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SCIENCE DIRECT

Journal of Sound and Vibration 263 (2003) 569–591

JOURNAL OF SOUND AND VIBRATION

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Design sensitivity analysis for sequential structural–acoustic problems

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Received 29 August 2001; accepted 30 June 2002

Abstract

A design sensitivity analysis of a sequential structural–acoustic problem is presented in which structural and acoustic behaviors are de-coupled. A frequency-response analysis is used to obtain the dynamic behavior of an automotive structure, while the boundary element method is used to solve the pressure response of an interior, acoustic domain. For the purposes of design sensitivity analysis, a direct differentiation method and an adjoint variable method are presented. In the adjoint variable method, an adjoint load is obtained from the acoustic boundary element re-analysis, while the adjoint solution is calculated from the structural dynamic re-analysis. The evaluation of pressure sensitivity only involves a numerical integration process for the structural part. The proposed sensitivity results are compared to finite difference sensitivity results with excellent agreement.

1. Introduction

Structure-induced noise and vibration control at low frequency is an important area of research for reducing the noise level generated by various structural parts. In automotive applications, for example, the noise level of a passenger compartment can be reduced by changing the structural design parameters. Design sensitivity analysis (DSA) is an essential process in the gradient-based

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PII: S0022-460X(02)01067-2

optimum control technique. Some research results have been reported in DSA of a structural–acoustic problem. Ma and Hagiwara [1, 2], Wang et al. [3], and Choi et al. [4] developed DSA of a coupled structural–acoustic problem using a finite element method (FEM). Either a direct or frequency method is used to solve the system of matrix equations. However, the excessive number of elements to represent the complicated three-dimensional acoustic cavity has been the major bottleneck of the finite element-based approach [5]. To avoid the problems associated with a large number of elements in an acoustic domain, Salagame et al. [6] presented an analytical sensitivity method using a Rayleigh integral [7]. The sensitivity of a surface velocity is obtained by differentiating the frequency-response matrix equation, and the pressure sensitivity is then calculated by differentiating the Rayleigh integral. This approach is limited to a flat plate problem. Recently, Scarpa [8] proposed a parametric sensitivity calculation method using a symmetric Eulerian formulation. The velocity potential is used instead of the pressure to represent the fluid's behavior.

Compared to FEM, the boundary element method (BEM) has an advantage in the modelling of the acoustic cavity: It is unnecessary to generate a complicated, three-dimensional acoustic model. Several research studies have been conducted for DSA using BEM. Assuming that the structure's velocity sensitivity is known, Smith and Bernhard [9] developed a semi-analytical design sensitivity formulation. Cunefare and Koopman [10], Kane et al. [11], Matsumoto et al. [12], and Koo [13] presented an analytical design sensitivity formulation using BEM. For the general structure-induced noise problem, however, the velocity sensitivity has to be calculated from the structural frequency-response analysis [14]. A structural acoustic sensitivity algorithm with respect to sizing design variables based on finite element and boundary element computations has been presented [15]. A structural acoustic sensitivity formulation based on boundary elements has been developed for structures subject to stochastic excitation [16].

In this paper, a design sensitivity analysis of a sequential structural–acoustic problem is presented in which structural and the acoustic behaviors are de-coupled. For the case of a harmonic excitation, the dynamic behavior of the structure is described using a frequency-response analysis. A boundary element method [17] is used to calculate the radiated noise (pressure) from the structural response (harmonic velocity). Instead of differentiating a discrete matrix equation, a continuous variational equation is differentiated with respect to the design parameter. In case of sizing design, the boundary integral equation does not contain any terms that are explicitly dependent on the design; only implicitly dependent terms exist through the state variables.

While the direct differentiation method in DSA follows the same solution process as the response analysis, the adjoint variable method follows a reverse process. One of the challenges of the adjoint variable method in sequential DSA is how to effectively and practically formulate this reverse process. For example, in the transient response DSA developed by Haug et al. [18], the adjoint problem becomes a terminal-value problem, whereas the original problem is an initial-value problem. Such an opposite solution process in the adjoint problem causes a significant amount of inconvenience and ineffectiveness in DSA. To overcome these difficulties, a sequential adjoint variable method is presented in which the adjoint load is obtained from boundary element re-analysis, and the adjoint variable is calculated from structural dynamic re-analysis. So far, no research results have been reported in the development of the adjoint variable method in a sequential problem. In addition, it is shown that the acoustic adjoint problem still uses the same coefficient matrix from the direct problem, even if the coefficient matrix is not symmetric.

2. Review of structural–acoustic analysis

2.1. Frequency-response analysis

Consider a structure under dynamic load $\mathbf{F}(\mathbf{x}, t)$. The differential equation that governs the behavior of this hyperbolic system can be written as

$$\rho \mathbf{y}_{,tt}(\mathbf{x}, t) + C\mathbf{y}_{,t}(\mathbf{x}, t) + L\mathbf{y}(\mathbf{x}, t) = \mathbf{F}(\mathbf{x}, t), \quad \mathbf{x} \in \Omega^S, \ \ t > 0, \tag{1}$$

where Ω^S is the structure's domain, $\mathbf{y}(\mathbf{x}, t)$ the displacement, $L(\mathbf{x})$ the linear partial differential operator, $\rho(\mathbf{x})$ the structural mass density, and $C(\mathbf{x})$ the viscous damping effect. The subscribed comma denotes the derivative with respect to time, i.e., $\mathbf{y}_{,t} = \partial\mathbf{y}/\partial t$ (velocity) and $\mathbf{y}_{,tt} = \partial^2\mathbf{y}/\partial t^2$ (acceleration). The initial conditions of the dynamic problem are given by

$$\mathbf{y}(\mathbf{x}, 0) = \mathbf{y}^0(\mathbf{x}), \qquad \mathbf{y}_{,t}(\mathbf{x}, 0) = \mathbf{y}^0_{,t}(\mathbf{x}), \quad \mathbf{x} \in \Omega^S, \tag{2}$$

where $\mathbf{y}^0(\mathbf{x})$ is the initial displacement, and $\mathbf{y}^0_{,t}(\mathbf{x})$ is the initial velocity.

For the steady state response, the time-dependent terms from Eq. (1) should be removed. Since the harmonic load is being considered, $\mathbf{F}(\mathbf{x}, t)$ can be expressed as

$$\mathbf{F}(\mathbf{x}, t) = \mathbf{f}(\mathbf{x})\mathrm{e}^{\mathrm{j}\omega t}, \tag{3}$$

where $\mathbf{f}(\mathbf{x})$ is the magnitude of the harmonic load and ω is the load frequency, which is considered a constant. In contrast, the steady state response has the same frequency as the applied load but may have a different phase angle. Using the complex variable method, the displacement $\mathbf{y}(\mathbf{x}, t)$ can be expressed as

$$\mathbf{y}(\mathbf{x}, t) = \mathbf{z}(\mathbf{x})\mathrm{e}^{\mathrm{j}\omega t}, \tag{4}$$

where $\mathbf{z}(\mathbf{x})$ is the complex displacement.

Time dependency of the dynamic problem can be eliminated by substituting Eqs. (3) and (4) into Eq. (1), to obtain the spatial state operator equation as

$$-\omega^2 \rho \mathbf{z}(\mathbf{x}) + \mathrm{j}\omega C\mathbf{z}(\mathbf{x}) + L\mathbf{z}(\mathbf{x}) = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} \in \Omega^S \tag{5}$$

with its appropriate boundary conditions.

The variational formulation of Eq. (5) is similar to the static problem. However, since the complex variable $\mathbf{z}(\mathbf{x})$ is used for the state variable, the complex conjugate $\bar{\mathbf{z}}^*$ is used for the displacement variation. By multiplying $\bar{\mathbf{z}}^*$ and integrating it over the domain Ω^S, the variational equation can be derived after integration by parts for differential operator L as

$$\begin{aligned} &\int\!\!\int_{\Omega^S} [-\omega^2 \rho \mathbf{z}^{\mathrm{T}} + \mathrm{j}\omega C\mathbf{z}^{\mathrm{T}}]\bar{\mathbf{z}}^* \,\mathrm{d}\Omega^S + \int\!\!\int_{\Omega^S} \boldsymbol{\sigma}(\mathbf{z})^{\mathrm{T}} \boldsymbol{\varepsilon}(\bar{\mathbf{z}}^*)\,\mathrm{d}\Omega^S \\ &\quad = \int\!\!\int_{\Omega^S} \mathbf{f}^{b^{\mathrm{T}}} \bar{\mathbf{z}}^* \,\mathrm{d}\Omega^S + \int_{\Gamma^s} \mathbf{f}^{s^{\mathrm{T}}} \bar{\mathbf{z}}^* \,\mathrm{d}\Gamma, \quad \forall \bar{\mathbf{z}} \in Z, \end{aligned} \tag{6}$$

where $\bar{\mathbf{z}}^*$ is the complex conjugate of the kinematically admissible virtual displacement $\bar{\mathbf{z}}$, and Z is the complex space of kinematically admissible virtual displacements. Eq. (6) provides the variational equation of the dynamic frequency response under an oscillating excitation with frequency ω. For derivational convenience, the following terms are defined:

$$d_{\mathbf{u}}(\mathbf{z}, \bar{\mathbf{z}}) = \int\int_{\Omega^S} \rho \mathbf{z}^{\mathrm{T}} \bar{\mathbf{z}}^* \, \mathrm{d}\Omega^S, \tag{7}$$

$$c_{\mathbf{u}}(\mathbf{z}, \bar{\mathbf{z}}) = \int\int_{\Omega^S} C \mathbf{z}^{\mathrm{T}} \bar{\mathbf{z}}^* \, \mathrm{d}\Omega^S, \tag{8}$$

$$a_{\mathbf{u}}(\mathbf{z}, \bar{\mathbf{z}}) = \int\int_{\Omega^S} \boldsymbol{\sigma}(\mathbf{z})^{\mathrm{T}} \boldsymbol{\varepsilon}(\bar{\mathbf{z}}^*) \, \mathrm{d}\Omega^S, \tag{9}$$

$$\ell_{\mathbf{u}}(\bar{\mathbf{z}}) = \int\int_{\Omega^S} \mathbf{f}^{b^{\mathrm{T}}} \bar{\mathbf{z}}^* \, \mathrm{d}\Omega^S + \int_{\Gamma^s} \mathbf{f}^{s^{\mathrm{T}}} \bar{\mathbf{z}}^* \, \mathrm{d}\Gamma, \tag{10}$$

where $d_{\mathbf{u}}(\bullet,\bullet)$ is the kinetic sesqui-linear form, $c_{\mathbf{u}}(\bullet,\bullet)$ is the damping sesqui-linear form, $a_{\mathbf{u}}(\bullet,\bullet)$ is the structural sesqui-linear form, and $\ell_{\mathbf{u}}(\bullet)$ is the load semi-linear form. The definitions of the sesqui-linear and semi-linear forms can be found in Ref. [19].

Since the structure-induced pressure within the acoustic domain is related to the velocity response, it is convenient to transfer displacement to velocity using the relation

$$\mathbf{v}(\mathbf{x}) = \mathrm{j}\omega \mathbf{z}(\mathbf{x}). \tag{11}$$

By using Eqs. (6)–(11), the variational equation of the frequency-response problem can be obtained as

$$\mathrm{j}\omega d_{\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) + c_{\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) + \frac{1}{\mathrm{j}\omega} a_{\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) = \ell_{\mathbf{u}}(\bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z. \tag{12}$$

The structural damping, a variant of viscous damping, is caused either by internal material friction or by the connection between structural components. It has been experimentally observed that for each cycle of vibration the dissipated energy of the material is proportional to displacement [20]. When the damping coefficient is small, as in the case of structures, damping is primarily effective at those frequencies close to the resonance. The variational equation with the structural damping effect is

$$\mathrm{j}\omega d_{\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) + \kappa a_{\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) = \ell_{\mathbf{u}}(\bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z, \tag{13}$$

where $\kappa = (1 + \mathrm{j}\phi)/\mathrm{j}\omega$, and ϕ is the structural damping coefficient.

After the structure is approximated using finite elements, and kinematic boundary conditions are applied, the following system of matrix equations is obtained:

$$[\mathrm{j}\omega \mathbf{M} + \kappa \mathbf{K}]\{\mathbf{v}(\omega)\} = \{\mathbf{f}(\omega)\}, \tag{14}$$

where [**M**] is the mass matrix and [**K**] is the stiffness matrix.

2.2. Acoustic boundary element method

From the structure's velocity result, the boundary element method is used to evaluate the pressure response in an acoustic domain. The standard wave equation is reduced to the Helmholtz equation [21] in the harmonic response problem as

$$\nabla^2 p + k^2 p = 0, \tag{15}$$

where p is the pressure, $k(=\omega/c)$ is the wave number, c is the velocity of the wave propagation, and ∇^2 is the Laplace operator.

For BEM, the structural behavior must first be computed, and then it can be used as a boundary condition to compute radiated noise p. If the acoustic domain is considered to be in R^3, then the boundary of this domain constitutes the structure's domain, Ω^S. By integrating over the domain and by using Green's theorem, the Helmholtz equation (15) constitutes the boundary integral equation [21] as

$$\int\int_{\Omega^S}\left[G(\mathbf{x},\mathbf{x}_0)\frac{\partial p}{\partial n}-p(\mathbf{x})\frac{\partial G}{\partial n}\right]\mathrm{d}\Omega^S=\alpha p(\mathbf{x}_0), \tag{16}$$

where $G(\mathbf{x},\mathbf{x}_0)$ is Green's function, $\mathbf{x}$ is the position of a reference point, $\mathbf{x}_0$ is the position of an observation point, $\partial/\partial n$ is the normal component of the gradient, and S is the acoustic boundary, which is again a structural domain. In Eq. (16), the constant α is equal to 1 for $\mathbf{x}_0$ inside the acoustic volume, 0.5 for $\mathbf{x}_0$ on a smooth boundary surface, and 0 for $\mathbf{x}_0$ outside the acoustic volume. Note that Eq. (16) can provide a solution for both radiation and interior acoustic problems.

On the surface of the acoustic boundary, the following relation between the pressure and the structural velocity is given:

$$\nabla p=-\mathrm{j}\rho\omega\mathbf{v}, \tag{17}$$

where ρ is the structural density and $\mathbf{v}$ is the acoustic velocity, which was computed from the frequency response in Eq. (13). If $\mathbf{x}_S$ is a point on the acoustic boundary surface, then the boundary integral equation (16) becomes

$$\int\int_{\Omega^S}\left[-\mathrm{j}\rho\omega G(\mathbf{x}_S,\mathbf{x}_0)v_n(\mathbf{x}_S)-\frac{\partial G}{\partial n}p(\mathbf{x}_S)\right]\mathrm{d}\Omega^S=\alpha p(\mathbf{x}_0), \tag{18}$$

where v_n is the normal component of surface velocity $\mathbf{v}$. For derivational convenience, Eq. (18) can be rewritten as

$$b(\mathbf{v}_0;\mathbf{v})+e(\mathbf{x}_0;p_S)=\alpha p(\mathbf{x}_0), \tag{19}$$

where $b(\mathbf{x}_0;\bullet)$ and $e(\mathbf{x}_0;\bullet)$ are linear integral forms that correspond to the left-hand side of Eq. (18). Note that unlike the structural forms in Eqs. (7)–(10), these integral forms are independent of the sizing design variable; thus no subscribed $\mathbf{u}$ is used in their definitions.

The boundary element method has two steps: first evaluating the pressure variable on the acoustic boundary using the structural velocity, and then calculating the pressure variable within the acoustic domain using the boundary pressure information. Let the acoustic boundary S be approximated by N number of nodes. If observation point $\mathbf{x}_0$ is positioned at every node, then the following linear system of equations is obtained:

$$[\mathbf{A}]\{\mathbf{p}_S\}=[\mathbf{B}]\{\mathbf{v}\}, \tag{20}$$

where $\{\mathbf{p}_S\}=\{p_1,p_2,\dots,p_N\}^{\mathrm{T}}$ is the nodal pressure vector, $\{\mathbf{v}\}$ is the $3N\times 1$ velocity vector, $[\mathbf{A}]$ is the $N\times N$ coefficient matrix, and $[\mathbf{B}]$ is the $N\times 3N$ coefficient matrix. Note that these vectors and matrices are all complex variables. The process of computing the boundary pressure $\{\mathbf{p}_S\}$ assumes domain discretization, and the condition in Eq. (19) is imposed in every node. However, for the purposes of DSA, let us consider a continuous counterpart to Eq. (20), defined as

$$A(p_S) = B(\mathbf{v}), \tag{21}$$

where the integral forms $A(\bullet)$ and $B(\bullet)$ correspond to the matrices [**A**] and [**B**] in Eq. (20), respectively. The boundary pressure can then be calculated from $p_S = A^{-1} \circ B(\mathbf{v})$.

Once $\{\mathbf{p}_S\}$ has been computed, Eq. (19) can be used to compute the acoustic pressure at any point $\mathbf{x}_0$ within the acoustic domain in the form of a vector equation as

$$p(\mathbf{x}_0) = \{\mathbf{b}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{v}\} + \{\mathbf{e}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{p}_S\}, \tag{22}$$

where $\{\mathbf{b}(\mathbf{x}_0)\}$ and $\{\mathbf{e}(\mathbf{x}_0)\}$ are the column vectors that correspond to the left-hand side of the boundary integral equation (18).

In a sizing design problem, in which panel thickness is a design variable, integral forms $b(\mathbf{x}_0; \bullet)$ and $e(\mathbf{x}_0; \bullet)$ in Eq. (19) are independent of the design variable. Only implicit dependence on the design exists through the state variable $\mathbf{v}$ and p, which will be developed in the following section. However, in a shape design problem, the acoustic domain changes according to the structural domain change, which is a design variable. Thus, integral forms $b(\mathbf{x}_0; \bullet)$ and $e(\mathbf{x}_0; \bullet)$ depend on the design. Such a problem, however, is not investigated in this study.

3. Design sensitivity analysis

The purpose of design sensitivity analysis (DSA) is to compute the dependency of performance measures on the design. In this study, only sizing design is considered, such as the thickness of a plate and the cross-sectional dimension of a beam.

3.1. Design sensitivity formulas

Assume that $\psi(\mathbf{u})$ is continuously differentiable with respect to design $\mathbf{u}$. If the design is perturbed in the direction of $\delta\mathbf{u}$ (arbitrary), and τ is a parameter that controls the perturbation size, then the variation of $\psi(\mathbf{u})$ in the direction of $\delta\mathbf{u}$ is defined as

$$\psi'_{\delta\mathbf{u}} \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}\psi(\mathbf{u} + \tau\delta\mathbf{u})\bigg|_{\tau=0} = \frac{\partial\psi^{\mathrm{T}}}{\partial\mathbf{u}}\delta\mathbf{u}. \tag{23}$$

Throughout this paper, prime "$'$" plays precisely the same role as the first variation in the calculus of variations. For convenience, subscribed $\delta\mathbf{u}$ will often be ignored. The term "derivative" or "differentiation" will often be used to denote the variation in Eq. (23). If the variation of a function is continuous and linear with respect to $\delta\mathbf{u}$, the function is differentiable (even more precisely, it is Fréchet differentiable).

It is also assumed that the solution to the frequency-response problem in Eq. (13) and the solution to the boundary integral equation (19) are differentiable with respect to the design. That is, the following forms of variation exist:

$$\mathbf{v}' = \frac{\mathrm{d}}{\mathrm{d}\tau}[\mathbf{v}(\mathbf{x}, \mathbf{u} + \tau\delta\mathbf{u})]\bigg|_{\tau=0} = \frac{\partial\mathbf{v}}{\partial\mathbf{u}}\delta\mathbf{u} \tag{24}$$

and

$$p' = \frac{\mathrm{d}}{\mathrm{d}\tau}[p(\mathbf{x}, \mathbf{u} + \tau\delta\mathbf{u})]\bigg|_{\tau=0} = \frac{\partial p^{\mathrm{T}}}{\partial \mathbf{u}}\delta\mathbf{u}. \tag{25}$$

3.2. Direct differentiation method

A direct differentiation method computes the variation of state variables in Eqs. (24) and (25) by differentiating the state equations (13) and (19) with respect to the design. Let us first consider the structural part, i.e., the frequency-response analysis in Eq. (13). The forms that appear in Eq. (13) explicitly depend on the design, and their variations are defined as

$$d'_{\delta\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}[d_{\mathbf{u}+\tau\delta\mathbf{u}}(\tilde{\mathbf{v}}, \bar{\mathbf{z}})]\bigg|_{\tau=0}, \tag{26}$$

$$a'_{\delta\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}[a_{\mathbf{u}+\tau\delta\mathbf{u}}(\tilde{\mathbf{v}}, \bar{\mathbf{z}})]\bigg|_{\tau=0} \tag{27}$$

and

$$\ell'_{\delta\mathbf{u}}(\bar{\mathbf{z}}) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}[\ell_{\mathbf{u}+\tau\delta\mathbf{u}}(\bar{\mathbf{z}})]\bigg|_{\tau=0}, \tag{28}$$

where $\tilde{\mathbf{v}}$ denotes state variable $\mathbf{v}$ with the dependence on τ being suppressed, and $\bar{\mathbf{z}}$ and its complex conjugate are independent of the design. The detailed expressions of $d'_{\delta\mathbf{u}}(\bullet, \bullet)$, $a'_{\delta\mathbf{u}}(\bullet, \bullet)$, and $\ell'_{\delta\mathbf{u}}(\bullet)$ will be developed in Section 3.3 using analytical examples.

Thus, by taking a variation of both sides of Eq. (13) with respect to the design, and by moving terms explicitly dependent on the design to the right side, the following sensitivity equation can be obtained:

$$\mathrm{j}\omega d_{\mathbf{u}}(\mathbf{v}', \bar{\mathbf{z}}) + \kappa a_{\mathbf{u}}(\mathbf{v}', \bar{\mathbf{z}}) = \ell'_{\delta\mathbf{u}}(\bar{\mathbf{z}}) - \mathrm{j}\omega d'_{\delta\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}) - \kappa a'_{\delta\mathbf{u}}(\mathbf{v}, \bar{\mathbf{z}}), \quad \forall \bar{\mathbf{z}} \in Z. \tag{29}$$

Presuming that velocity $\mathbf{v}$ is given as a solution to Eq. (13), Eq. (29) is a variational equation, with the same sesqui-linear forms for displacement variation $\mathbf{v}'$. Note that the stiffness matrices corresponding to Eqs. (13) and (29) are the same, and that the right-hand side of Eq. (29) can be considered a fictitious load term. If a design perturbation $\delta\mathbf{u}$ is defined, and if the right-hand side of Eq. (29) is evaluated with the solution to Eq. (13), then Eq. (29) can be numerically solved to obtain $\mathbf{v}'$ using the finite element method. By interpreting the right-hand side of Eq. (29) as another load form, Eq. (29) can be solved by using the same solution process as the frequency-response problem in Eq. (13).

Now the acoustic aspect will be considered, which is represented by the boundary integral equation (19). A direct differentiation of Eq. (19) yields the following sensitivity equation:

$$b(\mathbf{x}_0; \mathbf{v}') + e(\mathbf{x}_0; p'_S) = \alpha p'(\mathbf{x}_0). \tag{30}$$

Since integral forms $b(\mathbf{x}_0; \bullet)$ and $e(\mathbf{x}_0; \bullet)$ are independent of the design, the above equation has exactly the same form as Eq. (19). Thus, using the solution ($\mathbf{v}'$) of the structural sensitivity equation (29), Eq. (30) can be used by following the same solution process as BEM, to obtain the pressure sensitivity result. Thus, like Eq. (20), the following matrix equation has to be solved in the discrete system:

$$[\mathbf{A}]\{\mathbf{p}'_S\} = [\mathbf{B}]\{\mathbf{v}'\}. \tag{31}$$

Then, like Eq. (22), the pressure sensitivity at point x_0 can be obtained from

$$p'(\mathbf{x}_0) = \{\mathbf{b}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{v}'\} + \{\mathbf{e}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{p}'_S\}. \tag{32}$$

This sensitivity calculation process is the same as the BEM solution process described from Eq. (20) to Eq. (22).

3.2.1. Structural performance measure

A general performance measure that represents a variety of structural responses can be written in integral form as

$$\psi_1 = \int\int_{\Omega^S} g(\mathbf{v}, \mathbf{u})\, \mathrm{d}\Omega^S. \tag{33}$$

where function $g(\mathbf{v}, \mathbf{u})$ is assumed to be continuously differentiable with respect to its arguments. The integral form of a performance measure in the above equation is not restricted in representing a general function. For example, if a function value at a point is required, then a Dirac-delta measure may be used inside the integration. The reason for introducing the integral form of a performance measure is that in FEM the pointwise definition of a function is meaningless, since the variational formulation enforces the definition of a function value in the sense of a Sobolev norm [22]. This is different from BEM, in which a function can be defined at a point. Note that ψ_1 is a complex functional in frequency-response analysis.

The variation of ψ_1 with respect to the design variable becomes

$$\begin{aligned} \psi'_1 &= \frac{\mathrm{d}}{\mathrm{d}\tau}\left[\int\int_{\Omega^S} g(\mathbf{v}(\mathbf{x}; \mathbf{u} + \tau\delta\mathbf{u}), \mathbf{u} + \tau\delta\mathbf{u})\mathrm{d}\Omega^S\right]\Bigg|_{\tau=0} \\ &= \int\int_{\Omega^S} (g_{,\mathrm{v}}^{\mathrm{T}}\mathbf{v}' + g_{,\mathbf{u}}^{\mathrm{T}}\delta\mathbf{u})\, \mathrm{d}\Omega^S, \end{aligned} \tag{34}$$

where $g_{,\mathbf{v}} = \partial g/\partial \mathbf{v}$ and $g_{,\mathbf{u}} = \partial g/\partial \mathbf{u}$ are column vectors, and their expressions are known from the definition of the function g. The objective of DSA is to obtain an explicit expression of ψ'_1 in terms of $\delta\mathbf{u}$. If the structural design sensitivity equation (29) is solved for the variation $\mathbf{v}'$, then the sensitivity of ψ_1 can be calculated from Eq. (34) using the numerical integration process.

3.2.2. Acoustic performance measure

Consider a performance measure that is defined at point $\mathbf{x}_0$ within the acoustic domain as

$$\psi_2(\mathbf{x}_0) = h(p(\mathbf{x}_0), \mathbf{u}), \tag{35}$$

where the function $h(p, \mathbf{u})$ is assumed to be continuously differentiable with respect to its arguments. Note that acoustic performance ψ_2 is not defined in the integral form, as was the case for structural performance ψ_1.

The variation of the performance measure with respect to the design variable becomes

$$\psi_2' = \frac{\mathrm{d}}{\mathrm{d}\tau}[h(p(\mathbf{x};\mathbf{u}+\tau\delta\mathbf{u}),\mathbf{u}+\tau\delta\mathbf{u})]\bigg|_{\tau=0} = h_{,p}p' + h_{,\mathbf{u}}^{\mathrm{T}}\delta\mathbf{u}, \tag{36}$$

where the expression of $h_{,p} = \partial h/\partial p$ and $h_{,\mathbf{u}} = \partial h/\partial\mathbf{u}$ are known from the definition of the function h. Thus, from the solution to the acoustic design sensitivity equation (30), the sensitivity of ψ_2 can readily be calculated. However, the calculation of p' also requires the solution to the structural sensitivity equation (29).

3.3. Adjoint variable method

Since the number of design variables is larger than the number of active constraints in many optimization problems, the adjoint variable method is attractive [18]. However, the adjoint variable method is known to be limited to a symmetric operator problem. In this section, the adjoint variable method is further extended to non-symmetric complex operator problems. Since the adjoint variable method is directly related to the performance measure, structural and acoustic performance measures are treated separately. In case of an acoustic performance measure, a sequential adjoint variable method is introduced.

3.3.1. Structural performance measure

To obtain an explicit expression for ψ_1' in terms of $\delta\mathbf{u}$, it is necessary to rewrite the first term in Eq. (34) explicitly in terms of $\delta\mathbf{u}$. As with the static problem, an adjoint equation can be introduced by replacing $\mathbf{v}'$ in Eq. (34) with the complex virtual displacement $\bar{\boldsymbol{\lambda}}$ and by equating it to the variational equation (13) with respect to adjoint variable $\boldsymbol{\lambda}^*$ as

$$\mathrm{j}\omega d_{\mathbf{u}}(\bar{\boldsymbol{\lambda}},\boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\bar{\boldsymbol{\lambda}},\boldsymbol{\lambda}) = \int\!\!\int_{\Omega^S} g_{,\mathbf{v}}^{\mathrm{T}}\bar{\boldsymbol{\lambda}}\,\mathrm{d}\Omega^S, \quad \forall\bar{\boldsymbol{\lambda}}\in Z, \tag{37}$$

where an adjoint solution, $\boldsymbol{\lambda}\in Z$, or equivalently its complex conjugate $\boldsymbol{\lambda}^*$, is desired. Note that the forms $d_{\mathbf{u}}(\bullet,\bullet)$ and $a_{\mathbf{u}}(\bullet,\bullet)$ are not symmetric with respect to their arguments, because their arguments are complex variables. Since Eq. (37) is satisfied for all $\bar{\boldsymbol{\lambda}}\in Z$, and since $\mathbf{v}'\in Z$, Eq. (37) may be evaluated at $\bar{\boldsymbol{\lambda}} = \mathbf{v}'$, to obtain

$$\mathrm{j}\omega d_{\mathbf{u}}(\mathbf{v}',\boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\mathbf{v}',\boldsymbol{\lambda}) = \int\!\!\int_{\Omega^S} g_{,\mathbf{v}}^{\mathrm{T}}\mathbf{v}'\,\mathrm{d}\Omega^S. \tag{38}$$

In addition, since the sensitivity equation (29) is satisfied for all $\bar{\mathbf{z}}\in Z$, and since $\boldsymbol{\lambda}\in Z$, Eq. (29) may be evaluated at $\bar{\mathbf{z}} = \boldsymbol{\lambda}$ to obtain

$$\mathrm{j}\omega d_{\mathbf{u}}(\mathbf{v}',\boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\mathbf{v}',\boldsymbol{\lambda}) = \ell_{\delta\mathbf{u}}'(\boldsymbol{\lambda}) - \mathrm{j}\omega d_{\delta\mathbf{u}}'(\mathbf{v},\boldsymbol{\lambda}) - \kappa a_{\delta\mathbf{u}}'(\mathbf{v},\boldsymbol{\lambda}). \tag{39}$$

It becomes apparent that the left-hand side of Eqs. (38) and (39) are exactly the same. Thus, from these two equations we obtain

$$\int\!\!\int_{\Omega^S} g_{,\mathbf{v}}^{\mathrm{T}}\mathbf{v}'\,\mathrm{d}\Omega^S = \ell_{\delta\mathbf{u}}'(\boldsymbol{\lambda}) - \mathrm{j}\omega d_{\delta\mathbf{u}}'(\mathbf{v},\boldsymbol{\lambda}) - \kappa a_{\delta\mathbf{u}}'(\mathbf{v},\boldsymbol{\lambda}). \tag{40}$$

Therefore, the terms that are implicitly dependent on the design in Eq. (34) are explicitly expressed in terms of $\delta\mathbf{u}$. By substituting the relation in Eq. (40) into Eq. (34), ψ' is explicitly represented in terms of $\delta\mathbf{u}$ as

$$\psi_1' = \int\int_{\Omega^S} g_{,\mathbf{u}}^{\mathrm{T}}\delta\mathbf{u}\ \mathrm{d}\Omega^S + \ell'_{\delta\mathbf{u}}(\boldsymbol{\lambda}) - \mathrm{j}\omega d'_{\delta\mathbf{u}}(\mathbf{v},\boldsymbol{\lambda}) - \kappa a'_{\delta\mathbf{u}}(\mathbf{v},\boldsymbol{\lambda}). \tag{41}$$

Specific expressions of ψ' for different performance measures and different structural components will be developed in detail in the analytical example section.

3.3.2. Acoustic performance measure

The acoustic performance ψ_2 in Eq. (35) is defined at point $\mathbf{x}_0$, and its sensitivity expression in Eq. (36) contains p', which has to be explicitly expressed in terms of $\delta\mathbf{u}$. The objective is to express p' in terms of $\mathbf{v}'$ such that the adjoint problem defined in the previous section can be used. By substituting the relation in Eq. (30) into the sensitivity expression of Eq. (36), and by using the relation in Eq. (21), we obtain

$$\begin{aligned}\psi_2' &= h_{,\mathbf{u}}^{\mathrm{T}}\delta\mathbf{u} + h_{,p}p' \\ &= h_{,\mathbf{u}}^{\mathrm{T}}\delta\mathbf{u} + h_{,p}[b(\mathbf{x}_0;\mathbf{v}') + e(\mathbf{x}_0;A^{-1}\circ B(\mathbf{v}'))].\end{aligned} \tag{42}$$

In Eq. (42), $\alpha = 1$ is used since $\mathbf{x}_0$ is the interior point. Thus, ψ_2' is expressed in terms of $\mathbf{v}'$. The second term on the right-hand side of the above equation can be used to define the adjoint load by substituting $\bar{\boldsymbol{\lambda}}$ for $\mathbf{v}'$. Hence, the following form of the adjoint problem is obtained:

$$\mathrm{j}\omega d_{\mathbf{u}}(\bar{\boldsymbol{\lambda}},\boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\bar{\boldsymbol{\lambda}},\boldsymbol{\lambda}) = h_{,p}[b(\mathbf{x}_0;\bar{\boldsymbol{\lambda}}) + e(\mathbf{x}_0;A^{-1}\circ B(\bar{\boldsymbol{\lambda}}))], \quad \forall\bar{\boldsymbol{\lambda}}\in Z, \tag{43}$$

where an adjoint solution $\boldsymbol{\lambda}^*$ is desired. By following the same process that is described from Eqs. (37) to (41), the sensitivity of ψ_2 can be obtained as

$$\psi_2' = h_{,\mathbf{u}}\delta\mathbf{u} + \ell'_{\delta\mathbf{u}}(\boldsymbol{\lambda}) - j\omega d'_{\delta\mathbf{u}}(\mathbf{v},\boldsymbol{\lambda}) - \kappa a'_{\delta\mathbf{u}}(\mathbf{v},\boldsymbol{\lambda}). \tag{44}$$

It is interesting to note that even if ψ_2 is a function of pressure p, its sensitivity expression in Eq. (44) does not require the value of p; only the structural solution $\mathbf{v}$ and the adjoint solution $\boldsymbol{\lambda}^*$ are required in the calculation of ψ_2'.

Even if Eq. (44) looks similar to the structural performance measure in Eq. (41), a fundamental difference exists in the calculation of the adjoint load in Eq. (43). To illustrate, consider a discrete form of the adjoint load. Eq. (43) can be written in the discrete system as

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]\{\boldsymbol{\lambda}^*\} = \{\mathbf{b}\} + [\mathbf{B}]^{\mathrm{T}}[\mathbf{A}]^{-\mathrm{T}}\{\mathbf{e}\}, \tag{45}$$

where the right-hand side corresponds to the adjoint load in the discrete system. Instead of computing the inverse matrix, let us define an acoustic adjoint problem in BEM as

$$[\mathbf{A}]^{\mathrm{T}}\{\boldsymbol{\eta}\} = \{\mathbf{e}\}, \tag{46}$$

where the acoustic adjoint solution $\{\boldsymbol{\eta}\}$ is desired. Even if the coefficient matrix $[\mathbf{A}]$ is not symmetric, the adjoint equation (46) can still use the factorized matrix of the boundary element equation (20). By substituting $\{\boldsymbol{\eta}\}$ into Eq. (45), we obtain the structural adjoint problem, as

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]\{\boldsymbol{\lambda}^*\} = \{\mathbf{b}\} + [\mathbf{B}]^{\mathrm{T}}\{\boldsymbol{\eta}\}. \tag{47}$$

Note that the acoustic adjoint solution $\{\boldsymbol{\eta}\}$, which is obtained from BEM, is required to compute the structural adjoint load, and frequency-response re-analysis then provides the structural adjoint solution $\{\boldsymbol{\lambda}^*\}$. Thus, two different adjoint problems are defined: the first is similar to BEM, and is used to compute the adjoint load, while the second is similar to the structural frequency-response problem.

3.4. Analytical examples

In many structural–acoustic problems, a structural part is described by using a plate/shell component, and an acoustic domain is enclosed by the structure. A typical design problem would reduce sound pressure levels in the passenger position by changing the plate thickness. In such a problem, the design variable is the thickness of a plate/shell component, and the performance measure is the sound pressure level at selected points in the acoustic domain. Also, in order to reduce the radiated acoustic power from the structure, the structure's velocity can be also considered as a performance measure.

The structural variational equation of harmonic motion is given by Eq. (13). The objective is to derive explicit forms of $d_{\mathbf{u}}(\bullet,\bullet)$, $a_{\mathbf{u}}(\bullet,\bullet)$, and $\ell_{\mathbf{u}}(\bullet)$ for a plate/shell component. In general, a shear-deformable plate/shell has three translation degrees of freedom and two rotational degrees of freedom. Thus, the structural state variable $\mathbf{z}$ is defined by

$$\mathbf{z} = [z_1, z_2, z_3, \theta_1, \theta_2]^{\mathrm{T}}. \tag{48}$$

Strain is decomposed into membrane, bending, and transverse shear parts as

$$\boldsymbol{\varepsilon}^m = \begin{bmatrix} z_{1,1} \\ z_{2,2} \\ z_{1,2} + z_{2,1} \end{bmatrix}, \quad \boldsymbol{\kappa} = \begin{bmatrix} \theta_{1,1} \\ \theta_{2,2} \\ \theta_{1,2} + \theta_{2,1} \end{bmatrix}, \quad \boldsymbol{\gamma} = \begin{bmatrix} z_{3,2} - \theta_2 \\ z_{3,1} - \theta_1 \end{bmatrix}. \tag{49}$$

Note that the strain resultants given in Eq. (49) have the following properties: $\theta_{1,1}$ is the curvature in the x_1 direction, $\theta_{2,2}$ is the curvature in the x_2 direction, and $(\theta_{1,2} + \theta_{2,1})$ is the twisting curvature. In Eq. (49), $z_{3,2} - \theta_2$ and $z_{3,1} - \theta_1$ are the shear rotation in the 2–3 and 1–3 plane, respectively. By using the above definitions, the structural sesqui-linear form [23] is defined as

$$a_{\mathbf{u}}(\mathbf{z}, \bar{\mathbf{z}}) = \int\!\!\int_{\Omega^S} [h\boldsymbol{\varepsilon}^m(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\varepsilon}^m(\mathbf{z}) + \frac{h^3}{12}\boldsymbol{\kappa}(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\kappa}(\mathbf{z}) + h\boldsymbol{\gamma}(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{D}\boldsymbol{\gamma}(\mathbf{z})]\,\mathrm{d}\Omega^S, \tag{50}$$

where

$$\mathbf{C} = \frac{E}{1 - \nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1 - \nu)/2 \end{bmatrix}, \quad \mathbf{D} = \frac{E\xi}{2(1 + \nu)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{51}$$

and ξ is the shear correction factor, compensating for the assumption of constant shear strain along the cross-section. The three terms within the integral of Eq. (50) represent the membrane, bending, and transverse shear contribution. Since $a'_{\delta\mathbf{u}}(\bullet,\bullet)$ is the explicit derivative of $a_{\mathbf{u}}(\bullet,\bullet)$

with respect to h,

$$d'_{\delta\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} [\boldsymbol{\varepsilon}^m(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\varepsilon}^m(\mathbf{z}) + \frac{h^2}{4}\boldsymbol{\kappa}(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\kappa}(\mathbf{z}) + \gamma(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{D}\boldsymbol{\gamma}(\mathbf{z})]\delta h \,\mathrm{d}\Omega^S. \tag{52}$$

From the known structural solution $\mathbf{z}$ (or $\mathbf{v}$), the structural variation of Eq. (52) can be calculated using the numerical integration process.

For plate/shell components, the kinetic sesqui-linear form and its variation can be defined as

$$d_{\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} \rho h \mathbf{z}^{\mathrm{T}}\bar{\mathbf{z}}^* \,\mathrm{d}\Omega^S \tag{53}$$

and

$$d'_{\delta\mathbf{u}}(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} (\rho \mathbf{z}^{\mathrm{T}}\bar{\mathbf{z}}^*)\delta h \,\mathrm{d}\Omega^S. \tag{54}$$

If the applied load consists of externally applied pressure $\mathbf{F}(\mathbf{x})$ and the self-weight given by

$$\mathbf{f}(\mathbf{x}) = \mathbf{F}(\mathbf{x}) + \gamma \mathbf{g} h(\mathbf{x}), \tag{55}$$

where γ is the weight density of the plate, and $\mathbf{g}$ is a unit vector in the direction of gravity, then the load semi-linear form $\ell_{\mathbf{u}}(\bullet)$ and its variation can be defined as

$$\ell_{\mathbf{u}}(\bar{\mathbf{z}}) = \int\int_{\Omega^S} [\mathbf{F} + \gamma \mathbf{g} h]^{\mathrm{T}}\bar{\mathbf{z}}^* \,\mathrm{d}\Omega^S \tag{56}$$

and

$$\ell'_{\delta\mathbf{u}}(\bar{\mathbf{z}}) = \int\int_{\Omega^S} \gamma \mathbf{g}^{\mathrm{T}}\bar{\mathbf{z}}^* \delta h \,\mathrm{d}\Omega^S. \tag{57}$$

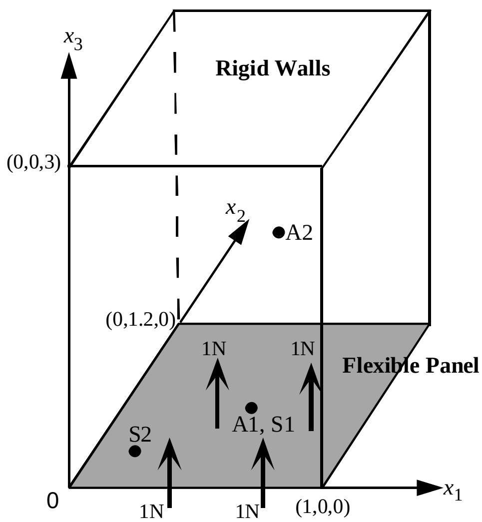

Fig. 1. Acoustic cavity with flexible wall.

Consider an acoustic cavity with a flexible panel, as illustrated in Fig. 1. The cavity is surrounded on all but one side by rigid walls, and the open side is closed by a clamped panel of linear elastic material with the structural damping coefficient φ. The panel's uniform thickness, h, is selected as the design variable, i.e., $\mathbf{u}(\mathbf{x}) = \{h\}$. Let us consider such performance measures as the acoustic pressure $p(\mathbf{x}^{\mathrm{a}})$ at point $\mathbf{x}^{\mathrm{a}}$ in the acoustic cavity, and the x_3 directional velocity $v_3(\mathbf{x}^s)$ at point $\mathbf{x}^{\mathrm{s}}$ on the structural panel. A harmonic force $f(\mathbf{x}, t)$ with frequency ω is applied to the plate. Here, $f(\mathbf{x}, t)$ is assumed to be independent of the design variable $\mathbf{u}(\mathbf{x})$.

3.4.1. Structural performance measure

The performance measure in this example is the vertical velocity at point $\mathbf{x}^s$, whose mathematical expression is

$$\psi_1 = v_3(\mathbf{x}^s) = \int\int_{\Omega^S} \delta(\mathbf{x} - \mathbf{x}^s) v_3 \, \mathrm{d}\Omega^S, \tag{58}$$

where $\delta(\bullet)$ is the Dirac-delta measure at zero. Eq. (58) is a simple form of Eq. (33), which is the general form for a structural performance measure. The variation of ψ_1 is

$$\psi_1' = v_3'(\mathbf{x}^s) = \int\int_{\Omega^S} \delta(\mathbf{x} - \mathbf{x}^s) v_3' \, \mathrm{d}\Omega^S. \tag{59}$$

Working from Eq. (37), the corresponding adjoint equation is obtained as

$$\mathrm{j}\omega d_{\mathbf{u}}(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) = \int\int_{\Omega^S} \delta(\mathbf{x} - \mathbf{x}^s) \bar{\boldsymbol{\lambda}}_3 \, \mathrm{d}\Omega^S, \quad \forall \bar{\boldsymbol{\lambda}} \in Z. \tag{60}$$

In Eq. (60), the term on the right-hand side is the adjoint load for the structural velocity. The physical meaning of the adjoint load, which corresponds to the harmonic velocity at a point, is a unit harmonic force applied at point x^s. The design sensitivity of ψ_1 is obtained from Eq. (41) as

$$\psi_1' = \ell'_{\delta\mathbf{u}}(\boldsymbol{\lambda}) - \mathrm{j}\omega d'_{\delta\mathbf{u}}(\mathbf{v}, \boldsymbol{\lambda}) - \kappa a'_{\delta\mathbf{u}}(\mathbf{v}, \boldsymbol{\lambda}). \tag{61}$$

The fact that the primary state equation (13) and the adjoint equation (60) represent the same structure with different loads provides an efficient method for numerical implementation, since only one finite element model is required to solve both the original and adjoint equations. Substituting the variations of those forms in Eqs. (52), (54), and (57), the design sensitivity expression becomes

$$\begin{aligned} \psi_1' = {} & \int\int_{\Omega^S} \gamma \mathbf{g}^{\mathrm{T}} \boldsymbol{\lambda}^* \delta h \, \mathrm{d}\Omega^S - \mathrm{j}\omega \int\int_{\Omega^S} (\rho \mathbf{v}^{\mathrm{T}} \boldsymbol{\lambda}^*) \delta h \, \mathrm{d}\Omega^S \\ & - \kappa \int\int_{\Omega^S} [\boldsymbol{\varepsilon}^m(\boldsymbol{\lambda}^*)^{\mathrm{T}} \mathbf{C} \boldsymbol{\varepsilon}^m(\mathbf{v}) + \frac{h^2}{4} \boldsymbol{\kappa}(\boldsymbol{\lambda}^*)^{\mathrm{T}} \mathbf{C} \boldsymbol{\kappa}(\mathbf{v}) + \boldsymbol{\gamma}(\boldsymbol{\lambda}^*)^{\mathrm{T}} \mathbf{D} \boldsymbol{\gamma}(\mathbf{v})] \delta h \, \mathrm{d}\Omega^S. \end{aligned} \tag{62}$$

Thus, ψ_1' is expressed in terms of δh.

3.4.2. Acoustic performance measure

Consider a pressure performance measure at point $\mathbf{x}^a$, given as

$$\psi_2 = p(\mathbf{x}^a). \tag{63}$$

Eq. (63) is a simple form of Eq. (35), a general form of the acoustic performance measure. The variation of the performance measure, corresponding to Eq. (42), is

$$\psi_2' = p'(\mathbf{x}^a) = b(\mathbf{x}^a; \mathbf{v}') + e(\mathbf{x}^a; A^{-1} \circ B(\mathbf{v}')). \tag{64}$$

The adjoint equation for ψ_2' is obtained using Eq. (43) as

$$\mathrm{j}\omega d_{\mathbf{u}}(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) + \kappa a_{\mathbf{u}}(\bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda}) = b(\mathbf{x}^a; \bar{\boldsymbol{\lambda}}) + e(\mathbf{x}^a; A^{-1} \circ B(\bar{\boldsymbol{\lambda}})), \quad \forall \bar{\boldsymbol{\lambda}} \in Z. \tag{65}$$

The term on the right-hand side of this equation is referred to as the acoustic adjoint load. In actual implementation, the adjoint load is calculated from the secondary adjoint problem defined in Eq. (46). The discrete adjoint problem in Eq. (47) can then be solved for $\boldsymbol{\lambda}^*$. From Eq. (41), the design sensitivity expression of the acoustic pressure becomes

$$\psi_2' = \ell'_{\delta\mathbf{u}}(\boldsymbol{\lambda}) - \mathrm{j}\omega d'_{\delta\mathbf{u}}(\mathbf{v}, \boldsymbol{\lambda}) - \kappa a'_{\delta\mathbf{u}}(\mathbf{v}, \boldsymbol{\lambda}). \tag{66}$$

Note that the design sensitivity expression in Eqs. (61) and (66) has identical forms. Thus, the same numerical integration process can be used for both structural and acoustic performance measures. However, in case of an acoustic performance measure, the secondary adjoint problem in Eq. (46) must be solved in order to define the structural adjoint load.

4. Numerical examples

4.1. Numerical method

A structural–acoustic system is solved using both finite element and the boundary element methods. The variational equation of the harmonic motion of a continuum model, Eq. (13), can be reduced to a set of linear algebraic equations by discretizing the model into elements and by introducing shape functions and nodal variables for each element. It is assumed that the structural finite element and the acoustic boundary element meshes match at their interfaces. Acoustic pressure $p(x)$ and structural velocity $v(x)$ are approximated using shape functions and nodal variables for each element in the discretized model as

$$\left.\begin{aligned} \mathbf{v}(\mathbf{x}) &= \mathbf{N}_s(\mathbf{x})\mathbf{v}^e \\ p(\mathbf{x}) &= \mathbf{N}_a(\mathbf{x})\mathbf{p}^e \end{aligned}\right\}, \tag{67}$$

where $\mathbf{N}_s(\mathbf{x})$ and $\mathbf{N}_a(\mathbf{x})$ are matrices of shape functions for velocity and pressure, respectively, and $\mathbf{v}^e$ and $\mathbf{p}^e$ are the element nodal variable vectors. Substituting Eq. (67) into Eq. (13) and carrying out integration yields the same matrix equation as Eq. (14) rewritten here as

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]\{\mathbf{v}(\omega)\} = \{\mathbf{f}(\omega)\}. \tag{68}$$

After obtaining the structural velocity, BEM is used to evaluate the pressure response on the boundary, as well as within the acoustic domain, as explained in Section 2.2.

Fig. 2 shows the computational procedure for the adjoint variable method with a structural FEA and an acoustic BEA code. Even if FEM and BEM are used to evaluate the acoustic performance measure, only the structural response $\mathbf{v}$ is required to perform design sensitivity analysis. The adjoint load is calculated from the transposed boundary element analysis, and the

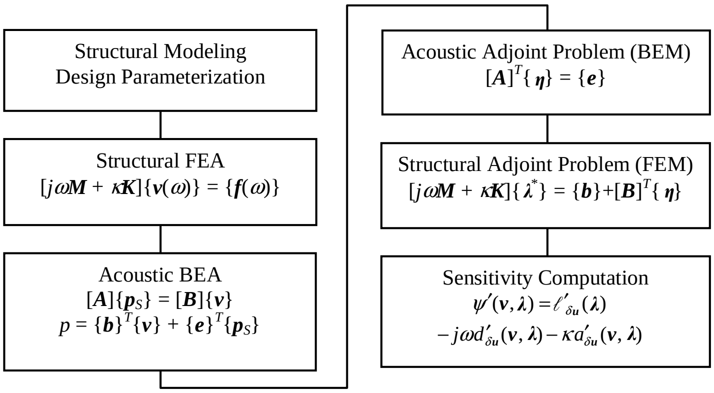

Fig. 2. Computational procedure of design sensitivity analysis.

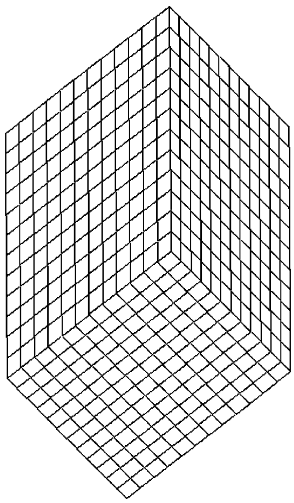

Fig. 3. An acoustic box model.

adjoint equations are then numerically solved using the FEA code with the same finite element model used for the original structural analysis.

Numerical solutions are used to compute the design sensitivity, and the integration of the design sensitivity expressions in Eq. (44) can be evaluated using a numerical integration method, such as the Gauss quadrature method [23]. The integrands are functions of the state variable, the adjoint variable, and gradients of both variables, as illustrated in Eq. (44).

4.2. Design sensitivity analysis of a box model

Fig. 3 depicts the acoustic cavity and the panel, previously discussed in Section 3.3. The acoustic medium in the cavity is air, with a mass density of $\rho_0 = 1.205\,\mathrm{kg/m^3}$ and wave propagation velocity of $c = 344\,\mathrm{m/s}$. The panel is an aluminum plate with thickness of 0.01 m, mass density of $\rho_s = 2700\,\mathrm{kg/m^3}$, Young's modulus of $E = 7.1 \times 10^{10}\,\mathrm{Pa}$, a Poisson's ratio of $\nu = 0.334$, and a structural-damping coefficient of $\varphi = 0.06$. A harmonic force $f = 1.0$ N in the x_3 direction is applied at four points on the plate as shown in Fig. 1. The whole structure is discretized by 864 elements and 866 nodes. In the frequency-response analysis, the five sides of the structure are fixed to simulate the rigid wall; only the bottom panel is allowed to move. In the acoustic analysis, the pressure value of each node is calculated from the structural velocity data and the pressure value in the acoustic cavity is then evaluated.

Panel thickness is chosen as the design variable, and only one design variable is considered in this example. The following design sensitivities are considered: the acoustic pressure at A_1 (0.5, 0.6, 0.), the interface point at the panel center; at A_2(0.5, 0.6, 1.5), the cavity center; and the panel velocity in the x_3 direction at point A_1. The MSC/NASTRAN program [24] is used for direct frequency analysis of the primary and adjoint structural problems, whereas BEM is used for the primary and adjoint acoustic problems. Fig. 4 provides the amplitude of pressure at points A_1 and A_2 for the frequency range between 1 and 140 Hz. The peak values of the pressure appear at frequencies corresponding to natural frequencies of the plate.

The design sensitivities are computed at 76 Hz, which is close to the resonant frequency, as shown in Fig. 4. The 2×2 Gauss quadrature formula is used for numerical integration over boundary elements. The design sensitivity results are shown in Table 1. Since the pressure ($p = p_r + \mathrm{j}p_i$) is a complex variable, the sensitivity of its amplitude can be calculated from the formula

$$|p|' = \frac{p_r p_r' + p_i p_i'}{|p|}, \tag{69}$$

where $p' = p_r' + \mathrm{j}p_i'$ is obtained from the design sensitivity analysis. The sensitivity of the velocity and displacement amplitudes can also be obtained using a similar method.

In Table 1, $\psi(u)$ and $\psi(u + \Delta u)$ are the frequency responses at designs u and $u + \Delta u$, respectively, where Δu is the amount of design perturbation. The forward finite difference design sensitivity is obtained by $\Delta\psi/\Delta u = (\psi(u + \Delta u) - \psi(u))/\Delta u$, and ψ' is the predicted design

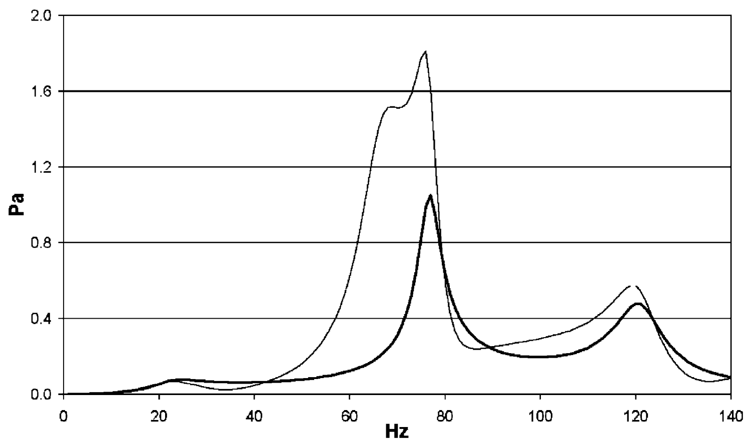

Fig. 4. Analysis results of acoustic cavity with flexible wall: —, A1 plate; —, A2 cavity.

Table 1
Sensitivity accuracy compared to the finite difference method

Performance type	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi\Delta u$	ψ'	$\Delta\psi/\Delta u/\psi' \times 100\%$
Displacement at A_1	3.27959E−5	3.27557E−5	−0.040403	−0.040181	100.55
Velocity at A_1	0.015668	0.0156416	−19.219	−19.187	100.17
Pressure at A_1	1.8118117	1.8096199	−2191.8	−2223.3	98.58
Pressure at A_2	0.9901643	0.9889635	−1200.8	−1198.4	100.20

sensitivity using the proposed method. A design perturbation of $\Delta u = 1.0 \times 10^{-6}$ m is used, and the predicted values are compared with the finite difference results. Table 1 presents design sensitivity results for the acoustic pressure, in Pascal (Pa), and for the structural velocity in the x_3 direction. Good agreement is obtained between ψ' and $\Delta\psi/\Delta u$. Since the applied load magnitude is fixed, an increase in panel thickness reduces plate vibration and radiated pressure. Consequently, all sensitivities are negative.

A major advantage of the adjoint variable method appears when a large number of design variables exist. In the early product development stage, for example, a design engineer may want to decide on the panel thickness for each section in order to minimize acoustic noise. To this end, the element sensitivity plot (Fig. 5) clearly shows the sensitivity of the pressure at the cavity center to the elements, and helps to determine new panel thicknesses. If the direct differentiation method is employed, then 144 design sensitivity equations must be solved in order to obtain such information, while with the adjoint variable method only one adjoint equation needs to be solved.

4.3. Design sensitivity analysis of a vehicle model

One of the important applications of this proposed method is structure-borne noise reduction of the commercial vehicle. Fig. 6 shows a concept design finite element model of a next generation hydraulic hybrid vehicle. In addition to power-train vibration and wheel/terrain interaction, a hydraulic pump is a source of vibration, considered as a harmonic excitation. Because of this additional source of excitation, vibration and noise is more significant than with a conventional power train. In this example, the noise level of the passenger compartment is chosen as the performance measure, and vehicle panel thicknesses are chosen as design variables. From the power train analysis and rigid-body dynamic analysis, the harmonic excitations at 12 locations are obtained. Frequency-response analysis is carried out using MSC/NASTRAN to obtain the velocity response at eight frequencies, which correspond to the peak values of the structure's velocity below 100 Hz.

After solving the structure's velocity response, an acoustic boundary element analysis is carried out using the cabin acoustic boundary element model, as shown in Fig. 6. Table 2 shows sound pressure levels at the driver's ear position. Since the sound pressure level at 93.6 Hz is significantly higher than at other frequencies, the design modification is carried out at that frequency. Fig. 7 shows the sound pressure level inside the cabin compartment. The maximum sound pressure level at the driver's ear is 77.8 dB when the reference pressure of 2×10^{-8} kg/mm s^2 is used.

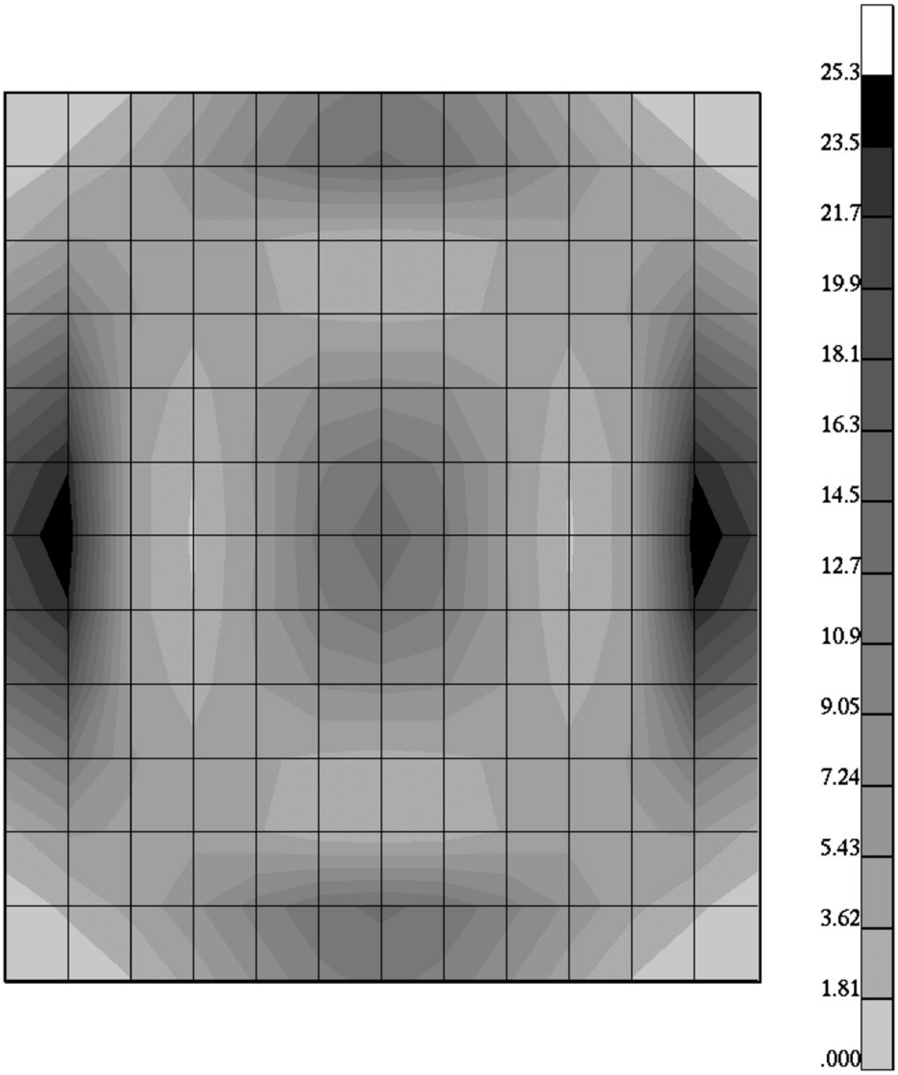

Fig. 5. (Negative of) element sensitivity for the pressure at the cavity center.

Forty design variables are selected in this example. First, the acoustic adjoint problem in Eq. (46) is solved, and the structural adjoint problem of Eq. (47) is then solved to obtain the adjoint response $\boldsymbol{\lambda}^*$. Using the velocity response $\mathbf{v}$ and the adjoint response $\boldsymbol{\lambda}^*$, the numerical integration process given in Eq. (66) calculates the sensitivity results for each structural panel, as shown in Table 3. The results show that a thickness change in the chassis component has the greatest potential for achieving a reduction in sound pressure levels. Since the numerical integration process is carried out on each finite element, the element sensitivity information can be calculated without any additional effort. Fig. 8 plots the sensitivity contribution of each element to the sound pressure level. Such graphic-based sensitivity information is very helpful to the design engineer to determine the direction of the design modification.

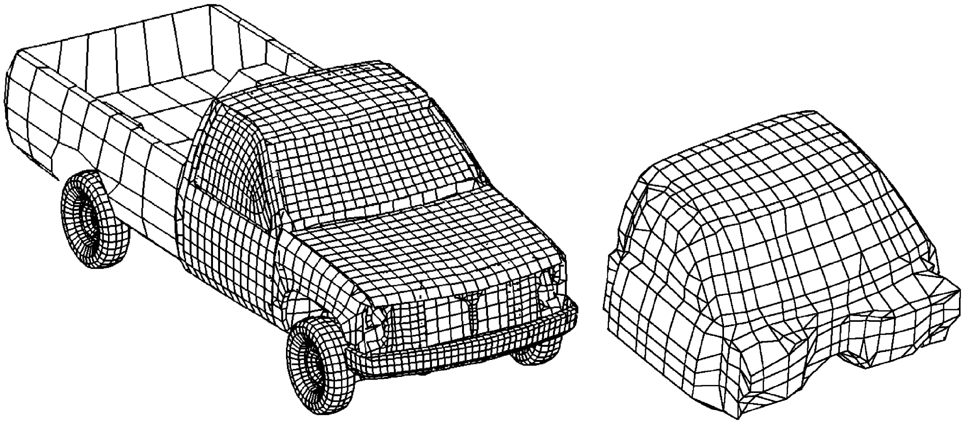

Fig. 6. Vehicle structure FE model and acoustic BE model of the cabin part.

Table 2
Sound pressure levels at the driver's ear position

Frequency (Hz)	Pressure (kg/mm s^2)	Phase angle (degree)
47.3	0.64275E−04	66.915
59.5	0.35889E−03	328.99
75.9	0.66052E−04	193.91
81.8	0.41081E−03	264.21
86.0	0.21629E−03	176.18
90.5	0.43862E−03	171.44
93.6	0.75627E−02	178.30
98.7	0.22676E−03	226.07

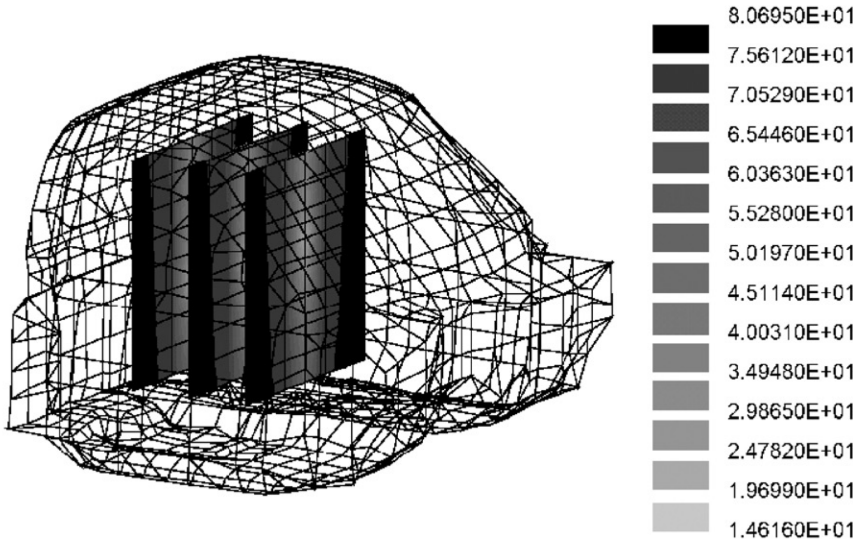

Fig. 7. Sound pressure level plot at the driver's position (max: 77.8 dB).

Table 3
Normalized sound pressure level sensitivity w.r.t. panel thickness

Component	Sensitivity	Component	Sensitivity
Chassis	−1.0	Chassis MTG	−0.11
Left wheelhouse	−0.82	Chassis connectors	−0.10
Right door	0.73	Right fender	−0.07
Cabin	−0.35	Left door	−0.06
Right wheelhouse	−0.25	Bumper	−0.03
Bed	−0.19	Rear glass	0.03

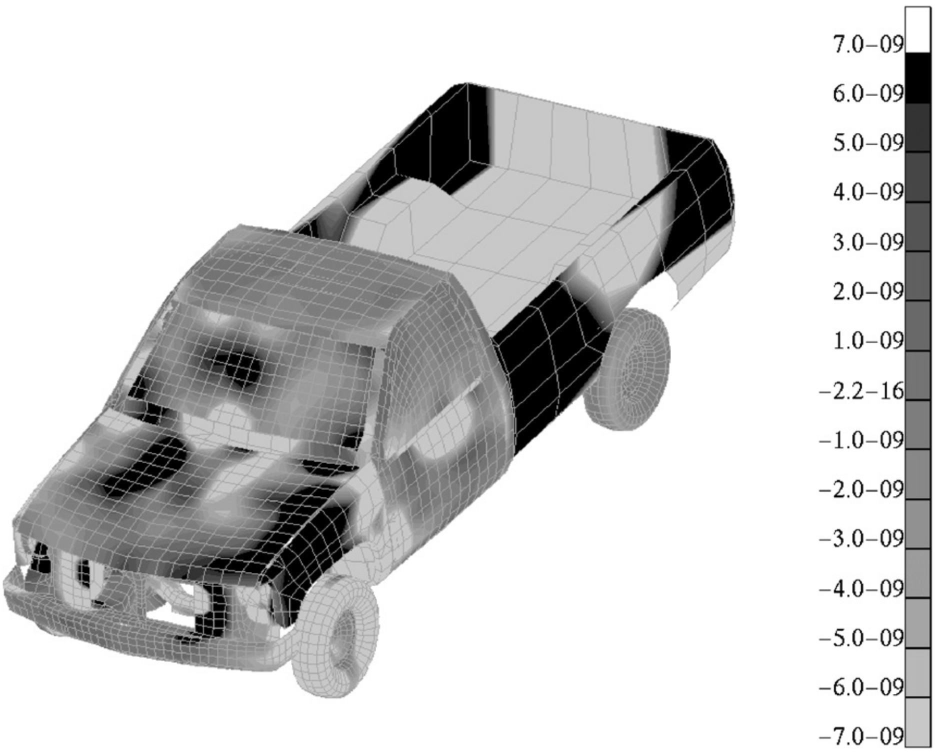

Fig. 8. Element design sensitivity plot with respect to panel thickness.

In Table 4, the accuracy of the proposed sensitivity result is compared to the sensitivity result calculated using the finite difference method. The vertical velocity at the center of the cabin roof is considered as a performance measure. The proposed sensitivity results agreed with the finite difference sensitivity results within a range of 10% when 0.1% of the thickness is perturbed.

As was shown in Table 3, the chassis component has the highest sensitivity for the sound pressure level, which means that a change in the thickness of the chassis component is the

Table 4
Design sensitivity result for v_z at the cabin roof center (initial value = 0.40293 mm/s, perturbation = 0.1%)

Design	Perturbed	FDM	DSA	Ratio (%)
Bumper	0.40292	−3.5739E−3	−3.9091E−3	91.43
Chassis	0.40196	−3.1287E−1	−3.0824E−1	101.50
Arm LL	0.40288	−9.8022E−3	−9.6368E−3	101.72
Arm LR	0.40250	−9.0502E−2	−9.6967E−2	93.33
Oil Box	0.40293	1.9519E−3	2.0538E−3	95.04
Brake FL	0.40289	−6.9373E−3	−6.4794E−3	107.07
Brake FR	0.40239	−1.0890E−1	−9.7718E−2	111.45
Chassis Conn	0.40274	−5.2836E−2	−5.2732E−2	100.20
Arm Conn UL	0.40293	−4.1533E−5	−4.1283E−5	100.60
Arm Conn UR	0.40293	−1.1367E−5	−1.0735E−5	105.89

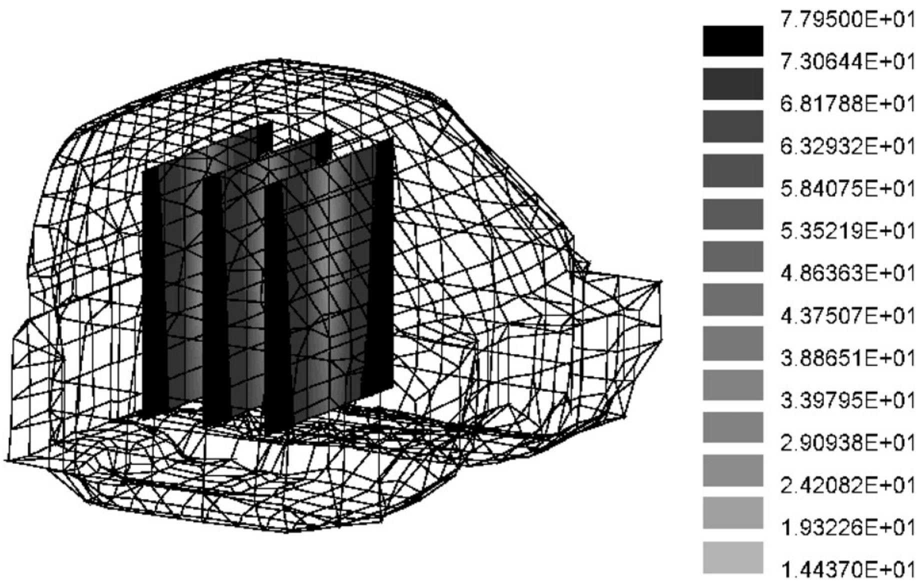

Fig. 9. Sound pressure level plot at updated design at driver's position (max: 75.0 dB).

most effective way to reduce the sound pressure level. The thickness of the chassis is therefore increased by 1.0 mm.The whole analysis process is repeated for the modified design. Fig. 9 shows sound pressure levels at the driver's ear when the excitation frequency is 93.6 Hz at the updated design. The maximum value of the sound pressure is reduced from 77.8 to 75.0 dB.

Structural–acoustic performance improvement at the updated design can be investigated further by considering the pressure results around the critical frequency. Fig. 10 plots the change in the level of sound pressure at the driver's ear for the initial and improved design.

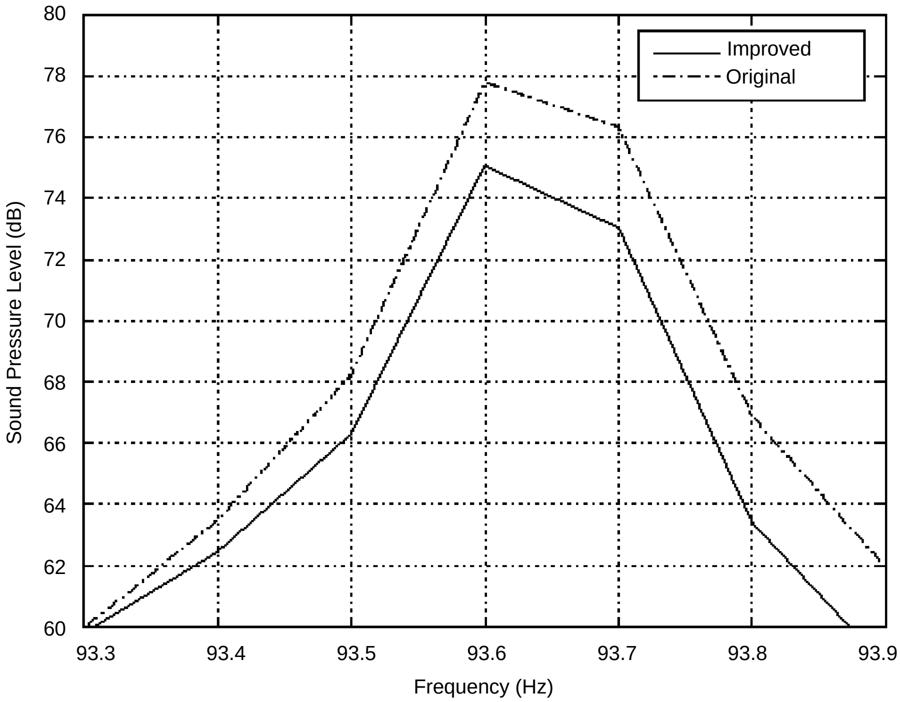

Fig. 10. Sound pressure level distribution for initial (–·–) and updated designs(—).

Thus, sound pressure levels are effectively reduced by increasing the thickness of the chassis component.

5. Conclusions

Based on the assumption that acoustic behavior does not influence structural behavior, a design sensitivity analysis of a sequential structural–acoustic problem is presented. Using the adjoint variable method, a sequential adjoint problem is presented in which the adjoint load is calculated by solving a boundary adjoint problem, and the adjoint solution is calculated from a structural adjoint problem. The sequential adjoint variable method significantly reduces computational costs compared to the direct differentiation method, as the number of performance measures is in general less than the number of design variables. In addition, the element sensitivity information provides valuable design information such as determining the location of stiffening ribs, instead of directly changing element thicknesses.

Acknowledgements

This research is supported by the Automotive Research Center that is sponsored by the US Army TARDEC under contract DAAE07-94-C-R094.

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