EAS4200C Aerospace Structures Homework #4 (Due: Oct. 2nd)

1. Show that there is no warping in the bar of circular cross-section

Hint: First, calculate the constant C in $\phi(x,y) = C\left(\frac{x^2}{a_2} + \frac{y^2}{a_2} - 1\right)$ from the compatibility equation. Second, calculate shear strains γ_{xz} and γ_{yz} from shear stresses τ_{xz} and τ_{yz}. Third, integrate $w(x,y)$ from the definition of shear strains: $\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$.

2. Consider a straight bar of a uniform elliptical cross-section. The semimajor and semiminor axes are a and b, respectively. (a) Show that the stress function of the following form:

$$\phi(x,y) = C\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)$$

provides the solution of the torsion of the bar. That is, show that the stress function satisfies equilibrium and boundary conditions. (b) Find the expression of C from the compatibility equation and show that

$$J = \frac{\pi a^3 b^3}{a^2 + b^2}, \quad \tau_{xz} = \frac{-2Ty}{\pi a b^3}, \quad \tau_{yz} = \frac{2Tx}{\pi a^3 b}$$

Hint: The area of the ellipse can be obtained by

$$\iint_A \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) dA = \pi ab$$