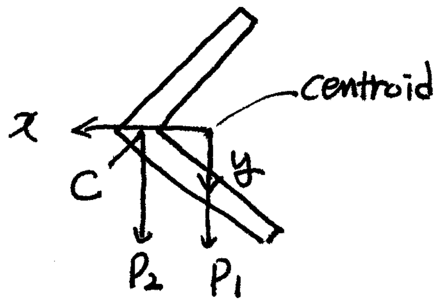
CH.8. Shear Center

~ Plane of loads must pass the shear center for the beam to experience only bending & shear.

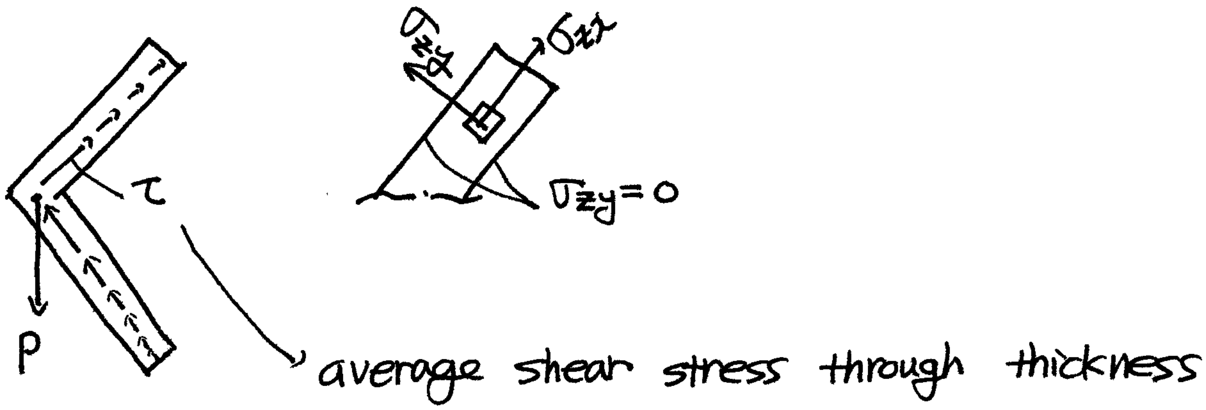
8.1. Approximation for shear

· Cross-section with more than 1 symmetry : shear center = centroid

· " 1 symmetry : shear center ≠ centroid.
but, both lie on the symmetry axis.



· In order to find shear center, shear stress of the section must be calculated.

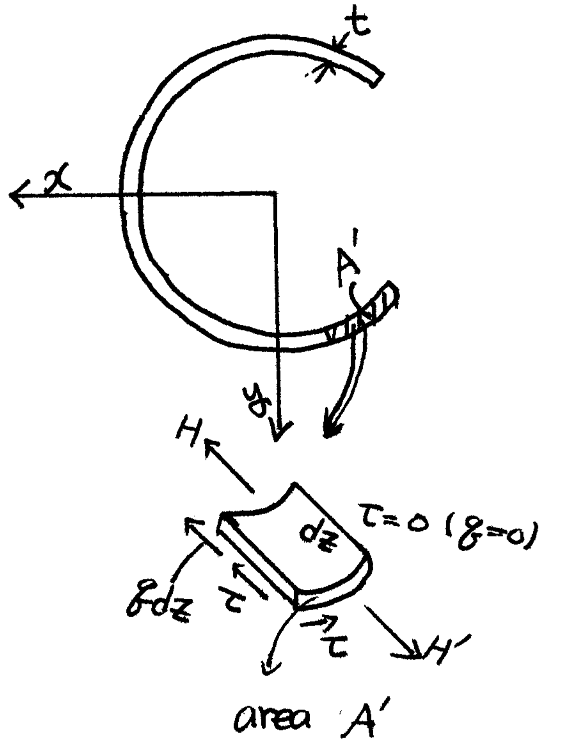


8.2. Shear Flow

∘ Shear flow $\boxed{}$

· Force Equilibrium

$$q\,dz = H' - H$$



$H =$

$H' =$

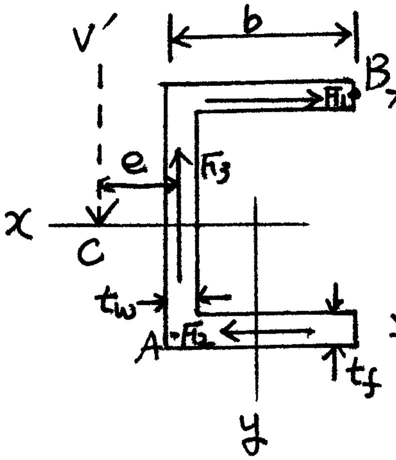
$\Rightarrow q =$

$-\cancel{x} \cdot \dfrac{dM_x}{dz} = V_y$. $\quad \displaystyle\int_{A'} y\,dA = A'\bar{y}'$

$\bar{y}'$: distance from x-axis to centroid of A'

$Q = A'\bar{y}'$

8.3. Shear Center for Channel Section

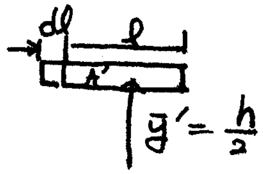


· Equilibrium

$$\begin{cases} \Sigma F_x = \\ \Sigma F_y = \\ \Sigma M_A = \end{cases}$$

· Idealize the cross-section to 3 narrow rectangles.

$$I_x = \frac{1}{12} t_w h^3 + \underbrace{2 \times \frac{1}{12} b t_f^3}_{\uparrow \text{ ignore}} + 2 \times b t_f \left(\frac{h}{2}\right)^2$$

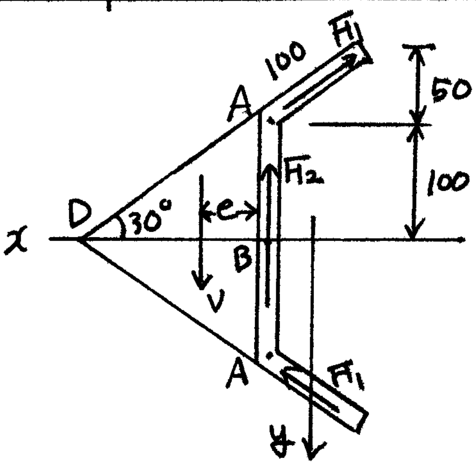


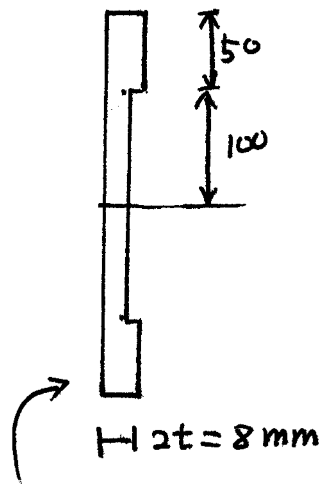
$$I_x = \frac{1}{12} t_w h^3 + \frac{1}{2} t_f b h^2$$

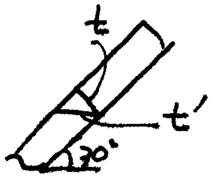
$$F_1 = \qquad\qquad = \frac{V_y}{I_x} \cdot \frac{h}{2} \int_0^b t_f \, l \, dl$$

$$= \frac{V_y t_f b^2 h}{4 I_x}$$

Example 8.1. Channel with Sloping Flanges







$$t' \sin 30 = t$$

$$t' = \frac{t}{\sin 30} = 2t$$

From the equivalent cross-section

$$I_x = \frac{1}{12} \times 8 \times 300^3 - \frac{1}{12} \times 4 \times 200^3 = 15.33 \times 10^6 \ mm^4$$

In order to skip the integration over the inclined flange, take moment equilibrium w.r.t. D.

$$q_A = \qquad\qquad = \frac{V}{I_x} \cdot (100 \times 4) \cdot 125 = 50{,}000 \frac{V}{I_x}$$

$$q_B = \qquad\qquad\qquad = 70{,}000 \frac{V}{I_x}.$$

Since the shear flow changes parabolically along A-B-A, the average shear flow is

$$q_{AVE} = \qquad\qquad = 63{,}330 \frac{V}{I_x}$$

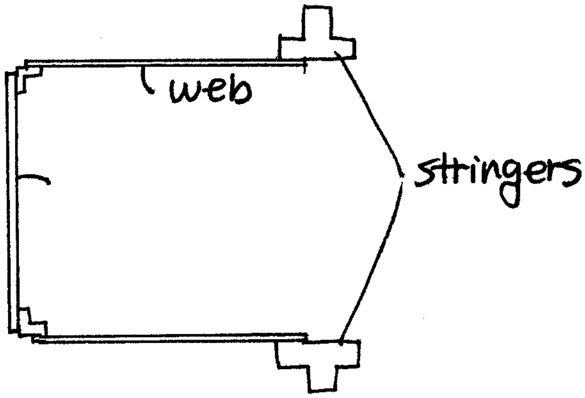
$$F_{12} = q_{ave} \cdot 200 = 12{,}670{,}000 \frac{V}{I_x}.$$

$$\Sigma M_D = 0 = V_y \cdot \left(\frac{100}{\tan 30} - e\right) - F_{12} \frac{100}{\tan 30}$$

$$\therefore e = \frac{100}{\tan 30} - 12{,}670{,}000 \frac{100}{I_x \cdot \tan 30} = 30.1 \text{ mm}$$

8.4. Shear Center for Composite Beams

~ Carry large bending load but small shear load.



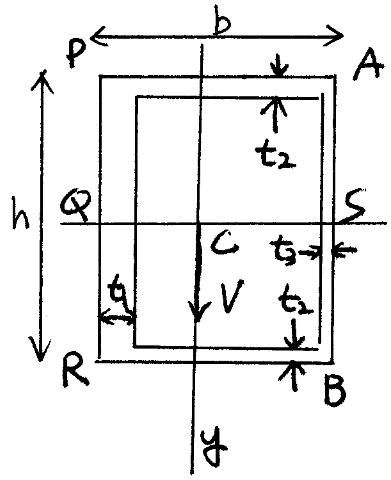
Assumption :

1. Web does not carry tensile and compressive stress
2. Shear flow is constant in a web.

$2n$: # of stringers

$\bar{y}_i$: distance from centroid to stringer i

8.5. Shear Center of Box Beam



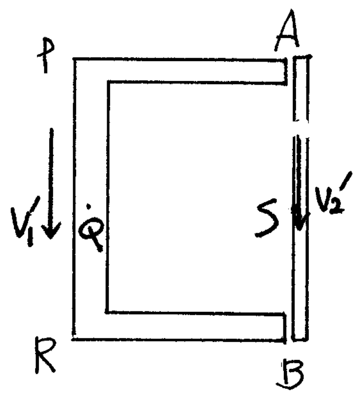
- x-axis : symmetry
- y-axis // plane of loading
- Shear center? ⇒ nee to know q.
- However there is no end of section that has $q=0$.
- Use the condition that no twist ⇒

from CH.6.

$T=0$.

$\therefore$

l: perimeter of the box, counterclockwise.



- Assume $q_A=0$ then due to symmetry $q_B=0$.
- Imaginary cut at A & B ($\because q_A=q_B=0$) but maintain continuity in displacement.

※

- Use $q=\frac{VQ}{I_x}$ to calculate q_1 & q_2
- After that add q_A and impose $\int_0^l \frac{q}{t}\,dl=0$ condition to calculate q_A.

Example 8.4. Box Beam

$b = 300 \quad h = 500 \quad t_1 = 20 \quad t_2 = t_3 = 10.$

$I_x = 687.5 \times 10^6 \ mm^4$

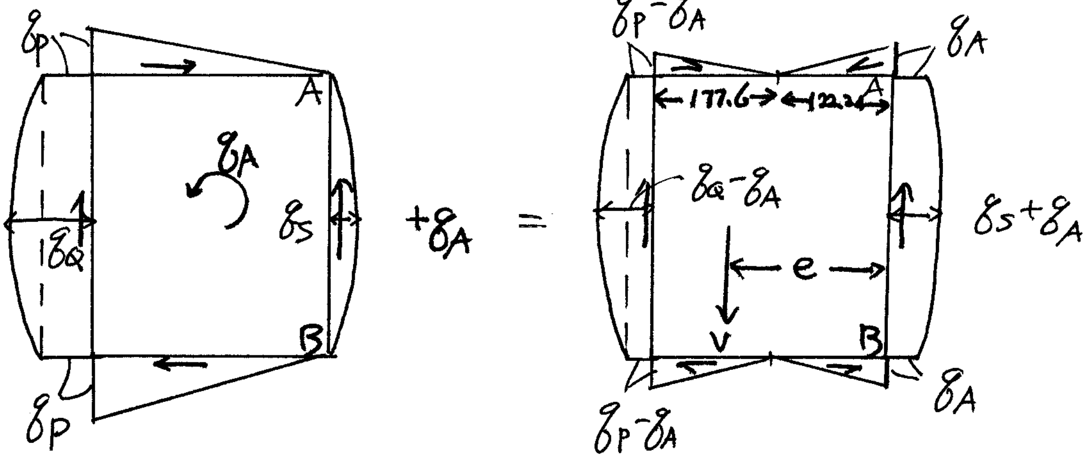
curvature is same.

Assume $V = I_x, \quad V_1 = I_{x1}, \quad V_2 = I_{x2} \quad (\because V \propto I)$

$q_P = \qquad = (bt_2)\cdot\frac{h}{2} = 750$ kN/mm

$q_Q = \qquad = 1,375$ kN/mm

$q_S = \qquad = 312.5$ kN/mm



Final result must satisfy $\qquad = 0.$

$$0 = \left[q_A - q_P - \frac{2}{3}(q_Q - q_P)\right]\frac{h}{t_1} + \left(q_A - \frac{q_P}{2}\right)\frac{b}{t_2} + \left(q_A + \frac{2}{3}q_S\right)\frac{h}{t_3}$$
$$+ \left(q_A - \frac{q_P}{2}\right)\frac{b}{t_2}$$

$\Rightarrow q_A = 305.6$ kN/mm

$$\Sigma M_B = 0 = V\cdot e - \left[\frac{2}{3}(q_Q - q_P) + q_P - q_A\right]\cdot 300\cdot 500 - \frac{q_P - q_A}{2}\cdot 177.6 \times 500$$
$$+ \frac{q_A}{2}\cdot 122.24 \times 500$$

$\Rightarrow e = 203$ mm.

$V_1 = [q_P - q_A + \frac{2}{3}(q_Q - q_P)] \cdot 500 = 430{,}533\ kN$

$V_2 = (q_A + \frac{2}{3}q_S) \cdot 500 = 256{,}967\ kN$

$V = V_1 + V_2 = 687{,}500\ kN.$

Solve Problems 8.5 8.12 8.25

8.5. Derive the relation for e for the circular arc cross section shown in Figure E of Table 8.1.

8.12. A 4-mm-thick plate of steel is formed into the cross section shown in Figure P8.12. Locate the shear center for the cross section.

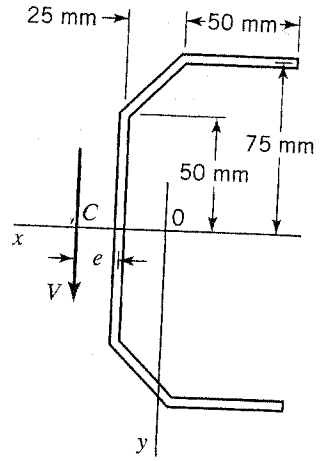


FIGURE P8.12

8.25. The channel shown in Figure P8.25 is subjected to nonsymmetric bending. The associated shear forces, which act through the shear center, are $V_x = -2400$ N and $V_y = 1800$ N. Determine the distribution of the shear stress throughout the cross section. Make a sketch, to scale, of the shear stress distribution in the channel walls.

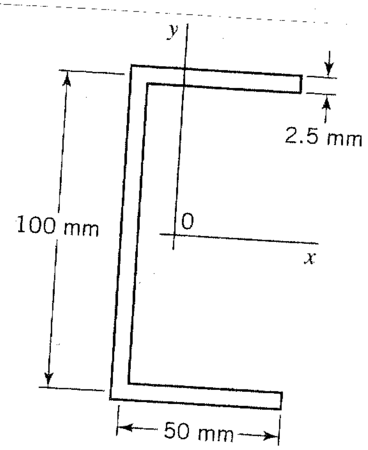


FIGURE P8.25