



A Bayesian approach for a damage growth model using sporadically measured and heterogeneous on-site data from a steam turbine

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ABSTRACT

Accurate prediction of the remaining useful life (RUL) of plant turbines is a major scientific challenge for effective operation and maintenance in the power plant industry. This paper proposes an RUL prediction methodology that incorporates a damage index into the damage growth model. A Bayesian inference technique is used to consider uncertainties while estimating the probability distribution of a damage index from on-site hardness measurements. A Bayesian approach is proposed for the damage growth model for use with aged steam turbines. The predictive distribution of the damage index is estimated using its mean and standard deviation. As a case study, real steam turbines from power plants are examined to demonstrate the effectiveness of the proposed Bayesian approach. The results from the proposed damage growth model can be used to predict the RULs of the steam turbines of power plants regardless of load types (peak-load or base-load) of the power plant.

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1. Introduction

The design life of steam turbines is typically 25 years or 200,000–250,000 h [1–3]. The power plant industry focuses significant effort on reducing operation costs and extending the service life of critical machines (e.g., steam turbines) to avoid premature failure. Condition-based maintenance (CBM) has drawn great attention as a strategy for cost-effective operation and maintenance (O&M) decisions. A CBM program consists of four main steps: data acquisition, data processing, health prognostics, and maintenance decision-making [4,5]. The health prognostics step includes not only diagnostics for fault detection, isolation, and identification; it also includes prognostics for predicting the remaining useful life (RUL) before failure [6–8]. There are increasing demands for engineering aftermarket services to manage steam turbines in a timely and proper manner [9]. RUL prediction for complicated and large-scale systems is a major scientific challenge and a significant issue for effective O&M. With respect to turbines that are already in service, an effective method is required to accurately predict RUL through the limited available resources [10].

Numerous elements of steam turbines in power plants are exposed to harsh thermal loading conditions. Theoretically, the RUL of key elements could be predicted by metallurgical or theoretical analysis of as-received and degraded elements [1]. Creep and fatigue life have been

shown to be associated with the material damage rate. Damage rate measurement methods, such as microscopic observation and indirect measurement of hardness, have been proposed [11]. However, in real-world applications, it is not easy to implement damage rate measurement methods since the specimens available from actual steam turbines are limited.

Non-destructive techniques, such as replication analysis and hardness tests, can also be used to evaluate the damage rate. Replication analysis has been widely adopted to evaluate the damage rate of in-service steam turbines. It can be used to classify the level of material degradation in accordance with guidelines such as Neubauer or Vereinigung der Großkesselbesitzer e.V (VGB) [9,10,12–16]. Damage rates for steam turbines with ferritic steel have been determined by investigating the degree of micro-structural phase evolution, micro-void formation of grain boundaries, and evolution of carbides from visual inspection via scanning electron microscope (SEM) images [12,17,18]. Several elements (e.g., tubes, turbines, and pipes) have been studied to quantify damage via replication analysis. However, the replication method is based on five or six states; thus, results of its RUL prediction are classified as five or six states. Quantitative and accurate RUL prediction is relatively difficult [19]. For example, there was little difference in microstructures between low-stress and high-stress locations (See Fig. 1). Likewise, the hardness at the locations shows relatively little difference

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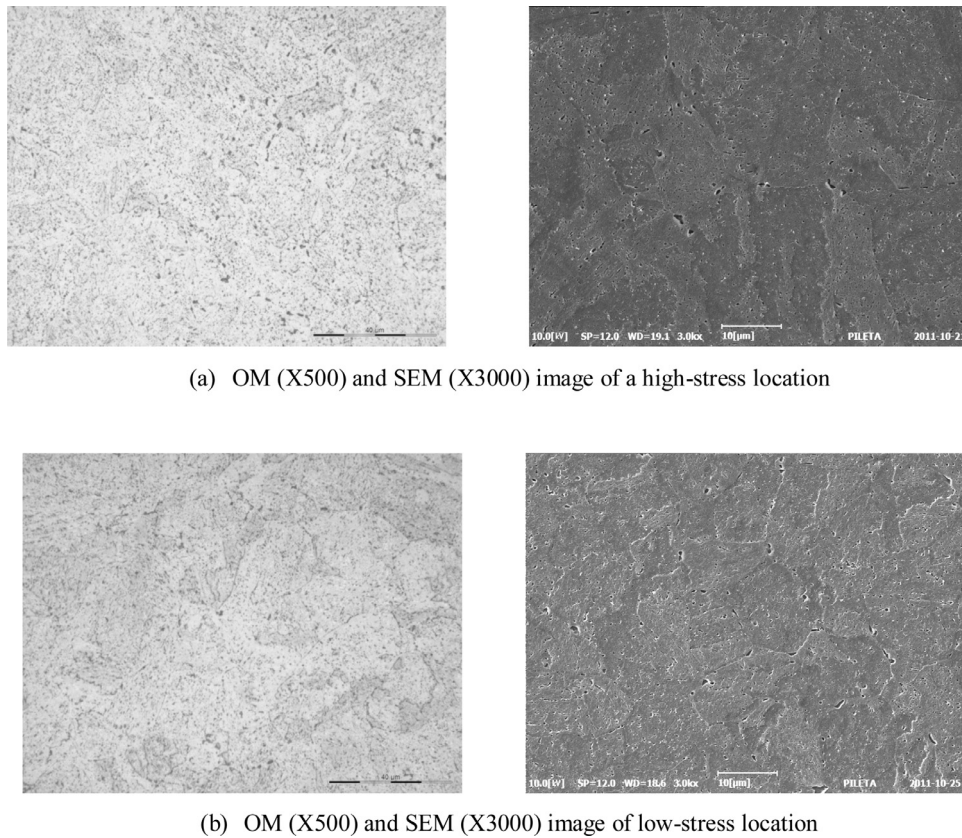


Fig. 1. Microstructures of 1Cr1Mo1/4V rotor steel after 146,708 h operation.

Table 1
Metallurgical analysis results.

Location		Averaged hardness	Degradation grade	Creep grade	Micro crack	Abnormal microstructure
High-stress	150°	251.5	Level 4	A	No	No
	330°	251.7	Level 5	A	No	No
Low-stress	150°	260.0	Level 3	A	No	No
	330°	257.6	Level 3	A	No	No

(see Table 1). Therefore, it is difficult to use these results to quantify the health conditions from visual inspection of optical microscope (OM) or SEM images. Thus, rather than qualitative measures, a quantitative measure is appropriate to predict RUL related to creep or fatigue damage.

Rebound hardness test methods provide quantitative measures to evaluate the relative damage rate. These measures can be easily implemented, and measured hardness values can be calibrated with results from the conventional Vickers hardness test, which is only available in a laboratory setting [20]. Fujiyama et al. [21,22] used hardness values as a correction factor to supplement empirical formulas (e.g., the Larson-Miller Parameter (LMP)) for RUL prediction that considers creep damage. Recently, Mukhopadhyay et al. [23] proposed a hardness-ratio-based creep life model that considers dislocation and precipitate phenomena. However, in this approach, the predicted creep life can significantly deviate due to variations in temperature, as hardness values are combined with the LMP relation. Recently, instrumented indentation test methods were developed to measure strength in-situ. The indentation test is a non-destructive technique that determines material properties – including elastic modulus, tensile strength, and residual stress – by analyzing the indentation load-depth curve [24]. Despite its potential advantages, to date, a very limited amount of scientific work has

been conducted in the research area of damage rate evaluation or RUL prediction of in-service components [25–27]. Also, there is almost no actual measurement data available from indentation testers that includes operating time.

In this study, the hardness measurement method that is most commonly and easily used in actual field settings is used for RUL prediction. Nonetheless, RUL predictions based on the rebound hardness test method are subject to uncertainties. Those uncertainties are due to aleatory and epistemic uncertainties in irregular and discontinuous measurement and non-homogeneous samples. In particular, discrepancy reduction and a damage growth model that considers uncertainties should be developed to accurately predict the RULs of aged components in power plants [28,29].

Bayesian approaches have been used to address uncertainty for model-based prognostics. For example, Guan et al. [30] proposed a general framework for probabilistic prognosis using maximum entropy approach with the classical Bayesian method for fatigue damage assessment. Dawn et al. [31] used Bayesian inferences with the MCMC algorithm to estimate fatigue and wear damage. More recently, Chiacchio et al. [32] presented a Bayesian approach to update model parameters of existing fatigue models for composites. Compare et al. [33] proposed a semi-Markov degradation model based on expert knowledge and few

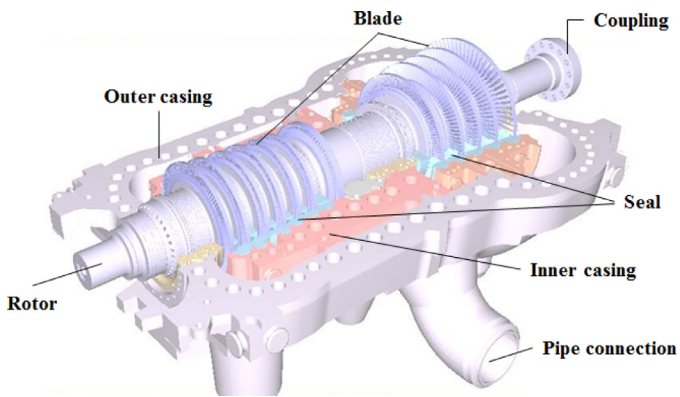


Fig. 2. Schematic of a steam turbine (high and intermediate pressure parts).

field data within the Bayesian statistical framework. In the previous Bayesian approaches for model-based prognostics, the correlation between hyper-parameters and uncertainties of damage model's parameters was not accounted for. To the best of our knowledge, this study is the first attempt to define a damage threshold for steam turbines to execute an RUL prediction.

To this end, this paper presents a Bayesian approach to a new damage growth model that can utilize sporadically measured and heterogeneous on-site data from steam turbines. A hardness-based damage index is selected as a damage indicator to evaluate the damage rate. Using this, a new damage growth model is proposed as a function of operating time. Sporadically measured hardness is random due to the uncertainty that arises from the heterogeneity of turbines in terms of manufacturers, sizes, operating conditions, sites, etc. Therefore, the mean and standard deviation of the damage index are predicted considering the parameters' correlation and the distribution can be identified simultaneously by using Bayesian inference [28] and MCMC simulation. The predicted damage growth results from the Bayesian and nonlinear regression method are compared and validated using actual field data from ten turbine units.

The remainder of the paper is organized as follows. Section 2 presents an overview of steam turbines and sporadically measured and heterogeneous on-site data with uncertainties. Section 3 focuses on the damage growth model using the Bayesian updating method and MCMC simulation. Section 4 presents the RUL prediction results of Bayesian and the nonlinear least square (Nlsq) method. A damage threshold is proposed to determine design life, the proposed methodology is validated, and the RUL distribution for an aged steam turbine is predicted based on the proposed damage growth model. Section 5 presents the conclusions of the research.

2. Overview of steam turbines and On-site measured data

2.1. Description of steam turbines and failure mechanisms

Steam turbines are one of the most popular power generating machines used in the power industry. They are widely used because water is prevalent, boiling points are moderate, and the operating cost is reasonable. Steam turbines are machines that convert thermal energy from hot and pressurized steam to mechanical (rotational motion) work. Steam turbines are designed to improve thermodynamic efficiency by adopting multiple stages to expand steam [34]. As shown in Fig. 2, high-pressure parts of steam turbines consist of (1) a casing or shell that is usually divided at the horizontal center line and contains the stationary blade system; (2) a rotor carrying the moving buckets (blades or vanes) either on wheels or drums, with journal bearings at the ends of the rotor; (3) a set of bearings attached to the casing to support the shaft; (4) a coupling to connect with the driven machine; and (5) pipe connections to

the steam supply at the inlet and to an exhaust system at the outlet of the casing or shell.

Failure mode and effect analysis (FMEA) of steam turbines was conducted over twenty years in the electric power industry; results are shown in Table 2. FMEA qualitatively shows the occurrence, severity, and risk of key components in a steam turbine. Among the many turbine components, this study looks specifically at the HIP rotor for RUL prediction due to its high risk. Creep and low/high cycle fatigue (LCF/HCF) are known to be the dominant failure mechanisms of steam turbines [35,36]. High temperatures and centrifugal force causes creep damage in high-stress regions, such as bore and wheel hooks. Thermo-mechanical fatigue damage from the thermal cyclic load causes cracking at the wheel corner [1,22,36]. Material degradation related to damage in the turbines, such as low-cycle fatigue and creep, leads to unexpected breakdown and economic losses in the electric industry.

2.2. Characteristics of on-site measurement data

It is extremely difficult to measure material degradation directly. Destructive analysis is available only in well-controlled laboratories, while non-destructive analysis (i.e., the replication method) is limited in its ability to accurately predict the damage rate or RUL.

This study employed a rebound hardness tester (Leeb hardness tester in accordance with DIN 50,156-1 and ISO/FDIS 16,859-1) because of the need for on-site and non-destructive measurement. The load of the handheld probe of the hardness tester was 10 kgf. This study used Vickers hardness values. The hardness data are subject to uncertainty due to inconsistency in turbine targets, measurement locations, and testing operators [37]. To take into consideration the uncertainty effect, a set of measurement data were collected from ten turbine units: five base-load and five peak-load units. More than five repeated data measurements from each turbine were used in this study, as prior work showed that between 3 and 10 measured hardness data points are generally acceptable [38].

It is well known that virgin rotors and the low-temperature regions of the retired rotors have almost the same microstructure, consisting of finely dispersed carbide precipitates and densely distributed dislocations [39]. Within the turbine rotor, therefore, the hardness in a low-temperature region can be used as a reference hardness. Fig. 3 shows two different types of a steam turbine; typical base-load and peak-load steam turbines. To acquire material hardness data of both low-stress and high-stress conditions from the same turbine, as shown in Fig. 3, the wheel corner of the 1st stage of the turbine rotor was selected as a high-stress location. The groove of the exhaust section was chosen as the low-stress location. Steam turbines have different overhaul periods and schedules. Ten sets of the hardness data set, which were sporadically measured at overhauls over 10 years, are shown in Table 3. Thus, hardness data sets from both low and high-stress locations were arranged according to the equivalent operating hours (EOH). For both base-load or peak-load turbines, EOH can be calculated by using actual operating hours, the number of starts, and life factor as [40]

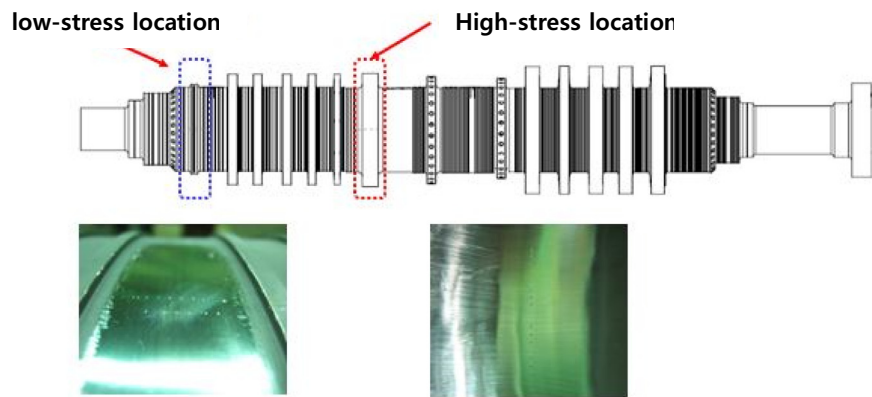
$$EOH = t_{op} + (LF \times N_{op}) \quad (1)$$

where t_{op} is the actual hours of operation, N_{op} is the number of starts, and LF is the life factor.

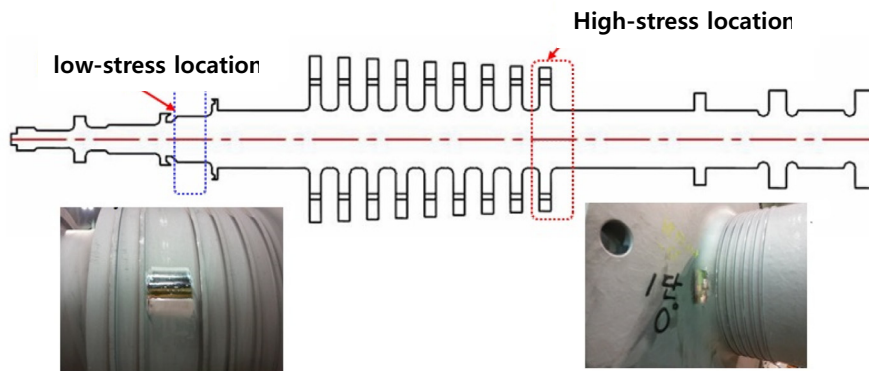
Development of a damage growth model that utilizes on-site hardness data encounters two major hurdles: (a) heterogeneity and (b) uncertainty in data. First, data can be collected during scheduled major overhauls in accordance with the maintenance strategy of the particular power generation company. Major overhauls are typically executed every four years and involve the complete disassembly, inspection, and reassembly of the steam turbine. In practice, sporadically measured data are also acquired from turbine units. Turbine units in coal power plants run at the base load continuously throughout a year, while peak-load turbines in a combined cycle power plant generally run only during periods of peak demand for electricity [41]. Based on these factors, turbine

Table 2
FMEA results for a steam turbine.

Comp.	Failure cause	Failure mechanism	Failure mode	Occurrence	Severity	Risk
<i>HIP(High-Intermediate Pressure) steam turbine</i>						
Rotor	Temp. cycling	Creep, LCF	Fracture	Not often	Very high	High
HP blade	Temp. cycling	LCF, HCF	Failure	Not often	High	Moderate
HP casing	Temp. cycling	CREEP, LCF	Crack	Not often	Moderate	Moderate
IP blade	Temp. cycling	LCF, HCF	Failure	Not often	High	Moderate
IP casing	Temp. cycling	CREEP, LCF	Crack	Not often	Moderate	Moderate
<i>LP(Low Pressure) steam turbine</i>						
Rotor	Wet. Cycling	Corrosion, LCF	Fracture	Not often	High	High
Blade	Wet. Cycling	LCF,HCF, Corrosion	Failure	Often	Moderate	Moderate
Bearing	Wear	Wear	Vibration	often	Low	Moderate



(a) Steam turbine of base-load power plant



(b) Steam turbine of peak-load power plant

Fig. 3. Measurement locations for material properties of turbines.

units operate with different fuel sources and power outputs, as shown in Table 3. Second, measurement data are subject to various sources of uncertainty, such as variability in material properties, measurement locations, surface conditions, and testing operators [42,43]. Even if turbines are made of the same material, the strengths of different turbines are different. In addition, the operator also represents a potential source of error related to testing conditions. Slightly mismatched measurement locations and/or different handling of the instrument may occasionally lead to deviations in the results.

In Table 3, H_a signifies the hardness at high-stress locations and H_l represents hardness at low-stress locations. These values are distributed, which means that uncertainty from sporadic measurements in heterogeneous turbines exists for each data set. Since the measured hardness is indirectly related to strength, the damage rate can be quantified and damage growth can be predicted for RUL calculation. In this paper, a damage growth model that uses a hardness-based damage index is proposed in Section 3.

To develop the damage growth model in this setting requires the use of heterogeneous and sporadic measurement data.

Table 3
Hardness data for ten turbine units.

Plant Unit	A1	B6	C2	B6 (replaced)	B3	D1 (retired)	E2	F1	G4 (retired)	H5 (retired)	
Output (MW)	500	500	500	500	500	200	182	350	200	200	
Load	Base					Peak					
Fuel	Coal	Coal	Coal	Coal	Coal	Oil	NG	Oil	NG	NG	
EOH (h)	74,327	95,097	115,671	146,708	157,995	186,478	201,671	212,522	213,175	255,288	
Ha	#	5	10	5	17	9	10	20	10	9	5
	Mean	260.0	241.1	260.5	251.6	241.7	260.1	222.0	237.8	243.5	222.0
	St. dev.	3.34	4.01	4.87	3.99	5.35	4.34	4.96	3.43	5.79	6.28
Hv	#	5	10	5	17	9	10	20	10	9	5
	Mean	263.0	245.7	265.8	258.8	249.3	271.6	236.1	261.9	267.8	272.8
	St. dev.	2.49	2.41	3.18	3.18	5.70	1.78	5.31	4.72	5.70	5.10

EOH: Equivalent Operating Hours.

Table 4
Damage index by direct damage measurement.

Damage measurement	Damage index	Creep	Fatigue	Etc
Hardness	$D = 1 - \tilde{H}/H$	Normal	Good	Non-destructive
Elasticity modulus	$D = 1 - \tilde{E}/E$	Good	Good	Non-destructive
Density	$D = (1 - \tilde{\rho}^2/\rho)^{2/3}$	Bad	Bad	Destructive
Ultrasonic waves	$D = (1 - \tilde{v}^2/v)^{2/3}$	Normal	Bad	Destructive
Cyclic stress amplitude	$D = (1 - \Delta\sigma^*/\Delta\sigma)$	Bad	Normal	Destructive
Tertiary creep	$D = 1 - (\dot{\epsilon}_p^*/\dot{\epsilon}_p)^{1/N}$	Good	Bad	Destructive
Electrical resistance	$D = 1 - \tilde{V}/V$	Normal	Bad	Destructive
Micrography	$D = \partial S_p/\partial S$	Normal	Bad	Destructive

2.3. Existing damage indices

Eight damage measurement methods and damage indices are compared in Table 4 [37]. Most damage measurements are destructive; thus, they are not suitable for in-service facilities and have limitations in their ability to consider both creep and fatigue damage. Among non-destructive methods, it is relatively easy to measure material hardness from actual steam turbines. Moreover, hardness is more sensitive to damage than the replication method due to the softening effect of damage [20]. From Table 4, thus, this study makes use of hardness data to define a hardness based damage index that takes into account both creep and fatigue damage as [44]

$$D = 1 - \frac{\tilde{H}}{H} = 1 - \frac{H_a}{H_v} \tag{2}$$

where H_a and H_v are the hardness values measured at aged (or damaged) and virgin (or undamaged) material states, respectively, using the Leeb hardness test. The hardness at the aged state is measured in a high-stress region (H_a), while the one at the virgin state is measured in a low-stress region (H_v), as shown in Fig. 3. Fig. 4 illustrates the box plots of measured hardness at different operating hours. Although the spread in the levels of hardness are not the same, the difference between high-stress and low-stress hardness increased. Due to previously mentioned uncertainties, hardness data are statistically distributed so that the probability density functions (PDF) of the damage index are shown in Fig. 5 at different operating hours. Since damage indices are able to track the progress of damage with operating hours, distributed damage indices based on hardness can be used to develop a damage growth model for RUL prediction.

3. Damage growth model using sporadically measured and heterogeneous on-site data

This section proposes a new damage growth model that utilizes the damage indices from hardness data. Bayesian inference and MCMC techniques are used to update the parameters of the damage growth model

in conjunction with the stochastic nature of the damage indices. The proposed model is applied to predict the RUL of steam turbines in a case study outlined in Section 4.

3.1. Proposed damage growth model

Damage growth models based on hardness data are rarely studied, even though damage growth or degradation models are needed for predicting the RUL of steam turbines. In general, model parameters can be estimated using expert knowledge and experimental data. Fig. 5 shows the histograms of the hardness-based damage index estimated from heterogeneous turbines with different operating times. The histograms provide an important observation. The damage index monotonically increases over operating time, although there is uncertainty that arises due to sporadic measurements from heterogeneous turbines. It is confirmed from observation that the hardness-based damage index can be used to represent damage growth.

A regression curve was built to understand damage growth behaviour over operating time, as shown in Fig. 6. Ten sets of hardness data measured at different operating times were used to estimate the damage indices plotted in the figure. The regression curve demonstrates the monotonic increase of the damage index over the entire lifetime. Moreover, the variability of the damage index increases with time. It is believed that greater variability over time mainly arises from sporadic measurements and the heterogeneity of turbines in terms of manufacturers and operating conditions. This necessitates the definition of a damage growth model in a Bayesian sense. This study thus proposes a Bayesian approach to the damage growth model as a function of operating times, as shown in Eq. (3). It is assumed that the time-varying damage index follows a Gaussian distribution, of which parameters can be updated with new hardness data using Bayesian inference. This assumption may not be ideal at the beginning of operation due to its biased nature. However, the normal distribution can represent the distribution of the damage index well at later operating times. This assumption is more important than at beginning times from the viewpoint of damage prediction, because the histograms become a uni-modal and symmetric

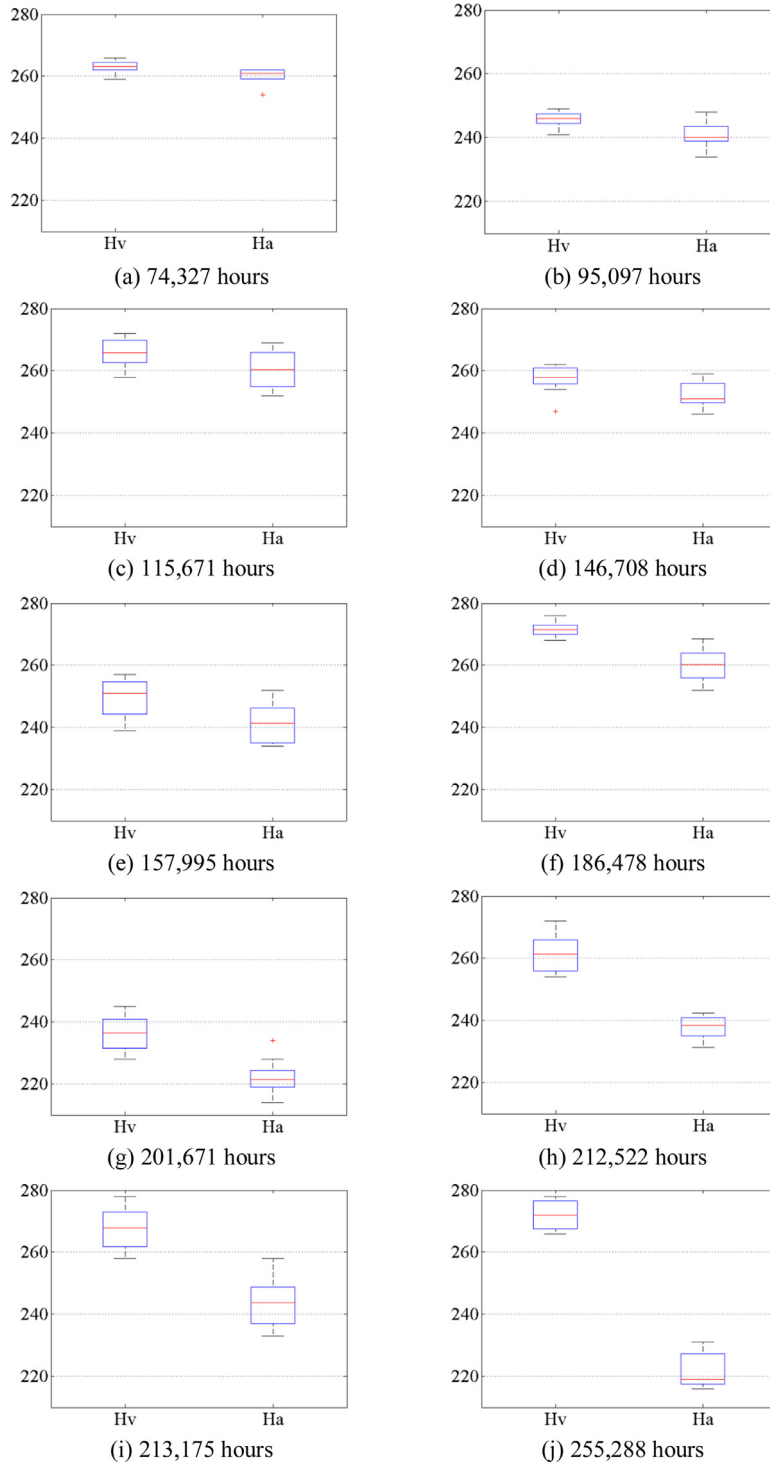


Fig. 4. Comparison of hardness; box plots by operation time.

distribution at later operating times. The damage growth model can be thus defined in the form of a distribution as

$$D(t) \sim N(\mu_D(t), \sigma_D(t)) \tag{3}$$

where the mean $\mu_D(t)$ and standard deviation $\sigma_D(t)$ of the time-varying damage exponentially increase over operating time, as shown in Fig. 7. They are thus modeled as $\mu_D(t) = \alpha_\mu \exp \beta_\mu t$ and $\sigma_D(t) = \alpha_\sigma \exp \beta_\sigma t$. The parameters of the mean and standard deviation of the damage are updated through Bayesian inference to reduce the uncertainty in remaining

useful life, which comes from the uncertainty in the hyper-parameters α, β for the mean and standard deviation of the damage indices.

The probability distributions of the damage index were independently developed using sporadic and heterogeneous experimental data measured at different operating (or service) times. However, it is still questionable whether hardness datasets measured from different sites at various operating times can be integrated to a single a damage growth model as a homogeneous dataset. To address the challenge, the U-pooling test is used to validate the adequacy of a distribution model of damage indices obtained under homogeneous conditions. This is

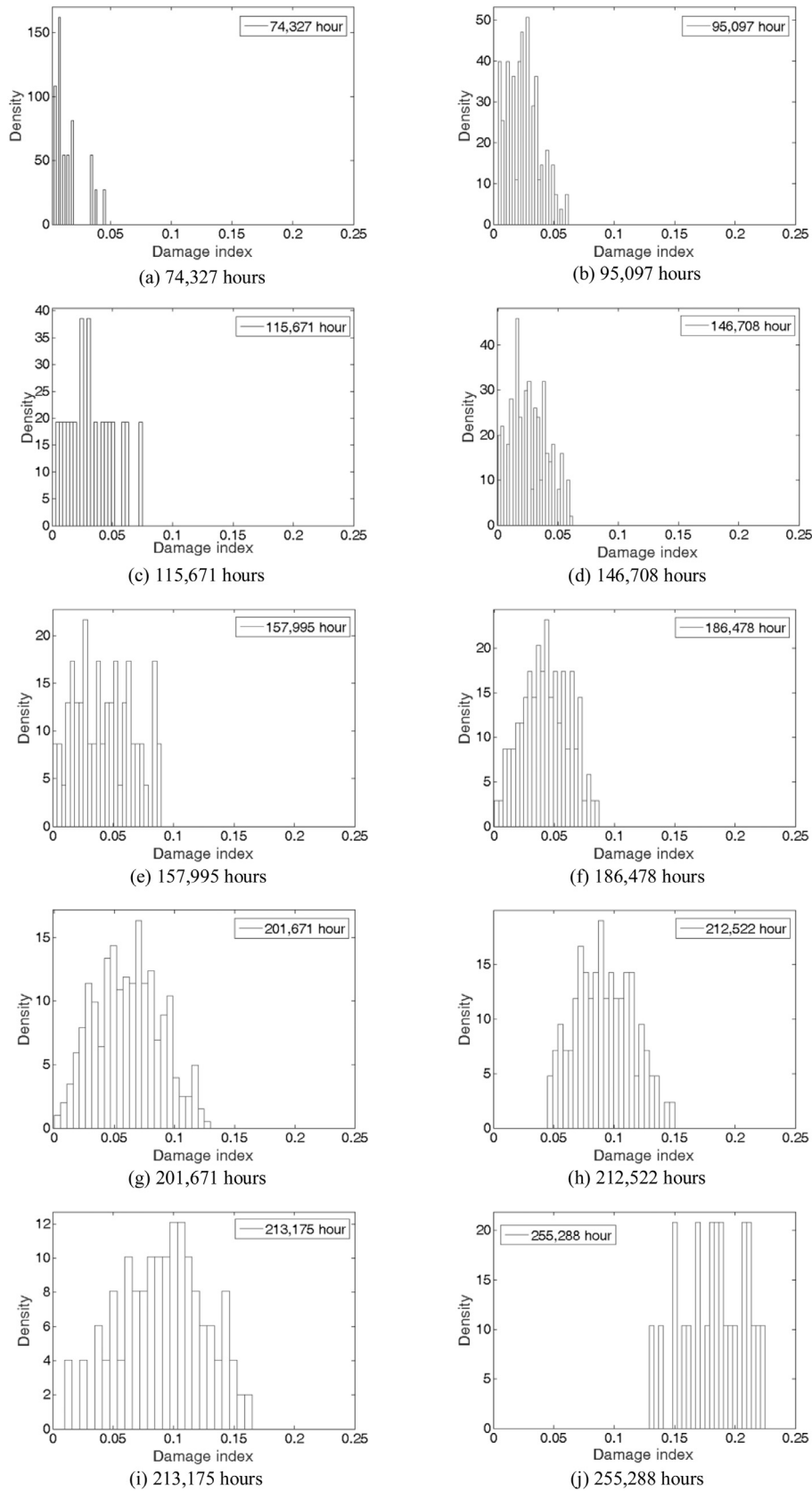


Fig. 5. Histograms based on the damage index.

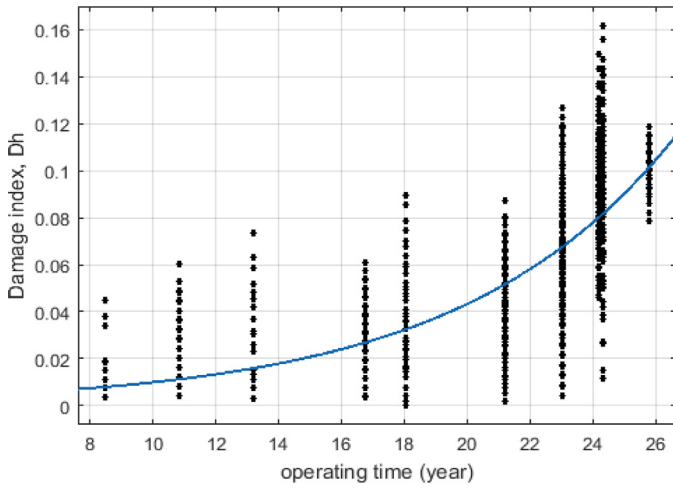
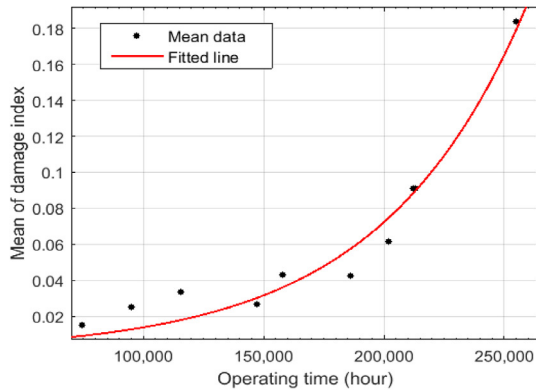
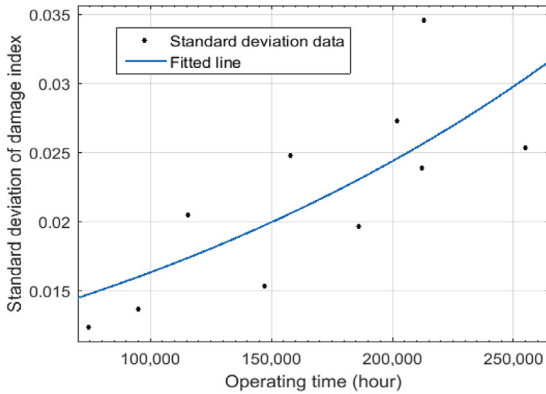


Fig. 6. Fitted line using regression methods.



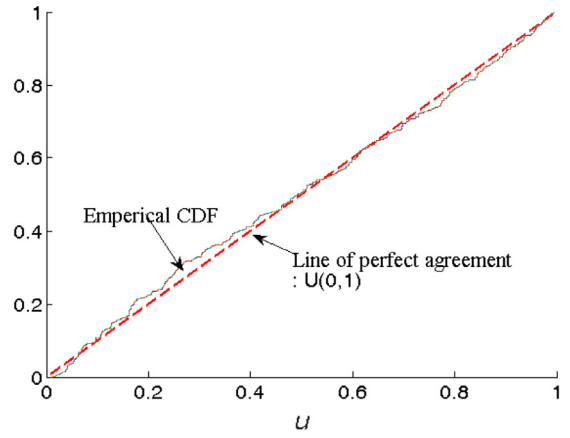
(a) Mean



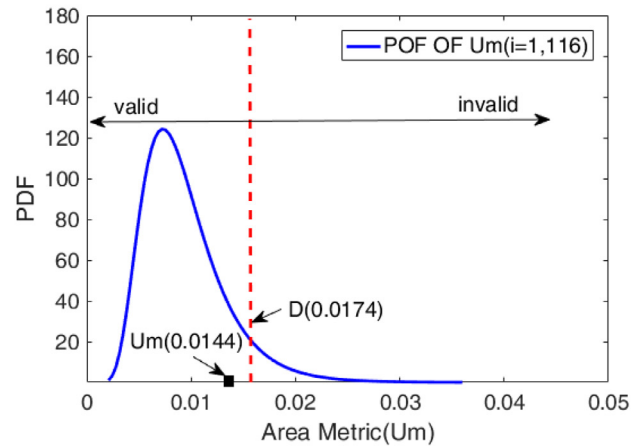
(b) Standard deviation

Fig. 7. Fitted line with mean and standard deviation of the damage index distribution.

generally used to check the degree of mismatch between the dispersion of experimental data and the distribution of predicted results by calculating the area between the CDF of the uniform distribution and the empirical CDF of u_i values corresponding to the experimental data [45–49]. If valid, damage indices obtained under heterogeneous conditions can be integrated into a single metric to assess the global predictive capability of a model. In order to develop a single metric, the goodness-of-fit is first evaluated for each damage index at each data set. For each sample 'k' of damage index, u_k is the value of goodness-of-fit. Then, the area metric is calculated by integrating the difference between the CDF



(a) Area metric



(b) Hypothesis testing based on the area metric with 5% confidence level

Fig. 8. Calculation of area metric, U_m .

of uniform distribution $U(0,1)$ and the experimental CDF of u_k . Therefore, the area metric based on damage indices is defined as:

$$U_m = \int_0^1 |F_u - F_{uni}| du \quad (4)$$

where F_u is the transformation of every damage index D_i into the CDF of responses from an assumed model; F_{uni} is the CDF of a uniform distribution $U(0,1)$.

In this study, there are ten damage indices D_i from experiments. The u_i of each damage index is calculated and the empirical CDF of each is shown in Fig. 8(a). Initial damage index values less than zero are physically impossible; therefore, they are excluded for data homogenization at an early stage. The calculated area after data homogenization is 0.0144; this is smaller than the threshold of 0.0175. The number of damage indices from data combination results is 1,116 and the significance level is 0.05. As a result, the null hypothesis of a normal distribution of the damage indices cannot be rejected, as shown in Fig. 8(b), and data homogenization enables integration of heterogeneous measured data from different turbines.

Though hardness data are obtained from heterogeneous situations, the homogeneity of the normally distributed damage index is validated and a developed damage growth model is available.

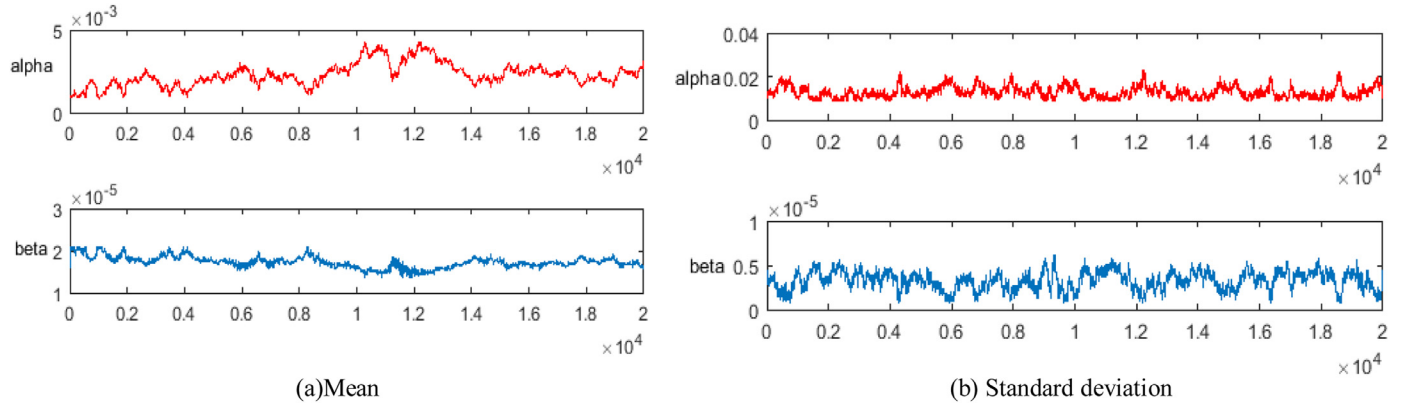


Fig. 9. Trace iteration of a random sample.

3.2. Bayesian updating scheme of the damage growth model

Damage growth can be predicted by the mean and standard deviation of the damage indices. The parameters of the mean and standard deviation can be estimated by a regression technique, such as the least squares method in Eq. (3). Since typical regression methods cannot consider the statistical correlation of hyper-parameters of the damage model, the accuracy of the life prediction results is low. One of the advantages of Bayes’ theorem over other parameter identification methods (e.g., the least squares method and maximum likelihood method) is its ability to identify the uncertainty structure of the identified parameters [31]. In this study, the Bayesian technique is employed to estimate the coefficients of mean, standard deviation, and statistical correlation for damage index distribution. Bayesian inference is based on Bayes’ theorem:

$$p(\theta|z) = L(z|\theta)p(\theta) \tag{5}$$

where $L(z|\theta)$ is the likelihood of the observed data, z is conditional on the given parameters θ ; $p(\theta)$ is the prior distribution of θ ; and $p(\theta|z)$ is the posterior distribution of θ conditional on z . We consider posterior distributions of the coefficients of the mean and standard deviation models in the same type. The posterior distributions of the coefficients are given as:

$$p(\alpha_\mu, \beta_\mu|\mu) \propto L(\mu|\alpha_\mu, \beta_\mu)p(\alpha_\mu, \beta_\mu) \tag{6}$$

In the Bayesian approach, the joint posterior distribution of the hyper-parameters α, β for the mean and standard deviation of the damage indices is obtained by multiplying likelihoods $L(\mu|\alpha_\mu, \beta_\mu), L(\sigma|\alpha_\sigma, \beta_\sigma)$ with prior distributions $p(\alpha_\mu, \beta_\mu), p(\alpha_\sigma, \beta_\sigma)$, respectively. The likelihood is the probability of obtaining the mean and standard deviation for given hyper-parameters α, β from measured hardness data. It has been shown previously that material hardness follows a normal distribution [38]. For simplicity, it is assumed that a non-conjugate Bayes model is used for the updating process of the damage growth model. Therefore, the likelihood also follows a normal distribution, with variances s_μ^2, s_σ^2 . The likelihood of the mean of the damage index can be expressed as:

$$L(\mu|\alpha_\mu, \beta_\mu) = \frac{1}{\sqrt{2\pi}s_\mu} \exp\left[-\frac{1}{2}\left(\frac{\mu - \mu_D(\alpha_\mu, \beta_\mu)}{s_\mu}\right)^2\right] \tag{7}$$

where $\mu_D(\alpha_\mu, \beta_\mu)$ is an estimated mean of the damage index equation derived from Eq. (3). The standard deviation of the damage index can be modeled, similar to Eq. (7). No prior information of the hyper-parameters α, β of the mean and standard deviation is available. For practical scenario, it is difficult to obtain the prior information for the actual steam turbine’s prognostics. In this paper, therefore, the prior distributions of the hyper-parameters are assumed to follow a uniform distribution whose ranges are twice larger than 90% confidence bound

of the estimated hyper-parameter α, β by the Nlsq method

$$p(\alpha_{\mu,\sigma}^0) \sim U(\alpha_{\mu,\sigma}^L, \alpha_{\mu,\sigma}^U), p(\beta_{\mu,\sigma}^0) \sim U(\beta_{\mu,\sigma}^L, \beta_{\mu,\sigma}^U) \tag{8}$$

where $\alpha^L, \alpha^U, \beta^L, \beta^U$ are the lower and upper bounds of the hyper-parameters of the mean and standard deviation, respectively.

Consequently, the posterior becomes a multiplication of the likelihood and prior distributions. The prior distribution and the likelihood function, respectively, are uniform and normal distribution, as introduced here to estimate parameters of the damage growth model using Bayesian inference.

3.3. Damage growth model updating

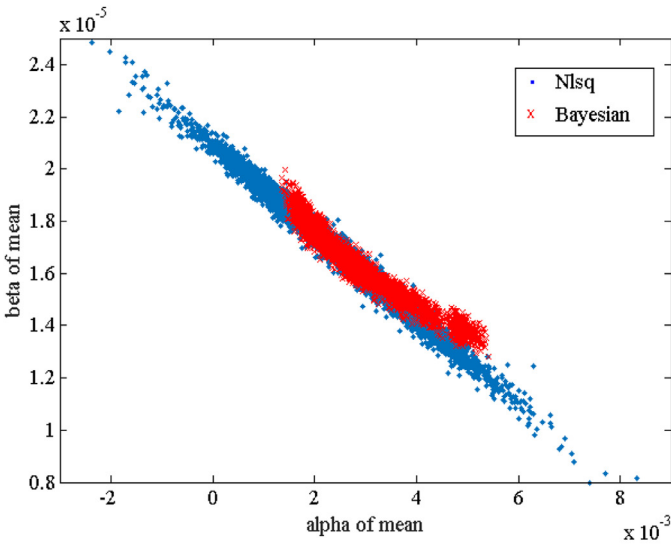
Since the expression of the posterior distribution of the mean and standard deviation of the damage index is available as a product of the likelihood and prior in Eq. (8), the shape of the posterior distribution can be estimated by calculating its parameters of mean and standard deviation at each time. The posterior distribution is complicated due to the correlation between multiple parameters in practical engineering applications; thus, a sampling method is effective to generate samples from an arbitrary posterior distribution. As a sampling method, Markov Chain Monte Carlo (MCMC) simulation is used to evaluate the posterior distribution after Bayesian updating [39,40]. MCMC simulation, in conjunction with the data augmentation technique, is computationally effective and useful to identify the correlation between hyper-parameters of the damage growth model [50,51]. This paper uses a general Metropolis-Hastings (M-H) algorithm to generate samples that simulate the posterior distribution of two hyper-parameters α and β of the damage index. As shown in Fig. 9, 20,000 samples for the hyper-parameters of the mean and standard deviation are generated to capture the nature of the distributions of the hyper-parameters. 4000 samples from the initial stage are discarded for data homogenization.

In general, the Nlsq method is easily used to estimate the mean and standard deviation of the damage index. Nonlinear models are more difficult to fit than linear models because hyper-parameters of the damage index cannot be estimated using regression. The Levenberg–Marquardt algorithm is used for solving the Nlsq problem [52], which estimates the distribution parameters of the damage growth model.

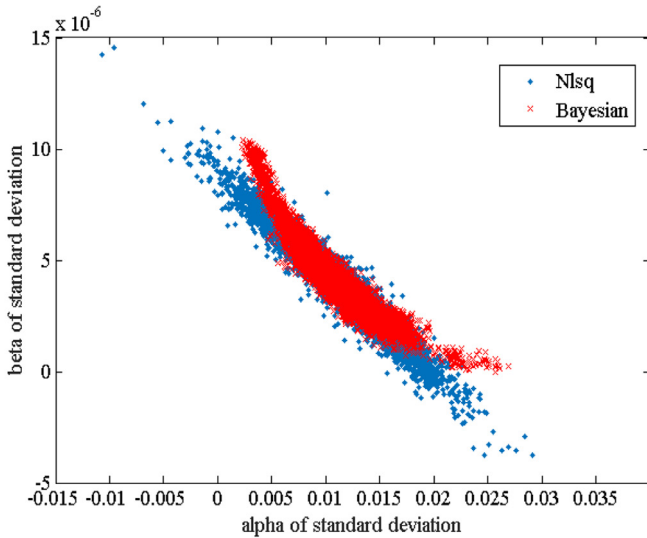
Figs. 10(a) and (b) show the joint random samples of the hyper-parameters (α and β) generated by using the Nlsq method and Bayesian method (BM). The Nlsq method yields the linear correlation of the random samples of the hyper-parameters with a constant correlation value. In contrast BM can reproduce the nonlinear correlation of the random samples. The correlation can be identified well; this is more important for accurately predicting damage growth and RUL. Table 5 shows the confidence intervals of the hyper-parameters of the damage growth model, along with the lower and upper bounds of the 90% intervals using both the Nlsq method and BM. Since the confidence bounds from BM

Table 5
Confidence intervals of parameters for damage growth model.

Mean	α_μ				β_μ			
	5%	50%	95%	Interval	5%	50%	95%	Interval
Bayesian	0.0018	0.0031	0.0041	0.0023	1.46E-05	1.58E-05	1.80E-05	3.4E-06
Nlsq	0.0010	0.00260	0.0041	0.0031	1.406E-05	1.66Ee0-5	1.93E-05	5.2E-06
Standard deviation	α_σ				β_σ			
	5%	50%	95%	Interval	5%	50%	95%	Interval
Bayesian	0.0059	0.0106	0.0162	0.0103	2.02E-06	4.14E-06	7.02E-06	5.0E-6
Nlsq	0.0057	0.0110	0.0163	0.0106	1.50E-07	3.98E-06	6.46E-06	6.3E-6

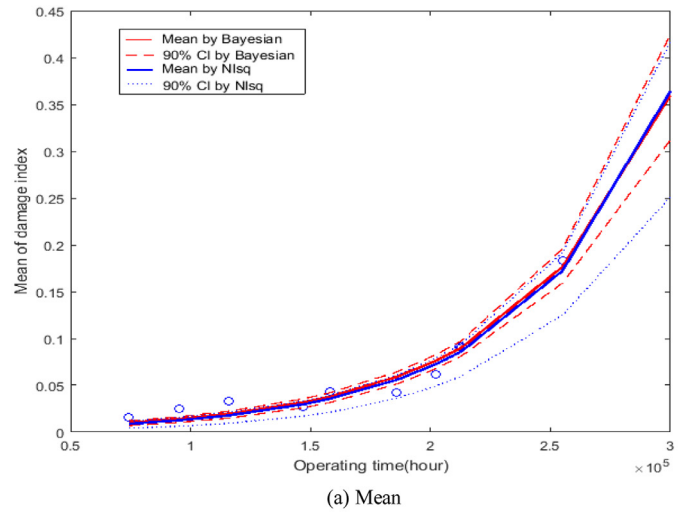


(a) Correlated random samples of the coefficients (α, β) of the damage index mean

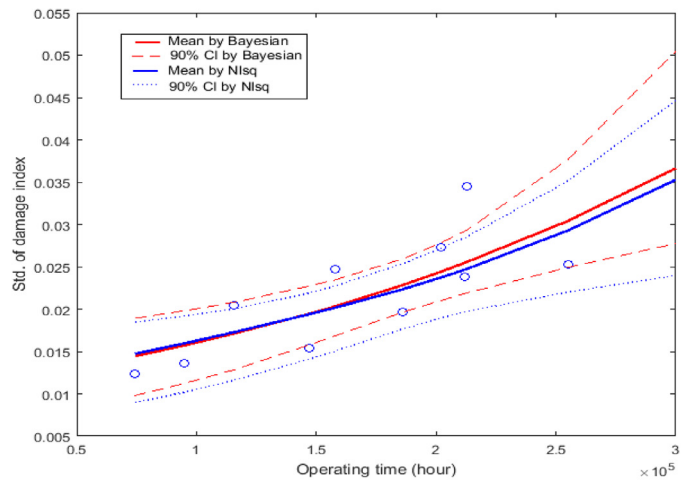


(b) Correlated random samples of the coefficient (α, β) of the damage index standard deviation

Fig. 10. Correlated random samples of the damage index.



(a) Mean



(b) Standard deviation

Fig. 11. Mean and standard deviation results obtained by performing the Bayesian updating.

are relatively narrower, the predicted mean and the standard deviation for the damage index hold less uncertainty. The probability distributions of the mean and standard deviation of the damage index can be obtained using Eq. (3) once the joint samples are obtained. To understand the effects of the correlation of the parameters in the damage growth model, the mean and standard deviation for the damage index are predicted using the Nlsq method and BM, as shown in Fig. 11. It is observed that the second quartile (50%) of the mean and standard deviation derived from the two methods are quite similar. However, BM gives a smaller

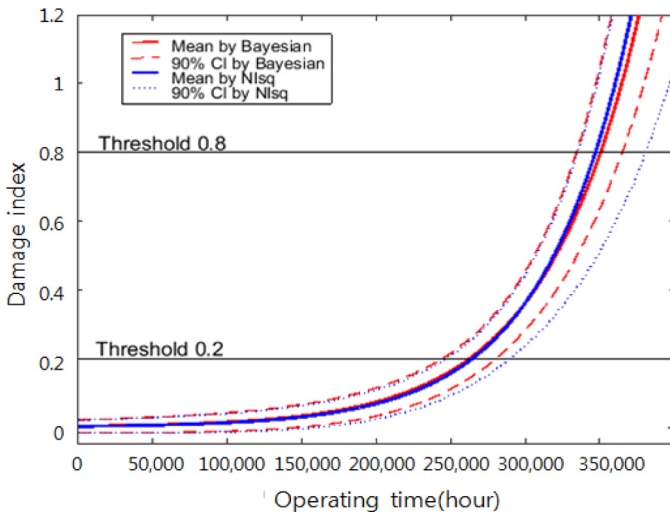


Fig. 12. Damage growth prediction curves with all ten data.

deviation of the mean and standard deviation. It is known that prediction accuracy is more sensitive to correlation than uncertainty type [53]. Each hyper-parameter is deterministically estimated and a linear correlation is added to the estimated results based on the covariance matrix in the Nlsq method. In the Bayesian method, on the other hand, the uncertainties in the unknown hyper-parameters are considered with a joint posterior distribution, and the parameters' correlation and the distribution can be identified simultaneously. As a result, it can be seen that the uncertainty of the damage growth model can be reduced by considering the nonlinear correlation of parameters for mean and standard deviation that constitute the damage growth model.

Even though measured hardness data are heterogeneous and random, a damage growth model can be constructed using the data homogenization process and Bayesian updating. Posterior distributions of the hyper-parameters (α and β) are used to predict the damage growth by using Eq. (3).

Fig. 12 shows the results of damage growth prediction by the Bayesian and Levenberg–Marquardt method, with the 10 data sets of damage indices shown in Fig. 5. The threshold lines of 0.2 and 0.8 are assumed and plotted as the typical ranges of the critical damage [30]. Even though both results are similar in the upper bound, differences of mean and lower bound gradually increase with time. Also, the Nlsq method clearly shows a different prediction with much wider uncertainty, even though the median is close to the true value. These results show that the Bayesian method that uses the mean and standard deviation of the damage index is applicable for predicting damage distribution and damage growth with uncertainty. Since damage growth is predicted with all ten data sets with operating times, the results from the two methods seem to have a similar trend.

4. Predicting the remaining useful life (RUL) of steam turbines

Once the parameters of the damage growth model are identified using Bayesian inference, the model can be used to predict the RUL, which is the remaining time until the damage indices grow to a threshold.

4.1. Damage threshold

Typically, RUL is expressed in terms of a damage index D and an operation hour t_{op} as $t_r = (1/D - 1)t_{op}$ [54]. Ideally, failure can be defined according to Eq. (2) when a damage index becomes 1. Once the hyper-parameters of the damage growth model are estimated, however, the future damage state and remaining useful life (RUL) can be pre-

Table 6
Comparison of RUL.

Operating Time(h)	Threshold 0.2		Threshold 0.8	
	B10	B50	B10	B50
0	240,000	250,000	325,000	335,000
200,000	40,000	50,000	125,000	135,000
250,000	-9,000	0	75,000	85,000

dicted by progressing the damage state until the damage index reaches a threshold [55].

A damage threshold is of great importance to RUL prediction. However, there is to date no study about a damage threshold for steam turbines. Since a steam turbine is a rotating machine under high speed, temperature, and pressure conditions, crack initiation or fracture in elastic-plastic stress fields should be considered to be the criteria to determine the end of life. Sumio [56] proposed that the value of critical damage D_c has been ascertained to be $0.2 < D_c < 0.8$ for elastic-plastic damage.

It is well known that the average design life of a steam turbine is approximately 200,000 h (around 25 years) [34,57–60]. RUL prediction has been carried out to decide between life extension or retirement. Approximately 25 to 30 years is generally accepted as an acceptable usage life time. However, experience shows that a turbine can operate beyond its design life because of its designed safety margin. Table 3 shows two units that were retired after operating 213,175 and 255,288 h; a service life beyond the average design life. In this study, the damage index and RUL are estimated for retired turbines to determine a threshold for damage growth of a steam turbine.

For the case of damage growth prediction shown in Fig. 13, there are large differences between Bayesian and Nlsq methods at 90% confidence intervals; this relates to the B-10 life. The B10 life metric, associated with 90% reliability, originated in the ball and roller bearing industry. This metric has become widely used in across a variety of industries [61,62]. The Bayesian methods predict damage growth accurately with relatively small uncertainty, compared with the Nlsq method, as shown in Fig. 13. This study conducted RUL prediction at three operating times (0, 200,000, and 250,000 h) to determine an appropriate failure criterion for the damage growth model. Table 6 shows comparison results from the damage growth model derived using Bayesian inference. By accepting the damage index, 0.2, as a failure criterion, the B50 life is 250,000 h and B10 life is 241,000 h. On the other hand, a failure criterion of the damage index 0.8 yields 325,000 and 335,000 h as the B10 and B50 life, respectively. By comparing these findings with the actual retirement history of steam turbines, it is concluded that a failure criterion of the damage index 0.2 gives a reasonable RUL for a steam turbine.

4.2. Validation of the proposed damage growth model

Since a new damage index distribution and damage growth model, based on sporadic and heterogeneous data, is proposed, it is necessary to validate the proposed model. We used the data sets in Table 3 to validate the proposed Bayesian method. The prior distribution is uniform distributions as discussed in Section 3.2. The posterior distributions of the hyper-parameters α , β of the mean and standard deviation for the damage growth model are obtained with seven sets of ten data (i.e., A1 to E2) without 8~10th data set (i.e., F1 and H5). For the purpose of comparison, the distributions of hyper-parameters α , β are also obtained using the Nlsq method. The mean value of the last data set is used to validate the prediction. The results of damage growth prediction using the Bayesian and Nlsq methods are given in Fig. 13(a). Next, the posterior distributions of the hyper-parameters α , β of the damage growth model are obtained with eight sets of ten data (i.e., A1 to F1) without 9~10th data set (i.e., G4 and H5). As shown in Fig. 13(b), the 8th

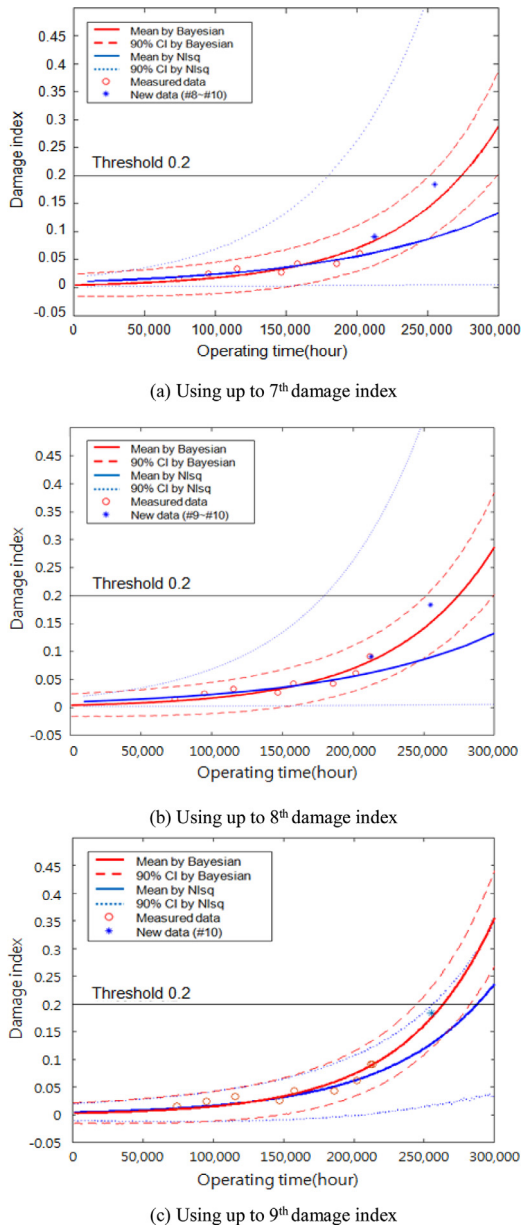


Fig. 13. Progressive damage growth predictions with variable number of training data.

data point overlaps the 9th data point because their operating times and damage indices are almost identical. Compared to the actual data, the mean value calculated by the Bayesian method shows good agreement and narrow confidence bounds, whereas the damage growth prediction by the Nlsq method does poor agreement and wider confidence bounds. In Fig. 13(c), posterior distributions of the hyper-parameters α , β are obtained with nine sets of data (i.e., A1 to G4) without the last data set (i.e., H5). As expected, the results with the proposed Bayesian method outperformed the Nlsq method.

Although the estimation results are fairly exact in the early stages, early stages are not of interest in terms of prognostics of a steam turbine. Thus, common Nlsq methods may not be suitable to predict the damage index distribution and the RUL of a steam turbine with limited and distributed data that does not follow a normal distribution. Additionally, it is observed that uncertainty in the mean and standard deviation is reduced with more data; thus, the confidence intervals are reduced from Figs. 12 and 13. Fig. 14 shows a comparison of the dam-

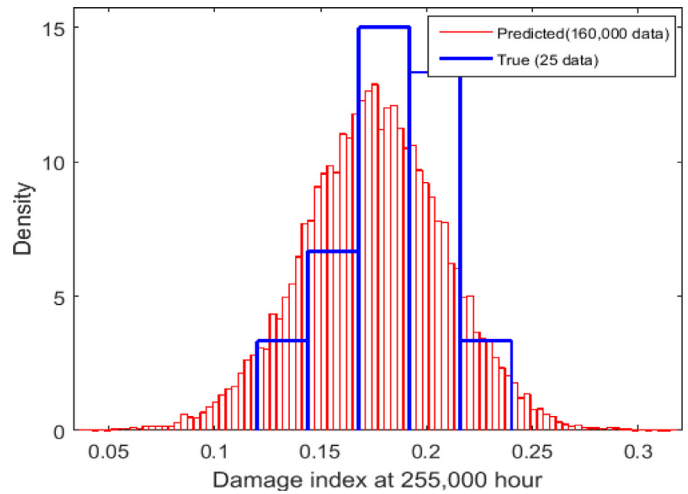


Fig. 14. Damage index distribution at 255,000 h operation.

age index distribution between the predicted one and the measured true one at 255,000 h operation. Even though the number of data in the true damage index distribution is quite small at 25, in Fig. 14, the damage distribution from the Bayesian method is very close to the true one. Additionally, the area metrics from the Bayesian and Levenberg-Marquardt method are calculated with the aggregated 25 data. The threshold was 0.11785 for the sample size of 25 and a significance level of 0.05. The area metric result of the Bayesian method of 0.09423 is less than the threshold; whereas, the Levenberg-Marquardt result is larger than the threshold. We can also conclude that the Bayesian approach can accept the assumption of a normally distributed distribution of the damage index in the validation process.

4.3. RUL prediction

Although advanced maintenance techniques are available in the literature, they have not been well implemented in the industry for various reasons, including lack of data, lack of an efficient model, and difficulty of implementation [4]. A common practice in condition-based maintenance for turbines in power plants is to analyze the condition of the equipment at regular or irregular intervals; the measurement of such condition information is then used in RUL prediction. RUL predictions of steam turbines can be used to determine the maintenance schedule of whole power plants. RUL can be predicted by subtracting the PDF of the damage index from the threshold by using the mean and standard deviation distribution. Since there is no information about actual failure data of steam turbines, in this study, operating times of 0, 200,000, and 250,000 h are used to predict RUL in the proposed damage growth model. These operating times represent the initial and average design life, respectively. PDFs and CDFs of the damage index with a 0.2 damage threshold at each operating time are shown in Fig. 15. The change of RUL with respect to operating times is shown in Fig. 16. In Fig. 16, the black solid line represents the true RUL. The true RUL is a negative slope line as the RUL decreases at every operating time. The red-dashed line is the predicted RUL using the damage growth model and threshold. It was clearly shown that the confidence bound become narrow with the increase of operating times. To show the differences between thresholds, additionally, distributions of RUL at 255,000 h are compared in Fig. 17 and in Table 6. By considering the average design life and actual retirement history of steam turbines, as a result, it is concluded that a damage threshold of 0.2 yields a reasonable RUL for a steam turbine. As a result, the RUL distribution of a steam turbine can be predicted using the Bayesian method, and B-lives can be determined by using the proposed damage threshold value of 0.2.

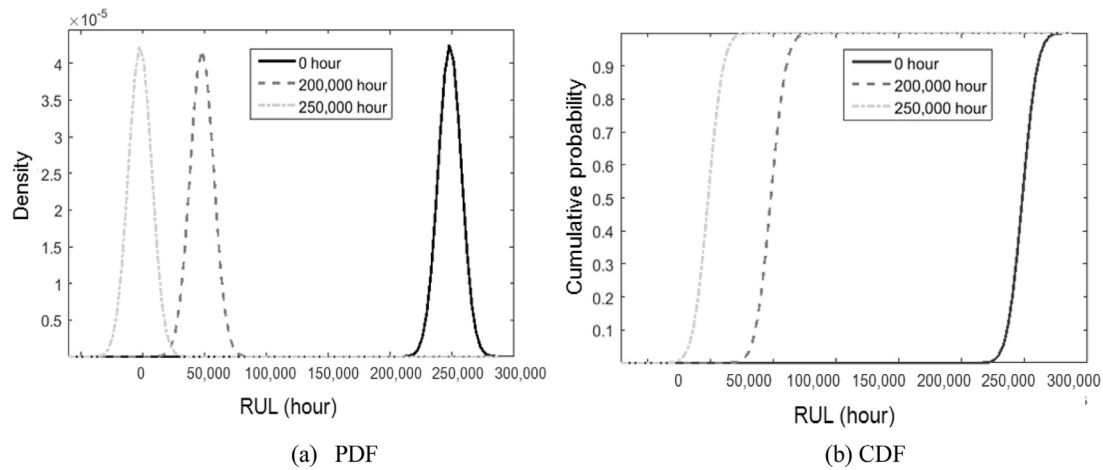


Fig. 15. RUL distributions with different operating times under damage threshold 0.2.

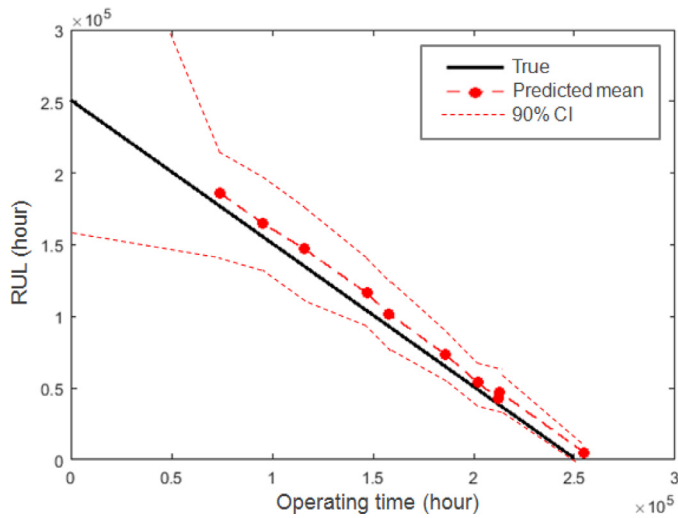


Fig. 16. RUL prediction at each operating time.

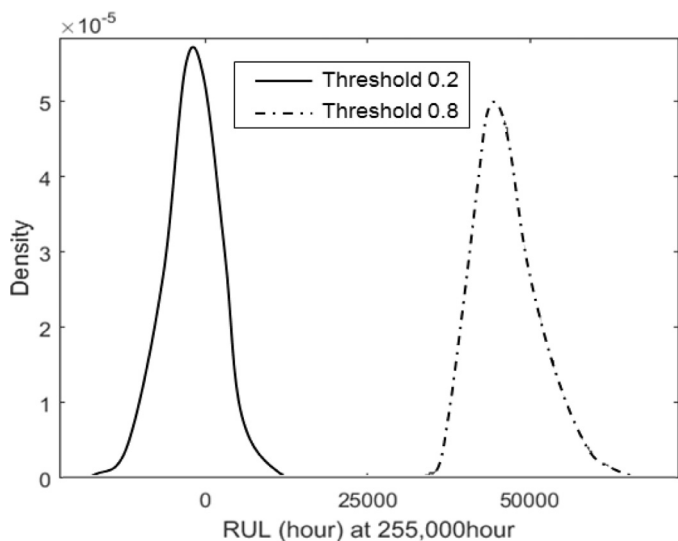


Fig. 17. RUL distribution under damage threshold 0.2 at 255,000 h operation.

5. Conclusions

This paper presented a damage growth model and an RUL prediction methodology for aged steam turbines by using Bayesian inference. Based on the study described in this paper, several conclusions can be drawn. First, RUL prediction methodologies developed in this research incorporate the damage index into damage growth model estimation. Since the damage index, as a function of hardness, is distributed due to various uncertainties, the mean and standard deviation from the damage index distribution are used to predict the damage growth. Second, the damage growth model for a steam turbine was proposed as a function of mean and standard deviation from the damage index distribution. A Bayesian inference technique was used to estimate the probability distribution of the damage index from on-site measurements. Hardness values of the damage index were measured using a rebound hardness tester. Third, the damage growth predicted using both Bayesian and Levenberg-Marquardt methods was compared and validated. It is well known that the ability to use prior information and to choose an appropriate statistical model are advantages of Bayesian inference over the Nlsqs method, especially in cases of nonlinear correlation of unknown parameters for a damage index. Also, as more measurement data are integrated into the updating process, uncertainties in prediction can be reduced. Fourth, by comparing predictions with the actual retirement history of steam turbines, it is concluded that a damage threshold of 0.2 gives a reasonable damage distribution and RUL for a steam turbine. Through the proposed methodology, it is expected that damage states and RULs of steam turbines can be predicted using the operating time, regardless of the type of turbine.

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References

- [1] Banaszkiwicz M. Multilevel approach to lifetime assessment of steam turbines. *Int J Fatigue* 2015;73:39–47.
- [2] Helmut Pollack, E.-W.P., N. Thamm, M.-A. Schwarz, Design and materials for modern steam turbines with two cylinder design up to 700 MW, 2004, SIEMENS AG.
- [3] Neil Wembridge, L.P., N. McGimpsey Noel, H. Thermal power station advice 007, PB Quality system. 2009.
- [4] Andrew KS, Jardine DL, Banjevic D. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mech Syst Signal Process* 2006;20:1483–510.
- [5] Khanh Le Son MF, Barros A., Levrat E, Benoit I. Remaining useful life estimation based on stochastic deterioration models: a comparative study. *Reliab Eng Syst Saf* 2013;112:11.
- [6] Wang P, ChaoHu BY. A generic probabilistic framework for structural health prognostics and uncertainty management. *Mech Syst Signal Process* 2012;28:622–37.

- [7] Hu C, Wang P, Youn BD, Lee W-R. Copula-based statistical health grade system against mechanical faults of power transformers. *IEEE Trans Power Deliv* 2012;27(4):1809–19.
- [8] Oh H, Jung JH, Jean BC, Youn BD. Scalable and Unsupervised Feature Engineering Using Vibration-Imaging and Deep Learning for Rotor System Diagnosis. *IEEE Trans Industrial Electronics* 2018;65(4):3539–49.
- [9] Furtado HC, Le May I. Metallography in life assessment of power plants. *Mater Charact* 1996;36(4–5):175–84.
- [10] Nurbanasari M, Suratman R. Creep damage assessment of a 50 MW steam turbine shaft. In: *Proceedings of MATEC Web of Conferences*, 34; 2015. p. 05002.
- [11] Choi WS, Song G, Koo J. Development and Implementation of i-LAS (indentation based turbine remaining life assessment system). In: Auerkari Juha, editor. *Proceedings of conference on life management and maintenance for power plants; Baltica VIII*, P.V.; 2010. p. 11.
- [12] Metallographic assessment of the temperature history and remanent life of power plant components, 2001, Nordtest, NT mat 002.
- [13] Sposito G, Ward C, Cawley P, Nagy PB, Scruby C. A review of non-destructive techniques for the detection of creep damage in power plant steels. *NDT & E Int* 2010;43(7):555–67.
- [14] Shibli IA, Holdsworth SR, Surrey D. Creep and fracture in high temperature components: design and life assessment issues. *DEStech Publications*; 2005.
- [15] Salonen J, Pertti A, Olli L, Mikko P. Experience on in-service damage in power plant components. *Eng Fail Anal* 2007;14(6):970–7.
- [16] Mazaheri M, Djevanroodi F, Nikbin KM. Creep life assessment of an overheated 9Cr-1Mo steel tube. *Int J Press Vessels Pip* 2010;87(12):746–52.
- [17] Holdsworth SR. Creep-fatigue properties of high temperature turbine steels. *Mater High Temp* 2001;18(4):261–5.
- [18] *Residual Life Assessment and Microstructure, ECCRC Recommendation*, 2005:6.
- [19] He Y, Dong J, Choi W, Jung J, Shin K. An Improved non-destructive replication metallography method for investigation of the precipitates in Cr-Mo-V turbine steel. *Surf Interface Anal* 2012;44(11–12):1411–14.
- [20] Tasan CC, Hoefnagels JPM, Geers MGD. A critical assessment of indentation-based ductile damage quantification. *Acta Mater* 2009;57:4957–66.
- [21] Fujiyama K, Kimura K, Muramatsu M, Yamada M. Evaluation of creep properties of degraded Cr-Mo-V turbine cast steels through hardness measurement and its application to life assessment. *ISIJ Int* 1990;30(10):869–74.
- [22] Fujiyama K, Nagai S, Akikuni Y, Fujiwara T, Furuya K, Matsumoto S, Takagi K, Kawabata T. Risk-based inspection and maintenance systems for steam turbines. *Int J Press Vessels Pip* 2004;81(10–11):825–35.
- [23] Mukhopadhyay SK, Roy H, Roy A. Development of hardness-based model for remaining life assessment of thermally loaded components. *Int J Press Vessels Pip* 2009;86:246–51.
- [24] Oliver WC, Pharr GM. Measurement of hardness and elastic modulus by instrumented indentation: advances in understanding and refinements to methodology. *J Mater Res* 2004;19(1):18.
- [25] Lee K-W, Kim K, Kim J-Y, Kim K-H, Choi B-H, Kwon D. Safety assessment based on mapping of degraded mechanical properties of materials for power plant pipeline using instrumented indentation technique. *J Loss Prev Process Ind* 2009;22(6):868–72.
- [26] Seok C-S, Koo J-M. Evaluation of material degradation of 1Cr-1Mo-0.25 V steel by non-destructive method. *Mater Sci Eng: A* 2005;395(1–2):141–7.
- [27] Choi W, Song G, Kim B, Chang S. Indentation based life assessment for boiler tubes of fossil power plants. *Procedia Eng* 2011;10:1035–40.
- [28] Wang P, Youn BD, Xi Z, Kloess A. Bayesian reliability analysis with evolving, insufficient, and subjective data sets. *J Mech Des* 2009;131(111009):1–11.
- [29] Oh H, Kim J, Son H, Youn BD. A systematic approach for model refinement considering blind and recognized uncertainties in engineered product development. *Struct Multidiscip Optim* 2016:1–15.
- [30] Guan X, Liu Y, Jha R, Saxena A, Celaya J, Geobel K. Comparison of two probabilistic fatigue damage assessment approaches using prognostic performance metrics. *Int J Progn Health Manag* 2011;5:11.
- [31] An D, Choi J, Kim NH, Pattabhiraman S. Fatigue life prediction based on Bayesian approach to incorporate field data into probability model. *Struct Eng and Mech* 2011;37(4):16.
- [32] Chiachío J, Chiachío M, Saxena A, Sankararaman S, Rus G, Goebel K. Bayesian model selection and parameter estimation for fatigue damage progression models in composites. *Int J Fatigue* 2015;70:12.
- [33] Compare M, Baraldi P, Bani I, Zio E, McDonnell D. Development of a Bayesian multi-state degradation model for up-to-date reliability estimations of working industrial components. *Reliab Eng Syst Saf* 2017;166(Supplement C):25–40.
- [34] Venkatesh K, Raju PVeera, Jayananda Kumar T. Residual life assessment of 60 MW steam turbine rotor. *Int J Sci Res Publ* 2012;2(8):11.
- [35] Choi W, Fleury E, Kim B, Hyun J-S. Life assessment of steam turbine components based on viscoplastic analysis. *J Solid Mech Mater Eng* 2008;2(4):478–86.
- [36] Fujiyama K. Development of risk-based maintenance planning program for power plant steam turbine. *KEPCO Research Institute*; 2007. p. 152.
- [37] *Guideline on the estimation of uncertainty in hardness measurement*. 2007, European Association of National Metrology Institute.
- [38] *Measurement uncertainty for Leeb hardness tests, Equotip Uncertainty Clarification Short E*, 2014.
- [39] Kimura K, Fujiyama K, Muramatsu M. Creep and fatigue life prediction based on the non-destructive assessment material degradation for steam turbine rotors. *High temperature creep-fatigue. Curr Jpn Mater Res* 1988;3:247–70.
- [40] Turtureanu N. Turbine tech: scheduling of steam turbine inspection intervals. In: *Proceedings of energy-tech*; 2008.
- [41] Kehlhofer R, Rukes B, Hannemann F, Stirnimann F. Combined-cycle gas steam turbine power plants. In: *Proceedings of PennWell*; 2009. p. 430.
- [42] Jung BC, Yoon HJ, Oh H, Lee GS, Yoo MJ, Youn BD, Huh YC. Hierarchical model calibration for designing piezoelectric energy harvester in the presence of variability in material properties and geometry. *Struct Multi Opt* 2016;53(1):161–73.
- [43] Kim DW, Oh H, Youn BD, Kwon DI. Bivariate Lifetime Model for Organic Light-Emitting Diodes. *IEEE Trans Ind Electron* 2017;64(3):2325–34.
- [44] Lemaître J, Dufailly J. Damage measurements. *Engineering fracture mechanics*. Springer; 1987.
- [45] Ferson S, Oberkampf WL, Ginzburg L. Model validation and predictive capability for the thermal challenge problem. *Comput Methods Appl Mech Eng* 2008;197(29–32):2408–30.
- [46] Oberkampf WL, Ferson S. Model validation under both aleatory and epistemic uncertainty. *National Nuclear Security Administration*; 2007.
- [47] Frenkel IB, Karagrigoriou A, Lisnianski A, Kleyner A. *Applied reliability engineering and risk analysis*. Wiley; 2013.
- [48] Wei Li ZL, Chen W, Jiang Z, Liu Y. New model validation metrics for correlated multiple response. In: *Proceedings of the 10th world congress on structural and multidisciplinary optimization*, Orlando, Florida; 2013.
- [49] Jung BC, Park J, Oh H, Kim J, Youn BD. A framework of model validation and virtual product qualification with limited experimental data based on statistical inference. *Struct Multidiscip Optim* 2015;51(3):573–83.
- [50] Qin H, Zhou W, Zhang S. Bayesian inferences of generation and growth of corrosion defects on energy pipelines based on imperfect inspection data. *Reliab Eng Syst Saf* 2015;144:334–42.
- [51] An D, Choi J, Schmitz TL, Kimb NH. *In situ* monitoring and prediction of progressive joint wear using Bayesian statistics. *Wear* 2011;270:11.
- [52] Gavin, H.P., *The Levenberg-Marquardt method for nonlinear least squares curve-fitting problems*, Department of Civil and Environmental Engineering, Duke University, 2015.
- [53] Namho Kim DA, Choi J. *Prognostics and health management of engineering systems: an introduction*. Springer; 2017. p. 347.
- [54] Woosung Choi GS, Bumshin K, Hyun J, Heo J. Development and application of life-assessment guidelines for fossil-fuel power plant facilities in Korea. *KSME-A* 2010;34(9):1265–72.
- [55] An D, Choi JH, Kim NH. Prognostics 101: a tutorial for particle filter-based prognostics algorithm using Matlab. *Reliab Eng Syst Saf* 2013;115:161–9.
- [56] Murakami S. *Continuum damage mechanics*. In: *Solid mechanics and its applications*. Netherlands: Springer; 2012. p. 402.
- [57] Speidel MO, Denk J, Scarlin B. Stress corrosion cracking and corrosion fatigue of steam-turbine rotor and blade materials: commission of the European Communities Report No EUR 13186 1991 65 pp. In: *Proceedings of the International Journal of Fatigue*; 1991. p. 430.
- [58] Shlyannikov VN, Yarullin RR. Method for assessment of the residual life of turbine disks. *Inorg Mater* 2010;46(15):1683–7.
- [59] Hans R, Clossen M, Schulz L, Opheys M, Siegel Michael. Advanced ultrasonic application for the inspection of turbine components. In: *Proceedings of the 10th European conference on non-destructive testing*, Moscow; 2010.
- [60] Kushwaha AD, Soni A, Garewal L. Critical review paper of steam turbine blades corrosion and its solutions. *Int J Sci Res Eng Technol* 2014;3(4):9.
- [61] Ryu D, Seogweon C. Novel concepts for reliability technology. *Microelectron Reliab* 2005;45:611–22.
- [62] Jongen R, Morshuis Peter HF, Smit J, Gulski E. A statistical approach to processing power transformer failure data. In: *Proceedings of 19th international conference on electricity distribution*, Vienna, CIRED; 2007.