※ How to solve cubic eq.?

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$$

$$\lambda_1 = g\cos\frac{\phi}{3} - \frac{C_1}{3}$$

$$\lambda_2 = g\cos\left(\frac{\phi+2\pi}{3}\right) - \frac{C_1}{3}$$

$$\lambda_3 = g\cos\left(\frac{\phi+4\pi}{3}\right) - \frac{C_1}{3}$$

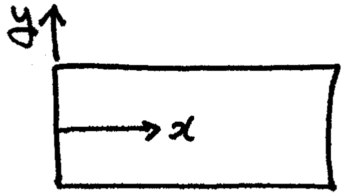
$$\phi = \cos^{-1}\left[-\frac{b}{2\sqrt{-a^3/27}}\right]$$

$$g = 2\sqrt{-a/3}$$

$$a = \frac{1}{3}(3C_2 - C_1^2)$$

$$b = \frac{1}{27}(2C_1^3 - 9C_1C_2 + 27C_3)$$

Example 2.11



given: $\varepsilon_{xx} = Cy(L-x)$, $\varepsilon_{yy} = Dy(L-x)$, $\gamma_{xy} = -(C+D)(A^2-y^2)$

B.C.: $u(0,0) = 0$, $v(0,0) = 0$, $\left.\frac{\partial u}{\partial y}\right|_{x=y=0} = 0$

Goal: $u = ?$ $v = ?$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = Cy(L-x) \Rightarrow u = Cy(Lx - \tfrac{1}{2}x^2) + Y(y)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = Dy(L-x) \Rightarrow v = \tfrac{1}{2}Dy^2(L-x) + X(x)$$

$$\gamma_{xy} = -(C+D)(A^2-y^2) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$= C(Lx - \tfrac{1}{2}x^2) + \frac{dY}{dy} - \tfrac{1}{2}Dy^2 + \frac{dX}{dx}$$

Move fn of x to the left & fn of y to the right

$$\underbrace{\frac{dX}{dx} + C(Lx - \tfrac{1}{2}x^2)}_{\text{depends on } x} = \underbrace{-(C+D)(A^2-y^2) + \tfrac{1}{2}Dy^2 - \frac{dY}{dy}}_{\text{depends on } y} \equiv \underset{\text{const.}}{E}$$

$$\therefore \left(\begin{array}{l} \frac{dX}{dx} + C(Lx - \frac{1}{2}x^2) = E \\ -\frac{dY}{dy} + \frac{1}{2}Dy^2 - (C+D)(A^2-y^2) = E \end{array}\right) \text{integ.}$$

$$\left(\begin{array}{l} X(x) = -C(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + Ex + J \\ Y(y) = \frac{1}{6}Dy^3 - (C+D)(A^2y - \frac{1}{3}y^3) - Ey + K \end{array}\right.$$

$$u(x,y) = Cy\left(Lx - \tfrac{1}{2}x^2\right) + Y(y)$$

$$= Cy\left(Lx - \tfrac{1}{2}x^2\right) + \tfrac{1}{6}Dy^3 - (C+D)\left(A^2y - \tfrac{1}{3}y^3\right) - Ey + K$$

$$v(x,y) = \tfrac{1}{2}Dy^2(L-x) + X(x)$$

$$= \tfrac{1}{2}Dy^2(L-x) - C\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) + Ex + J$$

B.C. $u(0,0) = K = 0$

$v(0,0) = J = 0$

$$\frac{\partial u}{\partial y} = C\left(Lx - \tfrac{1}{2}x^2\right) + \tfrac{1}{2}Dy^2 - (C+D)\left(A^2 - y^2\right) - E$$

$$\frac{\partial u}{\partial y}(0,0) = -(C+D)A^2 - E = 0 \qquad \therefore\ E = -(C+D)A^2$$

$$\therefore \boxed{\begin{aligned} u(x,y) &= Cy\left(Lx - \tfrac{1}{2}x^2\right) + \tfrac{1}{6}Dy^3 + \tfrac{1}{3}(C+D)y^3 \\ v(x,y) &= \tfrac{1}{2}Dy^2(L-x) - C\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) - (C+D)A^2x. \end{aligned}}$$