EXAM 2 KEY

1. (a) From KT cond.

$$-\nabla_x f = \sum_{i=1}^{2} \lambda_i \nabla g_i \Rightarrow \left.\begin{array}{l} -\frac{4}{3} = -2\lambda_1 - \lambda_2 \\ -\frac{4}{3} = -\lambda_1 - 2\lambda_2 \end{array}\right) \Rightarrow \lambda_1 = \lambda_2 = \frac{4}{9}$$

(b) It is a global min. because f is a convex fn and the feasible set by linear inequality constraints is convex.

2. $\sigma = \frac{P}{A}$ $\quad P \sim N(100, 10^2)N, \quad A \sim N(1, 0.1^2)\,mm^2$

$$\sigma_L = \underbrace{\sigma(\mu_P, \mu_A)}_{100} + \underbrace{\left.\frac{\partial \sigma}{\partial P}\right|_{\mu_P, \mu_A}}_{1}(P - \mu_P) + \underbrace{\left.\frac{\partial \sigma}{\partial A}\right|_{\mu_P, \mu_A}}_{-100}(A - \mu_A)$$

$$\sigma_L = 100 + (P - 100) - 100(A - 1) = P - 100A + 100$$

$$\mu_{\sigma_L} = \mu_P - 100\mu_A + 100 = 100$$

$$\sigma_{\sigma_L}^2 = [1 \ \ -100]\begin{bmatrix} 100 & 0 \\ 0 & 10^{-2} \end{bmatrix}\begin{bmatrix} 1 \\ -100 \end{bmatrix} = 200.$$

3. (a) $\mu_G = \mu_R - \mu_S = 200 \qquad \sigma_G^2 = \sigma_R^2 + \sigma_S^2 = 5000.$

(b) $\beta_{HL} = \frac{\mu_G}{\sigma_G} = \frac{200}{\sqrt{5000}} = 2.83$

(c) $\beta_t = \frac{\mu_G'}{\sigma_G} = 4 \qquad \therefore \mu_G' = 200\sqrt{2} = \mu_R - \mu_S'$

$$\therefore \mu_S' = \cancel{200\sqrt{2}} \ \mu_R - 200\sqrt{2} = 217 \text{ N}.$$