

**Tony L. Schmitz**  
Department of Mechanical Engineering and  
Engineering Science,  
University of North Carolina at Charlotte,  
Charlotte, NC 28223

**Jaydeep Karandikar**  
**Nam Ho Kim**

Department of Mechanical and  
Aerospace Engineering,  
University of Florida,  
Gainesville, FL 32611

**Ali Abbas**  
Department of Industrial and  
Enterprise Systems Engineering,  
University of Illinois at Urbana-Champaign,  
Urbana, IL 61820

# Uncertainty in Machining: Workshop Summary and Contributions

*A National Science Foundation-sponsored workshop was held from Feb. 24–26, 2010 in Arlington, VA. The purpose of this “Uncertainty in machining” workshop was to address uncertainty and risk in machining and related manufacturing operations. The application of decision theory, which defines how rational decision makers should make decisions in the presence of uncertainty, was discussed. A summary of the meeting outcomes is presented. To aid in the application of decision theory to manufacturing process modeling, an example of Bayesian inference for the well-known mechanistic turning force model is provided. The discrete grid method is presented, and updating is performed using force data from the Assessment of Machining Models study completed by the National Institute of Standards and Technology. [DOI: 10.1115/1.4004923]*

## 1 Introduction

Scientists and engineers often expend considerable effort in performing measurements and developing models to predict the results of future experiments. These are naturally significant activities in the manufacturing community, where the ability to achieve “first part correct” production and verify the dimensional accuracy and performance of these components is critical in today’s competitive global marketplace. Modeling activities for manufacturing processes generally include identifying the relevant inputs (in milling, for example, these include spindle speed, axial and radial depth of cut, feed per tooth, tool geometry, force model coefficients, and machine capabilities) and describing the relationship between these inputs to produce the desired outputs (e.g., part dimensions and surface/subsurface quality) using numerical and/or analytical algorithms. Whether considering a measurement or model prediction, however, the result is incomplete if it is not accompanied by a quantitative assessment of its uncertainty<sup>1</sup> [1,2].

Because the effects of uncertainty in modeling efforts for manufacturing processes, and measurements and modeling in general, is an issue of significant importance, a National Science Foundation-sponsored workshop was organized and held from Feb. 24–26, 2010 in Arlington, VA. The conference chair was Dr. Tony Schmitz, University of Florida, and the cochairs were Dr. Ali Abbas, University of Illinois at Urbana-Champaign, Dr. O. Burak Ozdoganlar, Carnegie Mellon University, Dr. K. Scott Smith, University of North Carolina at Charlotte, and Dr. John Ziegert, Clemson University. The workshop participants are listed in Appendix A.

## 2 Workshop Objective

The purpose of this “Uncertainty in machining” workshop was to discuss and address uncertainty and risk in machining and related manufacturing operations. Because the majority of models currently used to support manufacturing decision making are deterministic, results produced by these models often provide

<sup>1</sup>The uncertainty (of a measurement) is defined in Ref. [1] as a “parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand,” or a particular quantity subject to measurement.

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only approximate guidance for many manufacturing decisions and, therefore, many manufacturing decision makers rely heavily on rules of thumb and conservative choices. Such behavior can result in considerable economic loss to the manufacturer. The key notion underlying this workshop was that rigorous mathematical treatment of uncertainty and decision making under uncertainty and risk can provide substantial economic benefit to the industry and enhance the competitiveness of the manufacturing sector.

The proposed value of this workshop was based on its goal to broaden the perspective on engineering decision making from one of applied science and problem solving, which does not rely on the mathematics of prediction and decision theory, to one that does incorporate this mathematical framework. While this workshop was focused on manufacturing with a concentration on machining, it is believed that the findings of the workshop and follow-on research could have profound effects that permeate all of engineering practice. The proposed impact of this workshop included the economic benefit to the manufacturing sector, which will come about from improved decision making. Its aim was to provide a new perspective on engineering decision making, in general, that will eventually influence both engineering practice and engineering education.

## 3 Workshop Outcomes

Thirteen participants presented their perspectives on topics related to the workshop objectives. These presentations yielded extensive discussions and resulted in three primary group activities. First, the perceived needs required to move this research topic forward were identified. These included the following:

1. develop case studies at multiple scales (manufacturing processes to plant level) to teach Bayesian analysis and decision making within the context of manufacturing problems
2. develop guidelines for framing and specifying objective functions for manufacturing problems
3. introduce probability assessment concepts and applied Bayesian analysis into undergraduate engineering curricula
4. develop strategies for teaching decision analysis concepts to “deterministic” engineers
5. provide training for Bayesian updating
6. form partnerships to develop success stories from smaller-scale problems (e.g., tool wear)
7. formulate agenda to connect research to industry needs
8. establish a list of “big payoff” opportunities.

Second, a list of research opportunities was generated during a brainstorming session. These were framed as the following research questions:

1. What optimization tools are necessary and how can they be applied to more effectively couple design and manufacturing and, subsequently, motivate the in-sourcing of manufacturing capabilities?
2. An expensive part with critical tolerances was produced and then measured. The measurements show that the dimensions are close to the tolerances provided by the customer. Based on the measurement uncertainty, what guidelines should the manufacturer use in deciding if the part should be shipped?
3. How can a machinist decide if she/he should change a cutting tool while using it?
4. How can a manufacturer decide if she/he should make a part and, if so, set the selling price?
5. Given a *prior* (or initial belief), what is the value of new knowledge obtained by testing?
6. How does a manufacturer manage the potential conflict between the short and long term consequences of a decision? For example, a large equipment manufacturer may choose to outsource the manufacture of specific components to reduce cost today, but what effect would the decision have on future design/manufacturing capabilities?
7. How can a manufacturer compare different processes for producing a given part (e.g., machining versus sheet metal forming)?
8. How can a manufacturer select the correct machine to produce a selected part?
9. Given a set of machines and parts, how should the available resources be allocated?
10. When should production begin in order to meet a deadline?
11. How can expert information be used in a Bayesian decision making framework?
12. How can the yield for multi-operation parallel path machining systems (with redundant flow paths) be predicted in the presence of uncertainty?
13. How can multi-operation parallel path machining systems be controlled with uncertainty in part flow?
14. How can schedules be set for manufacturing facilities (at any scale) in the presence of uncertainty?

Third, a list of answers to the open question “Wouldn’t it be nice if...?” was compiled.

1. it was possible to determine the distribution in outputs of a process without performing experiments
2. it was possible to predict variability in a process in real-time
3. variables that most strongly drive cost could be identified
4. an integrated data system, including both the process and subsequent metrology, could be developed
5. the allowable variation in input parameters for a desired output uncertainty could be predicted
6. there were standardized data collection and sharing formats (including uncertainty) for machining data
7. undergraduate education emphasized probability theory (which describes the future), rather than just statistics (which describes only the past)
8. an improved understanding of uncertainty contributors to machining process performance could be achieved
9. the cost associated with each uncertainty contributor for a given process output could be determined.

Finally, a unifying theme, which developed over the course of the workshop, was the close coupling of design and manufacturing. It was the group consensus that manufacturing capabilities drive design options. In order to optimize designs, an understanding of manufacturing capabilities and limitations by the designer is necessary. For companies with significant research and development activities, developing new processes can lead to increased

design options provided the designer is aware of the research outcomes. It was emphasized that this is a strong argument against outsourcing manufacturing activities, particularly outside the United States. Research and development budgets for manufacturing naturally decrease as a result of outsourcing, which limits future design options.

#### 4 Decision Theory

Given the workshop objective and its focus on decision making, a primary topic of discussion was decision theory, which defines how rational decision makers should make decisions in the presence of uncertainty in all activities, including science and engineering; see Refs. [3–5], for example. The approach incorporates information, preferences, and available alternatives to derive the best decision alternative. Information is described in terms of a joint probability distribution that captures the uncertainty about the possible outcomes for each alternative and uses Bayesian analysis to improve knowledge when new information is revealed. The Bayesian approach uses a *prior* that captures the available information. As such, it can incorporate data and/or existing models; this makes it an attractive candidate to update information in experimental settings. The Bayesian approach can then be used in aggregating experimental results and can determine a *posterior* distribution based on all data or models that are collected.

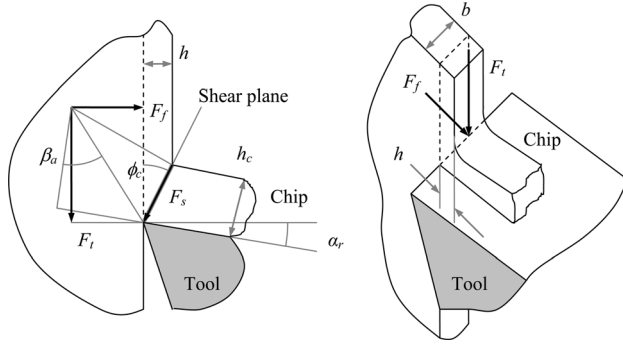
In decision theory, preferences are captured using a von Neumann–Morgenstern [6] utility function. Given a value model that converts parameter settings in a deterministic setting into dollar equivalents, a von Neumann–Morgenstern utility that captures preferences under uncertainty can be constructed over a monetary amount. The optimal decision is the one that maximizes the von Neumann–Morgenstern expected utility. While many milling decisions may involve multiple objectives, it is often preferred in manufacturing problems to formulate the selected optimization problem as a single objective function, profit. This requires only a single attribute utility function over profit. Decision theory also enables the value of a decision situation, given an arbitrary knowledge state to be determined when the outcomes are expressed in monetary terms. As a result, a value can be assigned to gaining knowledge, such as the outcome of an experiment. With this approach, optimal experiment design can be formulated as a sequential decision problem under uncertainty. The type and number of experiments that maximize expected utility are selected to determine the optimal sequence of experiments. The value of different available experiments can then be compared.

While decision theory fundamentals are well-established within the field, there are limited examples of its application to manufacturing problems. However, applications of decision theory to other domains are well-documented. For example, the theory has been extensively implemented to value oil (petroleum) properties and make decisions about new well drilling [7–15]. The similarities to machining are clear: there are significant uncertainties, new knowledge can be gained by testing, and profit drives the decisions. To aid manufacturing researchers in adopting decision theory, the application of Bayesian inference to the well-known mechanistic turning force model is presented in the following section.

#### 5 Bayesian Inference Example

During the workshop, several participants from industry expressed appreciation for the Bayesian methodology, but required some guidance for implementation. Therefore, they requested a simple and specific example that demonstrates the use of Bayesian analysis in manufacturing. After the workshop, the authors completed the following example in order to satisfy this request and convey the Bayesian approach to the manufacturing community. The simple turning force model described in Sec. 5.1 was selected because it is well-known in the machining field.

**5.1 Force Model Description.** To demonstrate Bayesian inference, the mechanistic turning force model for orthogonal cutting



**Fig. 1 Schematic of orthogonal cutting model (a positive side rake angle is shown; the tool's rake face is inclined above the horizontal for a negative rake angle)**

was selected [16]; see Fig. 1. This model describes the tangential (or cutting),  $F_t$ , and feed (or thrust),  $F_f$ , forces in turning as:

$$\begin{aligned} F_t &= K_t b h \\ F_f &= K_f b h \end{aligned} \quad (1)$$

where  $K_t$  and  $K_f$  are the (specific) cutting force coefficients,  $b$  is the chip width, and  $h$  is the uncut (commanded) chip thickness. The chip width and uncut chip thickness are specified by the machinist, but the coefficients must be determined.

These coefficients can be described as a function of three parameters selected by the machinist:  $b$ ,  $h$ , and the tool's side rake angle,  $\alpha_r$ ; and three measured values:  $F_t$ ,  $F_f$ , and the cut chip thickness,  $h_c$ . By collecting this data, the coefficients can be calculated [Eq. (2)] and used to predict forces for cuts performed under different conditions

$$\begin{aligned} K_t &= \tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin(\phi_c) \cos(\phi_c + \beta_a - \alpha_r)} \\ K_f &= \tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin(\phi_c) \cos(\phi_c + \beta_a - \alpha_r)} \end{aligned} \quad (2)$$

In Eq. (2),  $\tau_s$  is the shear stress along the shear plane (actually a thin zone where the uncut material is plastically deformed into the chip),  $\phi_c$  is the angle of the shear plane, and  $\beta_a$  is the average friction angle. The shear stress is defined by

$$\tau_s = \frac{F_s}{A_s} \quad (3)$$

where  $F_s$  is the shear force and  $A_s$  is the shear area. See Eqs. (4) and (5)

$$F_s = F_t \cos(\phi_c) - F_f \sin(\phi_c) \quad (4)$$

$$A_s = \frac{b h}{\sin(\phi_c)} \quad (5)$$

The average friction angle and shear plane angle are given by Eqs. (6) and (7), where  $r_c = \frac{h}{h_c}$  is the chip thickness ratio

$$\beta_a = \alpha_r + \tan^{-1} \left( \frac{F_f}{F_t} \right) \quad (6)$$

$$\phi_c = \tan^{-1} \left( \frac{r_c \cos(\alpha_r)}{1 - r_c \sin(\alpha_r)} \right) \quad (7)$$

Bayes' rule (or theorem) is used to update the user's beliefs about cutting forces given new information (such as a measurement result). It can be expressed as

$$p(A|BI) \propto p(A|I)p(B|AI) \quad (8)$$

where  $p(A|BI)$  is the posterior that gives the probability, or the degree of belief, that  $A$  is true given new data  $B$  and initial information  $I$ ,  $p(A|I)$  is the prior, and  $p(B|AI)$  is the likelihood function (or conditional probability). The product of the prior and likelihood function are used to determine the posterior; this is the process of learning, i.e., updating the prior beliefs given the new data  $B$  to obtain the posterior beliefs.

In the turning force model, there is uncertainty in the force coefficient values,  $K_t$  and  $K_f$ , due to uncertainty in  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$ . Bayes' rule can be used to update the probability distributions of  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  from prior distributions (i.e., initial beliefs), which, in turn, can be used to update distributions for  $K_t$  and  $K_f$ . In this study,  $p(A|I)$  is the user's initial beliefs about  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  so that Eq. (2) can be used to calculate the cutting force coefficients. The updating of the variables is completed using experimental values of  $h_c$ ,  $F_t$ , and  $F_f$ . Force predictions can then be made using the updated force coefficient distributions. For this case, Bayes' rule [as defined in Eq. (8)] is

$$f_{\tau_s, \beta_a, \phi_c}(\tau_s, \beta_a, \phi_c | F_{t,m}, F_{f,m}, h_{c,m}) \propto f_{\tau_s, \beta_a, \phi_c}(\tau_s, \beta_a, \phi_c) l(F_{t,m}, F_{f,m}, h_{c,m} | \tau_s, \beta_a, \phi_c) \quad (9)$$

where  $f_{\tau_s, \beta_a, \phi_c}(\tau_s, \beta_a, \phi_c | F_{t,m}, F_{f,m}, h_{c,m})$  is the posterior distribution of  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  given measured values<sup>2</sup> of the forces and chip thickness,  $F_{t,m}$ ,  $F_{f,m}$ , and  $h_{c,m}$ ,  $f_{\tau_s, \beta_a, \phi_c}$  is the prior distribution of  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$ , and  $l(F_{t,m}, F_{f,m}, h_{c,m} | \tau_s, \beta_a, \phi_c)$  is the likelihood of the measured force and chip thickness values given  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$ . The posterior (i.e., the new belief after updating) is the prior multiplied by the likelihood. For multiple measurements, the posterior after the first update becomes the prior for the second update and so on. Note that the posterior distributions must be normalized so that a unit area or volume under the probability distribution function (pdf) is obtained.

**5.2 Establishing the Prior.** It was decided for demonstration purposes to consider orthogonal cutting of 1045 steel based on the force and cut chip thickness data available from the Assessment of Machining Models (AMM) study completed by the National Institute of Standards and Technology (NIST) [17]. Data were collected for cutting speeds between 200 and 400 m/min, commanded chip thicknesses between 0.1 mm and 0.4 mm, and side rake angles between 5 and  $-7$  deg for coated and uncoated tools. An important consideration in applying Bayes' rule is the selection of prior distributions for the variables  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$ . In this case, the prior distributions were based on a literature review.

**5.2.1 Shear Stress,  $\tau_s$ .** Altintas [16, pp. 16–17] provided an example of orthogonal cutting of 1045 steel for a cutting speed of 110 m/min,  $h = 0.2$  mm,  $\alpha_r = 5$  deg, and  $r_c = 0.45$ . The shear stress was 693 MPa. Trent and Wright [18, Table 4.2] reported a shear stress of 480 MPa for 0.13%C steel with  $h = 0.5$  mm,  $\alpha_r = 0$ , and  $r_c = 0.27$  to 0.7. DeGarmo et al. [19, Fig. 21–21] gave a hardness-dependent value of 669 MPa for body-centered cubic (BCC) matrix steels with a Brinell hardness number (BHN) of 200. Tlustý [20, pp. 424–425] listed a shear stress of 764 MPa for 1035 steel using a cutting speed of 180 m/min,  $h = 0.2$  mm,  $\alpha_r = 10$  deg, and  $r_c = 0.49$ . Based on these results, a normal distribution was selected with a mean of 700 MPa and a standard deviation of 50 MPa.

**5.2.2 Shear Plane Angle,  $\phi_c$ .** Altintas [16, pp. 16–17] gave a value of 25 deg for 1045 steel with a cutting speed of 110 m/min,  $h = 0.2$  mm,  $\alpha_r = 5$  deg, and  $r_c = 0.45$ . Tlustý [20, pp. 424–425] provided a shear plane angle of 28 deg for 1035 steel with a cutting speed of 180 m/min,  $h = 0.2$  mm,  $\alpha_r = 10$  deg, and  $r_c = 0.49$ . Kalpakjian and Schmid [21, Fig. 8.15] reported data for mild steel that gives the relationship  $\phi_c = 28 - 0.52(\beta_a - \alpha_r - 8)$  deg. Iqbal et al. [22] provided data that described the variation in shear plane

<sup>2</sup>The subscript  $m$  denotes measured values from cutting experiments. The measured values were assumed to be statistically independent.

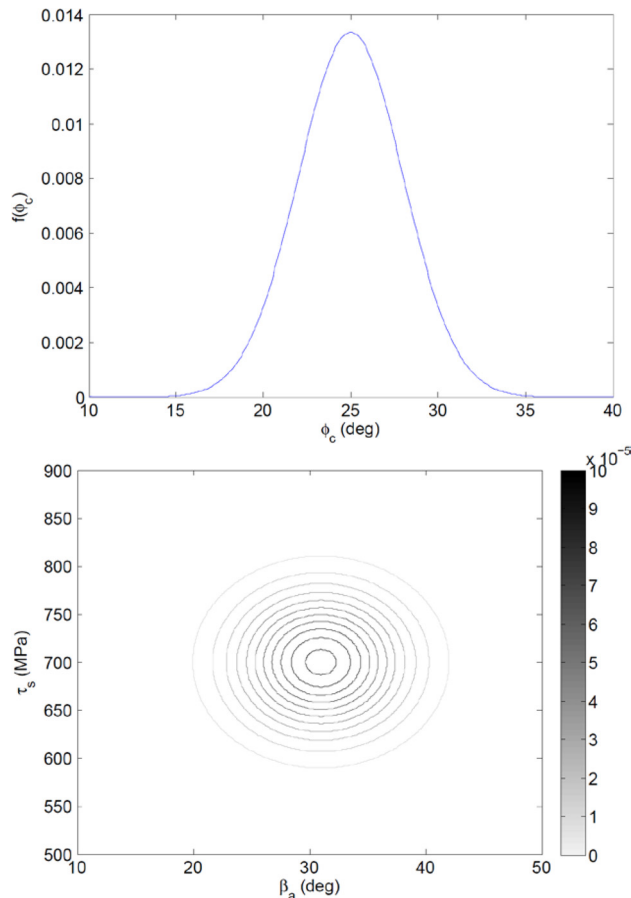
angle with cutting speed ( $v = 200\text{--}400$  m/min) for  $h = 0.1$  mm and  $\alpha_r = 0$  deg:  $\phi_c = 21.6 + 7.96 \times 10^{-3}(v - 198)$  deg. Based on these results, a normal distribution was selected with a mean of 25 deg and a standard deviation of 3 deg.

**5.2.3 Average Friction Angle,  $\beta_a$ .** Altintas [16, pp. 16–17] gave a value of 31.6 deg for 1045 steel with a cutting speed of 110 m/min,  $h = 0.2$  mm,  $\alpha_r = 5$  deg, and  $r_c = 0.45$ . Tlustý [20, pp. 424–425] reported an angle of 26.7 deg for 1035 steel with a cutting speed of 180 m/min,  $h = 0.2$  mm,  $\alpha_r = 10$  deg, and  $r_c = 0.49$ . Iqbal et al. [22] provided data that described the variation in shear plane angle with cutting speed ( $v = 200\text{--}400$  m/min) for  $h = 0.1$  mm and  $\alpha_r = 0$  deg:  $\beta_a = \tan^{-1}(0.73 - 2 \times 10^{-4}(v - 198))$  deg. Based on these results, a normal distribution was selected with a mean of 31 deg and a standard deviation of 5 deg.

To summarize, the initial beliefs based on the literature review were as follows:

1.  $\tau_s = 700$  MPa  $\pm$  50 MPa (one standard deviation, normal distribution)
2.  $\phi_c = 25 \pm 3$  deg (one standard deviation, normal distribution)
3.  $\beta_a = 31$  deg  $\pm$  5 deg (one standard deviation, normal distribution).

As shown in Eq. (7), the  $\phi_c$  values for the mechanistic model are dependent on  $r_c$  and, therefore, on  $h_c$ . From Eqs. (1) and (2), it seen that  $\tau_s$  and  $\beta_a$  values depend on force,  $F_t$  and  $F_f$ . Therefore, the  $\phi_c$  distribution was updated using  $h_c$  values, and the prior distribution of  $\tau_s$  and  $\beta_a$  was taken as a joint distribution to be updated using the measured force values,  $F_{t,m}$  and  $F_{f,m}$ . This ensures that the final posterior distributions of  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  take into account all three measured quantities. Also,  $\tau_s$  and  $\beta_a$  were



**Fig. 2** Prior distribution of  $\phi_c$  (top) and bivariate normal distribution of  $\tau_s$  and  $\beta_a$  (bottom)

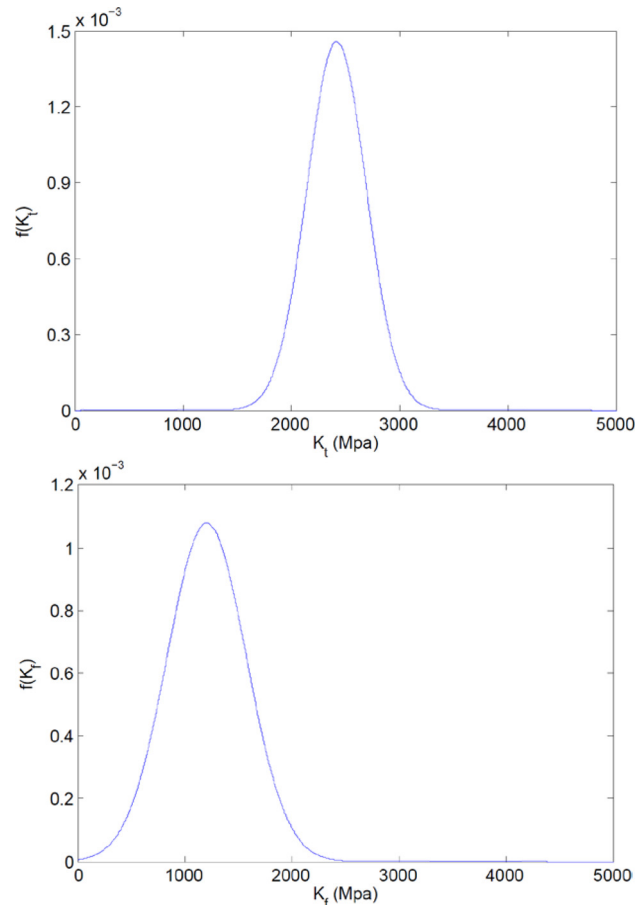
assumed to be independent of each other and their joint pdf was therefore a multiplication of the two individual distributions determined from the literature review. Figure 2 shows the prior distribution for  $\phi_c$  and the joint pdf for  $\tau_s$  and  $\beta_a$ .

A Monte Carlo simulation using these prior distributions was completed to calculate the corresponding distributions in  $K_f$  and  $K_t$  [using Eq. (2)] for  $\alpha_r = 5$  deg. For the simulation,  $1 \times 10^5$  random samples were drawn from the prior distributions, and the force coefficients were calculated at each sample point. No correlation among  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  was assumed. The mean values for  $K_t$  and  $K_f$  were 2414.3 and 1206.2 MPa, and the standard deviations were 272.9 and 377.4 MPa, respectively. The MATLAB command “normpdf” was used to compute the pdf (normal distribution) for  $K_t$  and  $K_f$  based on the mean and the standard deviation values; see Fig. 3. The pdf plots in this figure represent the priors for the two coefficients.

The prior distributions of  $\tau_s$ ,  $\phi_c$ , and  $\beta_a$  can be updated using the measured values for  $F_t$ ,  $F_f$ , and  $h_c$  listed in Table 1 from the NIST AMM database. All values in Table 1 were collected using  $\alpha_r = 5$  deg and  $b = 1.588$  mm. The cutting speed,  $v$ , is also included.

**5.3 Discrete Grid Method.** This section describes the discrete grid method used to determine the likelihood function required for Bayesian inference. The procedure was to: (1) update the  $\phi_c$  distribution (see Fig. 2) using measured  $h_c$  values; and (2) sample a random value of  $\phi_c$  from the updated distribution and use it, together with the measured force values, to update the joint distribution of  $\tau_s$  and  $\beta_a$ .

To determine the likelihood function for a measured value of  $h_c$ , the range of  $\phi_c$  values (10–40 deg) was first divided into a number of points (300). Second, the  $h_c$  value was

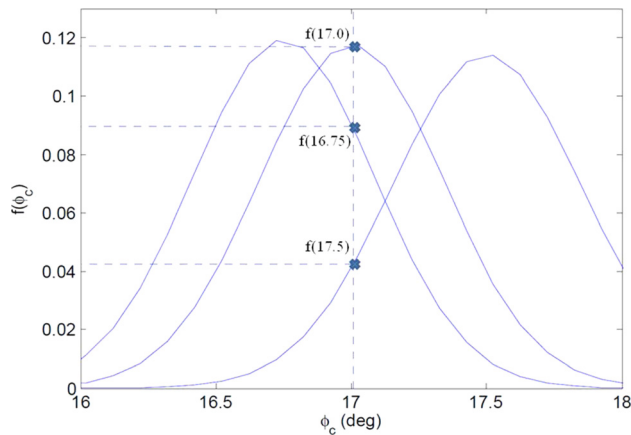


**Fig. 3** Prior distributions of  $K_f$  (top) and  $K_t$  (bottom)

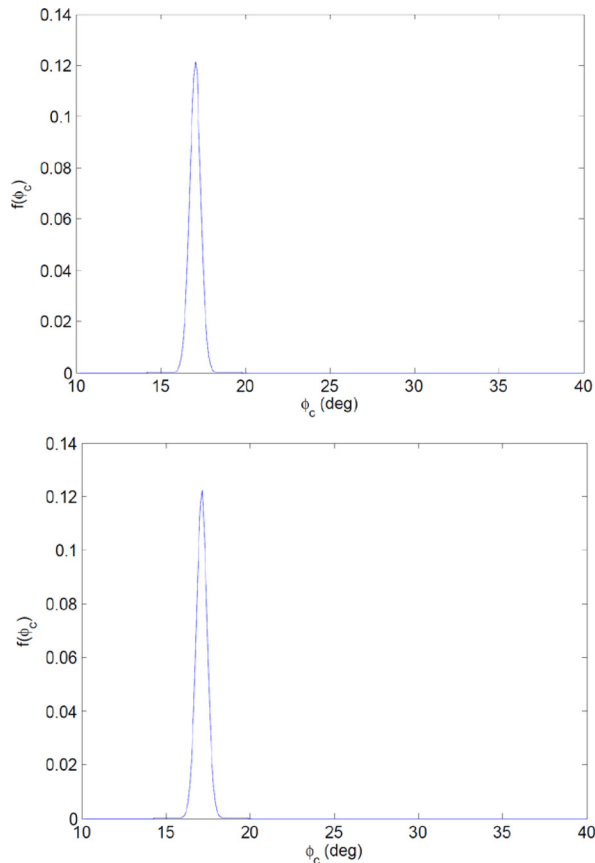


**Table 1 Measured  $F_b$ ,  $F_f$ , and  $h_c$  values used for Bayesian updating ( $\alpha_r = 5$  deg and  $b = 1.588$  mm)**

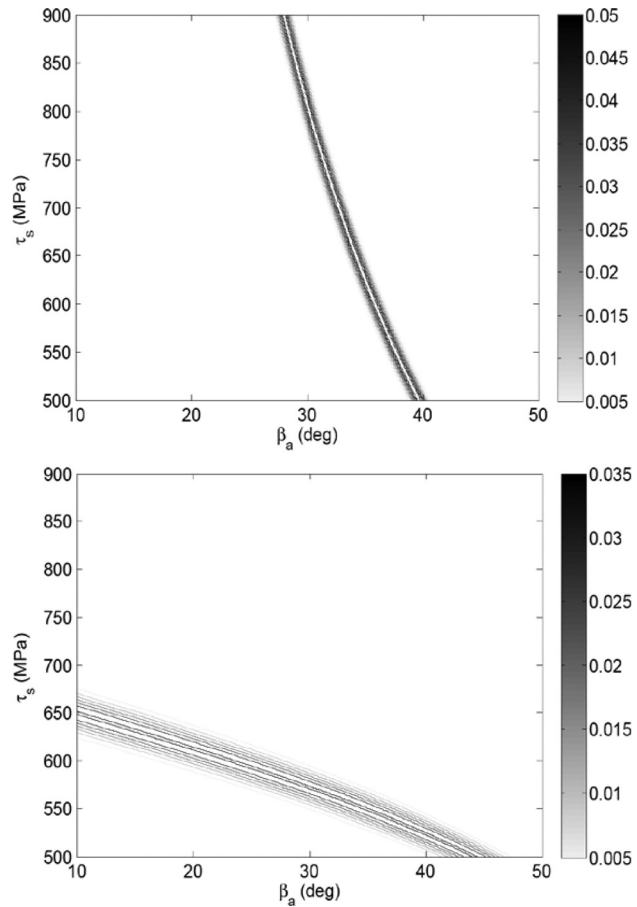
Test #	$v$ (m/min)	$h$ (mm)	$F_t$ (N)	$F_f$ (N)	$h_c$ (mm)
1	200	0.15	565.2	372	0.501
2	200	0.3	907.9	397.4	0.671
3	200	0.3	915.2	407.6	0.682
4	200	0.15	520.8	267.6	0.392
5	200	0.15	540	370	0.441



**Fig. 4 Figure to illustrate calculation of the likelihood function. The distributions (from left to right) have mean values of  $\phi_c = \{16.75, 17.0, \text{ and } 17.5\}$  deg. The value of the distributions for the  $\phi_c$  value (17.03 deg), which corresponds to the  $h_c$  measurement (0.501 mm) from Test #1, is the likelihood value.**



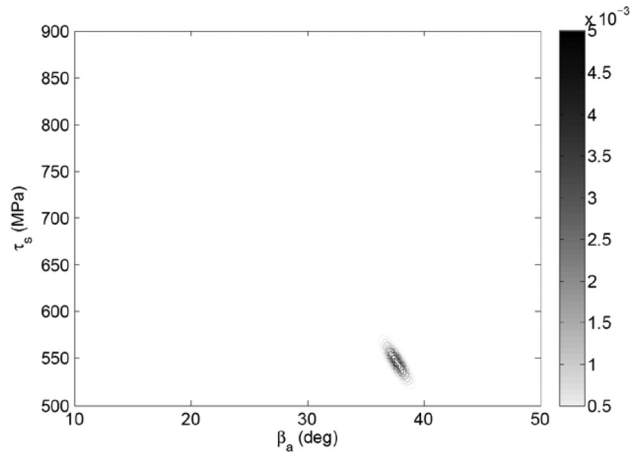
**Fig. 5 Likelihood distribution of  $\phi_c$  for  $h_c = 0.501$  mm measurement from Test #1 (top) and posterior distribution of  $\phi_c$  after the first update (bottom)**



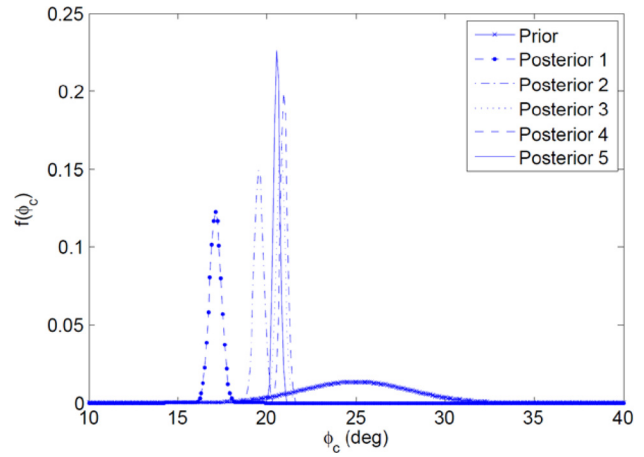
**Fig. 6 Likelihood joint distribution of  $\tau_s$  and  $\beta_a$  for force measurements from Test #1;  $F_t = 372$  N (top) and  $F_t = 565.2$  N (bottom)**

calculated at each point using Eq. (7). The calculated  $h_c$  value was taken as the mean value for each point. Third, the measurement uncertainty in  $h_c$  was defined. In this study, a normal distribution was assumed, and the standard deviation was selected to be 2% of the measured value. The calculated mean values and the assumed standard deviation defined a probability distribution for  $h_c$  at each point. Fourth, the likelihood at each point was taken to be the value of the selected  $h_c$  pdf at the measured value. The MATLAB command "normpdf" was again applied. The likelihood function gives the probability density that a selected  $\phi_c$  value would give the measured  $h_c$  value using the deterministic model in the presence of uncertainty.

To illustrate, the  $\phi_c$  value corresponding to  $h_c = 0.501$  mm (Test #1 from Table 1) is 17.03 deg from Eq. (7). The maximum value of the likelihood function would therefore be expected at  $\phi_c = 17.03$  deg. Figure 4 shows the method of calculating the likelihood for this measured value. The distributions at  $\phi_c = \{16.75, 17.0, \text{ and } 17.5\}$  deg (left to right in the figure) are provided. The distributions were determined using the calculated mean  $h_c$  value at that point and the assumed measurement uncertainty (represented by one standard deviation). The "x" symbols in the plot represent the value of the likelihood at  $\phi_c = \{17.5, 16.75, \text{ and } 17.0\}$  deg from bottom to top, which correspond to the measured  $h_c$  value (0.501 mm). The likelihood at these  $\phi_c$  sample points is the value of the pdf. Figure 5 shows the  $\phi_c$  likelihood and posterior for  $h_c = 0.501$  mm. As stated previously, the likelihood is maximum at  $\phi_c = 17.03$  deg. The posterior distribution of  $\phi_c$  after the first update was obtained using a point-by-point multiplication of the prior (see Fig. 2) and the likelihood. The prior and the likelihood distributions form a normal conjugate pair, and therefore,



**Fig. 7** Posterior joint distribution of  $\tau_s$  and  $\beta_a$  after the first update using force measurement data from Test #1;  $F_f=372$  N and  $F_t=565.2$  N. The posterior was obtained by a point-by-point multiplication of the prior (see Fig. 2) and the likelihood functions for  $F_f$  and  $F_t$  (see Fig. 6) and a normalization to obtain a unit volume under the surface.

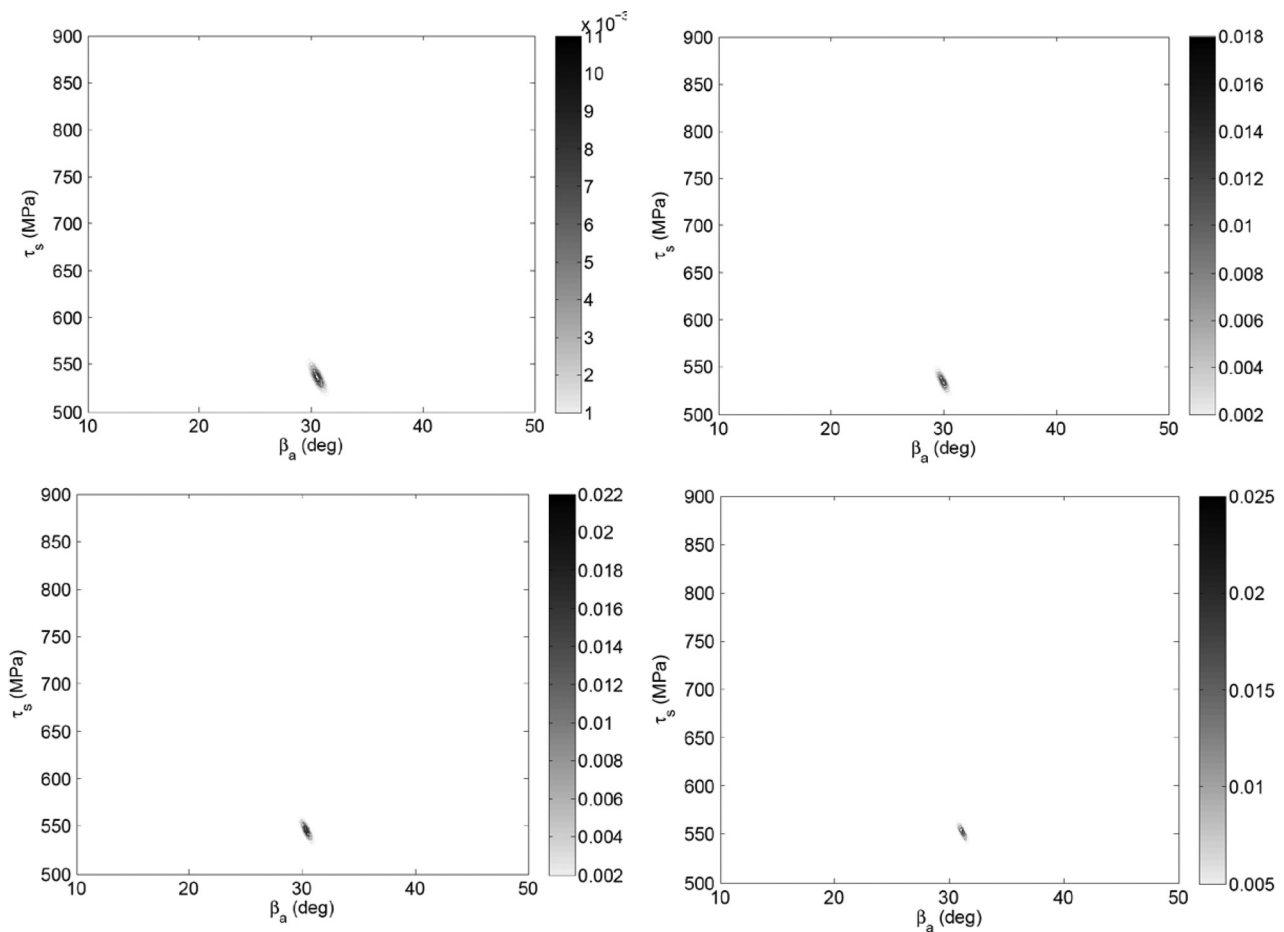


**Fig. 8** Posterior distributions of  $\phi_c$  for all updates and their comparison with the prior

the posterior distribution is also normal. The posterior distribution was normalized to obtain a unit area under the pdf.

The same procedure was followed to update  $\tau_s$  and  $\beta_a$  joint distribution prior using the measured force values. The selected range of possible  $\tau_s$  and  $\beta_a$  values was divided into a grid of points ( $300 \times 300$ ). The  $F_t$  and  $F_f$  values were calculated at each grid

point (i.e., the selected  $\{\tau_s, \beta_a\}$  pair) using first Eq. (2) and then Eq. (1). To compute the forces, a random value of  $\phi_c$  was sampled from the posterior distribution after the first update (see Fig. 5). The distribution of the forces at each grid point was assumed to be normal with an uncertainty (one standard deviation) equal to 2% of the measured force value. The mean values were set equal to the calculated forces. The likelihood at each grid point was taken to be the value of the pdf of force,  $F_t$  or  $F_f$ , at the measured value. The purpose of this approach was to evaluate the probability density that the selected  $\{\tau_s, \beta_a\}$  pair would yield the measured force



**Fig. 9** Posterior joint distributions of  $\tau_s$  and  $\beta_a$  after the second update (top left), third update (top right), fourth update (bottom left), and fifth update (bottom right)

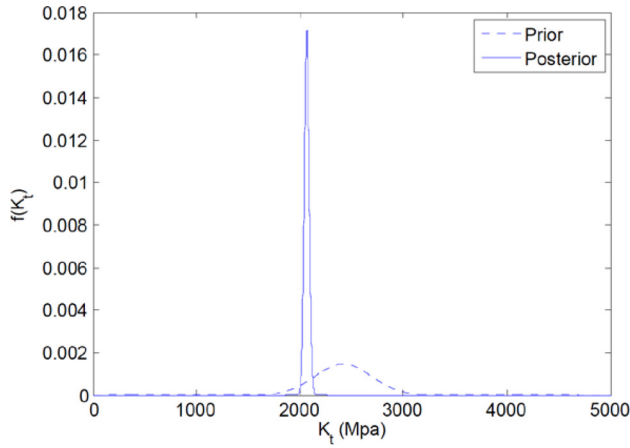


Fig. 10 Prior and posterior distributions for  $K_t$

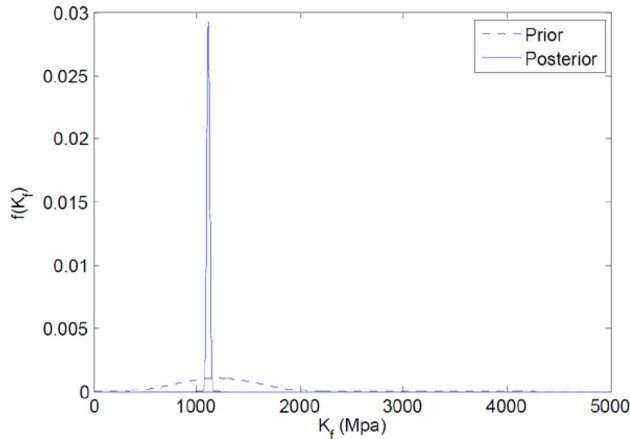


Fig. 11 Prior and posterior distributions for  $K_f$

Table 2 Measured  $F_t$ ,  $F_f$  and  $h_c$  values used for prediction ( $\alpha_r = 5$  deg and  $b = 1.588$  mm)

Test #	$v$ (m/min)	$H$ (mm)	$F_t$ (N)	$F_f$ (N)	$h_c$ (mm)
1	200	0.15	540	370	0.416
2	200	0.3	950	480	0.683
3	200	0.15	510	320	0.375
4	200	0.3	870	400	0.620

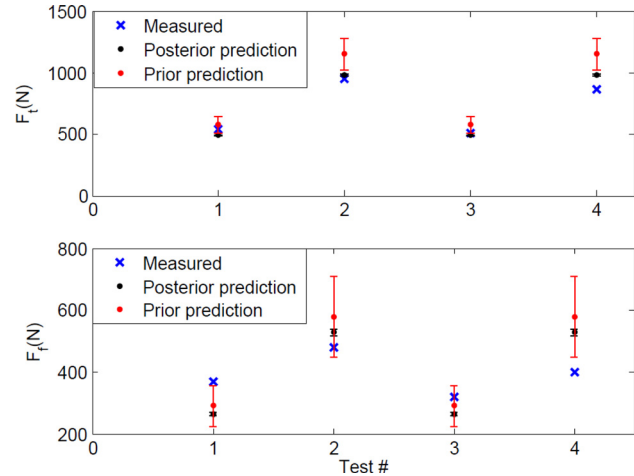


Fig. 12 Comparison of the measured and predicted forces,  $F_t$  and  $F_f$ , using the prior and posterior coefficient distributions ( $\alpha_r = 5$  deg)

Table 3 Summary of force data provided in Fig. 12 ( $\alpha_r = 5$  deg and  $b = 1.588$  mm)

Test #	Measured		Prior		Posterior	
	$F_t$ (N)	$F_f$ (N)	$F_t$ (N)	$F_f$ (N)	$F_t$ (N)	$F_f$ (N)
1	540	370	$574.9 \pm 65.0$	$287.3 \pm 88.9$	$491.3 \pm 5.5$	$263.8 \pm 3.1$
2	950	480	$1149.8 \pm 130.0$	$574.5 \pm 177.8$	$982.5 \pm 11.0$	$527.7 \pm 6.2$
3	510	320	$574.9 \pm 65.0$	$287.3 \pm 88.9$	$491.3 \pm 5.5$	$263.8 \pm 3.1$
4	870	400	$1149.8 \pm 130.0$	$574.5 \pm 177.8$	$982.5 \pm 11.0$	$527.7 \pm 6.2$

using the deterministic model in the presence of uncertainty. Figure 6 shows the likelihoods for the Test #1 data, where  $F_{f,m} = 372$  N and  $F_{t,m} = 565.2$  N; note that the color bar indicates the value of the pdf in these plots. The posterior distribution of  $\tau_s$  and  $\beta_a$  is obtained via a point-by-point multiplication of the prior with both likelihood functions (see Fig. 6) and normalization to obtain a unit volume under the surface. As with  $\phi_c$ , the prior and the likelihood form a conjugate normal pair, and therefore, the

Table 4 Measured  $F_t$ ,  $F_f$  and  $h_c$  values used for prediction ( $\alpha_r = -7$  deg and  $b = 1.588$  mm)

Test #	$v$ (m/min)	$H$ (mm)	$F_t$ (N)	$F_f$ (N)	$h_c$ (mm)
1	200	0.15	560	440	0.454
2	200	0.3	1070	660	0.803
3	200	0.15	550	370	0.387
4	200	0.3	950	520	0.66

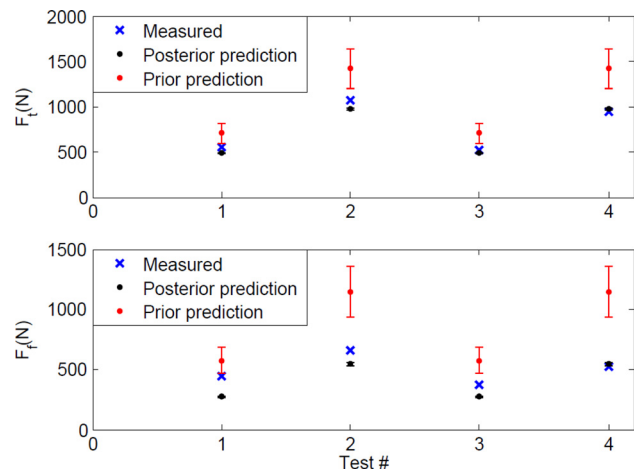


Fig. 13 Comparison of the measured and predicted forces,  $F_t$  and  $F_f$ , using the prior and posterior coefficient distributions ( $\alpha_r = -7$  deg)

**Table 5 Summary of the force data provided in Fig. 13 ( $\alpha_r = -7$  deg and  $b = 1.588$  mm)**

Test #	Measured		Prior		Posterior	
	$F_t$ (N)	$F_f$ (N)	$F_t$ (N)	$F_f$ (N)	$F_t$ (N)	$F_f$ (N)
1	560	440	709 ± 149.6	571 ± 189.7	490 ± 5.6	271.5 ± 3.4
2	1070	660	1419 ± 199.1	1143 ± 199.1	981.7 ± 11.2	543 ± 6.8
3	550	370	709 ± 149.6	571 ± 189.7	490 ± 5.6	271.5 ± 3.4
4	950	520	1419 ± 199.1	1143 ± 199.1	981.7 ± 11.2	543 ± 6.8

posterior is a bivariate normal distribution. Figure 7 shows the posterior joint distribution of  $\tau_s$  and  $\beta_a$  after the first update.

The same procedure was repeated to update the distributions for  $\phi_c$ ,  $\tau_s$ , and  $\beta_a$  for all five measurements in Table 1. As with the first update, all the remaining updates included two steps. Again,  $\phi_c$  was updated first. This time, however, the prior was set equal to the posterior from the previous update. The likelihood was again defined, but using the new value of  $h_c$  (from Test #2 for the second update). A random value from the resulting posterior  $\phi_c$  distribution was then used together with the Test #2 force data to calculate the new  $\tau_s$  and  $\beta_a$  likelihoods for  $F_t$  and  $F_f$ . The product of these likelihood distributions and the  $\tau_s$  and  $\beta_a$  posterior from the first update (now the prior) was the new posterior. This process was repeated for each test. Figure 8 shows the prior and the posterior distributions for all the  $\phi_c$  updates. Figure 9 shows the posterior distributions of  $\tau_s$  and  $\beta_a$  from the second to the fifth update. Note that the uncertainty (standard deviation) decreases with additional data. This indicates the improvement in knowledge with available information.

To determine the posterior distributions of the force coefficients, a Monte Carlo simulation was performed. Since the posterior distribution of  $\tau_s$  and  $\beta_a$  is a bivariate normal distribution, random samples were drawn from the distribution using the MATLAB command "mvnrnd." One input to this function is the covariance matrix. This matrix was identified using the MATLAB command "cov" based on the  $\tau_s$  and  $\beta_a$  values. Again,  $1 \times 10^5$  random samples were obtained from the posterior distributions, and the force coefficients were calculated at each sample point. Figures 10 and 11 show the prior and posterior distributions of  $K_t$  and  $K_f$ . The posterior mean values for  $K_t$  and  $K_f$  were 2064 and 1088.8 MPa, and the standard deviations were 23.2 and 13.6 MPa, respectively. Note that this approach provides not only expected (or mean) values but also the associated distribution (uncertainty).

**5.4 Prediction of Forces.** The posterior distributions for the force coefficients,  $K_t$  and  $K_f$ , were then used to predict forces using Eq. (1). Table 2 shows the measured  $F_t$ ,  $F_f$ , and  $h_c$  values obtained from the NIST AMM database ( $\alpha_r = 5$  deg and  $b = 1.588$  mm). Figure 12 shows the comparison of the measured and the predicted forces using the posterior distributions of  $K_t$  and  $K_f$ ; one standard deviation error bars are also included. Additionally, Fig. 12 includes the forces predicted using the prior distributions for the force coefficients; the results are also summarized in Table 3. Another set of prediction was performed with forces measured at  $\alpha_r = -7$  deg; see Table 4. Figure 13 shows the comparison of the measured and predicted forces using the posterior and prior distributions of  $K_t$  and  $K_f$ ; the results are also summarized in Table 5. The lack of overlap between the error bars for the inference results and the measured data is due either to underestimated uncertainty for the likelihood calculations, force measurement uncertainty, or an uncorrected bias in the model.

## 6 Conclusions

This paper reported on the National Science Foundation-sponsored workshop entitled "Uncertainty in machining." The purpose of the workshop was to address uncertainty and risk in machining and related manufacturing operations through the

application of decision theory to manufacturing models. To support this agenda, an example of Bayesian inference for the well-known mechanistic turning force model was presented. The beliefs about shear stress,  $\tau_s$ , shear plane angle,  $\phi_c$ , and average friction angle,  $\beta_a$ , and, therefore, the force coefficients  $K_t$  and  $K_f$ , were updated using measured values of the cut chip thickness,  $h_c$ , and the forces,  $F_t$  and  $F_f$ , from the Assessment of Machining Models database compiled by the National Institute of Standards and Technology. The prior (initial beliefs) was based on a literature review. The likelihood was determined using the discrete grid method, where the range of values for the variables of interest ( $\tau_s$ ,  $\phi_c$ , and  $\beta_a$ ) is divided into a grid of points. The chip thickness,  $h_c$ , and the forces,  $F_t$  and  $F_f$ , were calculated at each point using the deterministic equations. A normal distribution of the calculated values was assumed, and the uncertainty was specified. The value of the distribution at the measured value was taken to be the likelihood, which gives the probability that a selected variable value would give the measured value (using the deterministic model) in the presence of uncertainty. The posterior distribution was calculated by multiplying the prior and likelihood functions. This process was repeated for multiple tests to obtain a final posterior distribution of the force coefficients,  $K_t$  and  $K_f$ . The posterior distribution was then used to predict forces.

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## Appendix A: Workshop Participants

Ali Abbas	University of Illinois at Urbana-Champaign
Beth Allen	University of Minnesota
Robert D. Brown III	Incite! Decision Technologies
Satish Bukkapatnam	Oklahoma State University
Wei Chen	Northwestern University
Paul Collopy	DFM Consulting
Abhi Deshmukh	University of Massachusetts
Don Esterling	VeritasCNC
Dan Frayssinet	DP Technology
Rich Furness	Ford
Jerry Halley	Tech Manufacturing
George Hazelrigg	National Science Foundation
Rob Ivester	National Institute of Standards and Technology
Robert Jerard	University of New Hampshire
Jaydeep Karandikar <sup>3</sup>	University of Florida
Ranga Komanduri	Oklahoma State University
James Kong	Oklahoma State University
Dale Lombardo	GE Energy
Dan Luttrell	Moore Tools
Arif Malik	St. Louis University

<sup>3</sup>Graduate student supported by National Science Foundation funds.



Troy Marusich  
Laine Mears  
Shreyes Melkote  
Burak Ozdoganlar  
Zachary Piner  
Kamlakar Rajurkar  
Ralph Resnick

Walter Roy  
Don Saari  
Tony Schmitz

Albert Shih  
Scott Smith

John Snyder  
John Turner  
Roshan Vengazhiyil  
Ralph Wachter  
Paul Warndorf

Bob Winfough

Marty Wortman  
John Ziegert

Third Wave Systems  
Clemson University  
Georgia Tech  
Carnegie Mellon University  
DTL  
University of Nebraska  
National Center for Defense  
Manufacturing and Machining  
Army Research Laboratory  
University of California, Irvine  
University of North Carolina at  
Charlotte  
University of Michigan  
University of North Carolina at  
Charlotte  
TechSolve  
FAC&T  
Georgia Tech  
Office of Naval Research  
Association for Manufacturing  
Technology  
General Dynamics Ordnance and  
Tactical Systems  
Texas A&M University  
Clemson University

## References

- [1] Taylor, B., and Kuyatt, C., 1994, "NIST Technical Note 1297 1994 Edition Guidelines for Evaluating the Uncertainty of NIST Measurement Results," National Institute of Standards and Technology, Gaithersburg, MD.
- [2] International Organization for Standardization, *Guide to the Expression of Uncertainty in Measurement*, 1st ed. Geneva, Switzerland (1993).
- [3] Howard, R. A., and Hertz, D. B., 1966, *Proceedings of the Fourth International Conference on Operational Research*, Wiley-Interscience, New York.
- [4] Raiffa, H., 1968, *Decision Analysis: Introductory Lectures on Choices under Uncertainty*, Addison-Wesley, Reading, MA.
- [5] Howard, R. A., 1988, "Decision Analysis: Practice and Promise," *Manage. Sci.*, **34**, pp. 679–695.
- [6] Neumann, J., and Morgenstern, O., 1944, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.
- [7] Grayson, C. J., *Decisions Under Uncertainty, Drilling Decisions by Oil and Gas Operators* (Harvard Business School, Division of Research, Boston, MA, 1960).
- [8] Newendorp, P. D., 1975, *Decision Analysis for Petroleum Exploration*, PennWell Books, Tulsa, OK.
- [9] Brennan, M. J., and Schwartz, E. S., 1985, "Evaluating Natural Resource Investments," *J. Business*, **58**, pp.135–157.
- [10] Olsen, T. E., and Stensland, G., 1988, "Optimal Shut Down Decisions in Resource Extraction," *Econ. Lett.*, **26**, pp.215–218.
- [11] Paddock, J. L., Siegel, D. R., and Smith, J. L., 1988, "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases," *Q. J. Econ.*, **103**, pp. 479–508.
- [12] Clarke, H. R., and Reed, W. J., 1990, "Oil-Well Valuation and Abandonment with Price and Extraction Rate Uncertain," *Resour. Energ.*, **12**, pp. 361–382.
- [13] Keefer, D. L., 1991, "Resource Allocation Models with Risk Aversion and Probabilistic Dependence: Offshore Oil and Gas Bidding," *Manage. Sci.*, **37**, pp. 377–395.
- [14] Gibson, R., Schwartz, E., and Lund, D., 1991, "The Valuation of Long Term Oil-Linked Assets," *Stochastic Models and Option Values*, D. Lund and B. Øksendal, eds., Elsevier Science, Amsterdam.
- [15] Smith, J. E., and McCardle, K. F., 1998, "Valuing Oil Properties: Integrating Option Pricing and Decision Analysis Approaches," *Oper. Res.*, **46/2**, pp. 198–217.
- [16] Altintas, Y., 2000, *Manufacturing Automation Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design*, Cambridge University Press, New York, NY.
- [17] Ivester, R., Kennedy, M., Davies, M., Stevenson, R., Thiele, J., Furness, R., and Athavale, S., 2000, "Assessment Machining Models," *Progr. Rep.*, **4/3**, pp. 511–538.
- [18] Trent, E., and Wright, P., 2000, *Metal Cutting*, 4th ed., Butterworth-Heinemann, Woburn, MA.
- [19] DeGarmo, E. P., Black, J. T., Kohser, R., and Klamecki, B., *Materials and Processes in Manufacturing*, 9th ed. (Wiley, Hoboken, NJ, 2003).
- [20] Tlusty, J., 2000, *Manufacturing Processes and Equipment*, Prentice-Hall, Upper Saddle River, NJ.
- [21] Kalpakjian, S., and Schmid, S., 2008, *Manufacturing Processes for Engineering Materials*, 5th ed., Prentice-Hall, Upper Saddle River, NJ.
- [22] Iqbal, S., Mativenga, P., and Sheikh, M., 2007, "Characterization of Machining of AISI 1045 Steel Over a Wide Range of Cutting Speeds, Part I: Investigation of Contact Phenomena," *Proc. Inst. Mech. Eng., IMechE Conf.*, **221**, pp. 909–926.