Article

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Identification of correlated damage parameters under noise and bias using Bayesian inference

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Abstract

This article presents statistical model parameter identification using Bayesian inference when parameters are correlated and observed data have noise and bias. The method is explained using the Paris model that describes crack growth in a plate under mode I loading. It is assumed that the observed data are obtained through structural health monitoring systems, which may have random noise and deterministic bias. It was found that a strong correlation exists (a) between two parameters of the Paris model, and (b) between initially measured crack size and bias. As the level of noise increases, the Bayesian inference was not able to identify the correlated parameters. However, the remaining useful life was predicted accurately because the identification errors in correlated parameters were compensated by each other. It was also found that the Bayesian identification process converges slowly when the level of noise is high.

Keywords

parameters identification, damage growth parameters, correlated parameters, Bayesian inference

Introduction

Structural health monitoring (SHM) facilitates condition-based maintenance that provides a cost effective maintenance strategy by providing an accurate quantification of degradation and damage at an early stage without intrusive and time-consuming inspections.[1] Most SHM systems utilize on-board sensors/actuators to detect damage, find the location of damage, and estimate the significance of damage. Since SHM systems can assess damage frequently, they can also be used to predict the future behavior of the system, which is critically important for maintenance scheduling and fleet management. SHM systems can have a significant impact on increasing safety by allowing predictions of the structure's health status and remaining useful life (RUL), which is called prognostics.

In general, prognostics methods can be categorized into data-driven,[2] model-based,[3] and hybrid[4] approaches, based on the usage of information. The data-driven method uses information from collected data to predict the future status of the system without using any particular physical model. It includes least-square regression, Gaussian process regression, etc. The model-based method assumes that a physics model describing the behavior of the system is available. This method combines the physics model with measured data to identify model parameters and predict future behavior. Modeling the physical behavior can be accomplished at different levels, for example, micro- and macro-levels. Crack growth model[5] or fatigue life model[6] are often used for macro-level damage, and first principle models[7] are used for micro-level damage. The hybrid method combines the above-mentioned two methods, and includes particle filters[8–10] and Bayesian techniques.[11–14] Since the data-driven method identifies abnormality based on the trend of data, it is powerful in predicting near-future behaviors, while the model-based method has advantages in predicting long-term behaviors of the system. It is noted that in the

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model-based method for fatigue applications, the history of load is required in addition to the measured crack data.

In this article, a physics-based model for structural degradation due to fatigue damage is applied for prognostics, since damage grows slowly and the physics governing its behavior is relatively well-known. The main purpose of prognostics is to identify and repair those damages that threaten the system safety (condition-based maintenance) and to predict an appropriate maintenance schedule. Paris-family models are commonly used in describing the growth of cracks in aircraft panels under fatigue loading.[15] In general, these models have model uncertainty as they are not exact in describing crack growth. However, the objective of this article is to present how model parameters can be identified for a given model. Therefore, model uncertainty is ignored. In this article, the original Paris model[5] is used because it has the least number of parameters. It is assumed that the Paris model exactly describes the damage growth behavior if the model parameters are also exact. The main purpose of the article is to present the usage of Bayesian inference in identifying model parameters and predicting the RUL – the remaining cycles before maintenance. The study focuses on crack growth in a fuselage panel under repeated pressurization loading, which can be considered regular loading cycles. In this type of application, the uncertainty in applied loading is small compared to other uncertainties. Therefore, the crack growth behavior and the RUL can be predicted based on the identified model parameters before the crack becomes dangerous. The improved accuracy in these model parameters allows more accurate prediction of the RUL of the monitored structural component.

Identifying the model parameters and predicting damage growth, however, is not a simple task due to the noise and bias of data from SHM systems and the correlation between parameters, which is prevalent in practical problems. The noise comes from variability of random environments, while the bias comes from systematic departures of measurement data, such as calibration error. However, research for identifying model parameters under noise and bias, without mentioning correlated parameters, is limited.[9,16]

The main objective of this article is to demonstrate how Bayesian inference can be used to identify model parameters and to predict RUL, especially when the model parameters are correlated. In order to find the effects of noise and bias on the identified parameters, numerical studies utilize synthetic data; that is, the measurement data are produced from the assumed model of noise and bias. The key interest is how the Bayesian inference identifies the correlated parameters under noise and bias in data.

The article is organized as follows: in the second section, a simple damage growth based on Paris model is presented in addition to the uncertainty model of noise and bias; in the third section, parameter identification and RUL prediction using Bayesian inference and MCMC simulation method[17] are presented with different levels of noise and bias; and conclusions are presented in the final section.

Damage growth and measurement uncertainty models

Damage growth model

In this article, a simple damage growth model is used to demonstrate how to characterize damage growth parameters. Although some experimental data on fatigue damage growth are available in the literature,[18] they are not measured using SHM systems. Therefore, the level of noise and bias is much smaller than the actual data that will be available in SHM systems. In this article, synthetic damage growth data are used in order to perform statistical study on the effect of various levels of noise and bias. It is assumed that a through-the-thickness center crack exists in an infinite plate under the mode I loading condition. In aircraft structure, this corresponds to a fuselage panel under repeated pressurization loading (Figure 1), which is the main cause of fatigue in fuselage panels. Therefore, one flight corresponds to one cycle. In this approximation, the effect of finite plate size and the curvature of the plate are ignored. When the stress range due to the pressure differential is $\Delta\sigma$, the rate of damage growth can be written using the Paris model[5] as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m, \quad \Delta K = \Delta\sigma\sqrt{\pi a} \tag{1}$$

where a is the half crack size, N is the number of cycles (one cycle corresponds to one flight), ΔK is the range of stress intensity factor, and other parameters are shown in Table 1 for 7075-T651 aluminum alloy. Although the number of cycles, N, is an integer, it is treated as a real number in this model. The above model has two damage growth parameters, C and m, which are estimated to predict damage propagation and RUL. In Table 1, these two parameters are assumed to be uniformly distributed, whose lower and upper bounds were obtained from the scatter of experimental data.[19] They can be considered as the damage growth parameters of generic Al 7075-T651 material. In general, it is well-known that the two Paris parameters are strongly correlated,[20] but it is assumed initially that they are

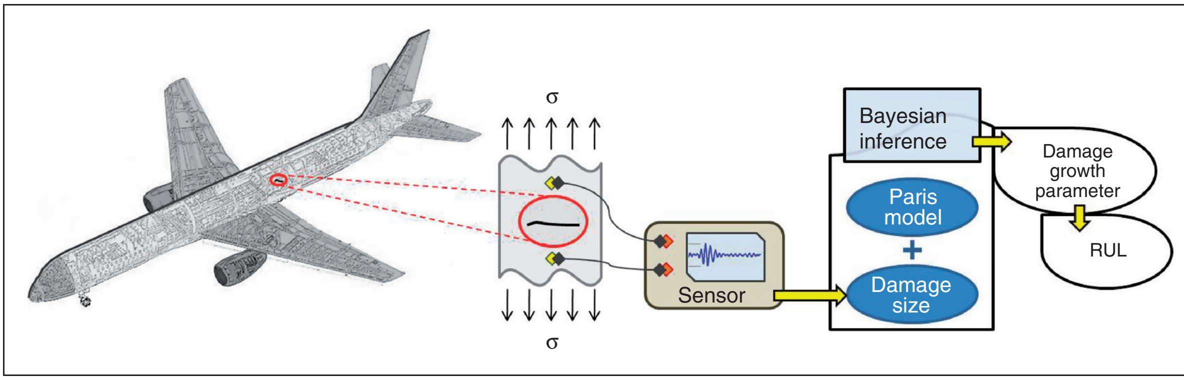

Figure 1. Through-the-thickness crack in a fuselage panel.

Table 1. Loading and fracture parameters of 7075-T651 Aluminum alloy

Property	Nominal stress $\Delta\sigma$ (MPa)		Fracture toughness K_{IC} (MPa$\sqrt{m}$)	Damage parameter m	Damage parameter log(C)
Value/Distribution	case 1:	86.5	Deterministic 30	Uniform (3.3, 4.3)	Uniform (log(5E-11), log(5E-10))
	case 2:	78.6			
	case 3:	70.8			

uncorrelated because there is no prior knowledge on the level of correlation. Using measured data of crack sizes, the Bayesian inference will show the correlation structure between these two parameters. Since the scatter is so wide, the prediction of RUL using the initial distributions of parameters is meaningless. The specific panel being monitored using SHM systems may have much narrower distributions of the parameters, or even deterministic values.

The half crack size a_i, after N_i cycles of fatigue loading, can be obtained by integrating Equation (1) and solving for a_i as

$$a_i = \left[N_i C\left(1-\frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_0^{1-\frac{m}{2}}\right]^{\frac{2}{2-m}} \tag{2}$$

where a_0 is the initial half crack size. In SHM, the initial crack size does not have to be the micro-crack in the panel before applying any fatigue loading. This can be the crack size that is detected by SHM systems the first time. In such a case, N_i should be interpreted as the number of cycles since the crack is detected. It is assumed that the panel fails when a_i reaches a critical half crack size, a_C. Here, we assume that this critical crack size is when the stress intensity factor exceeds the fracture toughness K_{IC}. This leads to the following expression for the critical crack size:

$$a_C = \left(\frac{K_{IC}}{\Delta\sigma\sqrt{\pi}}\right)^2 \tag{3}$$

Even if the crack growth model in Equation (1) is the simplest form, it requires identifying various parameters. First, the damage growth parameters, C and m, need to be identified, which can be estimated from specimen-level fatigue tests.[18] However, due to material variability, these parameters show different values for different batches of panels. In addition, the initial crack size, a_0, needs to be found. Liu and Mahadevan[21] used an equivalent initial flaw size, but in this case, it corresponds to error in the first data in SHM measurement. In addition, the fracture toughness, K_{IC}, also shows randomness due to variability in manufacturing.

Measurement uncertainty model

In SHM-based inspection, the sensors installed on the panel are used to detect the location and size of damage. Even if the on-line inspection can be performed continuously, it would not be significantly different from on-ground inspection because the structural damage will not grow quickly. In addition, the on-ground, compared to on-line, inspection will have much smaller levels of noise. The on-ground inspection may provide a significant weight advantage because only sensors, not measurement equipment, are on-board. Our preliminary study showed that there is no need to inspect at every flight because damage growth at each flight is extremely small.

A crack in the fuselage panel grows according to the applied load, pressurizations in this case. Then, the SHM systems detect the crack. In general, the SHM system cannot detect a crack when it is too small. Many SHM systems can detect a crack between the size of 5–10 mm. Therefore, the necessity of identifying the initial crack size becomes unimportant by setting a_0 to be the initially detected crack size. However, a_0 may still include noise and bias from the measurement. In addition, the fracture toughness, K_{IC}, is also unimportant because airliners may want to send the airplane for maintenance before the crack becomes critical.

The main objective of this article is to show that the measured data can be used to identify crack growth parameters, and then, to predict the future behavior of the cracks. Since no airplanes are equipped with SHM systems yet, we simulate the measured crack sizes from SHM. In general, the measured damage includes the effect of bias and noise. The former is deterministic and represents a calibration error, while the latter is random and represents noise in the measurement environment. The synthetic measurement data are useful for parameter study, that is, the different noise and bias levels show how the identification process is affected. In this context, bias is considered as two different levels, ± 2 mm, and noise is uniformly distributed between $-u$ mm and $+u$ mm. Four different levels of u are considered: 0, 0.1, 1, and 5 mm. The varying levels of noise represent the quality of SHM systems.

The synthetic measurement data are generated by (a) assuming that the true parameters, m_{true} and C_{true}, and the initial half crack size, a_0, are known; (b) calculating the true crack sizes according to Equation (2) for a given N_i and $\Delta\sigma$; and (c) adding a deterministic bias and random noise to the true crack size data including the initial crack size. Once the synthetic data are obtained, the true values of crack sizes as well as the true values of parameters are not used in the prognostics process. In this article, the following true values of parameters are used for all numerical examples: $m_{\text{true}} = 3.8$, $C_{\text{true}} = 1.5 \times 10^{-10}$, and $a_0 = 10$mm.

Table 1 shows three different levels of loading; the first two ($\Delta\sigma = 86.5$ and 78.6 MPa) are used for estimating model parameters, while the last ($\Delta\sigma = 70.8$) is used for validation purposes. The reason for using two sets of data to estimate damage growth parameters is to utilize more data having damage propagation information at an early stage. Theoretically, the true values of parameters can be identified using a single set of data because the Paris model is a nonlinear function of parameters. However, random noise can make the identification process slow, especially when parameters are correlated; that is, many different combinations of correlated parameters can achieve the same crack size. This property delays the convergence of Bayesian process such that meaningful parameters can only be obtained toward the end of RUL. Based on preliminary study, two sets of data at different loadings can help the Bayesian process converge quickly. This situation corresponds to two fuselage panel with different thickness.

Figure 2 shows the true crack growth curves for three different levels of loading (solid curves) and synthetic measurement data a_i^{meas}(triangles) with noise and bias. It is noted that the positive bias shifts the data above the true crack growth. On the other hand, the noises are randomly distributed between measurement cycles. It is assumed that the measurements are performed at every 100 cycles. Let there be n measurement data. Then, the measured crack sizes and corresponding cycles are represented by

$$\mathbf{a}^{\text{meas}} = \{a_0^{\text{meas}}, \quad a_1^{\text{meas}}, \quad a_2^{\text{meas}}, \ldots, a_n^{\text{meas}}\}$$
$$\mathbf{N} = \{N_0 = 0,\ N_1 = 100,\ N_2 = 200, \ldots, N_n\} \quad (4)$$

It is assumed that after N_n, the crack size becomes larger than the threshold and the crack is repaired.

Bayesian inference for characterization of damage properties

Damage growth parameters estimation

Once the synthetic data (damage sizes vs. cycles) are generated, they can be used to identify unknown damage growth parameters. As mentioned before, m, C, and a_0 can be considered as unknown damage growth parameters. In addition, the bias and noise are also unknown because they are only assumed to be known in generating crack size data. In the case of noise, the standard deviation, σ, of the noise and deterministic bias, b, are considered as an unknown parameters. The identification of σ will be important as the Bayesian process depends on it. Therefore, the objective is to identify (or, improve) these five parameters using the measured crack size data. The vector of unknown parameters is defined by $\mathbf{y} = \{m,\ C,\ a_0,\ b,\ \sigma\}$.

Parameter identification can be done in various ways. The least-squares method is a traditional way of identifying deterministic parameters. For crack propagation, Coppe et al.[22] used the least-square method to identify unknown damage growth parameters along with bias. However, in the least-squares

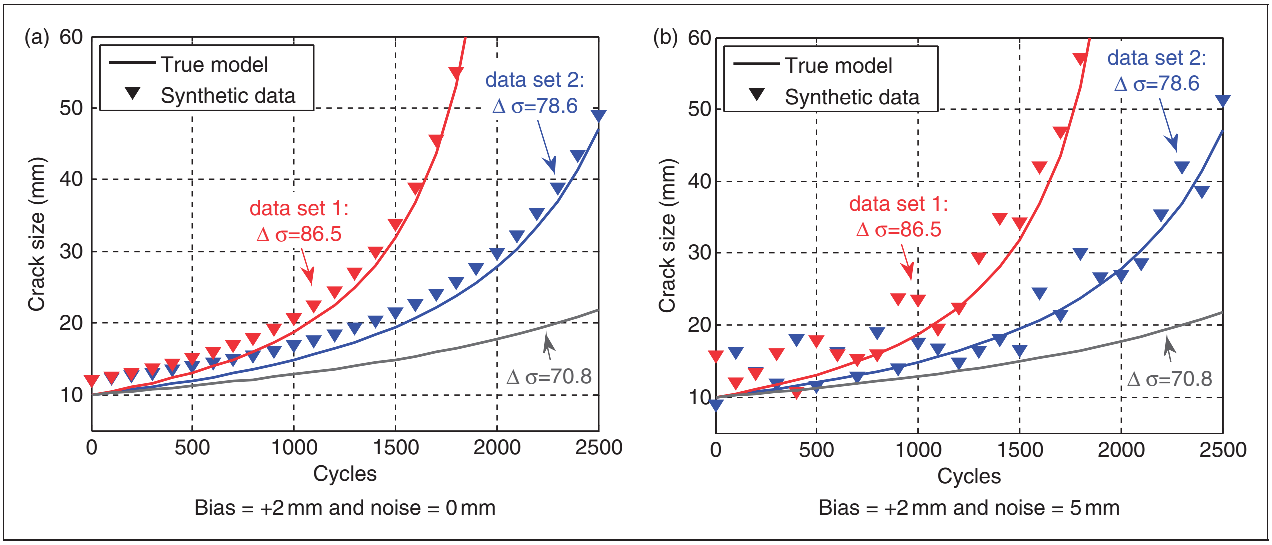

Figure 2. Crack growth of three different loading conditions and two sets of synthetic data.

method, it is non-trivial to estimate the uncertainty in the identified parameters. In this paper, Bayesian inference is used to identify the unknown parameters as well as the level of noise and bias. Coppe et al.[23] used Bayesian inference in identifying damage growth parameters, C or m. They used the grid method to calculate the posterior distribution of one variable and discussed that updating multi-dimensional variables can be computationally expensive. The grid method computes the values of PDF at a grid of points after identifying the effective range, and calculates the value of the posterior distribution at each grid point. This method, however, has several drawbacks such as the difficulty in finding correct location and scale of the grid points, spacing of the grid, and so on. In addition, it becomes computationally expensive when the number of updating parameters increases. Markov Chain Monte Carlo (MCMC) simulation is a computationally efficient alternative to obtain the probability density function (PDF) by generating a chain of samples.[17]

In Baye's theorem,[24] the knowledge of a system can be improved with additional observation of the system. More specifically, the joint PDF of $\mathbf{y}$ will be improved using the measured crack sizes $\mathbf{a}^{\text{meas}}$. The joint posterior PDF is obtained by multiplying the prior PDF with the likelihood as

$$p_{\mathbf{Y}}(\mathbf{y}|\mathbf{a}^{\text{meas}}) = \frac{1}{K} p_A(\mathbf{a}^{\text{meas}}|\mathbf{Y}=\mathbf{y})\, p_{\mathbf{Y}}(\mathbf{y}) \tag{5}$$

where $p_{\mathbf{Y}}(\mathbf{y})$ is the prior PDF of parameters, $p_A(\mathbf{a}^{\text{meas}}|\mathbf{Y}=\mathbf{y})$ is the likelihood or the PDF values of crack size at $\mathbf{a}^{\text{meas}}$ given parameter value of $\mathbf{y}$, and K is a normalizing constant. It is noted that the likelihood is constructed using n measured crack size data. For prior distribution, the uniform distribution is used for the damage growth parameters, m and C, as described in Table 1. For other parameters, no prior distribution is used; that is, noninformative. Therefore, the prior PDF becomes $p_{\mathbf{Y}}(\mathbf{y}) = p_{\mathbf{Y}}(m)\, p_{\mathbf{Y}}(C)$. The likelihood is the probability of obtaining the observed crack sizes $\mathbf{a}^{\text{meas}}$ given values of parameters. For the likelihood, it is assumed to be a normal distribution for the given five parameters including the standard deviation of the measured size, σ:

$$p_A(\mathbf{a}^{\text{meas}}|\mathbf{Y}=\mathbf{y}) \propto \left(\frac{1}{\sqrt{2\pi}\mathrm{y}_5}\right)^n \times \exp\left[-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{a_i^{\text{meas}} - a_i(\mathbf{y}_{1:4})}{\mathrm{y}_5}\right)^2\right], \mathbf{y} = \{m,\ C,\ a_0,\ b,\ \sigma\} \tag{6}$$

where

$$a_i(\mathbf{y}_{1:4}) = \left[N_i C\left(1-\frac{m}{2}\right)\left(\Delta\sigma\sqrt{\pi}\right)^m + a_0^{1-\frac{m}{2}}\right]^{\frac{2}{2-m}} + b \tag{7}$$

is the crack size from the Paris model with bias and a_i^{meas} is the measurement crack size at cycle N_i. In general, it is possible that the normal distribution in Equation (6) may have a negative crack size, which is physically impossible; therefore, the normal distribution is truncated at zero.

A primitive way of computing the posterior PDF is to evaluate Equation (5) at a grid of points after identifying the effective range. This method, however, has several drawbacks such as the difficulty in finding correct location and scale of the grid points, the spacing of the grid, and so on. Especially when a multi-variable joint PDF is required, which is the case in this article, the computational cost is proportional to M^5, where M is the number of grids in one-dimension. On the other hand, the MCMC simulation can be an effective solution as it is less sensitive to the number of variables.[17] Using the expression of posterior PDF in Equation (5), 5000 samples of parameters are drawn by using the Metropolis-Hastings (M-H) algorithm, which is a typical method of MCMC.

The effect of correlation between parameters

Since the original data of crack sizes are generated from the assumed true values of parameters, the objective of Bayesian inference is to make the posterior joint PDF converge to the true values. Therefore, it is expected that the PDF becomes narrower as n increases; that is, more data are used. This process seems straightforward, but the preliminary study shows that the posterior joint PDF may converge to values different from the true ones. It is found that this phenomenon is related to the correlation between parameters. For example, let the initially detected crack size be $\bar{a}_0^{\text{meas}}$ when the measurement environment has no noise. This measured size is the outcome of the initial crack size and bias:

$$\bar{a}_0^{\text{meas}} = a_0 + b \tag{8}$$

Therefore, there exist infinite possible combinations of a_0 and b to obtain the measured crack size. It is generally infeasible to identify the initial crack size and bias with a single measurement when the measured data is linearly dependent on multiple parameters. It was also well known that the two Paris model parameters, m and C, are strongly correlated.[25] This can be viewed from the crack growth rate curve, as illustrated in Figure 3. In this graph, the parameter m is the slope of the curve, while C corresponds to the y-intercept at $\Delta K = 1$. If a specific value of crack growth rate $\mathrm{d}a/\mathrm{d}N$ is observed, this can be achieved by different combinations of these two parameters. However, in the case of Paris model parameters, it is feasible to identify them because the stress intensity factor gradually increases as the crack grows. However, the embedded noise can make it difficult to identify the two model parameters because the crack growth rate may not be consistent with the noisy data. In addition, this can slow down the convergence in the posterior distribution, because when the crack is small, there is no significant crack growth rate. The effect of noise is relatively diminished as the crack growth rate increases, which occurs toward the end of life.

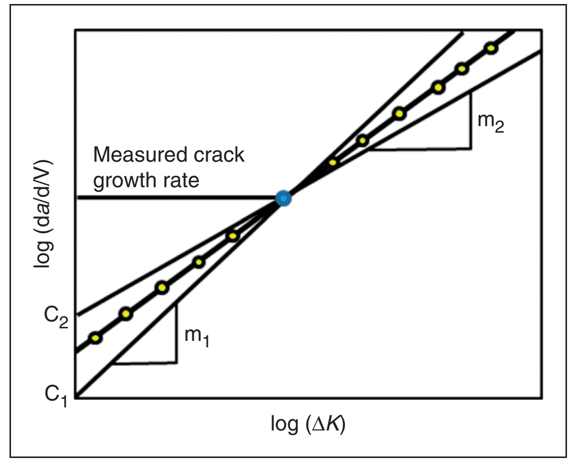

Figure 3. Illustration of showing the same crack growth rate with different combinations of parameters.

In order to overcome the above-mentioned difficulty in identifying correlated parameters, two different strategies are used in this article. There are many correlations between unknown parameters, but only the strongest correlated relationships are considered: the two Paris model parameters and a_0 and b. First, the two Paris model parameters are kept because they can be identified as the crack grows. Second, the relationship between a_0 and b has different characteristics from the model parameters (m, C). a_0 and b are time independent, and the sum of the two parameters are constant. Therefore, the bias is removed from the Bayesian identification process using Equation (8), assuming that the bias and initial crack size are perfectly correlated. This process seems straightforward, but the difficulty exists from the fact that the constant $\bar{a}_0^{\text{meas}}$ in Equation (8) is unknown. Below is the procedure of estimating $\bar{a}_0^{\text{meas}}$.

a. Assume that the measured initial crack size is $a_0 = a_0^{\text{meas}}$
b. With given a_0, use the Bayesian method to update the posterior joint PDF of m, C, b, σ
c. Calculate the maximum likelihood value b^* of b from the posterior joint PDF
d. Estimate $\bar{a}_0^{\text{meas}} = a_0 + b^*$
e. Eliminate b using $b = \bar{a}_0^{\text{meas}} - a_0$ and update the posterior joint PDF of m, C, a_0, σ

Figure 4 shows the posterior PDFs for the case of true bias of 2 mm (a) when $n = 13$ ($N_{12} = 1200$ cycles) and (b) when $n = 17$ ($N_{16} = 1600$ cycles). The posterior joint PDFs are plotted separately into three groups for the plotting purpose. In this case, it is assumed that there is no noise in the data. The true values of parameters are marked using a star symbol. Similar results were also obtained in the case with bias $= -2$ mm.

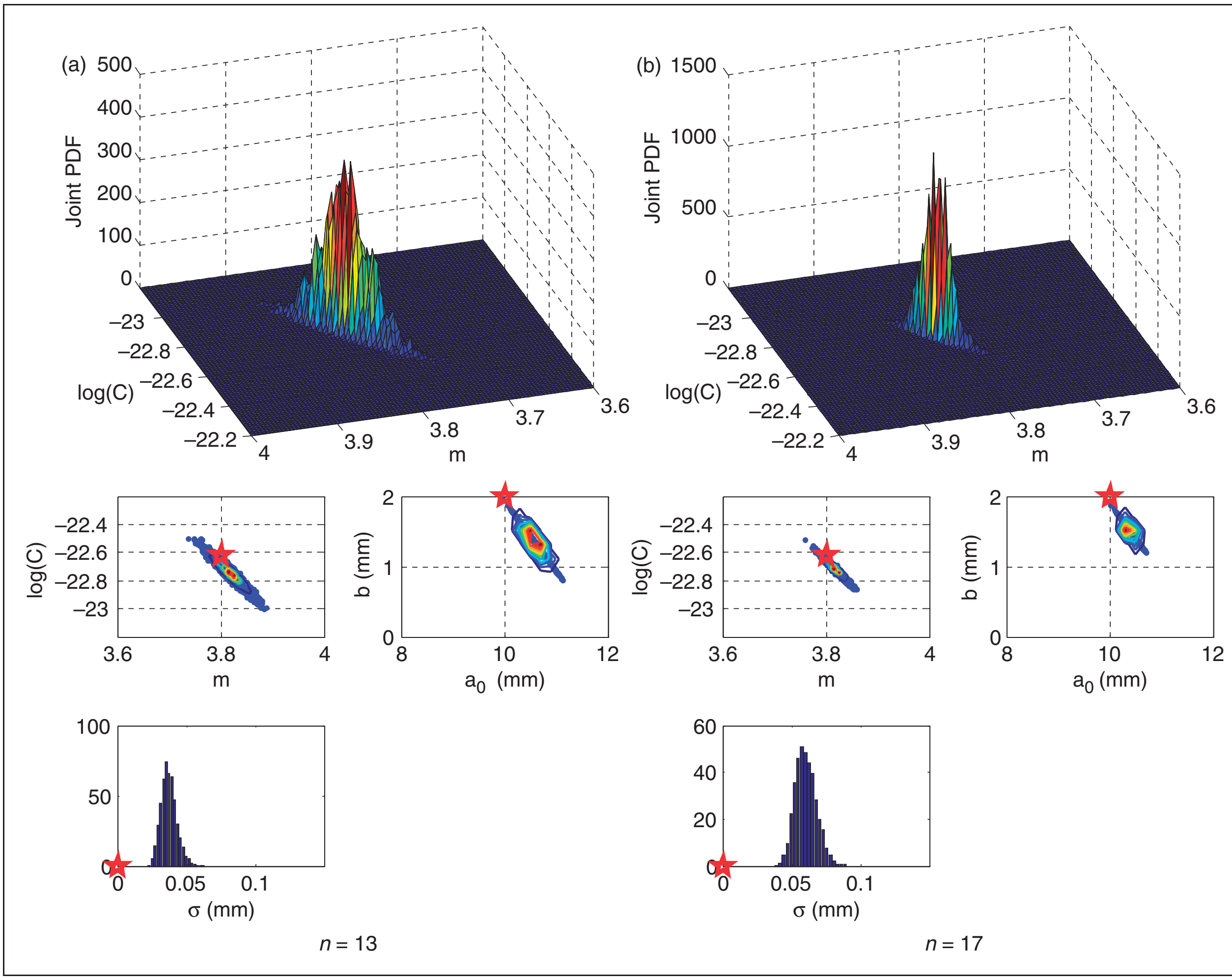

Figure 4. Posterior distributions of parameters with zero noise and true bias of 2 mm.

Firstly, it is clear that the two Paris model parameters are strongly correlated. The same is true for the initial crack size and bias; in fact, the PDF of bias is calculated from Equation (8) with the initial crack size. Secondly, it can be observed that the PDFs at $n = 17$ is narrower than that of $n = 13$, although the PDFs at $n = 13$ is quite narrow compared to the prior distribution. As the uncertainty in model parameters is reduced, the shape of distribution approaches a Gaussian distribution, regardless of the nonlinearity of the system. In addition, the central limit theorem[26] can be considered as a part of the reason why the joint distribution of (m, log C) appears Gaussian. Lastly, the identified results look different from the true values due to the scale, but the errors between the true values and the median of identified results are at a maximum of around 5%, except for bias. The error in bias looks large, but that is because the true value of bias is small. The error in bias is about 0.5 mm. The same magnitude of error exists for the initial crack size due to the perfect correlation between them. Table 2 lists all six cases considered in this paper, and all of them show a similar level of errors. It is noted that the identified standard deviation of noise, σ, does not converge to its true value of zero. This occurred because the original data did not include any noise. Zero noise can cause a problem in the likelihood calculation, as the denominator becomes zero in Equation (6). However, this would not happen for practical cases in which noise always exists.

The next example is to investigate the effect of noise on the posterior PDFs of parameters. The results of identified posterior distributions with different levels of noise were shown in Figure 5 when the true bias was 2 mm. Similar results were obtained when bias was −2 mm. The black, blue, and red colors represent noise levels of 0.1 mm, 1 mm, and 5 mm, respectively. The median location is denoted by a symbol (a circle for 0.1 mmnoise, a square for 1 mmnoise, and a star for 5 mmnoise). Each vertical line represents a 90% confidence interval (CI) of posterior PDF. The solid horizontal line is the true value of the parameter. In the case

Table 2. The median of identified parameters and the errors with the true values

		$n=13$				$n=15$				$n=17$			
		m	$\log(C)$	a_0	b	m	$\log(C)$	a_0	b	m	$\log(C)$	a_0	b
True values		3.8	−22.6	10	±2	3.8	−22.6	10	±2	3.8	−22.6	10	±2
$b=+2$mm	Median	3.82	−22.8	10.6	1.37	3.81	−22.7	10.4	1.53	3.82	−22.7	10.4	1.52
	error (%)	0.49	0.57	5.67	31.7	0.32	0.37	4.00	23.6	0.47	0.44	3.84	24.2
$b=-2$mm	Median	3.78	−22.5	9.50	−1.44	3.78	−22.5	9.51	−1.41	3.78	−22.5	9.49	−1.35
	error (%)	0.40	0.50	4.96	28.0	0.40	0.48	4.94	29.5	0.55	0.55	5.11	32.7

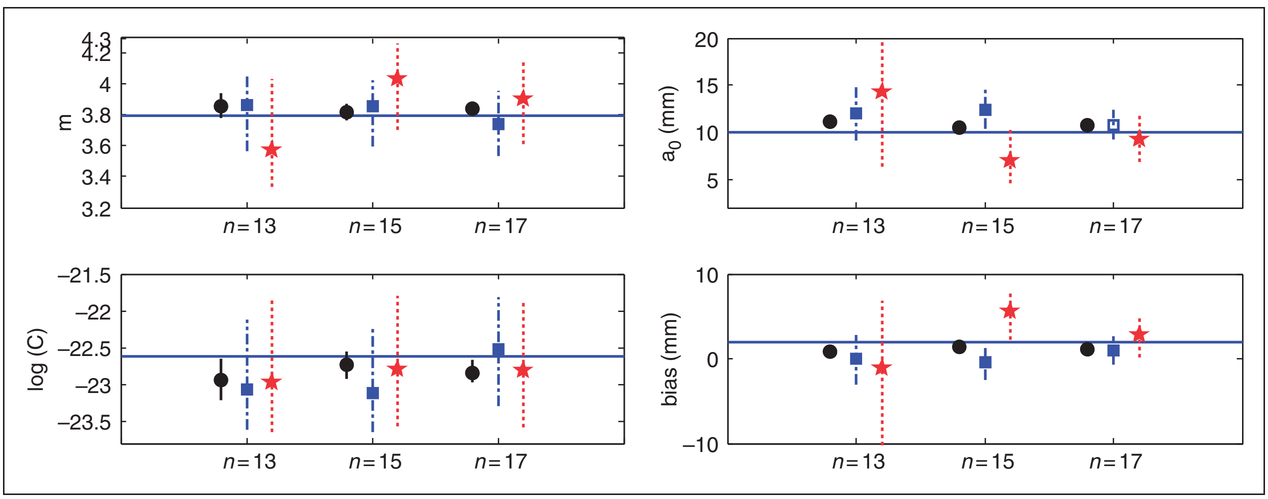

Figure 5. Posterior distributions with three different levels of noise (bias = 2 mm).

of noise level = 0.1 mm, all parameters were identified accurately with very narrow CIs. In the case of noise level = 1 mm, the initial crack size and bias were identified accurately as the number of data increased, whereas the CIs of two Paris parameters were not reduced. In addition, the median values were somewhat different from the true parameter values. Increasingly inaccurate results were observed as the level of noise increased to 5 mm. Therefore, it is concluded that the level of noise plays an important role in identifying correlated parameters using Bayesian inference. However, this does not mean that it is not able to predict RUL. Even if these parameters were not accurately identified because of correlation, the predicted RUL was relatively accurate, which will be discussed in detail next subsection.

Damage propagation and RUL prediction

Once the parameters are identified, they can be used to predict the crack growth and estimate RUL. Since the joint PDF of parameters are available in the form of 5000 sample, the crack growth and RUL will also be estimated using the same number of sample. First, using 5000 sets of parameters obtained from the MCMC method, Equation (2) is utilized to calculate 5000 numbers of crack size a_i after N_i cycles. Then, random measurement errors are added to the predicted crack sizes. For that purpose, 5000 samples of measurement errors are generated from normal distribution with zero mean and identified 5000 samples of σ. Then, the quality of prediction can be evaluated in terms of how close the median is to the true crack growth and how large the prediction interval (PI) is. The results of crack growth are shown in Figure 6 when the true bias is 2 mm. Different colors represent the three different loading conditions. The solid curves are true crack growth, while the dashed curves are medians of predicted crack growth distribution. The results are obtained as a distribution due to the uncertainty in parameters, but the medians of predicted crack growth are only shown in the figures for visibility. In addition, the critical crack sizes with different loadings use horizontal lines. Since the posterior distributions of

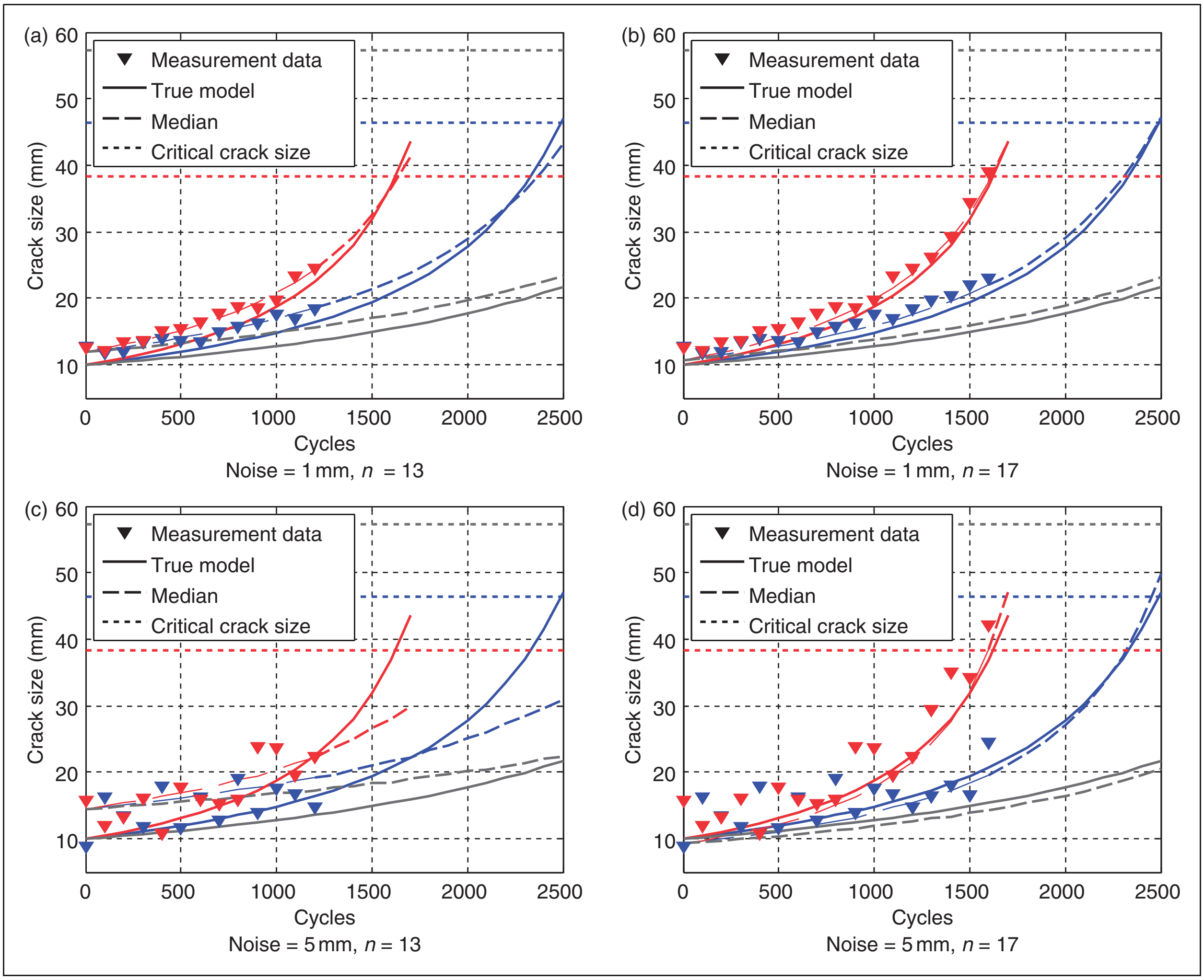

Figure 6. Prediction of crack growth with bias = +2 mm.

parameters are symmetric, there was no difference between the mean, mode, and maximum likelihood estimates.

Figure 6 shows that the results closely predicted the true crack growth when noise is less than 1 mm. Even if the level of noise is 5 mm, the results of predicted crack growth become close to the true one as the number of data increases. This means that if there are many data (much information about crack growth), the future crack growth can be predicted accurately, even if there is much noise. However, when the level of noise is large, the convergence is slow such that the accurate prediction happened almost at the end of life.

As can be seen from Figure 6, crack growth and RUL can be predicted with reasonable accuracy even though the true values of the parameters are not accurately identified. The reason is that the correlated parameters m and C work together to predict crack growth in Equation (2). For example, if m is underestimated, then the Bayesian process overestimates C to compensate for it. In addition, if there is large noise in the data, the distribution of estimated parameters becomes wider, which can cover the risk that comes from the inaccuracy of the identified parameters. Therefore it is possible to safely predict crack growth and RUL.

In order to see the effect of the noise level on the uncertainty of predicted RUL, Figure 7 plots the median and 90% prediction interval (PI) of the RUL and compared them with the true RUL. The RUL can be calculated by solving Equation (2) for N when the crack size becomes critical:

$$N_f = \frac{a_C^{1-m/2} - a_i^{1-m/2}}{C(1-\frac{m}{2})(\Delta\sigma\sqrt{\pi})^m} \quad (9)$$

The RUL is also expressed as a distribution due to the uncertainty of the parameters, which is obtained by replacing a_i and model parameters, m and C in

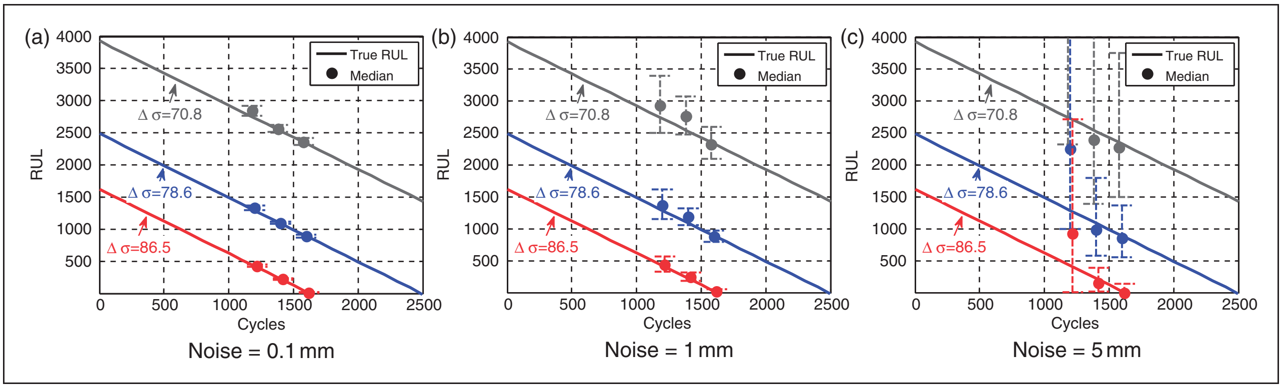

Figure 7. Median and 90% of prediction interval of the predicted RUL (bias = 2 mm).

Equation (9) with 5000 predicted crack growth and identified model parameters. In Figure 7, the solid diagonal lines are the true RULs at different loading conditions ($\Delta\sigma =$ 86.5, 78.6, 70.8). The precision and accuracy are fairly good when the noise is less than 1 mm, which is consistent with the crack growth results. In the case of a large noise, 5 mm, the medians are close to the true RUL, and the wide intervals are gradually reduced as more data are used. Therefore, it is concluded that the RULs are predicted reasonably in spite of large nose and bias in data.

Conclusions

In this article, Bayesian inference and the Markov Chain Monte Carlo (MCMC) method are used for identifying the Paris model parameters that govern the crack growth in an aircraft panel using SHM systems that measure crack sizes with noise and bias. Focuses have been given to the effect of correlated parameters and the effect of noise and bias. The correlation between the initial crack size and bias was explicitly imposed using analytical expression, while the correlation between two Paris parameters was identified through Bayesian inference. It is observed that the correlated parameter identification is sensitive to the level of noise, while predicting the remaining useful life is relatively insensitive to the level of noise. It is found that greater numbers of data are required to narrow the distribution of parameters when the level of noise is high. When parameters are correlated, it is difficult to identify the true values of the parameters, but the correlated parameters work together to predict accurate crack growth and RUL.

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