JOURNAL OF AIRCRAFT
Vol. 52, No. 5, September–October 2015

Probabilistic Manufacturing Tolerance Optimization of Damage-Tolerant Aircraft Structures Using Measured Data

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DOI: 10.2514/1.C032945

Manufacturing tolerances play an important role in designing low-cost and low-weight aircraft structures. A probabilistic cost model is developed that explicitly models performance cost along with quality and manufacturing costs to optimize manufacturing tolerances for damage-tolerant aircraft structures. The tolerance optimization is demonstrated for the design of a wing spar with fastener holes. The advantage of including performance cost is illustrated by comparing to an optimum found by ignoring the performance cost, showing substantial change in the optimum tolerance and total cost. The random aspect of quality cost is illustrated using measured manufacturing error data collected from the wing spar assemblies of a business jet. The paper also quantifies the uncertainties in the optimum due to finite samples of measured data used for modeling manufacturing error distributions, which were found to be small due to a large number of samples.

Nomenclature

C_{Al}	=	cost per pound of aluminum alloy U.S. dollars/lb
C_{CV}	=	constraint violation/scrap cost, U.S. dollars
C_M	=	manufacturing cost, U.S. dollars
C_{Mat}	=	material cost, U.S. dollars
C_{Prod}	=	production cost, U.S. dollars
C_Q	=	quality cost, U.S. dollars
C_{QR}	=	quality review cost, U.S. dollars
C_{QRPF}	=	quality review cost per fastener, U.S. dollars
C_{UL}	=	cost of useful load, U.S. dollars/lb
d	=	hole diameter, in.
e	=	edge distance, in.
L_P	=	performance loss, U.S. dollars
P_{CV}	=	probability of constraint violation
P_{HOS}	=	probability of hole oversize
P_{QR}	=	probability of quality review
P_{TE}	=	probability of tolerance exceedance
T	=	tolerance, in.
t	=	thickness, in.
W_p	=	weight of aluminum plate, lb
W_s	=	weight of wing spar, lb
w	=	width, in.
Δ	=	deviation/error/change
ρ_{Al}	=	density of aluminum alloy, lb/in.^{3}

Presented as Paper 2013-1465 at the 54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Boston, MA, 8–11 April 2013; received 10 April 2014; revision received 18 October 2014; accepted for publication 7 April 2015; published online 3 July 2015.

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I. Introduction

A MANUFACTURING tolerance is the permissible limit of variation in physical dimensions of a part and various geometric features (e.g., holes) comprising that part. Manufacturing tolerances play an important role in designing low-cost and low-weight damage-tolerant aircraft structures. Both objectives of low cost and weight can be interpreted as customer objectives, i.e., cheap (low cost) and high-performance (low weight) design. This paper develops a cost model that explicitly considers the low-weight objective by modeling performance cost along with the low-cost objective. Note that damage tolerance (DT) should not be confused with manufacturing tolerance. The DT structural design approach [1] has been successfully employed to safeguard against structural failures due to fatigue cracking through inspection programs. Therefore, manufacturing tolerance is specified to avoid compromising the DT capability of aircraft structures due to manufacturing variability/errors.

To explain the competing low-cost and low-weight objectives in the context of DT structural design, consider a fastener hole, shown in Fig. 1. A weight-optimal design (low design weight), shown in Fig. 1a, requires the hole axis to be exactly at a distance e from the edge of the plate in order to satisfy a prespecified fatigue crack growth life (FCGL) constraint. FCGL is the time taken by a preexisting crack (rogue flaw) at the hole to grow from an initial length a_{ini} to a critical length a_{cri}, and is used to derive the inspection intervals. The manufacturing variability (shown by a probability distribution in Fig. 1) in edge distance e of the hole may violate a prespecified FCGL constraint, which could lead to either repair or scrap. A simple solution to protect against such violations is to increase the width of the plate by allocating tolerance T to both edges of the initial weight-optimal design, as shown in Fig. 1b. So, manufacturing tolerance makes FCGL constraint insensitive to any variation within the tolerance band, shown in Fig. 1b. As a result, the quality (or rework) cost decreases with the decrease in the number of events that exceeds the tolerance band, i.e., the probability of tolerance exceedance decreases. Conversely, the structural weight and manufacturing costs (mainly material cost) increase due to an increase in the width of the plate from w to $w + 2T$, decreasing the value of an aircraft to a customer (i.e., performance loss/cost) that is similar to Taguchi et al.'s [2] philosophy of loss carried by customers after sale.

The first objective of this paper is to develop a probabilistic cost model that explicitly models performance cost along with quality and manufacturing costs to optimize manufacturing tolerances for DT components of aircraft structures (e.g., wing spar caps). The cost model developed in this study balances the conflicting low-cost and low-weight objectives simultaneously to design best-value damage-

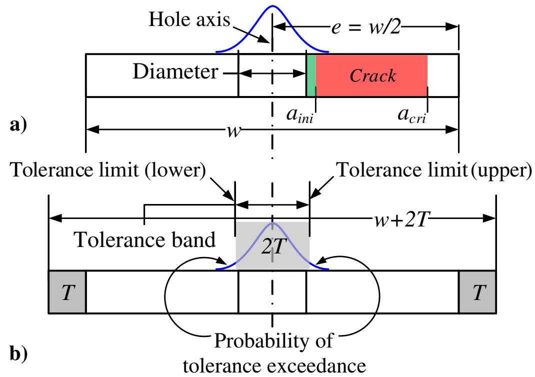


Fig. 1 Representations of a) weight-optimal design (without tolerance), and b) design after tolerance addition.

tolerant structures. The second objective of the paper is to use measured manufacturing error/variability data for identifying appropriate probability distributions of the random variables associated with various part dimensions. The third objective is to estimate the uncertainty in the optimum due to finite measured data available for probabilistic modeling of random variables.

The paper is organized in five sections. A review of cost-tolerance modeling literature is presented in Sec. II, which is followed by Sec. III that identifies the key features of the cost model developed in this study. Section IV.A introduces the concept of DT design of a wing spar at fastener holes in the context of the current practice of the aerospace industry. The paper then discusses the importance of manufacturing tolerances for a DT design in Sec. IV.B, and it identifies the two most common manufacturing error types (random variables) related to position and size dimensions of fastener holes in a wing spar. Section IV.C presents the data and probability distributions of both manufacturing errors measured from the wing assemblies of a business jet. The error data are further used in the calculation of quality cost. A detailed formulation of the cost model is presented in Sec. IV.D. The optimal tolerance is found by minimizing the expected total-cost objective, which is presented in Sec. IV.E. A comparison emphasizing the explicit modeling of the performance cost is also presented in Sec. IV.F. We then consider the effect of uncertainties in the cost-model inputs to estimate the uncertainty in the optimum in Sec. V.

II. Literature Review

Manufacturing tolerances first appeared on engineering drawings as plus/minus limits on nominal dimensions in the early 1900s. Since the early 1970s, numerous studies have been published that have advanced different areas of tolerance research. Hong and Chang [3] presented a comprehensive review of various tolerance research activities. They classified tolerance research activities into seven distinct categories: tolerancing schemes, tolerance modeling, tolerance specification, tolerance analysis, tolerance allocation, tolerance transfer, and tolerance evaluation. The tolerance activity undertaken in this paper can be classified under tolerance allocation. Dong [4] pointed out that the assignment of tolerance values is largely done by tolerance specification, which is based on a trial-and-error approach and, in some cases, based on one's design experience and knowledge of the manufacturing processes. Alternatively, tolerance synthesis/allocation presents a systematic approach that allocates optimal tolerances by employing detailed cost-modeling and optimization methods. Cost-model development is therefore a necessary first step that provides an objective function for tolerance optimization.

A. Cost-Tolerance Modeling with Manufacturing Cost

A majority of published studies model the cost-tolerance relationship deterministically with a regression equation (e.g., exponential relationship [5]) by considering the manufacturing cost (machining and assembly costs). Chase et al. [6] and Dong [4] compiled various general regression models that have been proposed to define the relationship between tolerance and the manufacturing cost. These models often fix the tolerance limits at a prespecified level (i.e., quality cost or out-of-specification proportion is fixed) for estimating the corresponding manufacturing cost. The tolerance limits (as shown in Fig. 1b) are traditionally set at either three standard deviations ($\pm 3\sigma$, six sigma) assuming normal variability in dimensions or by assuming a sure-fit tolerance (i.e., no out-of-specification dimension or no-quality cost). Decreasing the standard deviation σ shrinks the tolerance limits, which requires more accurate/costly manufacturing operations. However, if flexibility in tolerance limits ($\pm k\sigma$, where k is a parameter) is required, then one also needs to explicitly include quality cost in the formulation of cost-tolerance relationships.

B. Cost-Tolerance Modeling with Quality and Manufacturing Cost

Quality cost is encountered if functional performance (e.g., fatigue crack growth life constraint) is not met due to out-of-tolerance variations in a dimension. Many studies (e.g., [7–10]) performed tolerance optimization by balancing the quality loss/cost and manufacturing cost. Ye and Salustri [9] proposed a simultaneous tolerance synthesis method that balanced both manufacturing and quality costs. They modeled quality loss/cost using Taguchi et al.'s quadratic quality loss function [2] that assumes quality loss even if dimensions are produced within specified tolerance limits. Ye and Salustri [9] further assumed that the designer had knowledge of the process plan at the design stage. Lee and Woo [10] also included the cost due to the quality cost (rework) into the manufacturing cost. They formulated the tolerance allocation as a probabilistic optimization problem by treating various dimensions as random variables. The probabilistic (or quality cost) approach is flexible in allowing a certain proportion of assemblies to be out of tolerance. Lee and Woo [10] reasoned that explicit modeling of the quality cost often leads to less conservative tolerances as opposed to the deterministic approach (i.e., by modeling only the manufacturing cost) that does not explicitly consider quality cost.

C. Cost-Tolerance Modeling Including Performance Cost

The performance cost is another major component that plays an important role in achieving the maximum value aircraft design (e.g., low-cost and low-weight design). Like quality cost, one also needs to explicitly consider the cost benefit or loss to a customer due to loss in the performance due to tolerance allocation. One of the first attempts at explicit modeling of customers' objectives was by Soderberg [11] using a quality loss function. The total loss function for a customer was determined by including the component price. Recent studies by Kundu et al. [12,13] and Curran et al. [14] focused on the multidisciplinary optimization of tolerances at the aircraft surface by considering aircraft performance (parasite drag) and the manufacturing cost (inclusive of quality cost). They found that slight relaxation in tolerances led to considerable reduction in the assembly/manufacturing cost with minimal increase in parasite drag, which lead to overall reduction of the "direct operating cost" (a customer objective). Curran et al. [15] proposed a methodology that linked aircraft design (e.g., low structural weight) and manufacturing objectives (low production cost) through cost for achieving competitive aircraft design. Kundu et al. [16] stressed the important role of costing in multidisciplinary aircraft design to arrive at a best-value design. Castagne et al. [17] showed that effective design optimization can be achieved by linking manufacturing cost models with structural analysis models. Slack [18] proposed the value/worth of a product to a customer based on following: 1) the product's usefulness to a customer need, 2) the relative importance of a need being satisfied, 3) the availability of the product relative to when it is needed, and 4) the cost of ownership to the customer.

Murman et al. [19] further proposed that the value of a product can be measured by establishing a functional relationship between a product's performance, cost, and time.

III. Proposed Cost Model

We have identified the key components of the cost model from literature that are essential for optimization of manufacturing tolerances for damage-tolerant aircraft structures. The key cost components are manufacturing cost, quality cost, and performance cost. We consider the manufacturing cost to only include the material cost and assume the machining and assembly costs to be fixed due to a fixed process plan. A fixed process plan is assumed due to significant overlap in the manufacturing operations between the current (under production) and future (under design) aircraft, so only the material cost increases with the increase in manufacturing tolerance.

The quality cost/loss is considered as a loss to a manufacturer due to out-of-tolerance dimensions, so we employ a traditional quality cost function that, unlike a quadratic loss function, only assumes cost (due to rework or scrap) due to out-of-tolerance dimensions. Our formulation of a quality loss is probabilistic and does not assume a normal distribution to represent variations in dimensions. One of the two dimensions considered in this paper has a discrete multinomial distribution, and the other has continuous distribution. We use real industrial data to estimate the appropriate distributions to model manufacturing errors.

The final component is the performance cost that measures the loss to a customer due to loss in a product's performance. For our case, the increase in tolerance increases structural weight (loss in performance), which decreases the usefulness of an airplane to a customer. We measure the performance loss by multiplying the worth/value of a pound of useful load (cost of useful load) with the weight increase due to additional tolerance allocation. The cost of a useful load is estimated by dividing the sales price of an airplane with its useful load. The three cost components are summed to formulate a total-cost function. The optimal tolerance is found by minimizing the total-cost function that balances low-weight and low-cost objectives to achieve best-value structural design.

IV. Design Study: Probabilistic Manufacturing Tolerance Optimization of Wing Spar

The design study presented here demonstrates tolerance optimization by balancing low structural weight (performance cost) and low production cost (sum of material and quality cost) for a deterministic damage-tolerant design of a wing spar.

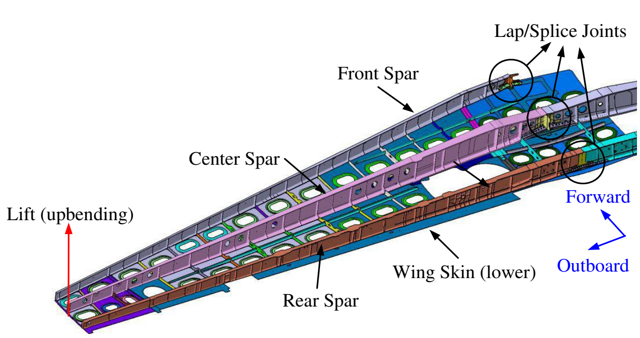


Fig. 2 Wing assembly of a business jet.

A. Deterministic Damage-Tolerant Design of a Wing Spar

The wing spars (beams) shown in Fig. 2 are primarily designed to take the vertical upbending loads (due to aerodynamic lift), which subject the lower spar caps (shown in Fig. 3) to axial tensile loads. Cracks/damage often originate at fastener holes drilled to attach spar caps, wing skin, and straps. These cracks grow under cyclic flight loads to a critical length that causes failure by fracture.

An objective of damage-tolerant design is to maintain flight safety by inspection and replacement/repair of cracking parts before failure. The structural inspection intervals are prespecified by aircraft manufacturers and act as design constraints. The initial inspection is usually set at mid-life I^*_{ini}, i.e., half of the minimum desired service life of a component. Recurring inspections are then set between midlife and the minimum desired service life. The structure is designed/sized to meet these prespecified inspection interval constraints, which are accomplished by performing analytical crack growth analyses at several fatigue critical fastener locations (e.g., wing lap/splice joints, shown in Figs. 2 and 3). We only consider the prespecified initial inspection constraint I^*_{ini} that is set at 12,000 flight hours (FHs) for the wing spar fastener holes. We execute crack growth analysis only for the most critical fastener hole in the lap/splice joint (joint shown in Fig. 3a). That is, the inspection interval I_{ini} (half of the calculated crack growth life) calculated from analysis must be greater than the prespecified constraint of 12,000 FHs:

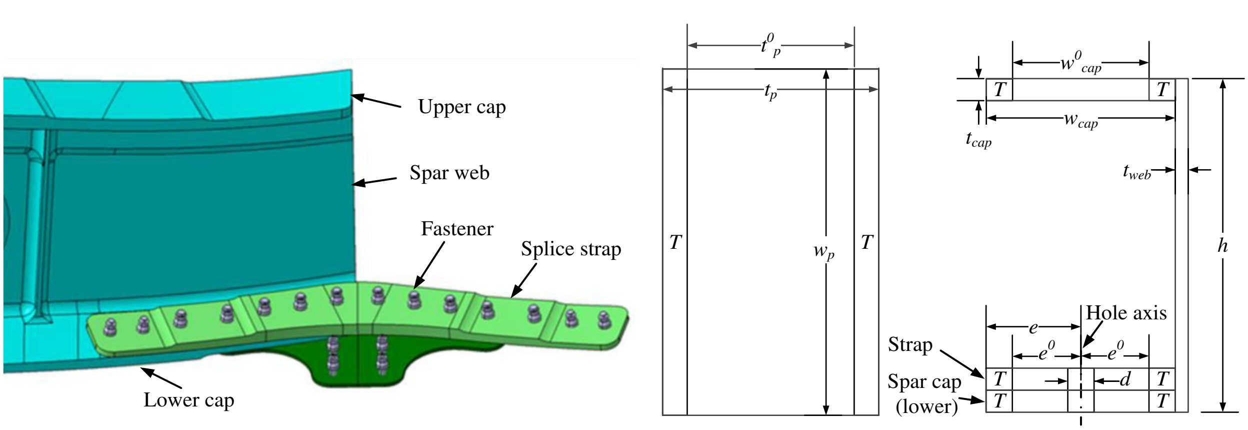
$$I_{\text{ini}} - I^*_{\text{ini}} > 0 \tag{1}$$

The inspection interval results are then extrapolated to entire wing spar fastener holes, which means that the entire wing spar will have the same optimal tolerance for the spar caps at every fastener hole. The details about crack growth analysis are presented in the Appendix.

An idealized cross-sectional geometry, shown in Fig. 3c, is used for performing crack growth analysis and tolerance optimization. The idealized geometry is representative of the real front spar, shown in Fig. 3a, which is 25 ft long. Other dimensions given in Table 1 are assumed to remain constant along the spar length. The strap is not included in the weight and cost calculations because its scrap cost and weight are negligible in comparison to the idealized wing spar. The idealized geometry has approximately the same weight (i.e., 60 lb) as that of the real spar, shown in Fig. 3a.

B. Fastener Hole Manufacturing Errors and Inspection Intervals

Crack growth life is sensitive to the distance of a fastener hole axis from the edge e and the diameter of the fastener hole d. Manufacturing errors (Δe and Δd) associated with these dimensions are



a) Front spar lap joint **b) Aluminum plate** **c) Machined spar**

Fig. 3 Representations of front spar lap joint and its idealized cross section.

Table 1 Dimensions of the idealized spar and aluminum plate

Aluminum plate		Machined spar	
Dimension	Value, in.	Dimension	Value, in.
w_p	10.10	w^0_{cap}	3.500
t^0_p	3.680	t_{cap}	0.165
l (spar length)	300.0	t_{web}	0.080
— —	— —	h	10.00
— —	— —	d	0.250
— —	— —	e^0	1.750

shown in Fig. 4. Manufacturers check if these errors cause a violation of inspection intervals by executing a crack growth analysis. An error qualifies for such a check if tolerance exceedance (TE) and hole-oversize (HOS) criteria, shown in Figs. 4a and 4b, are met. That is, if the edge deviation Δe exceeds the allocated tolerance T, then it is likely to cause a constraint violation. Similarly, a constraint violation could happen if a hole-diameter deviation occurs (i.e., $\Delta d > 0$). Note that the hole-diameter deviation means that the hole is oversized to the next available fastener size, i.e., fastener diameter sizes are available only in discrete increments of $1/64$ in.

C. Manufacturing Error Data, Distributions, Quality Review Process, and Probabilities of Interest

The edge distance deviation Δe is a continuous random variable, and the diameter deviation Δd is a discrete random variable. The Δe samples were collected from several wing assemblies of a business jet, shown in Fig. 5a, and they are used to estimate the representative distribution. The drilling operations were carried out manually with locations identified by the drill templates, as shown in Fig. 5b.

1. *Edge Distance Deviation Data*

The edge distances of 8164 fastener holes located in the lower spar caps of eight wing assemblies were measured using a digital caliper, as shown in Fig. 6. This data are further converted into the edge distance deviation Δe data by subtracting the edge distance defined on a drawing e from the measured edge distance e_m.

The edge distance deviation data are then modeled with a semiparametric distribution. More details on the semiparametric distribution are provided in [20]. The edge distance data can also be approximated by a logistic distribution with the following parameters: (location) $\mu = -5.5 \times 10^{-4}$ in. and (scale) $\sigma = 0.0138$ in. A normal fit to data has a mean of $\mu = -7.9 \times 10^{-4}$ in. and a standard deviation of $\sigma = 0.0248$ in.

2. *Hole-Oversize Data (Diameter Deviation)*

Hole-diameter deviation data Δd were collected from a quality database of 110 wing assemblies with a total sample size of 650,642. Aerospace fasteners are generally available in $\Delta d = 1/64$ in. increments, i.e., if a fastener of $\Delta d = 8/32$ in. gets oversized due to manufacturing error, then the next available fastener size is $8/32 + 1/64$ in., and so on. Therefore, Δd is a discrete random variable that can be represented by a histogram. The frequencies associated with 13 subsequent fastener increments are listed in Table 2, and the probability of oversizing a fastener is Prob. $(\Delta d > 0) = 1.724 \times 10^{-3}$ (1122 out of 650,642).

3. *Quality Review Process*

The process of identifying and resolving the manufacturing errors, identified in Fig. 4, is termed a quality review (QR; Fig. 7) process, and it incurs quality cost. A major task accomplished under QR is the analysis and resolution of a quality problem by engineers. If a crack growth analysis does not show an inspection constraint violation, then no repair is needed and only the QR cost C_{QR} is incurred. Violation of an inspection interval constraint may lead to scrapping, as few repair options are available, e.g., cold working a hole. Repair options are highly location dependent and would require complicated modeling that could be done if a full history of these repairs is available. However, we assume that constraint violation always leads to scrapping of the wing spar. This assumption is not expected to add

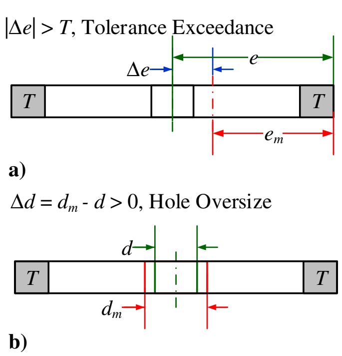
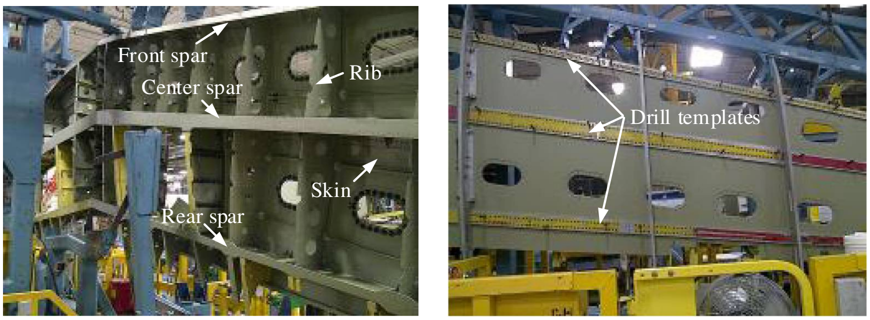


Fig. 4 Representations of a) tolerance exceedance event and b) hole-oversize event.

Fig. 6 Edge distance measurement of a fastener hole using a digital caliper.



a) b)

Fig. 5 Representations of drill templates for locating and drilling holes on wing spars.

Table 2 Fastener hole-oversize (diameter deviation) distribution

Δd, in.	Frequency	Δd, in.	Frequency
0/64	649520	7/64	17
1/64	100	8/64	7
2/64	666	>8/64	16
3/64	90	— —	— —
4/64	119	— —	— —
5/64	43	— —	— —
6/64	64	— —	— —

needless conservatism to the analysis because constraint violations are rare.

We estimate the quality cost by simulating the quality review process, shown in Fig. 7, for the design of an idealized wing spar geometry. We first generate manufacturing error samples (Δe, Δd) from the probability distributions identified in the previous section. These error samples are used to calculate the probability of QR P_{QR} and the probability of constraint violation P_{CV}, which are further used to calculate the quality cost. The calculation of these probabilities is discussed next.

4. Probability of Quality Review

The P_{QR} is estimated by assuming that P_{TE} (probability of tolerance exceedance) and P_{HOS} (probability of hole oversize) are independent so that

$$P_{\mathrm{QR}} = P_{\mathrm{TE}} + P_{\mathrm{HOS}} - P_{\mathrm{TE}} P_{\mathrm{HOS}} \tag{2}$$

where P_{TE} is estimated from the edge distance distribution, and P_{HOS} is estimated from the hole-diameter deviation distribution. Note that P_{TE} is a function of tolerance and P_{HOS} is not, because all the diameter deviations are reviewed. The P_{QR} gradually decreases with the increased tolerance T until it approaches 1.724×10^{-3}, which corresponds to the probability of oversizing a hole, as shown in Fig. 8.

5. Probability of Constraint Violation

The probability of constraint violation is estimated by Monte Carlo simulation, where a combination of randomly generated fastener deviation (Δe, Δd) samples (10 million) that pass the QR check are further checked for the possibility of constraint violation by executing crack growth analyses. The P_{CV} is approximated by dividing the number of samples failing to meet the constraint defined in Eq. (1) with the total number of samples. P_{CV} is observed to decrease more quickly than P_{QR} with the increase in tolerance, as shown in Fig. 8, and is an order of magnitude smaller than P_{QR}.

D. Cost Model for Tolerance Optimization

The cost used for tolerance optimization is the sum of the quality cost C_Q, manufacturing cost C_M, and performance loss L_P:

$$C_{\mathrm{total}} = \underbrace{C_Q + C_M}_{\text{Production Cost}} + L_P \tag{3}$$

where, the sum of C_Q and C_M measures the production cost, and the performance loss L_P measures the reduction in the value/worth of an aircraft to a customer due to addition of tolerance (i.e., due to weight increase).

1. Quality Cost

The C_Q is formulated as the sum of two components:

$$C_Q = C_{\mathrm{QR}} + C_{\mathrm{CV}} \tag{4}$$

The quality review cost C_{QR} captures the cost incurred due to the use of human resources for performing crack growth analysis and labor for altering the location and size of the discrepant hole for installation of the recommended fastener. It is given as

$$C_{\mathrm{QR}} = n_f P_{\mathrm{QR}} C_{\mathrm{QRPF}} = n_f P_{\mathrm{QR}} (C_{\mathrm{eng}} + C_{\mathrm{lab}}) \tag{5}$$

where, n_f is the total number of fastener holes (350) to be drilled in a spar, P_{QR} is the probability of a quality review, and C_{QRPF} is the quality review cost per fastener. The average engineering cost C_{eng} is the product of the average hourly engineering cost [100 U.S. dollars (USD)] and the average engineering time (3/4 h). The average labor cost is the product of the average hourly labor cost (65 USD) and the average labor time (1/2 h). Therefore, the expected value of $C_{\mathrm{QRPF}} = 107.5$ USD.

The cost of the constraint violation C_{CV} is the scrap cost that is estimated by

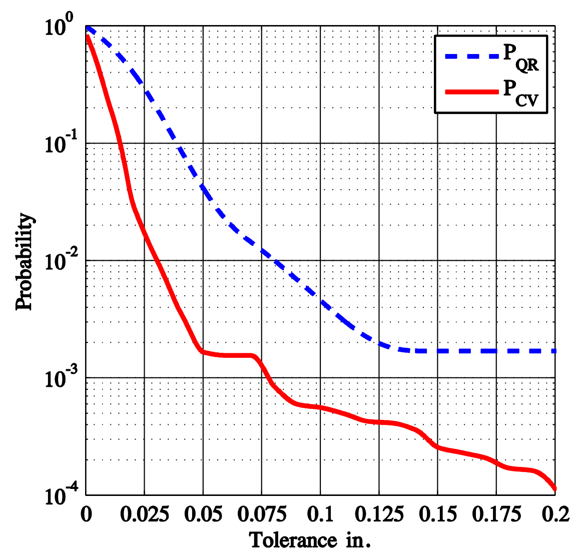


Fig. 8 Comparison between probability of quality review and constraint violation.

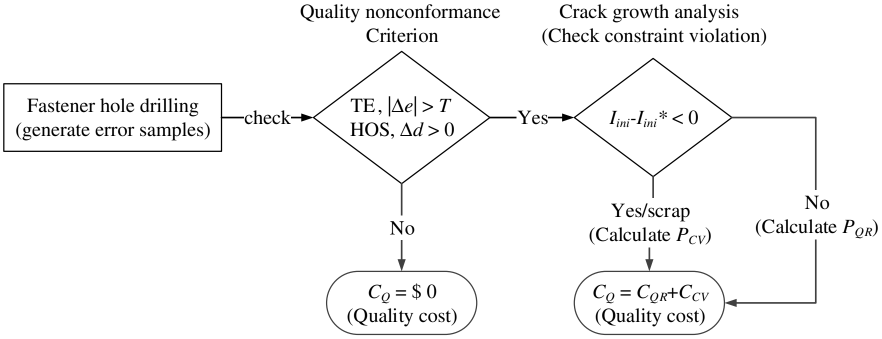


Fig. 7 Quality review process.

$$C_{CV} = 2P_{CV}W_pC_{Al} = 2P_{CV}(w_pt_pl\rho_{Al})C_{Al} \quad (6)$$

where C_{Al} is the per-pound cost of aluminum alloy (5 USD); W_p is the weight of the aluminum plate, and P_{CV} is the probability of violating an inspection interval constraint. A factor of two is used because the scrap cost is generally twice that of the raw material cost for high-valued parts such as a wing spar, which includes the indirect and machining costs.

2. Manufacturing Cost

The manufacturing cost C_M only includes the material cost C_{mat}, as machining and assembly costs are fixed due to assumption of a fixed process plan. The increase in material cost (i.e., due to tolerance addition) over the zero tolerance design is

$$C_M = C_{mat} = \Delta W_pC_{Al} = (2w_pTl\rho_{Al})C_{Al} \quad (7)$$

3. Performance Loss

The performance loss L_p represents the decrease in the value of an aircraft to a customer due to an increase in the structural weight due to tolerance allocation. The value is calculated by dividing the sales price of an aircraft with the useful load, as the useful load is an important characteristic that customers care about. The useful load is a proportion of the aircraft's maximum takeoff weight consisting of a full fuel load, crew, passengers, baggage, and contingency weight. The contingency weight acts as a buffer that manufacturers use to guarantee the promised payload and range in case of a structural weight increase. For example, an increase in operational empty weight $+W$ due to tolerance allocation will be offset by decreasing the contingency weight $-W$ to keep the payload unaffected.

Usually, the payload and range are promised to a customer by assuming that all contingency weight would be used up. So, it would be reasonable to calculate the value (cost of useful load C_{UL}) of weight to a customer based on the guaranteed payload that is estimated by dividing the sales price of an aircraft with a useful load:

$$C_{UL} = \frac{S_{price}}{W_{useful}} \quad (8)$$

The C_{UL} ranged between 800 and 1600 USD/lb for various business jets, as shown in Fig. 9 (aircraft arranged in increasing order of useful load capacity). The weight data for the airplanes were extracted from their official websites. The sample cost and weight of a business jet are given in Table 3.

For the idealized wing spar considered here, the performance loss L_p is the product of the increase in the weight of the idealized spar ΔW_s (measured with respect to zero-tolerance weight) and the cost of the useful load as

$$L_P = \Delta W_sC_{UL} = (4t_{cap}lT\rho_{Al})\left(\frac{S_{price}}{W_{useful}}\right) \quad (9)$$

where C_{UL} is assumed to be 1200 USD for the idealized spar (i.e., midpoint of the range shown in Fig. 9).

E. Tolerance Optimization

The optimal tolerance is found by minimizing the expected value of the total cost:

$$\text{Min } C_{total} = C_Q + C_{mat} + L_P \quad \text{subject} \times \text{ to } \; 0 \leq T \leq 0.2 \quad (10)$$

The optimization is simple and is solved graphically, as shown in Fig. 10. The total-cost curve C_{total} has a minimum/optimal value of 2476 USD at the optimal tolerance value of 0.0643 in. Notice that the C_{total} curve shows a nonlinear decrease up to the optimal tolerance, indicating the initial dominance of the quality cost C_{QR}, followed by an almost linear increase showing the dominance of performance loss L_P. At the optimum, the contributions of the quality cost C_Q, manufacturing cost C_M, and performance loss L_P toward the expected total cost are 27.6, 9, and 63.4%, respectively. The spar weight increases by $\Delta W_s = 1.3$ lb, i.e., a 2.17% increase from the zero-tolerance weight of 59.8 lb. Also, notice from Fig. 11 that C_{QR} is much larger than C_{CV} (i.e., on the average, about 30 times larger), and L_P is about seven times larger than C_{mat}. The optimal tolerance of 0.0643 in. translates to $\pm 2.6\sigma$ for Δe distribution, where $\sigma = 0.0248$ in. is the standard deviation of the Δe distribution sample.

Table 3 Sample weight and cost data for a business jet

Variable	Value
MTOW, lb	17110
Full fuel payload, lb	972
Full fuel load, lb	5828
Useful load, lb	6800
Sales price,[a] USD	8.76[b]
Cost of useful load, USD/lb	1288

[a]Data available online at http://www.aircraftcompare.com/ [retrieved 2015].
[b]In millions.

F. Tolerance Optimization by Ignoring Performance Cost

The effect on optimal tolerance due to lack of quantitative modeling of a customer's low-weight objective (i.e., low performance cost) is shown here. The optimization is performed by minimizing the sum of the quality and the material cost. The optimization results are shown graphically in Fig. 12, which shows the optimum at $T = 0.1145$ in. with a corresponding cost of 500 USD. The current tolerance used for the real wing spar was much closer to 0.1145 in. than to 0.0643 in.. This is because, in the actual design process, the low-weight objective was implicitly considered without quantitative modeling. The optimal tolerance of 0.1145 in. translates to $\pm 4.6\sigma$ for Δe distribution, where $\sigma = 0.0248$ in. is the standard deviation of the Δe sample. The production cost (sum of quality and material cost) is only reduced from 906 USD (36.6% of 2476 USD) for $T = 0.0643$ in. to 500 USD for $T = 0.1145$ in., but performance cost has also increased from 1570 to 2760 USD from the full cost optimization. So, the importance of including customer objectives (low weight and cost) for designing a competitive aircraft is illustrated with this comparison.

V. Uncertainty Analysis

In the previous section, the expected value/average of the total cost C_{total} was minimized to find the optimum tolerance, but C_{total} is uncertain due to uncertainties arising from finite manufacturing errors and cost data. We start uncertainty analysis by identifying the input variables that need to be treated as uncertain. Consider the following expanded form of C_{total} to identify key uncertainty sources:

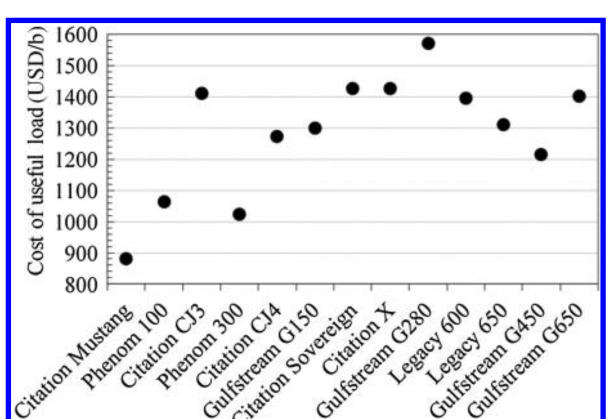


Fig. 9 Cost of useful load for various business.

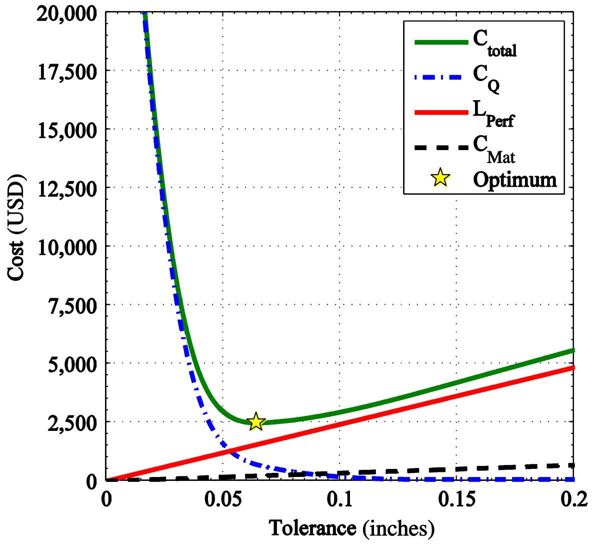


Fig. 10 Expected total cost and its various cost components as a function of tolerance.

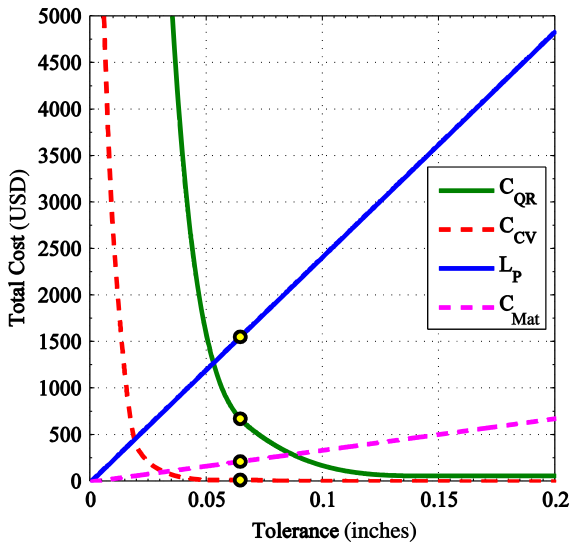


Fig. 11 Various subcomponents of cost model.

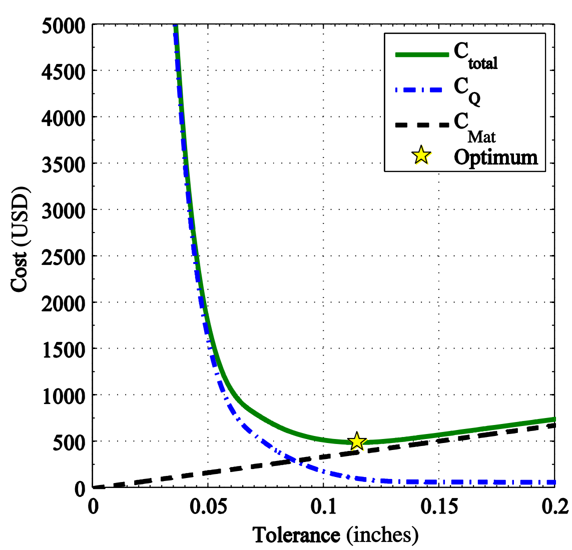


Fig. 12 Expected total cost and its cost components after ignoring performance cost.

$$C_{\text{total}} = \overbrace{\underbrace{n_f P_{\text{QR}} C_{\text{QRPF}}}_{C_{\text{QR}}} + \underbrace{2W_p P_{\text{CV}} C_{\text{Al}}}_{C_{\text{CV}}}}^{C_Q} + \underbrace{\Delta W_s C_{\text{UL}}}_{L_P} + \underbrace{\Delta W_p C_{\text{Al}}}_{C_M} \tag{11}$$

where P_{QR} (probability of quality review), C_{QRPF} (cost of quality review per fastener), P_{CV} (probability of constraint violation), and C_{Al} (per pound cost of aluminum) are uncertain. We elect to treat the cost of aluminum C_{Al} as deterministic because the optimum tolerance and total cost are not sensitive to C_{Al}, e.g., a 1% change in C_{Al} only leads to a 0.1% change in C_{total} and a 0.02% change in the optimal tolerance (refer to [20] for more details). The cost of a useful load C_{UL} (ratio of sales price to useful load) is also treated as deterministic because exact values of the promised sales price and promised useful load are fully known at the time of tolerance design. This results in the performance loss L_P and manufacturing/material cost C_M being deterministic. Further, P_{CV} is an order of magnitude smaller than P_{QR} (see Fig. 8) and, as a result, C_{CV} is very small in comparison to C_{QR} near the optimum (see Fig. 11). This indicates C_{CV} can also be treated as deterministic, which means that the only uncertain quantity left in Eq. (11) is the cost of quality review:

$$C_{\text{total}} = C_{\text{QR}} = n_f P_{\text{QR}} C_{\text{QRPF}} = n_f (P_{\text{TE}} + P_{\text{HOS}} - P_{\text{TE}} P_{\text{HOS}}) C_{\text{QRPF}} \tag{12}$$

where P_{QR} is the function of P_{TE} (probability of tolerance exceedance), and P_{HOS} (probability of hole oversize) is expressed by Eq. (2). The uncertainty in P_{HOS} and P_{TE} is due to finite samples of data available for estimating their mean values, e.g., mean values of $P_{\text{HOS}} = 1.72 \times 10^{-3}$ and $P_{\text{TE}} = 1.63 \times 10^{-2}$ at an optimum tolerance of 0.0643 in. The mean value of P_{HOS} is calculated by using a relatively large sample of data, with the number of samples as $n = 650,642$. The uncertainty in the mean value of a probability P can be calculated by using the following equation:

$$\sigma(P) = \sqrt{\frac{P(1-P)}{n}} \approx \sqrt{\frac{P}{n}} \tag{13}$$

This gives an uncertainty of $\sigma = 5.14 \times 10^{-5}$ in the mean value of $P_{\text{HOS}} = 1.72 \times 10^{-3}$ [i.e., a coefficient of variation (COV) of 3%]. On the other hand, uncertainty in the mean value of $P_{\text{TE}} = 1.63 \times 10^{-2}$ ($n = 8,164$) is $\sigma = 1.4 \times 10^{-3}$ (COV = 8.6%). The uncertainty in quality review cost C_{QR} due to uncertainties in P_{HOS} and P_{TE} individually can be estimated by propagating uncertainties in Eq. (12).

The uncertainty (standard deviation σ) in C_{QR} given the uncertainty in P_{HOS} can be calculated by propagating the uncertainty in Eq. (12) as

$$\begin{aligned}(\sigma_{C_{\text{QR}}} | \sigma_{P_{\text{HOS}}}) &= n_f \sigma_{P_{\text{HOS}}} C_{\text{QRPF}} \sqrt{\left(1 - P_{\text{TE}}\right)^2} \\ &= 350 \times 5.14 \times 10^{-5} \times 107.5 \times 0.984 \cong 2 \text{ USD}\end{aligned} \tag{14}$$

where P_{TE} and C_{QRPF} are treated as deterministic. Similarly, the uncertainty (standard deviation σ) in C_{QR} given the uncertainty in P_{TE} can be calculated by propagating the uncertainty in Eq. (12) as

$$\begin{aligned}(\sigma_{C_{\text{QR}}} | \sigma_{P_{\text{TE}}}) &= n_f \sigma_{P_{\text{TE}}} C_{\text{QRPF}} \sqrt{\left(1 - P_{\text{HOS}}\right)^2} \\ &= 350 \times 1.4 \times 10^{-3} \times 107.5 \times 0.998 \cong 53 \text{ USD}\end{aligned} \tag{15}$$

where P_{HOS} and C_{QRPF} are treated as deterministic. This uncertainty is an epistemic uncertainty, due to lack of knowledge resulting from the finite sample size (8164), and can be reduced by taking more samples. However, doubling the number of samples will only reduce the uncertainty by a square root of two or about 30%.

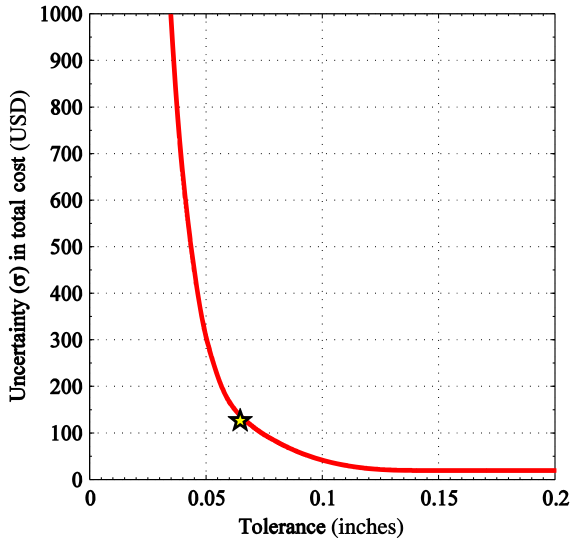


Fig. 13 Epistemic uncertainty in the expected total-cost curve.

Equations (14) and (15) show that uncertainty in C_{QR} due to P_{HOS} is negligible compared to the uncertainty due to P_{TE}. Therefore, P_{HOS} can be dropped for further uncertainty analysis, which reduces Eq. (12) to

$$C_{total} = C_{QR} = n_f P_{TE} C_{QRPF} \tag{16}$$

The quality review cost C_{QRPF} has both aleatory (variability/randomness) and epistemic (lack of knowledge) uncertainties. The aleatory uncertainty is modeled by probability distributions as was initially done to model Δe and Δd data, i.e., a probability distribution function (PDF) describes the inherent randomness or variability in the C_{QRPF}. The mean $\mu = 107.5$ USD and $\sigma = 18$ USD, which define the PDF of C_{QRPF}, were estimated from finite data. The mean of 107.5 USD was based on approximately 1000 samples of C_{QRPF}, and the epistemic uncertainty in the mean of 107.5 USD (due to finite data) can be estimated as follows:

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{1000}} \cong 0.50\ \text{USD} \tag{17}$$

Hence, the epistemic uncertainty in the mean of C_{QRPF} due to finite samples is negligible and ignored. However, there is another source of uncertainty in the mean of C_{QRPF} due to uncertainty in pay rates and labor time that will be used to resolve quality reviews in the future. A standard deviation of 20 USD in the mean of C_{QRPF} was assumed. Now, epistemic uncertainty in P_{TE} and C_{QRPF} can be propagated in Eq. (16) by using the following formula:

$$(\sigma_{axy}|\sigma_x, \sigma_y) = a\sqrt{\mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2} \tag{18}$$

where σ_{axy} is the uncertainty in the product axy of two random variables x and y, with constant a; σ_x is the uncertainty in x; σ_y is the uncertainty in y; μ_x is the mean/expected value of x; and μ_y is the expected value of y. Similarly, Eq. (16) becomes

$$\begin{aligned}(\sigma_{C_{QR}}|\sigma_{P_{TE}}, \sigma_{C_{QRPF}}) &= n_f\sqrt{\mu_{P_{TE}}^2\sigma_{C_{QRPF}}^2 + \mu_{C_{QRPF}}^2\sigma_{P_{TE}}^2 + \sigma_{P_{TE}}^2\sigma_{C_{QRPF}}^2} \\ &= 350\sqrt{0.0163^2(400) + 107.5^2(0.0014)^2 + (0.0014)^2(400)} \\ &= 126\ \text{USD}\end{aligned} \tag{19}$$

Note that P_{TE} is a function of tolerance that decreases with the increase in tolerance (refer to Sec. IV.C), and so is the uncertainty in C_{QR}, as shown in Fig. 13. Notice that the uncertainty is very large when the tolerance is small and decreases nonlinearly with the increase in tolerance. At optimal tolerance (i.e., $T = 0.0643$ in.), the uncertainty about the optimum total cost of 2476 USD is 126 USD (coefficient of variation of 5%). However, the epistemic uncertainty is solely due to C_{QR}, which is 659 USD (26.6% of 2476 USD) at the optimum, and 126 USD uncertainty (COV = 19%) may not be small. Note that about 53 USD out of these 126 USD are due to uncertainty in P_{TE}, so collection of more edge distance deviation data Δe would further reduce the epistemic uncertainty in C_{QR}. For example, increasing the sample size by a factor of two (i.e., a sample size of $2 \times 8,164 = 16,328$) would reduce the uncertainty due to P_{TE} from 53 to 37 USD (i.e., about a 30% decrease) that would further reduce the uncertainty in C_{QR} from 126 to 120 USD (i.e., only 5% decrease).

However, we can neglect the aleatory uncertainty (variability) in the cost of quality review per fastener C_{QRPF} for the 2400 spars expected for 400 airplanes. The expected number of quality reviews for 2400 wing spars is about 13,692 (5.7 per spar), and uncertainty in the average per quality review will be $18\ \text{USD}/\sqrt{13,692}$ (where 18 USD is the standard deviation of the C_{QRPF}), which is 0.15 USD and is negligible.

VI. Conclusions

A cost-based tolerance optimization method that balanced the multiple objectives of design, quality, and manufacturing teams was developed and illustrated with an example of a wing spar. It was found that systematic modeling of the performance cost is important for achieving low-cost and low-weight customer objectives. It was found that the explicit modeling of performance cost reduced the optimal tolerance by about 44% (from 0.1145 to 0.0643 in.) and the performance cost by about 43% (from 2760 to 1570 USD) with only a 32% increase (from 500 to 659 USD) in the quality cost. Uncertainty analysis showed that uncertainty in the quality cost due to finite samples of hole-diameter deviation data Δd was almost negligible due to the large dataset (650,643 samples) available for calculating P_{HOS}. It was only 2 USD out of the quality cost of 659 USD, or 0.3%. On the other hand, uncertainty due to edge distance deviation data Δe was small but certainly not negligible. It was 53 USD out of the quality cost of 659 USD (i.e., 8%) at the optimum tolerance of 0.0643 in. Therefore, if more Δe data are collected, the uncertainty could be reduced to a point where the designer could neglect it.

Appendix A: Crack Growth Analysis

The crack growth analysis is performed by using a free version of the Air Force Crack Growth software (AFGROW; Version 4.0012.15). The structure shown in Fig. 3c was further idealized into the joint shown in Fig. 14, with only the lower spar cap and strap needed to execute a crack growth analysis. The lap joint transfers load P lb (i.e., applied reference gross stress of $\sigma_{\text{ref-gross}} = P/wt$ ksi) m with each fastener picking up R_i lb (causing bearing stress $\sigma_{\text{brg}i}$) and bypassing P-$\sum R_i$ lb of load (causing bypass stress $\sigma_{\text{by}i}$). The proportion of the total load picked by each fastener depends upon fastener flexibility, which is calculated by modifying the analytical relationships given in [21] from the double-shear to the single-shear case. These relationships yield reasonably accurate results for the fastener loads, i.e., within 10% of the finite element analysis results. The resulting fastener load distribution shows that $R_2 = R_4$ and $R_1 = R_5$; it is clear that end fasteners R_1 and R_5 are fatigue critical, as they pick up the most load. Therefore, crack growth analyses are only executed for the first fastener, and then it is assumed that all the fastener holes on the wing spar have similar crack growth characteristics. We have used the single corner crack model to execute the crack growth analysis, as shown in Fig. 15. The initial crack length of $A = C = 0.05$ in. is used. The crack is grown under a variable-amplitude stress spectrum taken from the wing spar location of a business jet. It grows steadily until a critical crack length a_c is reached, causing fast fracture. The wing stress spectrum used in the analysis was 100 flight hours long, and initial the inspection interval was estimated by dividing the number of flight hours it takes to grow the crack to a critical length a_c by a factor of two.

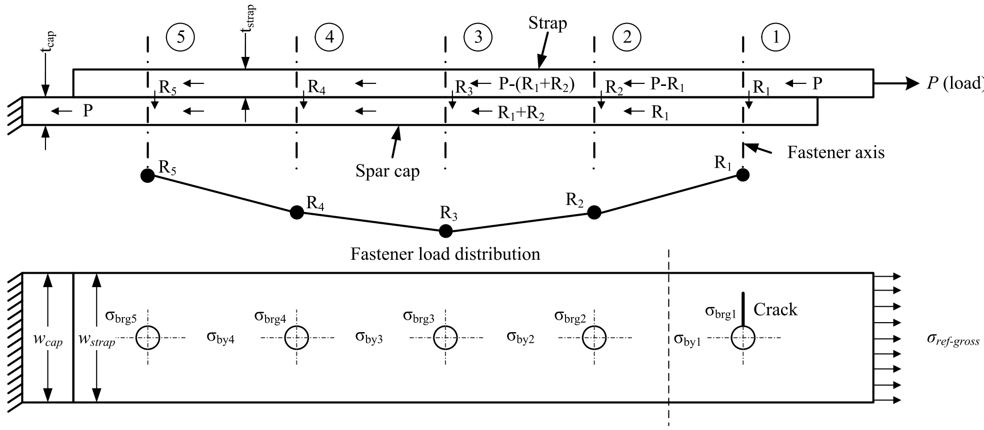


Fig. 14 Simplified lap joint geometry, load transfer, and stresses.

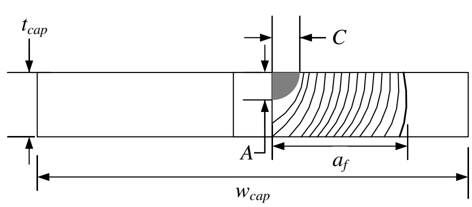


Fig. 15 Corner crack model used in the AFGROW software.

The loads calculated by analytical relationships are then used to find the bearing η_{brg} and bypass/tension stress η_{TR} ratios that serve as inputs to the AFGROW analysis. These ratios are calculated for the end fasteners by the following equations:

$$\eta_{\text{TR1}} = \frac{\sigma_{\text{by1}}}{\sigma_{\text{ref-gross}}} = \frac{P - R_1}{P} \tag{A1}$$

$$\eta_{\text{Brg}R1} = \frac{\sigma_{\text{brg1}}}{\sigma_{\text{ref-gross}}} = \frac{R_1}{P}\left(\frac{w}{d}\right) \tag{A2}$$

The probability of constraint violation (i.e., probability of violating an initial inspection constraint of 12,000 flight hours due to manufacturing errors) is estimated by a Monte Carlo simulation that requires millions of crack growth simulations. The actual AFGROW crack growth runs for the simulation were substituted by a two-dimensional (2-D) interpolation, i.e., crack growth analyses were performed using the grid combination of the two manufacturing error types followed by 2-D interpolation in between. The root mean square error of the interpolation ranged between 16 and 19 flight hours. To check the impact of this interpolation error on the P_{CV} calculation, 50,000 AFGROW analyses were performed that resulted in the same value of P_{CV} as that by the interpolation function. So, interpolation provided accurate estimates of P_{CV}. For more details, see [20].

Acknowledgments

We are thankful to the National Science Foundation grant CMMI-1131103, the Resource Allocation for Airliner Safety, and the Cessna Aircraft Company for supporting and helping us with this study. The invaluable guidance and feedback from Daniel Kalal (Engineering Manager, Fatigue and Damage Tolerance), and Bobbie Kroetch (Engineer-Sr., Fatigue and Damage Tolerance) at Cessna Aircraft Company are given special thanks.

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