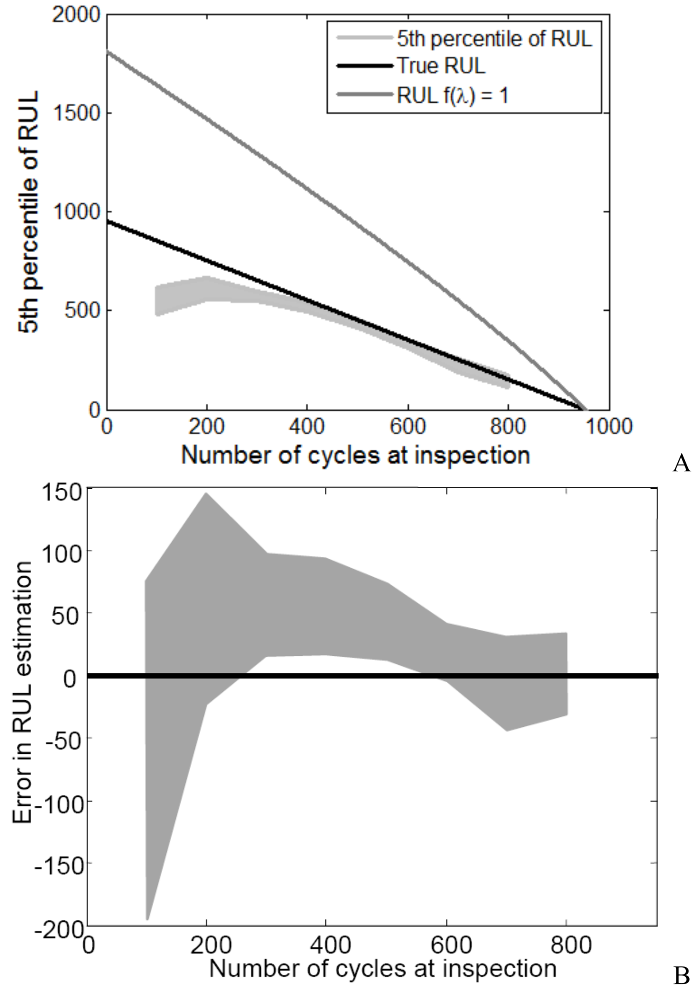

Figure 8. ESTIMATED RUL AND ERROR FOR AN EDGE CRACK IN A FINITE PLATE. (A) 68% CONFIDENCE INTERVAL OF 5TH-PERCENTILE CONSERVATIVE RUL ESTIMATES, (B) 68% CONFIDENCE INTERVAL OF ERROR BETWEEN THE TRUE RUL AND THE MAXIMUM LIKELIHOOD OF THE ESTIMATED RUL DISTRIBUTION.

Center crack in a plate with holes

The final example considers differences between the actual and predicted model that may be caused by localized stress concentrations in the plate. Four holes are inserted into the plate as shown in Figure 9(a). Only the right half of the plate was modeled with XFEM through the use of symmetry. Different from the two previous examples, there is no analytical expression of the correction factor; therefore it is obtained from XFEM is shown in Figure 9(b). The effect of holes is converted into 30 – 39% error in the stress intensity factor. Due to such a large stress intensity factor, the crack grew fast and reached the threshold size at 625 cycles. Therefore, only six updates were available for the LSFB method.

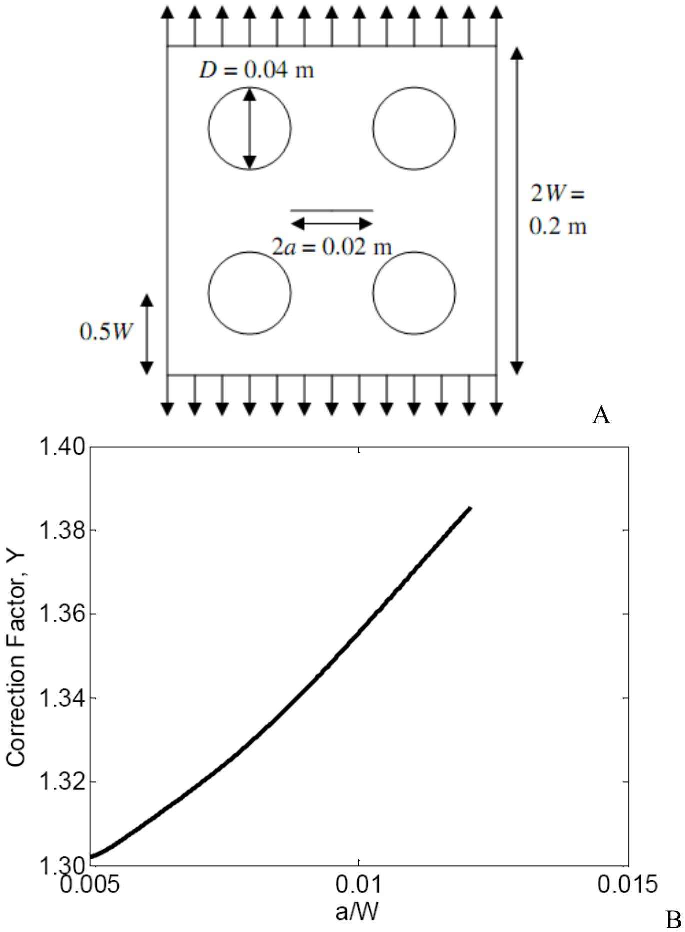

Figure 9. THE CENTER CRACK IN A FINITE PLATE WITH HOLES AND THE CORRECTION FACTOR. (A) INITIAL CRACK GEOMETRY, (B) CORRECTION FACTOR AS A FUNCTION OF CRACK SIZE.

Figure 10(a) shows the updated distribution of m using LSFB at cycle 600. As expected it compensates for the error in ΔK by over estimating m by 8.7%. The same conclusion can be drawn as previously, the larger the error in ΔK the more m is overestimated to compensate for it. Due to this error, the standard deviation of m becomes about 0.03, which is still a significant reduction from the initial standard deviation of 0.29.

As the LSFB analysis results in a final distribution of m, the predicted crack lengths for this distribution are plotted and compared directly to the XFEM data in Figure 10(b). The XFEM data fall within the bounds of the LSFB identification. The identified crack size distribution is wider than the previous two examples, which is because the model is increasingly far away from the center crack in an infinite plate mode. In addition, the fact that only six inspections have been performed before reaching the threshold size may also contribute to the relatively wide distribution.

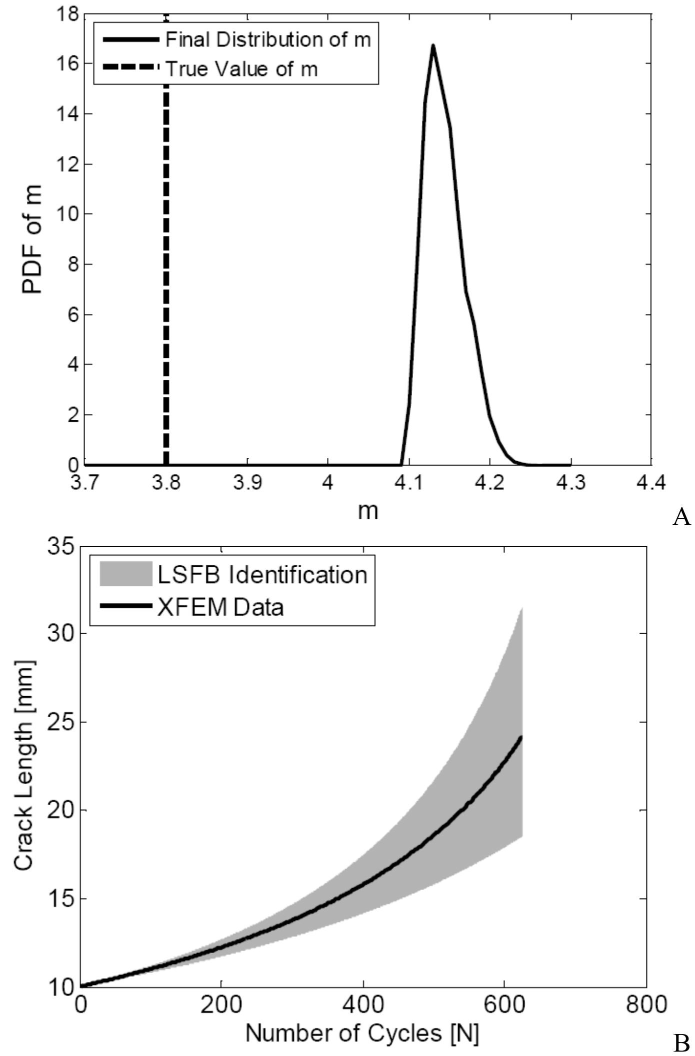

Figure 10. COMPARISON OF XFEM CRACK GROWTH DATA WITH CRACK GROWTH PREDICTED FROM LSFB ANALYSIS FOR A PLATE WITH HOLES. (A) UPDATED PDF OF PARAMETER M AT CYCLE 600, (B) DISTRIBUTION OF IDENTIFIED CRACK SIZE (68% CONFIDENCE INTERVAL).

Figure 11(a) shows the 5th-percentile conservative estimates of RUL, similar to Figure 5(a). Since there is no analytical approximation of correction factor is available, only the true RUL and the 68% confidence interval of the conservative estimated RUL using LSFB method (gray area) are plotted. Note that the predicted RUL stays close to the true one from the conservative side. Again, the large error in the correction factor has successfully been compensated by identifying equivalent parameter m that is about 8.7% larger than the true one.

Figure 11(b) shows the error between the maximum likelihood of the estimated distribution of the RUL and the true RUL. As observed previously, despite the fact that a simplistic model is used in which the range of stress intensity factor does not account for the complexity of the geometry, the LSFB method was able to estimate the RUL not only with accuracy but also fairly conservatively.

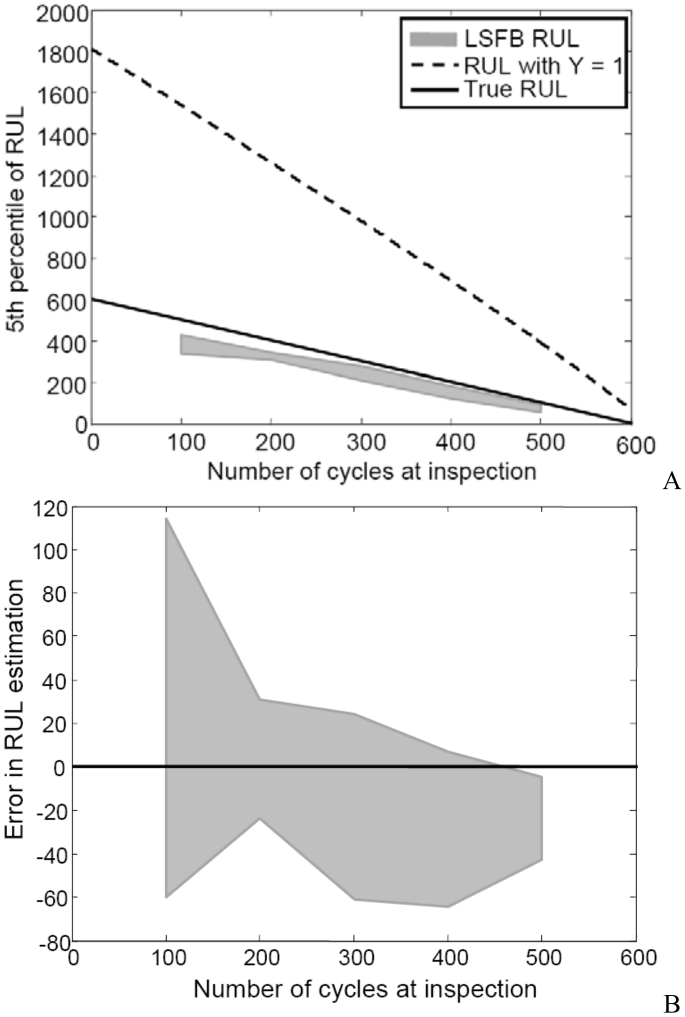

Figure 11. ESTIMATED RUL AND ERROR FOR A PLATE WITH HOLES. (A) 68% CONFIDENCE INTERVAL OF 5TH-PERCENTILE CONSERVATIVE RUL ESTIMATES, (B) 68% CONFIDENCE INTERVAL OF ERROR BETWEEN THE TRUE RUL AND THE MAXIMUM LIKELIHOOD OF THE ESTIMATED RUL DISTRIBUTION.

CONCLUDING REMARKS

In this paper, equivalent damage growth parameters were identified, which can compensate for complex geometric effects for SHM prognosis. The error in stress intensity factor was moved to the equivalent damage growth parameter, such that the prediction of remaining useful life is accurate. Three numerical examples showed that the deviation of damage growth parameter is proportional to the error in stress intensity factor. All three examples, however, showed that the estimated conservative remaining useful life converges to the true one from the safe side. Therefore, it is concluded that a simple model can be used to predict the behavior of complex problems by calculating equivalent parameters.

The least-square-filtered Baysian (LSFB) method was proposed in identifying unknown model parameters. This method took advantages from both least-square and Bayesian methods. The whole idea was to use the least-square method to reduce the level of noise and to identify deterministic parameters, and then Bayesian method is used to identify statistical parameters. In this way, it was possible to reduce the number of variables that need to be updated in the Bayesian method.

The method is demonstrated here updating only one parameter m of Paris model the same idea can be applied to the parameters m and C together. This should allow for even more accurate results because it would allow for more flexibility in fitting the equivalent model. The feasibility of using XFEM in the calculation of the likelihood function will also be explored which may identify the true m and C.

ACKNOWLEDGEMENT

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