

Numerical analysis and parameter study of a mechanical damper for use in long slender endmills

Nam H. Kim*, Dongki Won, John C. Ziegert¹

Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611-6250, USA

Received 20 November 2004; accepted 4 July 2005

Available online 19 August 2005

Abstract

A mechanical damper has been introduced to reduce tool vibration during the high-speed milling process. The mechanical damper is composed of multi-fingered cylindrical inserts placed in a matching cylindrical hole in the center of a standard end-milling cutter. Centrifugal forces during high-speed rotation press the flexible fingers against the inner surface of the tool. Bending of the tool/damper assembly due to cutting forces or chatter vibration causes relative axial sliding between the tool inner surface and the damper fingers, and dissipates energy in the form of friction work. In this paper, a simple numerical method is presented to estimate the amount of friction work during tool bending. Non-linear static finite element analysis is used to estimate normal and frictional contact forces due to centrifugal forces and cutting forces, and calculate the amount of frictional work dissipated by the damper. The numerical results are compared with analytical results, and show a similar trend. Parameter studies are also carried out using the numerical model to identify the best configuration to maximize the amount of friction work.

© 2005 Elsevier Ltd. All rights reserved.

Keywords: Chatter; Mechanical damper; Finite element analysis; Parameter study; Optimization

1. Introduction

Milling is a widely used process for manufacture of discrete mechanical components. Numerous efforts have been made to improve the efficiency of milling [1] by reducing the machining time. The rate at which material can be removed in a milling process is limited by one of three factors:

- (a) Torque or power limitations of the drive motors on the machine,
- (b) Tool failure due to excessive wear or breakage, or
- (c) Chatter vibration.

For parts made from free-machining materials and with features requiring long, slender endmills, the process efficiency is nearly always limited by the onset of chatter,

a self-excited vibration. The addition of a mechanical damper to the cutting tool can potentially help to stabilize the system against chatter and allows higher productivity.

Various methods of detecting and preventing chatter have been proposed for machine tool systems. Cobb [2] developed two different types of dampers for boring bars: shear and compression dampers. These dampers use a viscoelastic element that deforms during vibration to dissipate energy. Tlustý et al. [3] and Soliman and Ismail [4] developed a chatter recognition system using a microphone to detect the frequency of chatter when it occurs. The system then selects a new spindle speed (according to the parameters of the system) with higher chatter stability. Recently, the fuzzy system is utilized in suppressing chatter vibration [5,6].

Much work in the field of structural damping has been done by Slocum [7,8], who uses layered beams with viscoelastic materials between the layers. When the beam vibrates, viscoelastic layer experiences an axial shear deformation, which causes dissipation of vibration energy. Nagaya et al. [9] introduced the auto-tuning magnetic damper and vibration absorber to control micro-vibration of milling machine heads.

* Corresponding author. Tel.: +1 352 846 0665; fax: +1 352 392 7303.
E-mail address: nkim@ufl.edu (N.H. Kim).

¹ Tel.: +1 352 392 9930.

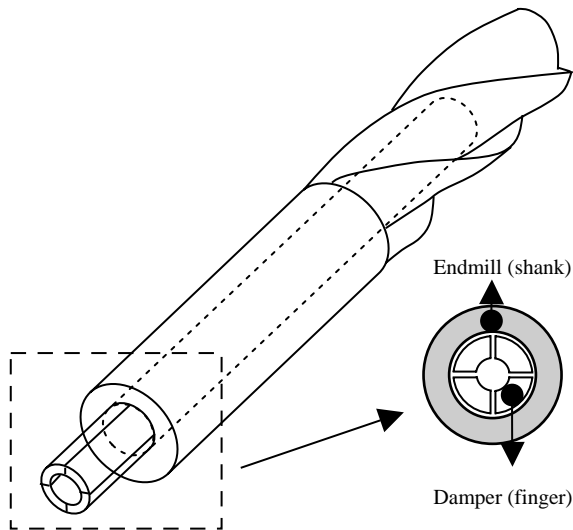


Fig. 1. Geometry of endmill and four-fingered mechanical damper.

Recently, Ziegert et al. [10,11] explored the possibility of deploying a mechanical damper directly inside a rotating cutting tool. The damper consists of a multi-fingered cylindrical insert placed inside a matching axial hole along the centerline of the milling cutter. During high-speed rotation, centrifugal forces press the outer surface of the insert fingers against the inner surface of the tool. During lateral (bending) vibrations of the tool, relative sliding occurs at the interface between the damper and tool inner surface, and the resulting frictional work in the contact interface dissipates energy and reduces vibration amplitude. They developed a simplified analytical model for the multi-fingered cylindrical damper and performed experiments.

In this paper, non-linear finite element analysis with frictional contact is used to study the mechanical damper, and calculate the amount of friction work during lateral bending of the tool. Although chatter vibration is a dynamic phenomenon, the amount of damping in the proposed system is directly dependent on the energy dissipated during lateral vibrations. If we assume that the contact pressure between the damping elements inside the tool is primarily due to centrifugal forces, i.e. the bending stiffness of the damping elements is small, then static finite element analysis is sufficient to predict frictional work during static bending. Although this is not a prediction of the damping in the dynamic system, additional energy dissipation will result in enhanced damping. Therefore, static finite element analysis can be used to optimize the design of the damper.

The system is first analyzed with the centrifugal force to compute the contact forces between the damper and tool. Next, a static lateral force is applied and the amount of friction work is calculated, giving a qualitative metric to evaluate damping performance and examine the effect of system design parameters on damper performance. Since the contact region and the contact force are unknown a priori, the problem is non-linear and an iterative solution

procedure (Newton–Raphson method) must be employed. The finite element analysis results are compared with the analytical results from Ziegert et al. [10].

The major advantage of the numerical method is that it does not make many simplifying assumptions of the analytical model and therefore can provide more realistic and accurate results [12,13]. Using the numerical model, one can easily change the system configuration and search for the optimum parameters that yield the best system performance. In this paper, the number of fingers on the damper insert and the inner radius of the damper are selected as design parameters. A parameter study is performed and the optimum configuration that maximizes friction work is identified based on the response surface.

The organization of the paper is as follows. In Section 2, finite element modeling techniques related to calculating friction work are introduced, including frictional contact modeling, sequential load application, friction work calculation, etc. The finite element analysis results are summarized in Section 3 along with the convergence of the solution and the effect of the initial tool position. In Section 4, the parameter study is presented and the best system configuration is identified, followed by conclusions in Section 5.

2. Finite element modeling

The finite element analysis procedure of the endmill system is presented in this section. Although, in reality, the cutting process is dynamic, static finite element analysis is performed using centrifugal forces to compute contact pressures, and a lateral force at the tip to induce bending into the tool. Thus, the friction work obtained must be interpreted as a qualitative measure.

2.1. Endmill and damper geometry

The cutting tool (endmill) analyzed in this paper has 19.05 mm (0.75 in.) outer diameter and 101.6 mm (4.0 in.) length. Although conventional endmills have a solid cylindrical cross section, the proposed mechanical damper requires an axial hole along the tool centerline [10] (see Fig. 1). When the multi-fingered cylindrical damper is inserted into the hollow tool, centrifugal forces from the high-speed spindle rotation cause high contact pressures between the damper fingers and the inner surface of the tool. When lateral bending of the system occurs, it causes a relative sliding motion between the damper and the tool due to their differences in neutral axis locations. This relative motion in conjunction with the contact pressure causes a friction stress at the interface, which dissipates the vibration energy. In this paper, this damping mechanism will be referred to as a mechanical damper.

While the geometry of the cutting edges of the tool is very important for cutting performance, it does not affect damper performance. Therefore, the tool can be simplified as

a hollow cylinder, which we refer to as the *shank*. The simplified ‘damper’ is also modeled as a hollow cylinder, slit along its length to form individual ‘fingers’. The inner diameter of the tool shank is set to 9.525 mm (0.375 in.) and it cannot be made larger because enough material must be left on the shank to allow cutting teeth to be formed. Thus, although a larger inner diameter of the tool might provide better damper performance, this is not considered as a design variable since these dimensions could not be used to produce the actual cutting tool. The *damper* has an outer diameter of 9.525 mm (0.375 in.). The inner diameter of the *damper* can be changed to maximize the frictional energy dissipation.

Although Fig. 1 shows only four fingers, the number of fingers can also be altered to improve damping performance. The parameter study detailed in Section 4 will examine the effect of varying the number of fingers as well as the damper inner diameter. Because a damper with one finger will not work in the manner described above, this case will not be considered.

2.2. Finite element model

The first step of finite element analysis is to build a computational model using the simplified geometry of the endmill. A commercial program, ANSYS [14], is used to perform non-linear finite element modeling and analysis for this research. Twenty-node solid elements (SOLID95) are used to build the shank and damper model, as illustrated in Fig. 2. The shank is modeled using two elements through the radial direction and 36 elements in the circumferential direction. The same number of elements is used for the damper, except that the elements are not connected along the slits, thus forming the individual fingers. A total of 4320 elements are used to model the shank and damper with 24,826 nodes. This number of elements was selected based on results of the convergence study in Section 3.1. The same material properties, Young’s modulus $E=206.78$ GPa, Poisson’s ratio $\nu=0.3$, and density $\rho=7820$ kg/m³, are assumed for both the shank and the damper for this analysis.

In the computational model shown in Fig. 2, the shank and damper are not connected. Instead, contact elements are created on the interface to prevent the parts from penetrating each other. The surface-to-surface contact algorithm is employed in this paper, with one surface denoted as

a ‘contact surface’ and the other surface as a ‘target surface’. The contact condition is then checked for each node on the contact surface and target surface. If any penetration is detected, either the penalty method [15] or the Lagrange multiplier method [16] is applied to the contact region to eliminate the penetration. In the computational model, 1080 contact elements and 1080 target elements are defined on the interface.

The Coulomb friction model is used to describe the frictional behavior at the interface. Although the friction coefficient is not measured experimentally, a conservative value of $\mu_f=0.15$ is used in this analysis.

Since the contact surface is circular, linear finite elements (eight-node solid) can cause significant error in approximating the geometry. When linear elements are used for the shank and the finger, the maximum error in the inner radius is 0.036 mm, which is inadmissible because the deformation in the radial direction due to the centrifugal force is about 0.0003 mm. Thus, the error in geometry interpolation would be larger than the deformation of the structure. The analysis result using linear finite elements confirms this observation as the distribution of the contact pressure is not similar to that estimated from the Hertz contact model [17]. In this paper, quadratic elements are used to represent the geometry of the tool and the contact surface so that the error in geometric interpolation is significantly reduced. Since the contact element is defined on the surface of the solid element, consistent order must be used for the tool (20-node solid element) and the contact surface (eight-node contact element).

In this paper, CONTA174 and TARGE170 in ANSYS are used for contact and target elements, respectively. CONTA174 is used to represent the contact and sliding between 3D ‘target’ surfaces and a deformable surface, defined by this element. This element is located on the surfaces of 3D solid or shell elements with mid-side nodes. It has the same geometric characteristics as the solid or shell element face with which it is connected. Contact occurs when the element surface penetrates one of the target segment elements on a specified target surface. TARGE170 is used to represent various 3D target surfaces for the associated contact elements. The contact elements themselves overlay the solid elements describing the boundary of a deformable body and are potentially in contact with the target surface. This target surface is discretized by a set of target segment elements (TARGE170) and is paired with its associated contact surface. In this paper, the damper is selected as a contact surface and the shank is selected as a target surface. It is important to note that the contact pressure and friction force data can only be calculated on the contact element and not the target element.

2.3. Boundary and load conditions

Fig. 2 illustrates that one end of the shank and damper is fixed throughout the analysis, approximating the clamping

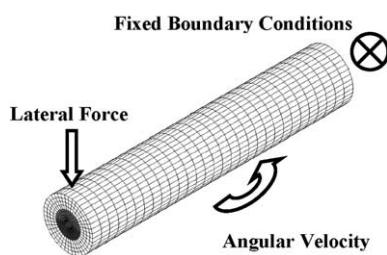


Fig. 2. Finite element model of the endmill and damper using 20-node solid elements and 8-node contact elements with boundary conditions.

of the tool in the holder. Since the end of the damper is not allowed to move in the radial direction, there will be no contact near the clamped end. This is one of the differences between the analytical and numerical models. The analytical model [10] assumes that contact occurs along the entire length of the damper, which is not the actual situation.

Even for small deformations and linear elastic behavior, the contact constraints introduce non-linearity into the problem. In ANSYS, a Newton–Raphson iterative method is employed to solve the non-linear system of equations, which is expensive but fast in convergence. All default parameters in ANSYS are used for this non-linear analysis.

The tool is subject to two different loads: centrifugal force due to tool rotation, and lateral forces from machining. If both these loads are applied simultaneously for the analysis, the computed frictional energy may be in error. Due to radial growth of the shank at high rotational speeds, there will be some sliding of the damper fingers along the inner surface of the shank. However, this sliding does not dissipate energy during machining. Only sliding resulting from bending of the tool should be used to evaluate damper energy dissipation. Therefore, the two loads are applied sequentially. First the tool is rotated with a constant angular velocity (26,000 rpm for this analysis), generating a contact force at the interface. Next, a lateral force is applied at the tip to simulate cutting forces during machining. In order to remove the artificial stress concentration, the lateral force is distributed to four nodes at the tip (see Fig. 2). Although the magnitude of the actual force depends on the workpiece material and machining parameters, a representative force of 100 N is used here because the objective of this paper is relative quantification of friction energy dissipation when various design parameters are altered.

Fig. 3 illustrates the sequence of loading conditions. First, the centrifugal force due to tool rotation is increased linearly with five sub-steps (Load Step 1) and then, the lateral force is applied gradually with five sub-steps (Load Step 2). During Load Step 2, the centrifugal force is maintained at a constant value. Any relative displacements

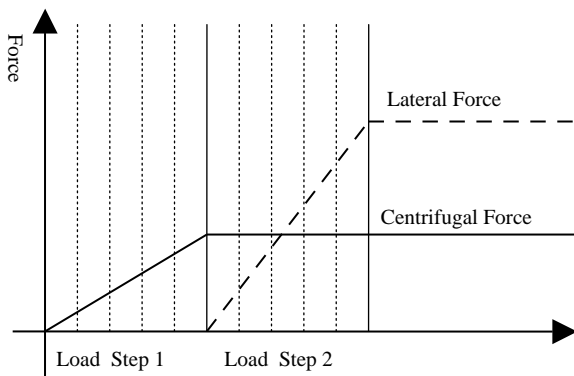


Fig. 3. Applied load conditions in each load step. Each load step is divided into five sub-steps.

at the contact interface during Load Step 1 are not used to calculate friction work. Only relative displacements during Load Step 2 are used to compute friction work, simulating the situation during machining. The use of additional loading sub-steps increases the computational time, but improves the convergence of non-linear analysis and the accuracy of the friction work computation.

3. Finite element analysis results

3.1. Effect of the element size

It is very important to determine the proper element size to obtain accurate simulation results. A fine mesh will usually yield an accurate result, but requires a large amount of computational cost. Since the finite element analysis needs to be repeated 45 times during the parameter study, and since the non-linear problem requires multiple matrix solutions, computational cost is an important issue. Because a reliable analytical solution does not exist, the convergence study is performed in order to decide the reasonable size of elements.

Fig. 4 plots the change of computed friction work as the number of elements is varied. A configuration with two fingers and 1.0 mm inner radius on the damper is used. When a small number of elements are used, the analysis underestimates the amount of friction work. The computed friction work appears to converge when 4320 are used. All subsequent modeling in this paper uses this number of elements.

3.2. Finite element analysis results

The work done by the friction force occurring between the inner surface of the endmill and the outer surface of the damper causes the damping effect that helps to stabilize

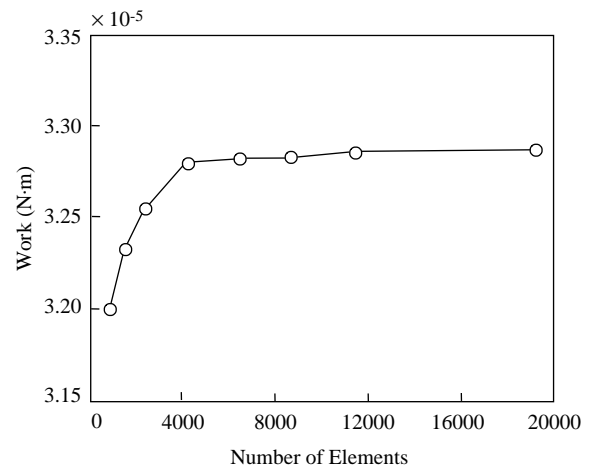


Fig. 4. Convergence with respect to the number of elements. Number of fingers is equal to two and the inner radius R_1 is equal to 1.0 mm

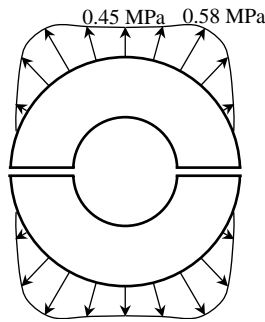


Fig. 5. Contact pressure distribution due to the centrifugal load.

the tool against chatter vibrations. According to the Coulomb friction model [16], the friction force and friction work can be expressed as

$$F_f = \mu_f \times N, \quad W_f = F_f \times U_f, \quad (1)$$

where F_f is the friction force, μ_f the friction coefficient, N the normal contact force, W_f the friction work, and U_f the relative displacement between the two contact surfaces. The normal force N is mainly caused by the centrifugal force created by the rotation of the endmill. The relative displacement U_f is mainly caused by the lateral deflection of the tool when the lateral force is applied at the tip.

When the centrifugal force is applied in the finite element model, contact occurs on the interface because the fingers are not constrained in the radial direction. Fig. 5 shows the distribution of the contact pressure along the circumference of the fingers. Note that the contact pressure is not uniformly distributed on the contact surface and the maximum value does not occur in the middle of the top and bottom fingers. This can be explained from the fact that both the shank and finger are deformable bodies.

The distribution of the contact pressure in Fig. 5 is different from the results in the analytical approach [10], which assumes a uniform contact pressure equal to the centrifugal acting on the finger divided by the projected contact area. The maximum contact pressure obtained from finite element analysis is 0.58 MPa, which is more than twice the constant contact pressure of 0.27 MPa that is used in the analytical model.

During Load Step 1, there is a relative motion at the contact surface due to the diameter change of the shank. However, this relative motion in Load Step 1 is not used in the computation of friction work, since it is not related to bending of the tool due to cutting forces.

In Load Step 2, a lateral force (simulating the cutting force) is applied in addition to the centrifugal force. The bending deformation of the shank and finger generates a relative displacement at the interface because of the difference in the location of the neutral axes of the elements. A maximum relative displacement of 0.0008 mm occurs at the bottom surface of the finger. No relative displacement is observed near the neutral axis of the shank. This is due to the fact that even for non-zero friction force, there will be no

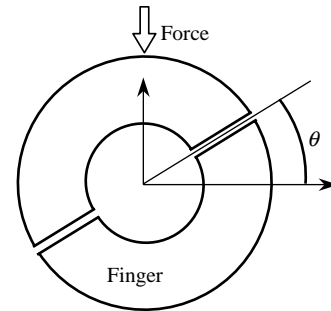


Fig. 6. Initial position of the finger with respect to the neutral axis of bending.

relative displacement until it becomes greater than $\mu_f \times N$ in the Coulomb model.

The amount of frictional work for Load Step 2 is calculated using the scalar product between the friction force vector and the relative displacement vector. Since the load step is divided into five sub-steps, friction work at each sub-step must be summed. In the case of the two-fingered damper, the total amount of friction work during Load Step 2 is 3.3426×10^{-5} N m.

3.3. Effect of finger position

In the numerical model, the tool shank and damper are assumed to be stationary, i.e. are not rotating, during the finite element analysis. Instead, the centrifugal force corresponding to a rotational speed of 26,000 rpm is imposed, followed by a static bending force. It has a particular direction relative to the axial slits in the damper, which form the fingers, because the bending force is static, and therefore, the calculated friction work depends on the start angle θ of the first finger (see Fig. 6). Changes in the start angle can change the amount of friction work even when all other design parameters are identical. Thus, it is necessary to find the maximum and the minimum friction work that can be generated based on the rotational angle θ .

Fig. 7 shows the maximum and the minimum values of friction work for different numbers of fingers when the inner radius is 1.0 mm. The result shows a large difference between the minimum and the maximum values when the number of fingers is small. However, the difference is reduced as the number of fingers is increased. Since the relative displacement has its maximum value at the bottom surface, the maximum value of friction work occurs for the configuration that has highest contact pressure at the bottom surface.

3.4. Comparison between the analytical and numerical results

The simplified analytical model developed by Ziegert et al. [10] assumes that the centrifugal force of the finger is uniformly distributed over the contact surface. Based on this

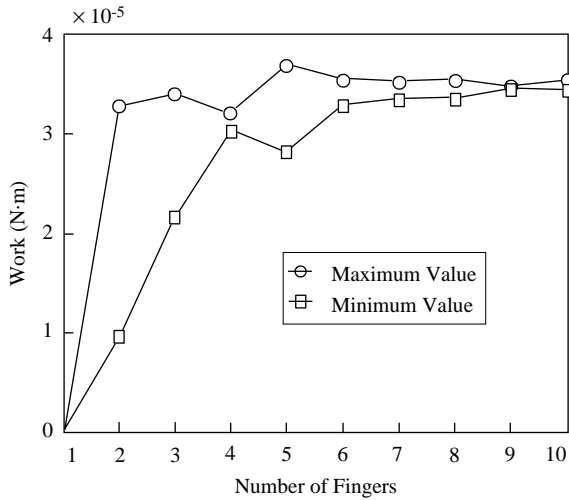


Fig. 7. The maximum and the minimum values of the damping work for the given number of fingers.

assumption and the finger geometry, the contact pressure can be analytically calculated as

$$P_c = \frac{M\omega^2}{A_c} R = \frac{M\omega^2}{A_c} \frac{2(R_2^3 - R_1^3)}{3(R_2^2 - R_1^2)} \frac{\sin \alpha}{\alpha} \quad (2)$$

where M is the mass of the finger; ω the angular velocity; A_c the contact area; R the distance between the rotation center and centroid of the finger; and α half of the angle of the arc that the finger occupies, i.e. two fingers, α is equal to $\pi/2$ (90°).

If the number of fingers is increased, the angle α approaches to zero, and the $(\sin \alpha)/\alpha$ term approaches one. Since the magnitude of friction work is proportional to the contact pressure, friction work is proportional to R , the distance from the rotational center to the centroid of the finger, which increases as the number of fingers increases. Thus, friction work increases along with the number of fingers, but its effect is reduced as the number of fingers increases.

Fig. 8 compares the analytical and numerical results for friction work as a function of the number of fingers. Friction work obtained from the finite element analysis is about 2.5 times less than that obtained from the analytical method. One possible explanation of the discrepancy is the assumption of uniform contact pressure in the analytical model.

In reality, the contact pressure is not constant and some portion of the finger does not contact with the shank. In addition, the largest relative displacement occurs at the bottom part of the endmill; this is because the lateral force is applied at the top and the non-linearity associated with the centrifugal force contributes to the asymmetry between the top and bottom fingers. During the non-linear analysis, ANSYS automatically updates the geometry and refers to the deformed configuration, which means the body force is calculated for the deformed geometry. Even though

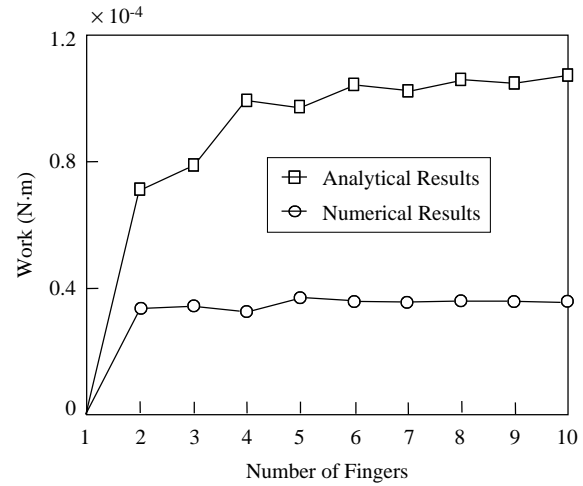


Fig. 8. Comparison of friction work estimated using analytical and numerical methods. The former is 2.5 times larger than the latter.

the analytical and numerical results show different magnitudes, the general trends are very similar to each other.

4. Parameter study

Parameter studies are performed to investigate the effect of damper design parameters on the friction work. Fig. 9 shows the effect of the first design parameter (the inner radius of the finger), and Fig. 10 shows that of the second design parameter (the number of fingers). In Fig. 9, the inner radius is changed from 1.0 to 3.5 mm for the two-fingered case. It is noted that work done by the friction force is initially increased and then decreases rapidly when the radius exceeds 2.0 mm. The initial increase can be

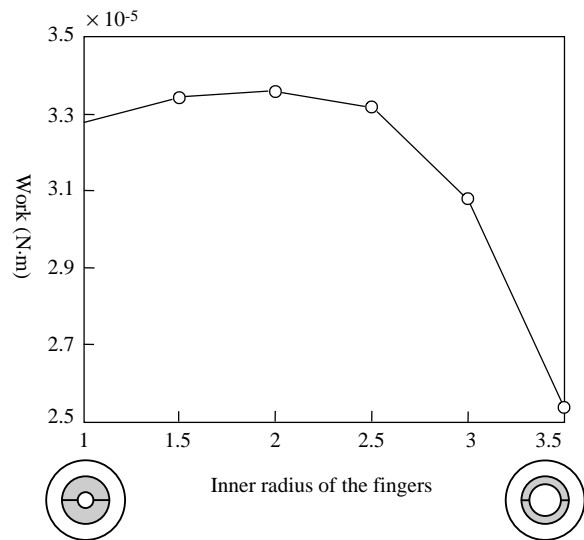


Fig. 9. Change of friction work with respect to the inner radius of the fingers.

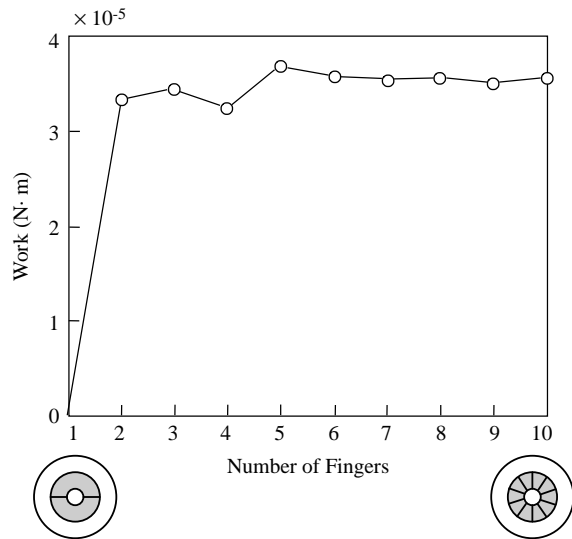


Fig. 10. Change of friction work with respect to the number of fingers.

explained from Eq. (2), the increase in the centroidal distance, R . However, as the number of fingers increases further, the rate of increase in R slows, and the decrease of the finger mass M becomes dominant. The analytical results in [10] showed a similar trend.

In Fig. 10, the number of fingers changed from two to ten for the case where the inner radius is fixed with 1.5 mm. In this graph, a line is shown joining the points for clarity, even though the number of fingers can only take on integer values. The initial angular position of the finger is chosen so that the damper produces the maximum friction work. It is noted that friction work for the four-fingered case is smaller than other cases, and the five-fingered performs the maximum friction work.

In order to find the configuration that yields the maximum amount of friction work, the first design parameter (inner radius of the finger, R_1) is varied over six different values and the second design parameter (number of fingers) by nine different values, simultaneously. Table 1 shows the results in 6×9 matrix form. In each configuration, the start angle is chosen such that the maximum friction work can be occurred. Fig. 11 plots the response surface of friction work with respect to two design

parameters. Again, although the second design parameter (number of fingers) is discrete, a continuous surface is plotted for illustration purposes. It is noted that the local peak, when the number of fingers is five, is maintained for all values of the inner radius. The general trend of the response surface is consistent. Based on the response surface, it can be concluded that friction work has its maximum value when the damper has five fingers and the inner radius is 1.5 mm. However, the large difference in the maximum and minimum damping values in this configuration, as shown in Fig. 7, may reduce the significance of this choice of design. The response surface is the most sensitive when the inner radius is large and the number of fingers is small. The effect of damping work increases as the number of fingers is increased and the inner radius is decreased.

Experimental verification of the FEA results is complicated by the fact that the interface pressure between the damper elements and the inner surface of the tool is generated by centrifugal forces occurring during high-speed rotation. However, we have performed experiments, which are reported in [10,11], detailing methods for measurement of frequency-response functions for rotating systems, methods for predicting chatter stability for systems where the dynamics changes with rotational speed; and cutting stability tests for endmills with internal dampers of the type described here. The cutting tests showed that up to 65% increase in stable depth of cut can be obtained for an endmill with an internal damper when compared to a conventional endmill with identical external dimensions.

5. Conclusions and discussions

In this paper, a numerical procedure for qualitatively estimating the amount of friction work in a mechanical damper is presented using finite element analysis. The dynamic problem is simplified to a static problem by applying a centrifugal force followed by a constant lateral force at the tool tip. Through non-linear finite element analysis and a parameter study, the amount of friction work as a function of inner radius and the number of fingers is computed, and the best configuration is identified.

Table 1
Results of the parameter study ($\times 10^{-5}$ N m)

Radius (mm)	Fingers								
	2	3	4	5	6	7	8	9	10
1.0	3.28	3.41	3.21	3.69	3.55	3.54	3.54	3.48	3.51
1.5	3.34	3.43	3.24	3.72	3.58	3.55	3.57	3.52	3.56
2.0	3.36	3.41	3.24	3.69	3.54	3.47	3.52	3.45	3.50
2.5	3.31	3.31	3.18	3.57	3.36	3.33	3.39	3.35	3.42
3.0	3.08	3.10	3.02	3.32	3.10	3.08	3.12	3.04	3.16
3.5	2.53	2.62	2.65	2.83	2.61	2.63	2.65	2.60	2.68

Each column represents friction work for a different number of fingers and each row for a different inner radius.

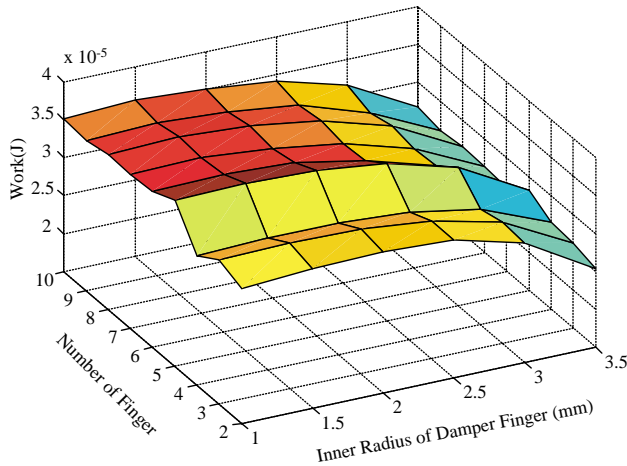


Fig. 11. Response surface of friction work.

The comparison between the numerical and simplified analytical methods shows that the friction work from the numerical approach is smaller than that from the analytical approach (about 2.5 times). The assumption of the constant contact pressure from the analytical approach contributes to the difference in results. In fact, the finite element results show that some portion of the finger is not in contact with the shank, particularly in the region of the clamped boundary condition at the end of the tool/damper. However, the trends from both approaches are consistent except for the magnitude.

The amount of friction work is found to strongly depend on the initial angle of the fingers. This effect is especially large when the number of fingers is small (two or three). However, in practice the endmill is under continuous rotation, and the friction work will be changed between two extreme values.

A design parameter study is carried out by changing the inner radius and the number of fingers. The inner radius is varied from 1.0 to 3.5 mm, and the number of fingers varied from two to ten. The results show general trends of friction work according to the change of the two design parameters. The magnitude of friction work increased as the inner radius decreased and the number of finger increased. The maximum value 3.72×10^{-5} N m of friction work is observed when the inner radius is 1.5 mm and the number of fingers is five. The parameter study also shows that when the number of the fingers is small, friction work is affected by the position of the fingers, but this dependence gradually decreases as the number of fingers is increased.

References

- [1] J. Tlustý, *Manufacturing Processes and Equipment*, Prentice-Hall, Englewood Cliffs, NJ, 2000.
- [2] W.T. Cobb, *Design of Dampers for Boring Bars and Spindle Extensions Master's Thesis*, University of Florida, Gainesville, FL, 1989.
- [3] J. Tlustý, K.S. Smith, W. Winfough, Techniques for the use of long slender end mills in high-speed machining, *Annals of the CIRP* 25 (1) (1996) 393–396.
- [4] E. Soliman, F. Ismail, Chatter suppression by adaptive speed modulation, *International Journal of Machine Tools & Manufacture* 37 (3) (1997) 355–369.
- [5] E.G. Kubica, F. Ismail, Active suppression of chatter in peripheral milling. 2. application of fuzzy control, *International Journal of Advanced Manufacturing Technology* 12 (4) (1996) 236–245.
- [6] M. Liang, T. Yeap, A. Hermansyah, A fuzzy system for chatter suppression in end milling, *Proceedings of the Institution of Mechanical Engineers Part B: Journal of Engineering Manufacture* (2004) 403–417.
- [7] A.H. Slocum, *Precision Machine Design*, Prentice-Hall, Englewood Cliffs, NJ, 1992.
- [8] E.R. Marsh, A.H. Slocum, An integrated approach to structural damping, *Precision Engineering* 18/2/3 (1996).
- [9] K. Nagaya, J. Kobayasi, K. Imai, Vibration control of milling machine by using auto-tuning magnetic damper and auto-tuning vibration absorber, *International Journal of Applied Electromagnetics and Mechanics* 16 (1–2) (2002) 111–123.
- [10] J.C. Ziegert, C. Stanislaus, T. Schmitz, R. Sterling, Enhanced Damping in Long Slender End Transactions of the 2004 North American Manufacturing Research Institute of SME, vol. 32 2004 pp. 1–8.
- [11] T. Schmitz, J.C. Ziegert, C. Stanislaus, A Method for Predicting Chatter Stability for Systems with Speed-Dependent Spindle Dynamics SME Technical Paper TP04PUB182, Transactions of the 2004 North American Manufacturing Research Institute of SME, vol. 32 2004 pp. 17–24.
- [12] T. Qu, D.C. Lin, A. Khajepour, K. Behdinan, Finite element modeling and stability analysis of chatter in end milling machining, *Transactions of the Canadian Society for Mechanical Engineering* 27 (3) (2003) 205–221.
- [13] R.P.H. Faassen, N. van de Wouw, J.A.J. Oosterling, H. Nijmeijer, Prediction of regenerative chatter by modelling and analysis of high-speed milling, *International Journal of Machine Tools & Manufacture* 43 (14) (2003) 1437–1446.
- [14] ANSYS Inc., 2004, *ANSYS User's Manual*, ANSYS Inc., Southpointe, 275 Technology Drive, Canonsburg, PA.
- [15] N.H. Kim, K.K. Choi, J.S. Chen, Y.H. Park, Meshless shape design sensitivity analysis and optimization for contact problem with friction, *Computational Mechanics* 25 (2/3) (2000) 157–168.
- [16] Z.H. Zhong, *Finite Element Procedures for Contact-Impact Problems*, Oxford University Press, New York, NY, 1993.
- [17] Johnson, K.L., 1985, *Contact Mechanics*, Cambridge University Press, Cambridge.