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Deciding Optimal Number of Fatigue Crack Growth Tests for Damage-Tolerant Design

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Fatigue crack growth coupon tests play an important role in designing safe and lightweight aircraft structures. How many fatigue crack growth coupon tests are enough is an important question to be answered. With few coupon tests, there will be a substantial uncertainty in design weight, with a significant chance of getting a heavier design, and a considerable chance of overestimating fatigue crack growth design life. A simulation study is presented to determine the optimal number of fatigue crack growth coupon tests by trading off testing cost against structural weight cost. It also gives a designer an idea on how increasing the number of tests reduces the risk of overestimating design life. The tradeoff is demonstrated for the damage-tolerant design of the lower spar caps of a business jet's wing. With eight coupon tests, there is a 26% chance of incurring at least 2.4% weight penalty and 18% chance of overestimating the fatigue crack growth life by more than 10%. For 64 tests, the chances reduce to 3 and 0.5%, respectively.

Nomenclature

a	=	crack length, in.
C	=	Paris constant
C_{sct}	=	cost of single coupon tests, U.S. dollars
C_t	=	cost of testing, U.S. dollars
c	=	effective crack length, in.
d	=	hole diameter, in.
K	=	stress intensity, ksi$\sqrt{\text{in.}}$
l_s	=	length of wing spar, in.
N_d	=	fatigue crack growth design life, flight hours
N_t	=	number of load cycles in a coupon test, cycles
n	=	Paris exponent
n_a	=	number of aircraft
n_{ct}	=	number of fatigue crack growth coupon tests
p	=	cost penalty for a pound of weight, U.S. dollars per pound
t	=	thickness, in.
W_{lc}	=	weight of lower cap, lb
W_{wing}	=	weight of wing due to lower caps of six spars, lb
w	=	width, in.
μ	=	mean
σ	=	standard deviation

Subscripts

a	=	aircraft
d	=	design
s	=	spar
lc	=	lower cap
t	=	test
$t-\mathrm{ow}$	=	thickness of overweight samples
$W-\mathrm{lc}$	=	weight of lower caps

Superscripts

*	=	true parameter value
ˆ	=	estimated parameter value

I. Introduction

MATERIAL testing is a key task that approximates material properties (e.g., yield strength, crack growth rate) needed for designing safe and low-weight aircraft structures [1]. In general, material properties are random and can be modeled with statistical distributions [2]. For static strength design, the Federal Aviation Administration (FAA) requires the use of *A*-basis or *B*-basis material allowable to compensate for material variability and sampling uncertainty arising from limited number of coupon tests [3]. The use of B-basis material properties could lead to heavier designs if a small number (e.g., less than 30) of tests are performed. As a result, aircraft companies end up performing a sufficiently large number of coupon tests (more than 30) to characterize the strength property of a given material, which usually results in sufficient weight savings by reducing sampling uncertainty [4,5].

In contrast, for damage-tolerant (DT) designs [6] (i.e., design based on fatigue crack growth life), the FAA does not mandate the use of basis values to compensate for material variability and sampling uncertainty due to the smaller number of fatigue crack growth (FCG) coupon tests. Furthermore, FCG coupon tests tend to be more time-consuming and expensive (about $1000–2000 per test) than material strength coupon tests (about $300 per test). Consequently, aircraft companies may be performing fewer FCG coupon tests than needed. It seems that some of the fatigue literature provides useful guidelines [7–9] to determine the sufficient number of fatigue tests for understanding crack initiation behavior but not propagation behavior. This is perhaps due to the concern for larger scatter in the crack initiation life than crack growth life [10]. However, the question of how many FCG coupon tests are needed appears to be largely unaddressed, even though it might be of practical interest to airframe manufacturers. Therefore, the main objective of this paper is to help designers in deciding the optimal number of FCG coupon tests.

The current DT design practice for the metallic airframe is deterministic and is mostly based on predicting the mean FCG life (i.e., part sizing is based on mean FCG rate properties). At first glance, this may seem a risky design approach, but risk of failure is minimized through the use of various conservative assumptions (some are discussed in Appendix A) in predicting the FCG design life of a component. In some cases, gross conservative safety factors are applied to the calculated FCG life. The tradeoff study presented in this paper assumes that components are designed using the mean FCG life as a design constraint. We present an example problem of

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designing lower caps of wing spars to decide the optimal number of FCG tests. In general, with the increase in the number of FCG tests, the mean design weight approximately remains the same, but there is substantial decrease in the uncertainty (due to finite sampling/number of tests) about the mean design weight. That is, the likelihood of getting a too-heavy design and the risk of underestimating the desired mean FCG life by some amount decrease with the increase in number of tests. The optimal number of tests is found by studying the tradeoff between costs of weight penalty (due to underestimation of FCG life) and FCG testing cost. We also find the risk of overestimating the FCG life by 5 and 10%, which could also be used by the designer to decide the number of tests in certain situations.

The paper is organized as follows. Sections II and III briefly introduce the FCG coupon testing and life prediction for designing components. Section IV introduces a procedure to simulate testing and life prediction. It also presents the distributions of predicted FCG life. These distributions are translated into distributions of design thickness (or weight) that are given in Sec. V. Next, Sec. VI extrapolates the weight penalties from simulation geometry to aircraft wing spars. Finally, Sec. VII presents the cost tradeoff results to determine the optimal number of FCG coupon tests.

II. Fatigue Crack Growth Coupon Testing

An important step in designing damage-tolerant aircraft structures is the estimation of the crack growth behavior of a material through coupon testing. The testing is usually repeated at different stress ratios (e.g., $R = 0.1, 0.5$) to capture the stress ratio effect for complete material characterization. For simplicity, we assume that multiple tests are repeated for only one stress ratio. The estimated material properties are further used to predict the FCG life of a component that is discussed in the next section.

The crack growth behavior of a material is quantified by measuring the rate of crack propagation from multiple FCG coupon tests. The ASTM standard document [11] gives guidelines for FCG testing procedures and data reduction techniques. The crack growth rate and stress intensity range data ($\mathrm{d}a/\mathrm{d}N_t$ versus ΔK) obtained via FCG coupon testing is fitted with a crack growth model as shown in Fig. 1. Many crack growth models (e.g., Walker equation, Nasgro equation) have been proposed to model the crack growth rate data [12,13] but are basically the extensions of the Paris law:

$$\frac{\mathrm{d}a}{\mathrm{d}N_t} = C(\Delta K)^n \tag{1}$$

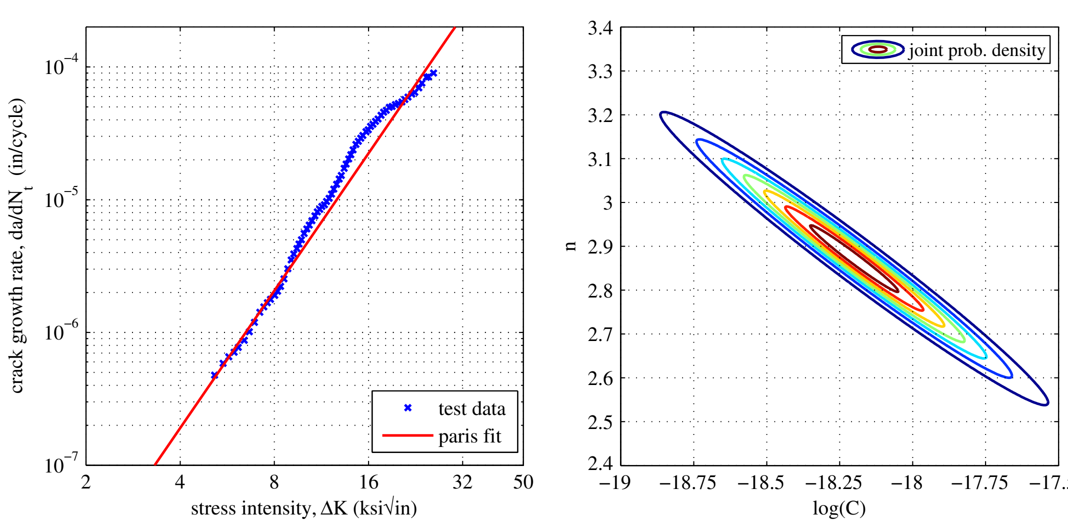
where $\mathrm{d}a/\mathrm{d}N_t$ is the crack growth rate measured in inches per cycle; a is the measured crack length; N_t is the number of test load cycles corresponding to each crack length measurement; C is the Paris constant; n is the Paris exponent; and ΔK is the range of stress intensity factor that drives crack growth.

Fitting the Paris law to data (as shown in Fig. 1a) gives a single sample of C and n. Generally, these material constants are treated as material properties that define the rate of crack growth for a particular material. In practice, multiple tests are usually performed for a given stress ratio (e.g., $R = 0.1$) and Paris fit to all the data gives a mean value of C and n. These mean values are further used to predict a mean FCG life for a component, which serves as a constraint for sizing a component.

However, the mean values of C and n are uncertain due to limited number of coupon tests (i.e., if the same number of tests is repeated, one would get different mean values). This leads to uncertainty in the mean FCG life of a component and further leads to uncertainty in the design weight. Such an uncertainty could be substantial with smaller number of coupon tests and is often treated as epistemic uncertainty [14,15]. That is, there would be substantial chances of overestimating and underestimating the mean FCG design life by some amount. The overestimation of life would lead to a lighter (thinner) design that increases the risk of failure. On the other hand, underestimation of life would lead to a heavier design that is also undesirable. Increasing the number of tests would considerably reduce the chances of overestimating the mean FCG life, and the weight penalty due to underestimation would also reduce.

To study the effect of increasing the number of tests on uncertainty, we simulate testing by generating finite samples of C and n (each depicting a coupon test) from assumed true (fully sampled) probability distributions. These samples are further used to calculate corresponding FCG life samples for a given loading and component geometry. The mean of these FCG life samples serves as a mean life of a component. The mean FCG life is then used as a constraint for sizing a component, which would give a corresponding design weight. Such simulation is outlined in Sec. IV.

The simulation requires an assumption of true probability distributions of C and n. These material constants are known to exhibit strong negative correlation [16,17], and so these are usually modeled with a joint probability distribution. Annis [18] and Akkaram et al. [19] modeled C with a log-normal distribution, or $\log(C)$ as normal, and n with a normal distribution using data from 68 tests (2024-T3 aluminum alloy) performed by Virkler et al. [16]. The joint probability distribution function (PDF) parameters estimated from 68 tests is given in Table 1. For simulation, we assume these parameters to represent a true/fully sampled joint distribution (usually unknown because it requires infinite number of tests) shown in Fig. 1b. The joint normal PDF is given in Eqs. (2) and (3):



a) b)

Fig. 1 Representations of a) Paris model fit to the crack growth rate data, and b) assumed true joint normal PDF of $\log(C)$ and n.

Table 1 Assumed true (*) parameters of the bivariate normal distribution

Material constant	Distribution	True mean μ^*	True standard deviation σ^*	True correlation ζ^*
Log(C)	Normal	−18.20	0.328	−0.982
n	Normal	2.872	0.165	−0.982

$$P(\log(C), n) = \frac{1}{2\pi\sigma_{\log(C)}\sigma_n\sqrt{1-\zeta^2}}\exp\left[-\frac{z}{2(1-\zeta^2)}\right] \quad (2)$$

$$z \equiv \frac{(\log(C)-\mu_{\log(C)})^2}{\sigma^2_{\log(C)}} - \frac{2\zeta(\log(C)-\mu_{\log(C)})(n-\mu_n)}{\sigma_{\log(C)}\sigma_n} + \frac{(n-\mu_n)^2}{\sigma^2_n} \quad (3)$$

III. Design of Lower Wing Spar Caps Using Fatigue Crack Growth Material Properties

The samples of material constants generated to simulate testing are used for calculating mean FCG design life for the wing spars/beams shown in Fig. 2. Wing spars are primarily designed to take bending loads due to aerodynamic lift, which subjects the lower spar caps (e.g., shown in Fig. 2b) to axial tensile loads. Cracks often originate at fastener holes drilled to attach spar caps and wing skin and grow under cyclic flight loads to a length leading to fracture.

In the viewpoint of DT design, parts (e.g., a portion of the spar cap shown in Fig. 3) are sized such that a particular design life goal is achieved (e.g., 24,000 flight hours for a typical business jet spar cap [20–22]). The FCG design life is predicted by executing an analytical crack growth analysis for the given load conditions (e.g., using inputs given in Table 2). For simplicity, the loading is assumed to be of constant amplitude type and number of load cycles is assumed equal to flight hours. The load range ΔP value is selected such that the thickness of the part (shown in Fig. 3) is similar to the real wing spar cap (shown in Fig. 2b) that was designed by considering variable-amplitude loading. For illustration, we use the integral form of the Paris law to calculate the design life:

$$N_d = \frac{1}{C}\int_{a_i}^{a_f}\left(\Delta K\right)^{-n}\mathrm{d}a \quad (4)$$

where N_d is the FCG design life, a_i is the length of assumed preexisting flaw at a hole, and a_f is the crack length at which failure is assumed to occur. For the part and crack geometry (through thickness crack at a hole) assumed in this paper, a surrogate model that replaces Eq. (4) is given in Appendix B. The random samples of C and n generated to simulate coupon testing are propagated through Eq. (4) to estimate the samples of FCG life for the geometry shown in Fig. 3. A stepwise simulation procedure is outlined in Table 3.

As a first step, we estimate the true (fully sampled) FCG design life distribution via Monte–Carlo simulation (MCS). This is done by computing Eq. (4) for 10 million samples of C and n generated from the assumed true joint PDF given in Table 1. The thickness of the part shown in Fig. 3 is sized such that the true mean FCG design life is equal to 24,000 flight hours (FH). This gives the value of true thickness ($t^* = 0.3663$ in.) that is used to estimate the weight penalty. In general, lifetime failure data are modeled with distributions having a positive skew (i.e., a longer right tail). In [9,23,24], researchers modeled lifetime data with a log-normal distribution. We also model the FCG design life distribution with a log-normal distribution as shown in Fig. 4. The parameters of the true FCG life's log-normal PDF given in Eq. (5) are $\mu_{\ln(N_d)}$(location) $= 10.044$ and $\sigma_{\ln(N_d)}$(scale) $= 0.289$, where $\mu_{\ln(N_d)}$ and $\sigma_{\ln(N_d)}$ are the mean and standard deviation of the natural logarithm of N_d:

$$f_{N_d} = \frac{1}{N_d\sigma_{\ln(N_d)}\sqrt{2\pi}}\exp\left(-\frac{(\ln(N_d)-\mu_{\ln(N_d)})^2}{2(\sigma_{\ln(N_d)})^2}\right) \quad (5)$$

The mean FCG life is

$$\mu_{N_d} = \exp\left(\mu_{\ln(N_d)} + \frac{\sigma^2_{\ln(N_d)}}{2}\right) \quad (6)$$

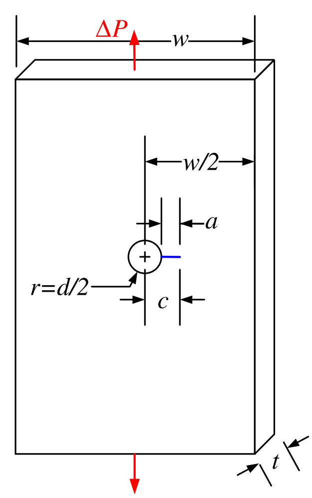
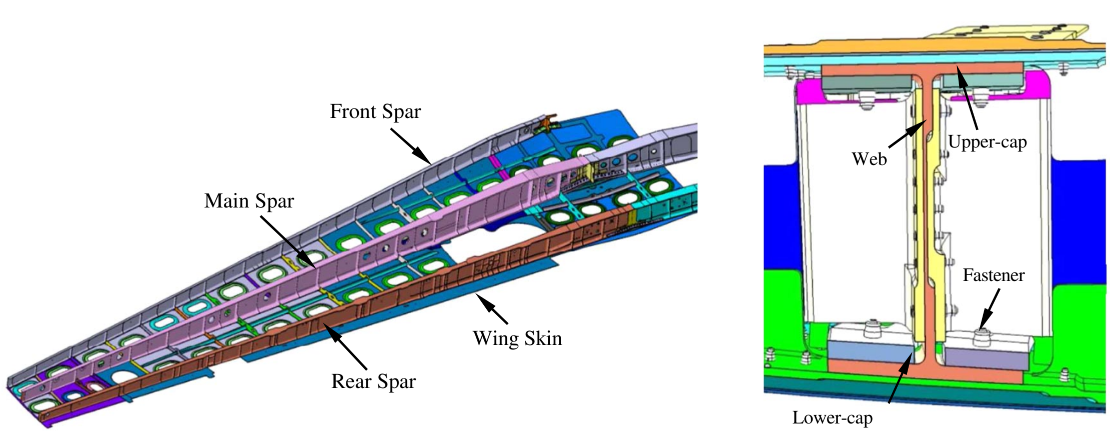


Fig. 3 Lower spar cap at critical fastener location subjected to axial tensile load.



a) b)

Fig. 2 Representations of a) wing assembly of a business jet, and b) cross-sectional view of wing assembly at main spar.

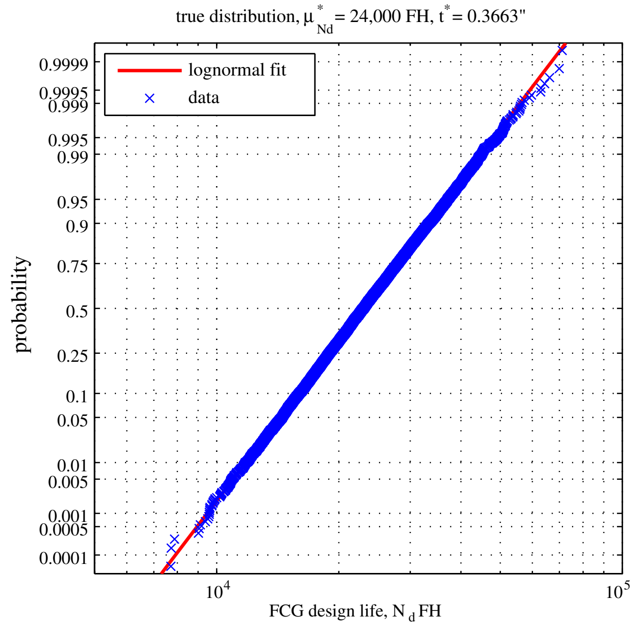


Fig. 4 Probability plot indicating log-normal distribution as a good fit to the FCG design life samples obtained via MCS.

Note that, for $t^* = 0.3663$ in., the true mean FCG life is exactly equal to the design goal of 24,000 FH. Thus, for a very large number of FCG tests, one would get design weight corresponding to the true thickness of $t^* = 0.3663$ in.

IV. Simulation of Crack Growth Testing and Distributions of Mean Design Life

In reality, only a handful of FCG coupon tests could be performed that introduce uncertainty in the mean values of C and n, which further lead to uncertain in the mean FCG design life value. This advances to uncertainty in the design thickness t or design weight W. For example, if one repeats n_{ct} number of coupon tests multiple times, the mean FCG design life based on repeated tests would be different each time, which would lead to different design weights. However, the uncertainty would decrease with the increase in the number of tests. To illustrate the effect of such uncertainty, the simulation procedure outlined in Table 3 is undertaken.

The simulation gives the distributions of mean FCG design life $\hat{\mu}_{N_d}$ that are shown as empirical cumulative distribution function (CDFs) in Fig. 5a for different n_{ct}. It can be noticed that all empirical CDFs approximately pass through the true mean life $\mu^*_{N_d} = 24{,}000$ FH at about 50% probability, and uncertainty/spread about te mean shows the possible values of $\hat{\mu}_{N_d}$ that could be obtained for a fixed true thickness of $t^* = 0.3663$ in. (estimated in the previous section), depending on the sample set of C and n obtained from FCG coupon testing. In Fig. 5a, the samples to the right of true mean life overestimate the mean FCG design life, which would lead to lighter designs but would erode the margin of safety. On the other hand, samples to the left of the true mean would lead to heavier designs, which is also undesirable. Furthermore, uncertainty about the true mean FCG life decreases rapidly with the increase in the number of coupon tests. This reduces the chances of overestimating and the chances of underestimating the FCG design life considerably.

Table 2 Dimensions and loads for the geometry shown in Fig. 3

Dimension	Value
Width w	2 in.
Diameter d	0.20 in.
Initial crack length a_i	0.05 in.
Failure crack length a_f	0.90 in.
Load range ΔP	1 kips
Stress ratio R	0

For example, consider the samples to the left of the true mean FCG life in Fig. 5a; for a fixed chance (e.g., 40 and 20%) of underestimating the FCG life (shown by horizontal arrows in Fig. 5a), the corresponding mean life increases with the increase in the number of tests. That is, the 20th percentile and 40th percentile mean FCG life values increase (see Fig. 5b) with increase in n_{ct}, which would eventually lead to the decrease in the corresponding values of design weight (discussed in the next section). On the other hand, considering samples to the right of the true mean FCG life in Fig. 5a, the probability of overestimating the true mean FCG life of 24,000 FH (e.g., at least by 5%, or 1200 FH, and 10%, or 1400 FH, shown by vertical lines in Fig. 5a) can be seen to decrease rapidly with the increase in the number of tests in Fig. 6. For example, the probability of being greater than 25,200 FH is about 35% for $n_{ct} = 8$ tests and about 10% for $n_{ct} = 64$ tests.

V. Distributions of Design Thickness

The mean FCG design life values estimated in Fig. 5a are either larger or smaller than the desired FCG design life goal of 24,000 FH. Thus, to check the effect of limited testing on design weight, thickness of the spar cap shown in Fig. 3 is changed in a way that each sample of $\hat{\mu}_{N_d}$ in Fig. 5a is equal to 24,000 FH. That is, the thickness for the geometry shown in Fig. 3 is found such that $\hat{\mu}_{N_d} \geq 24{,}000$ FH. This gives the distributions of design thickness, which are shown as empirical CDFs in Fig. 7. It can be noticed that all the CDFs pass through the true design thickness of 0.3663 in. at about 50% probability. The spread about true design thickness represents the uncertainty in the design thickness/weight values. That is, one could obtain a design from a range of possible design weights depending on the sample set of C and n obtained from coupon testing. However, spread/uncertainty decreases with the increase in the number of tests.

In Fig. 7a, the samples of thickness to the left of the true thickness represent designs that are lighter due to overestimation of FCG life as was shown in Fig. 5a. That is, the spar cap is thinner than it should be, which would increase the risk of failure. However, it is not easy to estimate how much of this decrease in design weight (due to overestimation of life) would offset the cost of fatigue problems due to increased risk of failure. On the other hand, the thickness samples to the right of the true thickness are heavier due to underestimation of FCG life. It is easier to analyze the cost benefit due to reduction in weight penalty due to these heavier designs with the increase in number of tests against the testing cost. Thus, the optimal number of tests will be based on the mean of all the samples to the right of the true thickness (0.3663 in.) shown in Fig. 7a. The mean of these overweight thickness samples ($\mu_{t-\mathrm{ow}}$, horizontal line in Fig. 7a) decreases with the increase in the number of tests, which is also shown in Fig. 7b. The thickness penalty Δt also reduces, which is measured with respect to true thickness of $t^* = 0.3663$ in.:

$$\Delta t = \mu_{t-\mathrm{ow}} - t^* \tag{7}$$

The thickness penalties corresponding to Fig. 7b (i.e., for the geometry shown in Fig. 3) are given in Table 4. Also, from Table 4, it

Table 3 Simulation of FCG coupon testing and calculation of mean FCG design life

Step	Description
1	Generate n_{ct} random samples of C and n from the true joint PDF given in Table 1.
2	Calculate samples of FCG design life N_d using Eq. (4), and estimate parameters $(\hat{\mu}_{\ln(N_d)}, \hat{\sigma}_{\ln(N_d)})$ by fitting the log-normal distribution to the FCG samples.
3	Use parameters$(\hat{\mu}_{\ln(N_d)}, \hat{\sigma}_{\ln(N_d)})$to estimate mean FCG design life using Eq. (6), i.e., $\hat{\mu}_{N_d}$.
4	Repeat steps 1–3 10,000 times by generating new sample sets of C and n to get a distribution of$\hat{\mu}_{N_d}$.
5	Repeat steps 1–4 for $n_{ct} = \{8, 16, 32, 64, 128, 256, 512, 1024\}$.

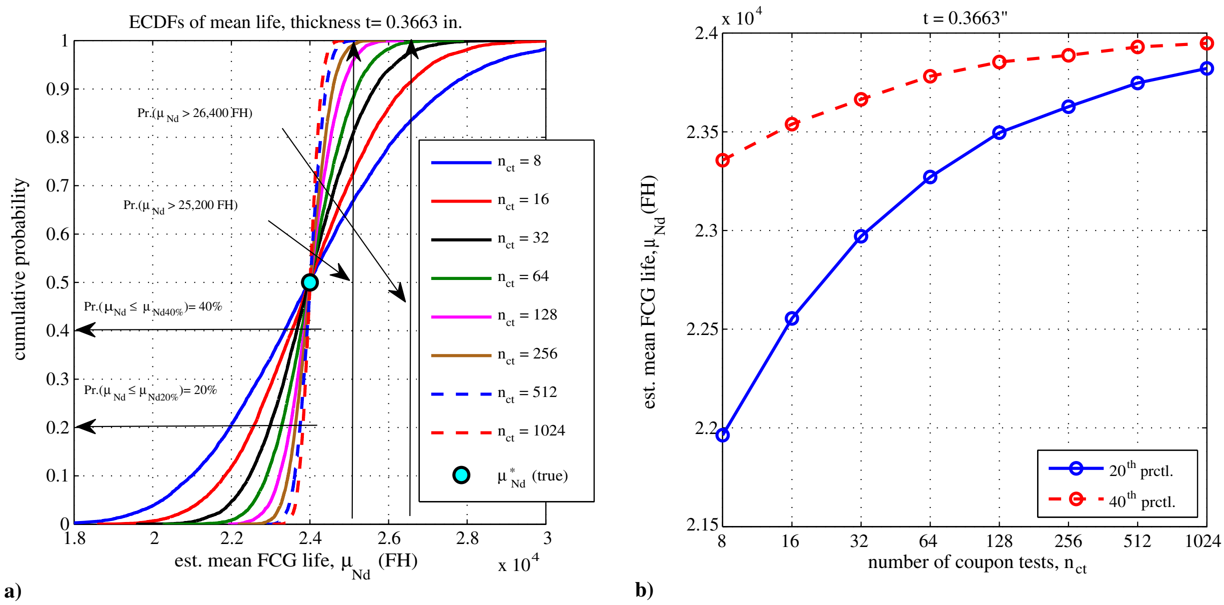


Fig. 5 Representations of a) empirical CDFs of mean FCG life for $t = 0.3663$ in., and b) 20th and 40th percentile mean FCG life as a function of number of coupon tests.

can be noticed that the probability of being heavier by more than 2.4% (shown by the vertical line in Fig. 7a) reduces with the increase in n_{ct}. For example, for $n_{\text{ct}} = 8$, there is 26% chance of being heavier by more than 2.4%, i.e., $Pr(t > 0.375$ in.), which reduces to 3% for $n_{\text{ct}} = 64$.

VI. Extrapolation of Weight to Wing Spar and Fleet

Typically, for a business jet's wing, approximately 50–70% of the wing spar's lower-cap sizing is dominated by the FCG life constraint. The thickness results listed in Table 4 are extrapolated to calculate the weight penalty for the entire lower cap of a spar. Thus, we consider that the wing spar is $l_s = 24$ ft long (a typical length of a midsized business jet spar [20]), and for the first half of spar's length, the lower cap's thickness is assumed to be t and width as $w_{\text{lc}} = 4$ in. Further, for the second half of the spar's length, the thickness and width are assumed to be $1/4t$ and $w_{\text{lc}}/2$. The mean weight for entire lower cap of a single spar corresponds to the mean thickness of overweight samples ($\mu_{t-\text{ow}}$):

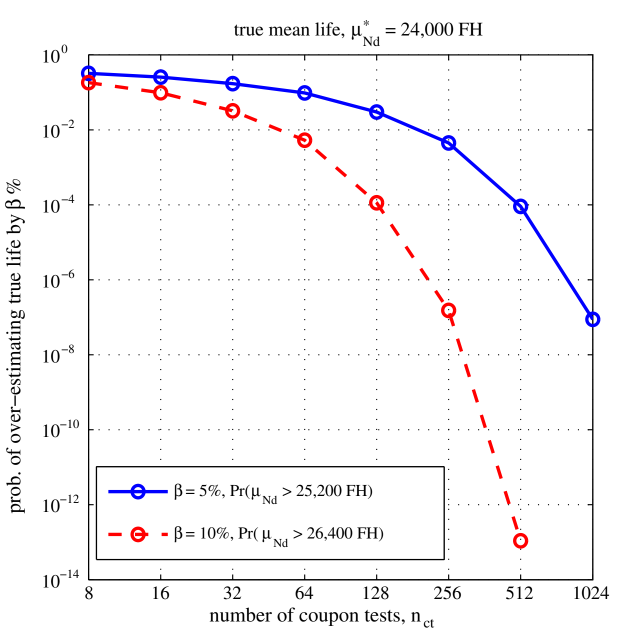


Fig. 6 Probability of overestimating FCG design life by 5 and 10% as a function of number of tests.

$$\mu_{W-\text{lc}} = \left(\frac{1}{2} w_{\text{lc}} l_s \mu_{t-\text{ow}} + \frac{1}{16} w_{\text{lc}} l_s \mu_{t-\text{ow}}\right)\rho_{\text{al}} = \frac{9}{16} w_{\text{lc}} l_s \mu_{t-\text{ow}} \rho_{\text{al}} \qquad (8)$$

where $\rho_{\text{al}} = 0.105$ lb/in.^{3} As $n_{\text{ct}} \to \infty$, $\mu_{t-\text{ow}} \to t^* = 0.3663$ in., and $\mu_{W-\text{lc}} \to W^*_{\text{lc}} = 24.92$ lb. This is about 70% of the weight of the real main spar's lower cap shown in Fig. 2. Next, a typical business jet's wings have six spars, and so their combined weight is approximated using the following equation:

$$\mu_{W-\text{wing}} = 2(\mu_{W-\text{ms}} + \mu_{W-\text{fs}} + \mu_{W-\text{rs}}) = 2(1 + 0.50 + 0.75)\mu_{W-\text{lc}} = \frac{9}{2}\mu_{W-\text{lc}} \qquad (9)$$

where $\mu_{W-\text{ms}}$ is the mean weight of the main spar, $\mu_{W-\text{fs}}$ is the mean weight of the front spar (about 50% of the main spar), and $\mu_{W-\text{rs}}$ is the mean weight of the rear spar (about 75% of the main spar). Therefore, the weight penalty for the wings due to six spar caps (ΔW_{wing}) is calculated as follows:

$$\Delta W_{\text{wing}} = \mu_{W-\text{wing}} - W^*_{\text{wing}} \qquad (10)$$

where W^*_{wing} is the true weight of the wing due to six lower caps corresponding to the true thickness of $t^* = 0.3663$ in. (i.e. $W^*_{\text{wing}} = 112.2$ lb). The corresponding weight penalties are listed in Table 5. It can be noticed that the weight penalty is about 3.53 lb (about 3% of W^*_{wing}) for $n_{\text{ct}} = 8$ and reduces to about 0.3 lb (0.3% of W^*_{wing}) for 1024 tests.

VII. Cost of Weight Penalty and Testing Cost Tradeoff

The weight penalties calculated in the previous section are based on the mean thickness of the overweight samples (from Fig. 7a) can now be used for calculating the cost penalty to the operators due to additional weight attributable to finite coupon testing. The worth of structural weight proposed by Curran et al. [25] is 300/kg (136/lb). Similarly, Kim et al. [26] referred to the U.S. National Materials Advisory Board report [27] that estimated the worth of weight saved as 200/lb for a civil transport aircraft. Acar et al. [5] varied the cost of weight penalty p between 200/lb and 1000/lb. Bhachu et al. [21] proposed a measure of worth based on useful load and found that p varies between \$800 and \$1600 for business jets. In this paper, we vary p between 200/lb and 1200/lb to see its effect on the optimum

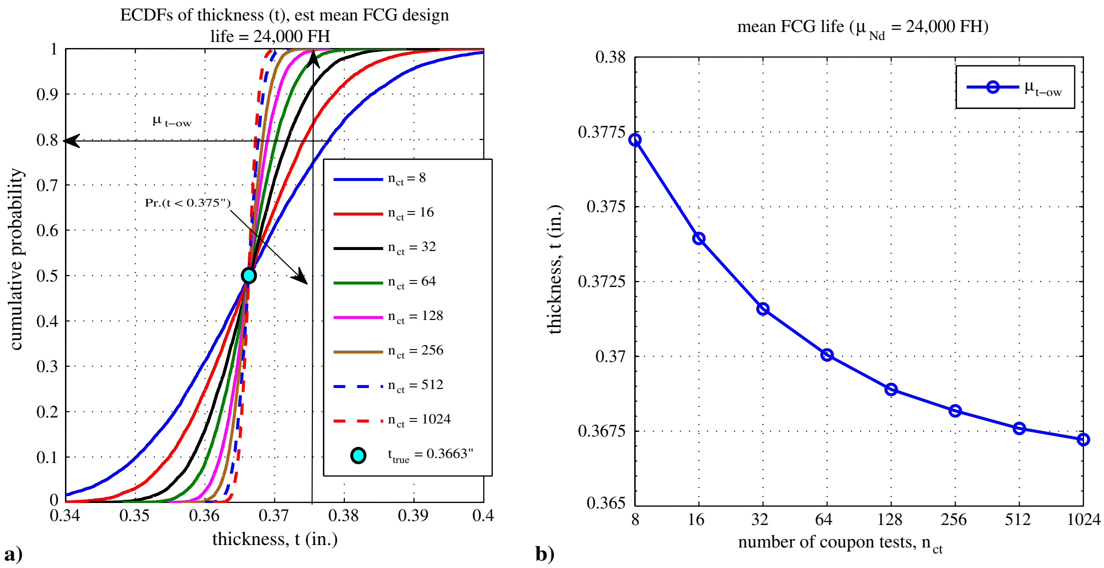


Fig. 7 Representations of a) CDFs of thickness, and b) mean of overweight thickness samples as a function of number of tests.

number of FCG coupon tests. The cost associated with weight penalty is calculated using the following formula:

$$C_{\Delta W-\text{fleet}} = n_a \Delta W_{\text{wing}} p \tag{11}$$

where n_a is the number of aircraft (400), i.e., the weight penalty is transformed into the cost penalty for the entire fleet of aircraft to be produced by a manufacturer. On the other hand, the cost of coupon testing increases linearly with the number of FCG coupon tests:

$$C_t = n_{\text{ct}} C_{\text{sct}} \tag{12}$$

where C_{sct} is the cost of single coupon tests that ranges between \$1000 and \$2000. These cost numbers are based on personal communications with engineers at Cessna Aircraft Company. We assume the mean cost of \$1500 per test for our analysis. Notice that the cost of FCG coupon testing is much higher than \$300 for material yield strength tests mentioned in [5].

Table 4 Mean thickness of overweight samples, thickness penalties, and probability of overdesigning

n_{ct}	$\mu_{t-\text{ow}}$, in.	Δt, in.	$Pr(t > 0.375\ \text{in.})$, %
8	0.3778	0.0115	26
16	0.3743	0.0080	18
32	0.3719	0.0056	9.5
64	0.3702	0.0039	3
128	0.3690	0.0027	0.6
256	0.3683	0.0020	0.03
512	0.3676	0.0013	— —
1024	0.3672	0.0009	— —
→∞	0.3663	— —	— —

Table 5 Weight penalty for wing spars based on mean thickness of overweight samples

n_{ct}	$\mu_{W-\text{wing}}$, lb	ΔW_{wing}, *lb*
8	115.7	3.53
16	114.6	2.45
32	113.9	1.71
64	113.4	1.20
128	113.0	0.82
256	112.8	0.61
512	112.6	0.41
1024	112.4	0.29
→∞	112.2	— —

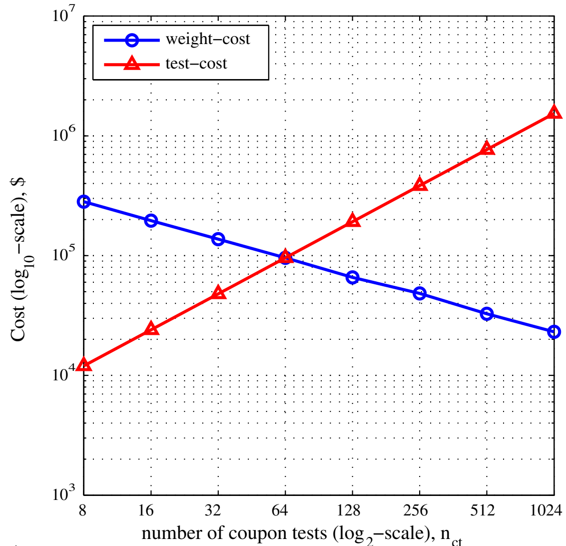
The tradeoff results are shown in Fig. 8. It can be noticed from Fig. 8a that, for weight penalty of $p = 200/\text{lb}$, the optimal number of FCG coupon tests is around 64 (i.e., the point where both costs are balanced), and for $p = 1200/\text{lb}$, it is about 210. Thus, one could justify 64–210 FCG tests for designing damage-tolerant components (e.g., wing spar caps). On the other hand, if the designer is too concerned about eroding the margins of safety on the FCG design life, then Fig. 6 could be used to qualitatively decide the number of tests. For example, if the designer is okay with the 1% risk of overestimating the FCG life by 5%, then the optimal number of tests would range from 128 to 256 (refer to Fig. 6). However, if the designer is fine with 1% risk of overestimating the desired mean FCG life by 10%, then the optimal number of tests would range from 32 to 64. In any case, the current amount of FCG coupon testing that typically ranges from 8 to 32 seems to be somewhat inadequate.

The results presented in this paper are based on the assumption of the Paris law, but in reality, models that take stress ratio into account are employed (e.g., Walker equation). In those cases, the testing is repeated for multiple stress ratios, and so the optimal number of tests could be distributed equally among those testing conditions. For example, if testing is repeated at four different stress ratios, then the optimal number of tests (e.g., 64) may be distributed among four stress ratios (i.e., 16 tests per stress ratio). However, such an assumption should be verified by performing a simulation similar to the one presented in this paper with the improved model.

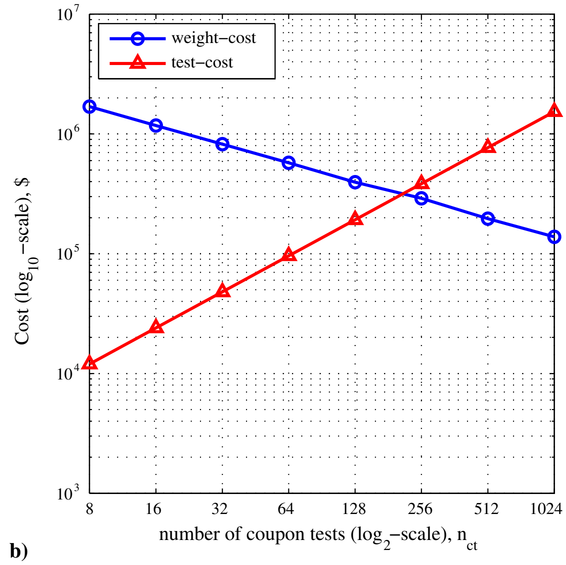
VIII. Conclusions

From the preceding analysis, it was demonstrated that performing more FCG coupon tests reduces the risk of both overestimating the mean FCG design life and overdesigning the structure, although it is not easy to decide the optimal number of tests by calculating the cost associated with the reduction in the chances of fatigue problems (due to overestimation of life). But one could make an informed decision by looking at an acceptable probability (e.g., 1%) of overestimating the life by some percentage (e.g., 5 or 10%). Such an approach may be useful for deciding the optimal number of tests for designing damage-tolerant components (e.g., stringers) that does not drive much weight into the airframe.

On the other hand, for structures like wing spars, a decision on the optimal number of tests could be made by studying the tradeoff between costs of weight penalty and testing cost. It was shown that about 64–256 coupon tests could be justified in comparison to the current practice of about 8–32 tests, depending upon the associated weight penalty that was assumed to range from \$200/lb to \$1200/lb. Therefore, we conclude that the current amount of FCG coupon testing that ranges from 8 to 32 could be increased to maximize the



a)



b)

Fig. 8 Tradeoff between weight penalty cost and testing cost for a) $p = 200/\mathrm{lb}$, and b) $p = 1200/\mathrm{lb}$.

chances of weight savings and reducing the chances of fatigue problems due to overestimation of life.

Appendix A: Conservative Assumptions of Crack Growth Analysis

The deterministic crack growth analysis makes many conservative assumptions (implicit safety factors) and explicit safety factors for conservatively predicting the FCG design life. Some implicit safety factors/conservative assumptions are as follows.

1) Conservative assumption about the size of preexisting rogue flaw (e.g., 0.05 in.). The probability of such a flaw to exist in a pristine structure is about 10^{-6} [28].

2) Mean crack growth rate curves are multiplied with a safety factor to derive the upper bounds.

3) The material testing is usually performed under a high-humidity and high-temperature environment that is known to accelerate the crack growth rate. Thus, material constants obtained by fitting crack growth models to such data are conservative.

4) Crack growth models often only model the linear Paris region (i.e., where crack growth rates are between 10^{-6} and 2×10^{-5} inches per cycle, or between $\Delta K = 6 - 16$ ksi $\sqrt{\text{in.}}$). That is, ignoring near-threshold crack growth data tends to underpredict the crack growth life.

5) Sometimes a factor of safety (e.g., 1.5 to 2) is used to knock down the calculated FCG life.

Therefore, in the presence of such conservative assumptions, one hopes to cover for the uncertainty arising from flight load spectrum and material randomness.

Appendix B: Fatigue Crack Growth Life Based on Geometry, Load Conditions, and Stress Intensity Solution

The FCG life equation given in Eq. (4) can be expanded as follows:

$$N_d = t^n \left[C^{-1} \left(\Delta P \sqrt{\pi} / w \right)^{-n} \int_{a_i}^{a_f} \left(G_{\text{cf}} \sqrt{a} \right) \mathrm{d}a \right] \tag{B1}$$

$$G_{\text{cf}} = F_{\text{wc}} G_{\text{cf}}^{\infty} = \left(0.7071 + 0.7548 \left(\frac{r}{r+a} \right) + 0.3415 \left(\frac{r}{r+a} \right)^2 + 0.642 \left(\frac{r}{r+a} \right)^3 + 0.9196 \left(\frac{r}{r+a} \right)^4 \right) \times \sqrt{\sec\left(\frac{\pi r}{w} \right) \sec\left(\frac{\pi (r + a/2)}{w - a} \right)} \tag{B2}$$

where F_{wc} is the finite width correction, and G_{cf}^{∞} is the infinite plate solution. The integrand in Eq. (B1) requires numerical integration that is computationally expensive in the simulation. Therefore, it is approximated by the surrogate given in Eq. (B3) that is valid for the geometry given in Table 2:

$$N_d = t^n \left[C^{-1} \left(\Delta P \sqrt{\pi} / w \right)^{-n} \times \left(1.64 - 1.22n + 1.14n^2 - 0.3n^3 + 0.047n^4 \right) \right] \tag{B3}$$

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