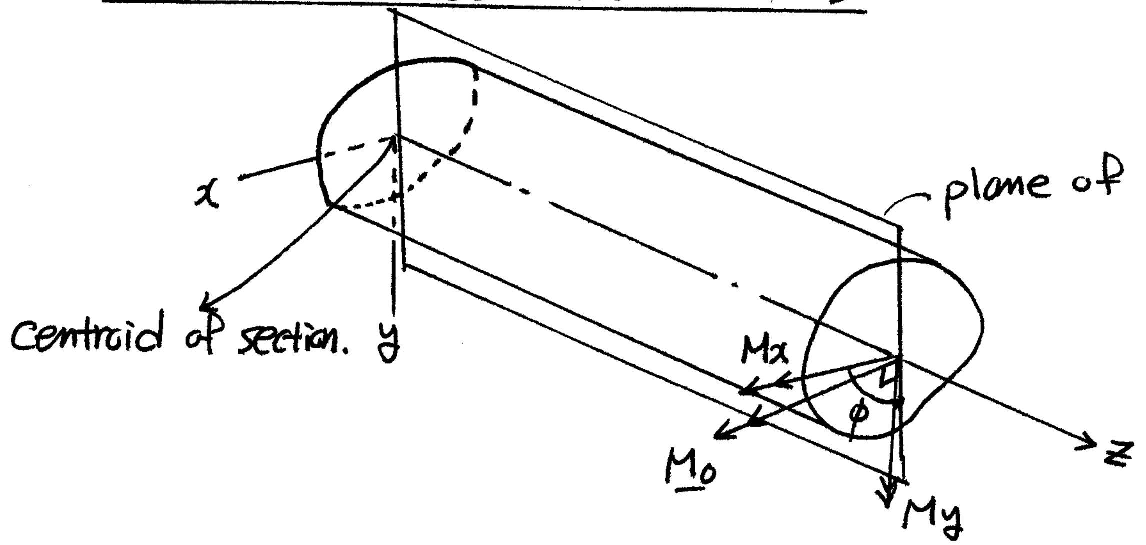
CH 7. Bending of Straight Beams

7.1. Fundamentals

1. Centroidal Coordinate A es



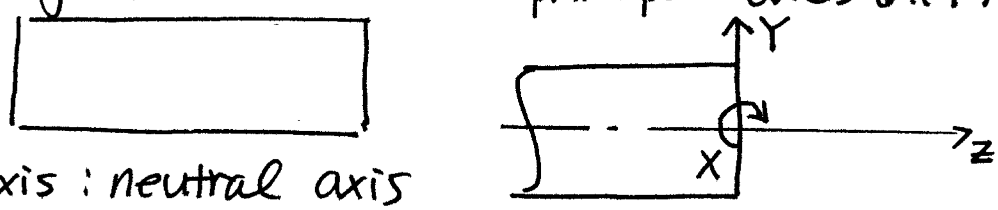
- plane of load : plane that is perpendicular to bending moment.

2. Shear Load & Shear Center

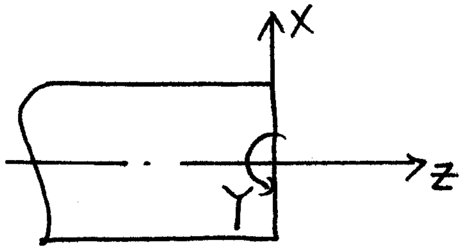
- Shear center ; when the shear force passes through the shear center, no torque will be generated.
- bending axis ; passes through the shear center.
- cross-section has symmetry axis or anti-symmetry axis, shear center locates on that axis.

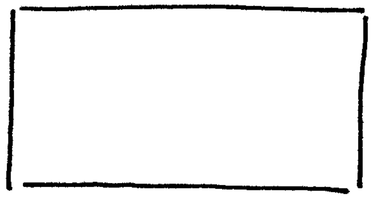
3. Symmetric Bending

- Every cross-section has principal axes (X, Y) ; $I_{XY} = 0$.



X-axis : neutral axis

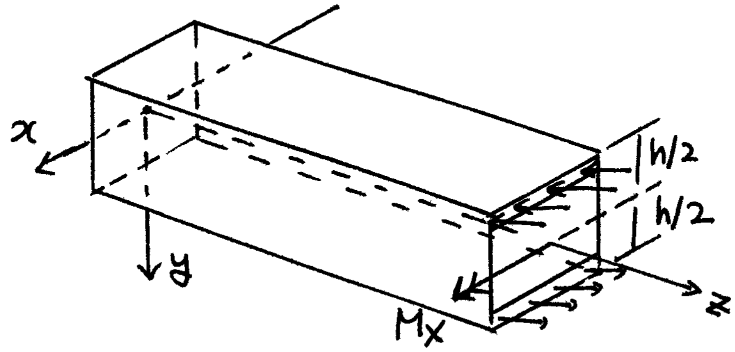




Y-axis : neutral axis.

- Symmetric bending – bending occurs about a neutral axis that coincides with the principal axis.

Ex)



- Rectangular cross-section.
- shear center = centroid = bending axis

$$\sigma_{zz} = \frac{M_x Y}{I_x}$$

- Max stress occurs at $Y = \pm h/2$, $I_x = bh^3/12$

$$\sigma_{max} = \frac{6|M_x|}{bh^2} = \frac{|M_x|}{S_x}$$

; elastic section modulus.

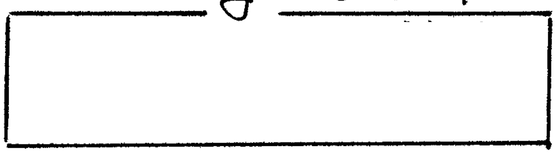
$C_{max} = \max\left(\left|\frac{h}{2}\right|, \left|-\frac{h}{2}\right|\right)$

- $\sigma_{max} \propto \frac{1}{S_x}$.

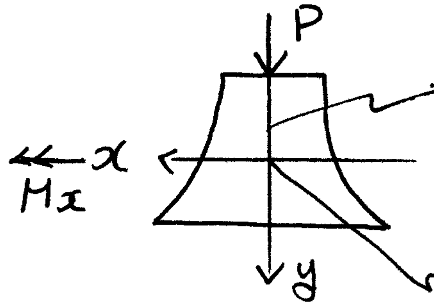
Good to design a cross-section that has large S_x.

4. Non-Symmetric Bending

- axis of bending is different from principal axis.



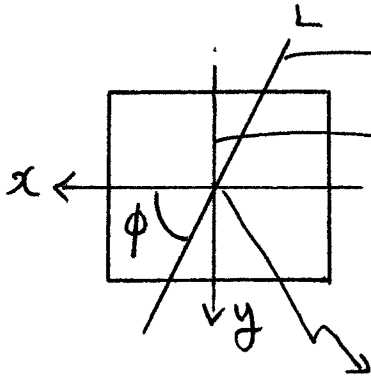
5. Plane of Loads



plane of symmetry
principal axis
plane of loads

shear center, centroid.

symmetric bending



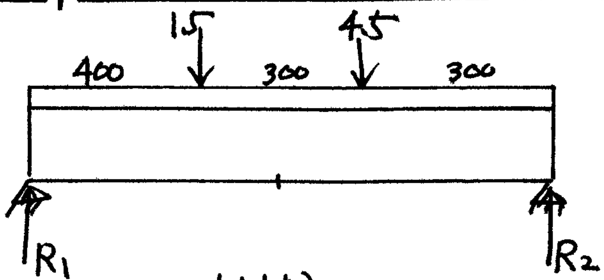
plane of loads

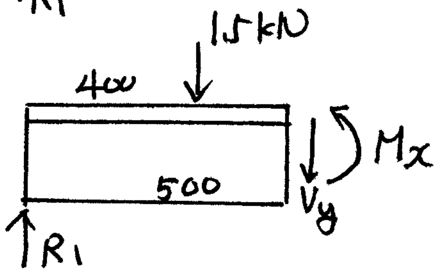
principal axis
plane of symmetry

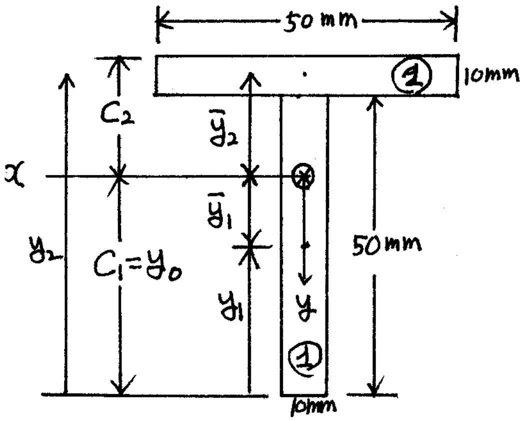
centroid, shear center

Nonsymmetric bending

Example 7.2. T-section Beam







- Centroid

- Equilibrium

$R_1 = 2250\,N, \quad V_y = 750\,N, \quad M_x = 975\,N\cdot m$

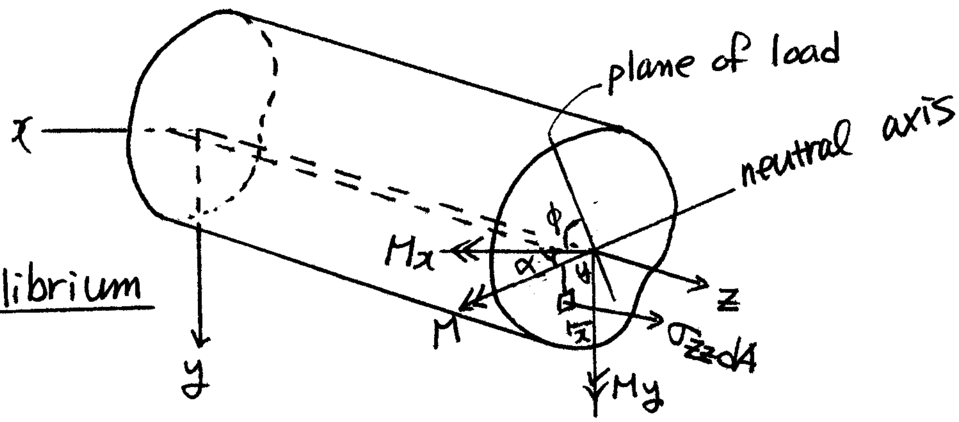
- Max. stress at bottom surface (c_1)

$$\sigma_{zz} = \frac{M_x c_1}{I_x}$$

Max. comp. stress at top surface ($-c_2$)

$$\sigma_{zz} = \frac{M_x(-c_2)}{I_x}$$

7.2. Non-Symmetric Bending Stress



1. Equilibrium

2. Deformation

– plane section remains plane after deformation.

$$e_{zz} = a'' + b''x + c''y$$ extension (linear in x & y)

$$\varepsilon_{zz} = \frac{e_{zz}}{\Delta z} = \frac{a''}{\Delta z} + \frac{b''}{\Delta z}x + \frac{c''}{\Delta z}y$$

$a' = \frac{a''}{\Delta z}$ $\quad b' = \frac{b''}{\Delta z}$ $\quad c' = \frac{c''}{\Delta z}$

$|100$

3. Stress - Strain Relations

$$\sigma_{zz} = E\varepsilon_{zz} = Ea' + Eb'x + Ec'y$$

$\boxed{}$ $a = Ea'$ $b = Eb'$ $c = Ec'$

4. Load - Stress Relation.

$$\begin{cases} \int \sigma_{zz} dA = a\int dA + b\int x dA + c\int y dA = \\ \int y\sigma_{zz} dA = a\int y dA + b\int xy dA + c\int y^2 dA = \\ \int x\sigma_{zz} dA = a\int x dA + b\int x^2 dA + c\int xy dA = \end{cases}$$

$$\int x dA = \int y dA = 0 \quad \because$$

$$\Rightarrow \begin{cases} aA = 0. \\ bI_{xy} + cI_x = M_x \\ bI_y + cI_{xy} = -M_y \end{cases}$$

$\Rightarrow$ $a = 0.$

$$\begin{Bmatrix} b \\ c \end{Bmatrix} = \frac{1}{I_{xy}^2 - I_x I_y} \begin{bmatrix} I_{xy} & -I_x \\ -I_y & I_{xy} \end{bmatrix} \begin{Bmatrix} M_x \\ -M_y \end{Bmatrix}$$

$$b = \frac{M_x I_{xy} + M_y I_x}{I_{xy}^2 - I_x I_y} \qquad c = \frac{-M_x I_y - M_y I_{xy}}{I_{xy}^2 - I_x I_y}$$

$\Rightarrow$ $\boxed{}$

5. Neutral Axis

- angle ϕ with positive x-z plane. = plane of load.

$$\begin{cases} M_x = M\sin\phi \\ M_y = -M\cos\phi \end{cases} \qquad \frac{M_y}{M_x} = -\cot\phi.$$

- Neutral axis $\Rightarrow$ $\sigma_{zz} = 0$.

$$\Rightarrow \quad y = \frac{M_x I_{xy} + M_y I_x}{M_x I_y + M_y I_{xy}}\, x = x \tan\alpha.$$

∴

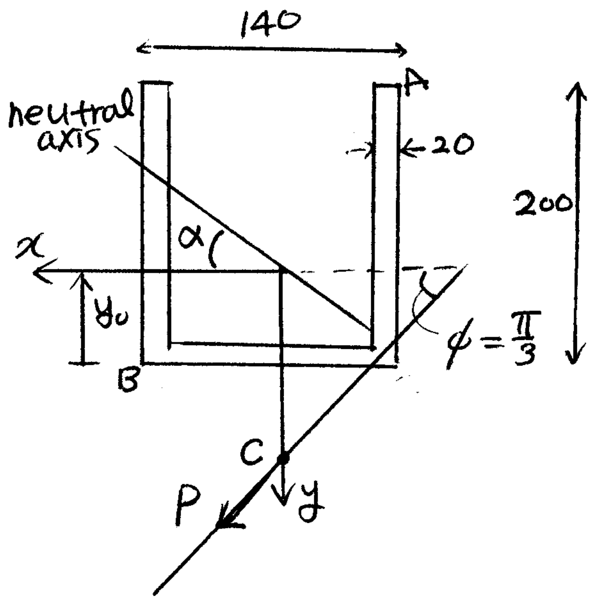
Example 7.3. Channel Section Beam

$P = 12\,\text{kN}$ at 3 m long beam

$A = 10^4\ \text{mm}^2$, $y_0 = 82\ \text{mm}$

$I_x = 39.69 \times 10^6\ \text{mm}^4$

$I_y = 30.73 \times 10^6\ \text{mm}^4$, $I_{xy} = 0$



Max tensile stress at A. Max. comp. stress at B

$M =$

$M_x =$ $\qquad\qquad$ $M_y =$

At A, $(x, y) = (-70, -118)$

$$\sigma_A = -\left(\frac{M_y I_x + M_x I_{xy}}{I_x I_y - I_{xy}^2}\right) x_A + \left(\frac{M_x I_y + M_y I_{xy}}{I_x I_y - I_{xy}^2}\right) y_A = 133.7 \text{ MPa}$$

At B, $(x, y) = (70, 82)$

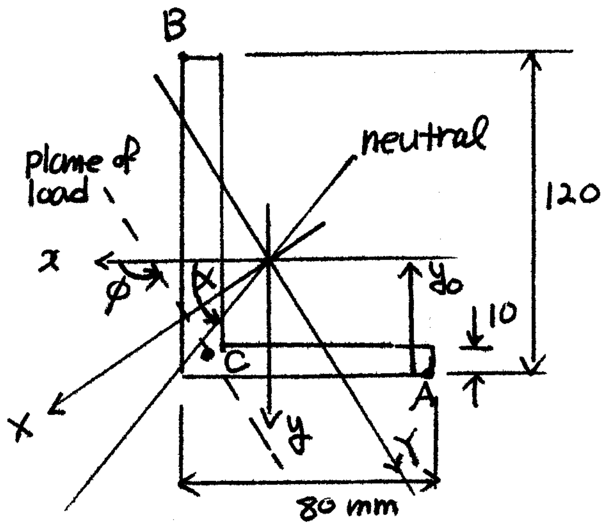
$$\sigma_B = -\left(\frac{M_y I_x + M_x I_{xy}}{I_x I_y - I_{xy}^2}\right) x_B + \left(\frac{M_x I_y + M_y I_{xy}}{I_x I_y - I_{xy}^2}\right) y_B = -105.4 \text{ MPa}$$

Example 7.4. Angle Beam

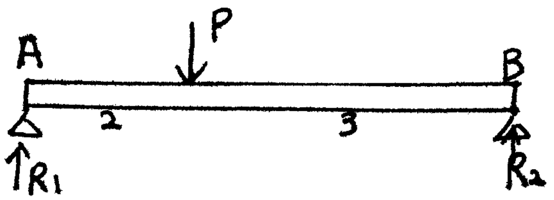
$A = 1900 \text{ mm}^2$, $I_x = 2.783 \times 10^6 \text{ mm}^4$

$x_0 = 19.74 \text{mm}$, $I_y = 1.003 \times 10^6 \text{mm}^4$

$y_0 = 39.74 \text{mm}$, $I_{xy} = -0.973 \times 10^6 \text{mm}^4$.



$\phi = \frac{2}{3}\pi$



$\Sigma M_B = P \times 3 - R_1 \times 5 = 0$ $\therefore R_1 = 0.6P$.

$M_{max} =$ at the load appl. point.

$M_x =$

$M_y =$

$(x_A, y_A) = (-60.26, 39.74) \Rightarrow \sigma_A =$

$(x_B, y_B) = (19.74, -80.26) \Rightarrow \sigma_B =$

※ Convenient form of bending stress

$$\sigma_{zz} = -\frac{M_y I_x + M_x I_{xy}}{I_x I_y - I_{xy}^2}\, x + \frac{M_x I_y + M_y I_{xy}}{I_x I_y - I_{xy}^2}\, y$$

$\tan\alpha =$

$$\Rightarrow \quad M_y I_x + M_x I_{xy} = (M_x I_y + M_y I_{xy}) \tan\alpha$$

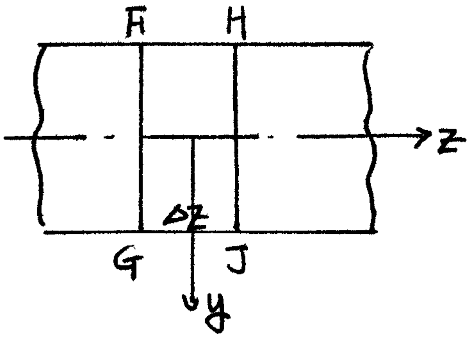
$$\Rightarrow \quad \sigma_{zz} = \underbrace{\frac{M_x I_y + M_y I_{xy}}{I_x I_y - I_{xy}^2}}_{=} (y - x\tan\alpha)$$

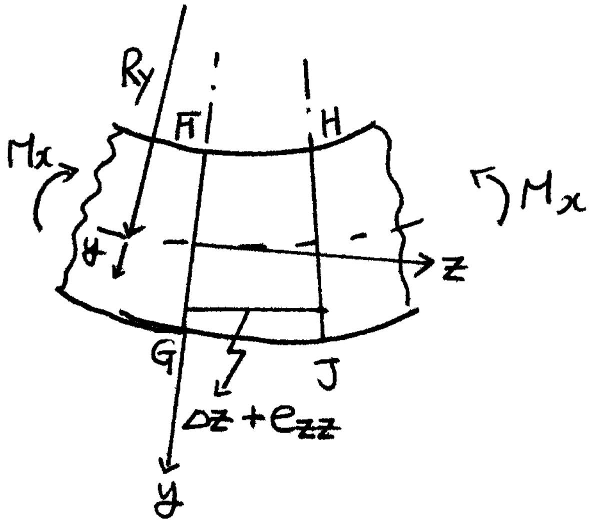
$\therefore$ $\boxed{}$

α: angle b/w x-axis & neutral axis.

Max. Min. stresses occur at the farthest distance from N.A. Put (x, y) into σ_{zz} expression

7.3. Deflection of Non-Symmetric Beam

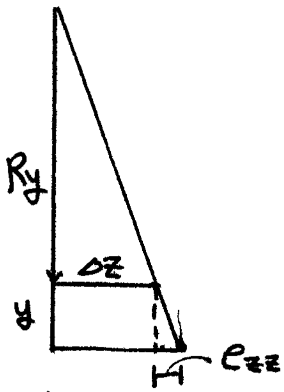




- R_y : radius of curvature (negative)

$$\frac{1}{R_y} \approx \frac{d^2V}{dz^2}$$: small deformation

- Elongation at location y

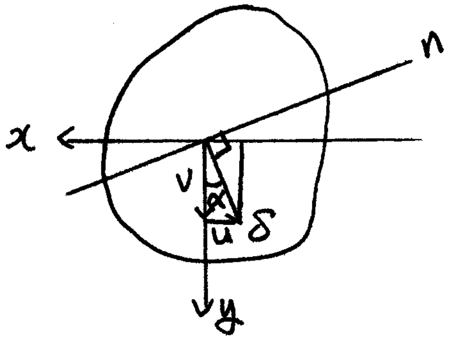


similarity of triangle

$$\Rightarrow -\frac{1}{R_y} = \frac{\varepsilon_{zz}}{y} \approx -\frac{d^2V}{dz^2}.$$

$$\frac{\varepsilon_{zz}}{y} =$$

symmetric $\Rightarrow I_{xy} = 0 \Rightarrow \dfrac{d^2V}{dz^2} = -\dfrac{M_x}{E I_x}.$



- Total deflection δ is normal to the neutral axis

$\therefore$

$\therefore$

Example 7.6. Channel section in Ex. 7.3

$P = 35$ kN, $\phi = \frac{5}{9}\pi$ $E = 72$ GPa

$\tan\alpha =$

$M = \frac{PL}{4} = 26.25$ kN·m

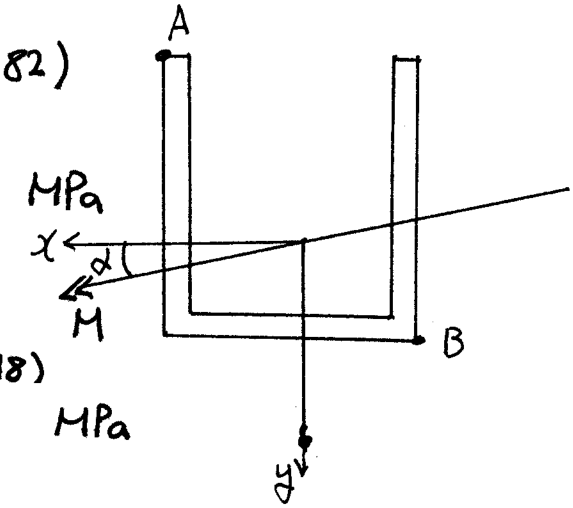
$M_x = M\sin\phi = 25.85$ kN·m

Max. tensile stress at B(−70, +82)

$\sigma_{zz} =$ MPa

Max. compressive stress at A (70, −118)

$\sigma_{zz} =$ MPa



Using $M_x = M\sin\phi \Rightarrow$

$v =$

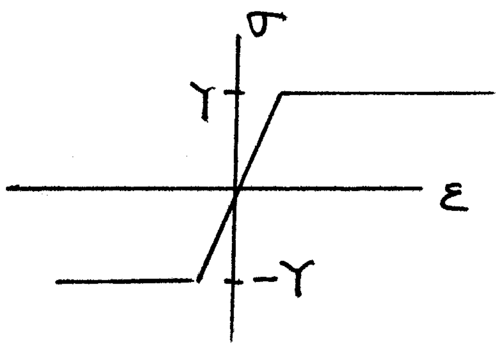
$u =$

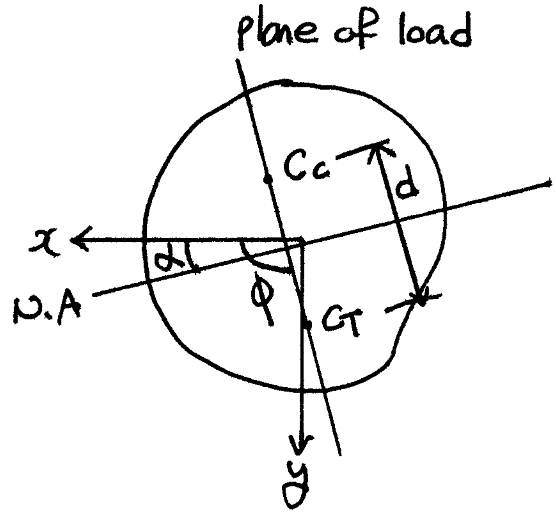
$\delta =$

7.5. Fully Plastic Load for Nonsymmetric Load

~ plane of load is specified.

- Location & direction of N.A. must be determined by trial and error
- Assumed α does not necessarily pass through
- N.A. must divide the cross-section into two equal area.





- Resultant tensile force located at G_T
- Resultant compressive force located ~~at~~ C_c.

$P_T = P_c$

$\therefore A_T = A_c = \frac{1}{2}A$

- Fully plastic moment

$\boxed{}$

- If line $\overline{C_cG_T}$ does not coincide with the plane of load, new α is assumed and repeats.

H.W.7: Solve problems 7.3, 7.11, 7.20, 7.33, 7.40

7.3. The cross section of a modified I-section beam is shown in Figure P7.3. A positive bending moment causes a maximum compressive flexural stress in the beam of magnitude 50 MPa. Determine the magnitude of M_x and the maximum tensile flexural stress for the cross section.

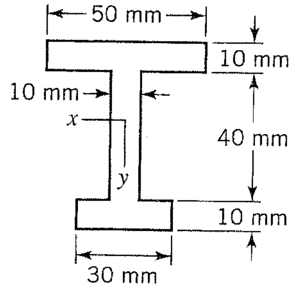


FIGURE P7.3

7.11. For the overhang beam shown in Figure P7.11, determine the maximum tensile flexural stress on sections just to the left and to the right of the section on which the 16.0 kN • m couple acts and their locations in the cross section. The flanges and the web of the cross section are all 20 mm thick.

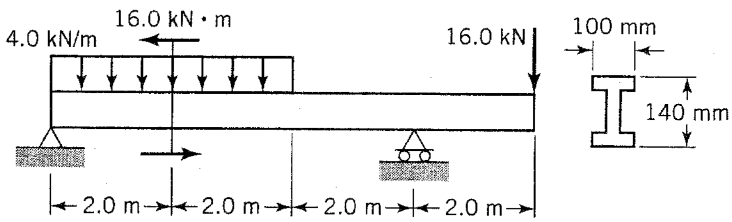


FIGURE P7.11

7.20. In Figure P7.20 let b = 300 mm, h = 300 mm, t = 25.0 mm, L = 2.50 m, and P = 16.0 kN. Calculate the maximum tensile and compressive stresses in the beam and determine the orientation of the neutral axis.

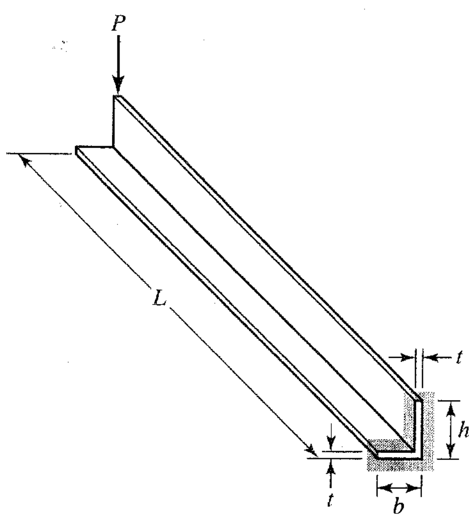


FIGURE P7.20

7.33. The beam shown in Figure P7.33 has a cross section of depth 60 mm and width 30 mm. The load P and reactions R_1 and R_2 all lie in a plane that forms an angle of 20° counterclockwise from the y axis. Determine the point in the beam at which the maximum tensile flexural stress acts and the magnitude of that stress.

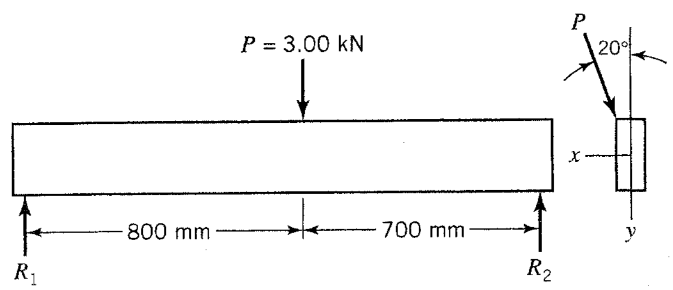


FIGURE P7.33

7.40. A wood beam of rectangular cross section 200 mm by 100 mm is simply supported at its ends (Figure P7.40). Determine the location and magnitude of the maximum flexural stress in the beam.

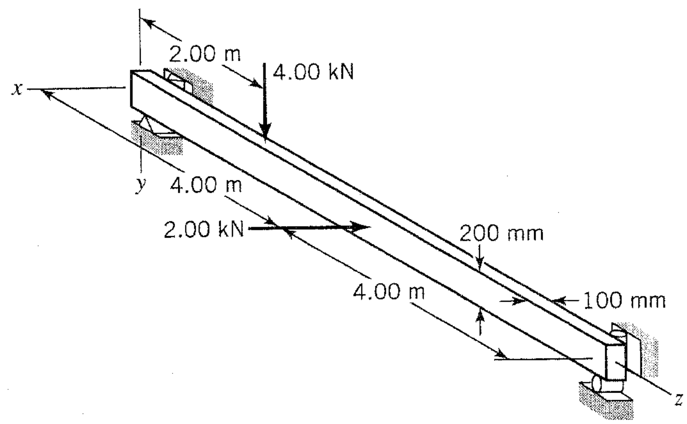


FIGURE P7.40