EAS6939 Homework #4 (Due: 3/22)

1. Consider the following design optimization problem:

$$\begin{aligned} &\text{Minimize} && f(\mathbf{x}) = x_1^2 + x_2^2 - 4x_1 + 4 \\ &\text{Subject to} && g_1(\mathbf{x}) = -x_1 \le 0 \\ & && g_2(\mathbf{x}) = -x_2 \le 0 \\ & && g_3(\mathbf{x}) = x_2 - (1 - x_1)^3 \le 0 \end{aligned}$$

- (i) Find the optimum point graphically
- (ii) Show that the optimum point does not satisfy K-T condition. Explain

2. An engineering design problem is formulated as:

Minimize $f(\mathbf{x}) = x_1^2 + 2x_2^2 - 5x_1 - 2x_2 + 10$

Subject to the constraints

$$h_1 = x_1 + 2x_2 - 3 = 0$$

$$g_1 = 3x_1 + 2x_2 - 6 \le 0$$

In all of the following questions, justify your answers.

- (i) Write K-T necessary conditions
- (ii) How many cases are there to be considered? Identify those cases.
- (iii) Find the solution for the case where g_1 is active. Is this acceptable case?
- (iv) Regardless of the solution you obtained in (iii), suppose the Lagrange multiplier for the constraint $h_1 = 0$ is $\lambda_1 = -2$ and the Lagrange multiplier for the constraint $g_1 \le 0$ is $\lambda_2 = 1$. If the equality and inequality constraints are simultaneously changed to $h_1 = x_1 + 2x_2 - 3.2 = 0$ and $g_1 = 3x_1 + 2x_2 - 6.2 \le 0$, what will be the new optimum cost?

3. A design problem is formulated as an unconstrained optimization problem to minimize

$$f(x_1, x_2, x_3) = x_1^3 + 2x_2^2 + 2x_3^2 + 4x_1x_3 + 2x_2x_3$$

- (i) Calculate the gradient of the cost function at (1, 1, 1)
- (ii) Calculate Hessian at the point (2, 1, 1)
- (iii) Is the cost function $f(\mathbf{x})$ a convex function? Why or Why not?
- (iv) Is the cost function $f(\mathbf{x})$ convex for the region $x_1 > 1$? Why or Why not?
- (v) Show that (0, 0, 0) is a stationary point. Is this a minimum, maximum, or inflection point? Why?